The Theory of Nonleptonic B Decays

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NPPC, 2003
Outline

- Nonleptonic Decays - the theoretical challenge
- A bit of history & the Physics we’re after
- Factorization and the Soft-Collinear EFT
- Decays
  \[ B \to J/\Psi K_s \quad B \to \phi K_s \]
  \[ B \to D\pi \quad B \to D\rho \quad B \to D^* K \]
  \[ \Lambda_b \to \Lambda_c \pi \quad \Theta_b \to \Theta_c \pi \]
  \[ B \to \pi\pi \quad B \to K\pi \quad B \to \rho\pi \]
- Conclusion and Outlook
Two body nonleptonic decays. Simple?

\[ \Gamma = \frac{|\vec{p}_\pi|}{8\pi m_B^2} |A|^2 \]

\[ A = \langle \pi\pi|H_{\text{weak}}|B\rangle \]

- Weak decay of quarks: \( b \rightarrow u(\bar{u}d) \)
- QCD bound states: \((b\bar{d}) \rightarrow (u\bar{d})(\bar{u}d)\)
  \[ \bar{B}^0 \rightarrow \pi^- \pi^+ \]
Electroweak Hamiltonian

\[ m_W, m_t \gg m_b \]

\[
H_{\text{weak}} = \frac{G_F}{\sqrt{2}} \sum_i \lambda^i C_i(\mu) O_i(\mu)
\]

trees

\[
O_1 = (\bar{u}b)_{V-A}(\bar{d}u)_{V-A}
\]

\[
O_2 = (\bar{u}_i b_j)_{V-A}(\bar{d}_j u_i)_{V-A}
\]

penguins

\[
O_3 = (\bar{d}b)_{V-A} \sum_q (\bar{q}q)_{V-A}
\]

\[
O_{4,5,6} = \ldots
\]

\[
O_{7,\gamma,8G} = \ldots
\]

\[
O_{ew} = \ldots
\]

\[
\lambda^i = \text{CKM factors}
\]

\[
\lambda^1 = V_{ub}V_{ud}^* \quad \lambda^3 = V_{tb}V_{td}^*
\]
QCD

- Confinement, Hadronization
- a tough problem

Note: Nonleptonic B-decays are not Gold Plated Observables for Lattice QCD

- two final state hadrons
- energetic pions, slow B
- very hard

HPQCD ‘03
Theory 1986: Factorization

\[ \langle \pi \pi | (\bar{u}b)_{V-A} (\bar{d}u)_{V-A} | B \rangle \simeq \langle \pi | (\bar{u}b)_{V-A} | B \rangle \langle \pi | (\bar{d}u)_{V-A} | 0 \rangle \]

If that is all there is ....

3) EXCLUSIVE NONLEPTONIC DECAYS OF D, D(S), AND B MESONS.
Published in Z.Phys.C34:103,1987

TOPCITE = 1000+

• Justified by \( N_c \to \infty \) in some cases
• No rescattering, strong phases are zero
• Ok for some decays, fails for others
• Inconsistent with QCD anomalous dimensions
Quarks

<table>
<thead>
<tr>
<th>Quarks</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>(\sim 4) Mev</td>
</tr>
<tr>
<td>d</td>
<td>(\sim 7) MeV</td>
</tr>
<tr>
<td>s</td>
<td>(\sim 120) MeV</td>
</tr>
<tr>
<td>c</td>
<td>(\sim 1.4) GeV</td>
</tr>
<tr>
<td>b</td>
<td>(\sim 4.5) GeV</td>
</tr>
<tr>
<td>t</td>
<td>174 GeV</td>
</tr>
</tbody>
</table>

\[\alpha_s(\mu), \mu \text{ resolution}\]
\[\alpha_s(\Lambda) \text{ non-perturbative}\]
\[\rightarrow \text{ long distance}\]
\[\alpha_s(m_b) \text{ perturbative}\]
\[\rightarrow \text{ short distance}\]

Expansion parameters are useful

\[\alpha_s(m_b) \approx 0.2, \quad \frac{\Lambda}{m_b} \approx 0.1, \quad \frac{m_{u,d}}{\Lambda}\]
B-meson

\[ \Lambda_{QCD} \ll m_b \quad \nu^\mu \text{ conserved} \]

Heavy Quark Effective Theory

eg. Inclusive Decay:

\[ B \rightarrow X_c \ell \bar{\nu}_\ell \]

\[ X_c = D, D^*, D\pi, D\rho\pi, \ldots \]

Operator Product Expansion in

\[ \frac{\Lambda_{QCD}}{m_b} \approx 0.1 \]

- \( m_b \rightarrow \infty \) is free quark decay, \( \alpha_s(m_b) \) computable
- No \( \frac{\Lambda_{QCD}}{m_b} \) corrections uses HQET
- At \( \frac{\Lambda^2_{QCD}}{m_b^2} \) two hadronic parameters \( \lambda_1, \lambda_2 \)

"Brown muck" = \( \bar{q} + \text{glue}, q\bar{q} \)

Isgur (90's)

\( \Lambda_{QCD} \approx 0.1 \)
Calculate differential rates: \[
\frac{d\Gamma}{dE_\ell}, \quad \frac{d\Gamma}{dm^2_X}, \quad \ldots
\]

Fit moments to simultaneously extract \( V_{cb}, m_b(\bar{\Lambda}), \lambda_{1,2} \)

eg. CLEO \( m_X \) moments

\[
|V_{cb}| = (42.1 \pm 1.0 \pm 0.7) \times 10^{-3}
\]

from S. Stone, EPS '03

from M. Artuso, Beauty '03
When you have to descend into the brown muck, you abandon all pretense of doing elegant, pristine, first-principles calculations. You have to get your hands dirty with uncontrolled approximations and models. When you are finished with the brown muck, you should wash your hands.”

Georgi (1991)

“... drinking the nonlep tonic ...”

Lipkin
Why Bother?

- Need nonleptonic decays to measure CP violation
- Baryon asymmetry of the universe wants more
- MSSM has 43 new CP violating parameters

- Study rare nonleptonic decays
- Loop dominated, look for new physics
  - $Br < 10^{-5}$, $B \to \phi K_s$, $B \to K\pi$

- Measure fundamental hadronic parameters & improve our understanding of QCD
eg. Measure $\sin(2\alpha)$ with $B^0(t) \to \pi^+\pi^-$, $\bar{B}^0(t) \to \pi^+\pi^-$

\[
A_{CP}(t) = -S_{\pi\pi} \sin(\Delta m_B t) + C_{\pi\pi} \cos(\Delta m_B t)
\]

\[
S_{\pi\pi} = \frac{2 \text{Im}\lambda}{1 + |\lambda|^2}, \quad C_{\pi\pi} = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}, \quad \lambda = e^{2i\alpha} \frac{1 + ei\gamma P/T}{1 + e^{-i\gamma P/T}}
\]

$T = \text{tree}, \quad P = \text{penguin}$

$P = 0$ then $S_{\pi\pi} = \sin(2\alpha)$

$P/T \neq 0$, need information from QCD, or isospin analysis

can remove $P$ with
$B^0 \to \pi^0\pi^0$, $\bar{B}^0 \to \pi^0\pi^0$, $B^+ \to \pi^+\pi^0$ plus isospin

\[
S_{\pi\pi} = \frac{2|\lambda|}{1 + |\lambda|^2} \sin(2\alpha_{\text{eff}})
\]
Some Clean Nonleptonic Info

CP asymmetry in $b \rightarrow c\bar{c}s$ ($B^0, \bar{B}^0 \rightarrow J/\Psi K_s, \Psi' K_s, J/\Psi K_L, \ldots$)

- A dominant weak phase, $V_{cb}V_{cs}^* \sim \lambda^2$, $V_{ub}V_{us}^* \sim \lambda^4$
- QCD $\simeq$ CP even
- strong phase cancels in $\bar{A}/A$
- $a_{CP}(t) \propto \sin(2\beta)$

$\lambda \sim 0.2$
More Clean Nonleptonic Info

CP asymmetry in \( b \rightarrow s\bar{s}s \) (\( B^0, \bar{B}^0 \rightarrow \phi K_s, \eta' K_s, \ldots \))

- Dominant weak phase:
  \( V_{cb}V_{cs}^* \sim \lambda^2 \), \( V_{ub}V_{us}^* \sim \lambda^4 \)
- Penguin Dominated
- At \( \sim 5\% \) level we expect
  \( \sin(2\beta)_{\phi K_s} \simeq \sin(2\beta)_{J/\Psi K_s} \)
- WA: 2.7\( \sigma \) deviation

Belle: \( S_{\phi K} = -0.96 \pm 0.51 \)
BaBar: \( S_{\phi K} = 0.45 \pm 0.43 \)
WA: \( S_{\phi K} = -0.14 \pm 0.33 \)
WA: \( S_{J/\Psi K} = 0.739 \pm 0.048 \)
Measuring CP violation in other decays requires

1. A fancier analysis strategy. Use SU(2) or SU(3) to relate amplitudes so data can be used to reduce uncertainties.
   - Flavor symmetries of QCD, \( m_u, m_d, m_s \ll \Lambda_{\text{QCD}} \)

2. Factorization from QCD to reduce the amplitudes to simple universal nonperturbative parameters.
   - Expand in \( E_\pi \gg \Lambda_{\text{QCD}} \)

These two possibilities are not exclusive.

The important thing to keep in mind is “what are the uncertainties”.

In 1999 Beneke, Buchalla, Neubert, Sachrajda proposed a QCD factorization theorem for $B \rightarrow \pi \pi$. Builds on earlier proposal by Politzer & Wise for $B \rightarrow D \pi$, which in turn built on Brodsky - Lepage type exclusive QCD factorization.

Amplitude is reduced to simpler matrix elements

$$\langle \pi \pi | \ldots | B \rangle \rightarrow \langle \pi | \ldots | B \rangle, \quad \langle \pi | \ldots | 0 \rangle, \quad \langle 0 | \ldots | B \rangle$$

At LO in $\frac{\Lambda_{QCD}}{E_{\pi}}$ strong phases are perturbative, $i \alpha_s(m_b)$, and therefore small.

Is it right?
Soft - Collinear Effective Theory

Bauer, Pirjol, Stewart
Fleming, Luke

• An effective field theory for energetic hadrons, $E \gg \Lambda_{\text{QCD}}$
Effective Field Theory

- Separate physics at different momentum scales
- Power expansion
- Make symmetries explicit
- Model independent, systematically improvable

<table>
<thead>
<tr>
<th>Effective Theories</th>
<th>Expansion Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Electroweak (Fermi) Hamiltonian</td>
<td>$m_b/m_W \ll 1$</td>
</tr>
<tr>
<td>(2) Heavy Quark Effective Theory (HQET)</td>
<td>$\Lambda/m_b \ll 1$</td>
</tr>
<tr>
<td>(3) Chiral Perturbation Theory, SU(3)</td>
<td>$m_{u,d,s}/\Lambda \ll 1$</td>
</tr>
</tbody>
</table>

All designed to separate hard $p_h \sim Q$ and soft $p_s$ momenta, $Q^2 \gg p_s^2$

Allow for energetic hadrons $\Rightarrow$ collinear $p_c$, new class of processes

\[ Q \gg \Lambda_{\text{QCD}} \]

\[ Q = E_H \]
Soft Collinear Effective Theory

eg. \[ \pi \rightarrow B \rightarrow D \]

Pion has:
\[ p_\pi^\mu = (2.3 \text{ GeV}) n^\mu = Q n^\mu \]
\[ n^2 = \bar{n}^2 = 0, \quad (n \cdot p = p^-) \]

Soft brown muck:
\[ p_s^\mu = (p^+, p^-, p_\perp) \sim (\Lambda, \Lambda, \Lambda) \]

Collinear constituents:
\[ p_c^\mu = (p^+, p^-, p_\perp) \sim \left( \frac{\Lambda^2}{Q}, Q, \Lambda \right) \sim Q(\lambda^2, 1, \lambda) \]
\[ \lambda = \frac{\Lambda}{Q} \]

\[ n^\mu \]
Degrees of freedom in SCET

Introduce fields for infrared degrees of freedom (in operators)

<table>
<thead>
<tr>
<th>modes</th>
<th>$p^\mu = (+, -, \perp)$</th>
<th>$p^2$</th>
<th>fields</th>
</tr>
</thead>
<tbody>
<tr>
<td>collinear</td>
<td>$Q(\lambda^2, 1, \lambda)$</td>
<td>$Q^2 \lambda^2$</td>
<td>$\xi_n, A_{n\mu}$</td>
</tr>
<tr>
<td>soft</td>
<td>$Q(\lambda, \lambda, \lambda)$</td>
<td>$Q^2 \lambda^2$</td>
<td>$q_s, A_{s\mu}$</td>
</tr>
<tr>
<td>usoft</td>
<td>$Q(\lambda^2, \lambda^2, \lambda^2)$</td>
<td>$Q^2 \lambda^4$</td>
<td>$q_{us}, A_{us\mu}$</td>
</tr>
</tbody>
</table>

SCET_1 → Energetic jets

$\Lambda \ll \Lambda Q \ll Q^2$

usoft $p^\mu \sim \Lambda$

collinear $p_c^2 \sim Q \Lambda$, $\lambda = \sqrt{\Lambda/Q}$

SCET_II → Energetic hadrons

soft $p^\mu \sim \Lambda$

collinear $p_c^2 \sim \Lambda^2$, $\lambda = \Lambda/Q$
Factorization

\( \bar{B}^0 \rightarrow D^+ \pi^- , \ B^- \rightarrow D^0 \pi^- \)

\( B, D \) are soft, \( \pi \) collinear

\( \mathcal{L}_{SCET} = \mathcal{L}_s^{(0)} + \mathcal{L}_c^{(0)} \)

Factorization if \( \mathcal{O} = \mathcal{O}_c \times \mathcal{O}_s \)

Bauer, Pirjol, I.S.

\[ \langle D\pi | (\bar{c}b)(\bar{u}d)|B \rangle = N \xi (v \cdot v') \int_0^1 dx \ T(x, \mu) \ \phi_{\pi}(x, \mu) \]

Universal functions:

\[ \langle D^{(*)} | O_s | B \rangle = \xi (v \cdot v') \]

\[ \langle \pi | O_c(x) | 0 \rangle = f_{\pi} \phi_{\pi}(x) \]

Calculate \( T, \ \alpha_s(Q) \)

\( Q = E_\pi, m_b, m_c \)

corrections will be \( \Lambda/m_c \sim 30\% \)

\( \text{LO} = \lambda^5 \text{ graphs} \)
### Universal hadronic parameters

<table>
<thead>
<tr>
<th>Process</th>
<th>Degrees of Freedom $(p^2)$</th>
<th>Non-Pert. functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \rightarrow D^+\pi^-$, ...</td>
<td>$c (\Lambda^2), s (\Lambda^2)$</td>
<td>$\xi(w), \phi_\pi$</td>
</tr>
<tr>
<td>$\bar{B}^0 \rightarrow D^0\pi^0$, ...</td>
<td>$c (\Lambda^2), s (\Lambda^2), c (Q\Lambda)$</td>
<td>$S(k_+^j), \phi_\pi$</td>
</tr>
<tr>
<td>$B \rightarrow X_{\text{endpt}}^s \gamma,$</td>
<td>$c (Q\Lambda), us (\Lambda^2)$</td>
<td>$f(k^+)$</td>
</tr>
<tr>
<td>$B \rightarrow X_{\text{endpt}}^u \ell\nu$</td>
<td>$c (Q\Lambda), s (\Lambda^2), c (Q\Lambda)$</td>
<td>$\phi_B(k^+), \phi_\pi(x), \zeta_{\pi}(E)$</td>
</tr>
<tr>
<td>$B \rightarrow \pi \ell\nu, ...$</td>
<td>$c (\Lambda^2), s (\Lambda^2), c (\Lambda^2)$</td>
<td>$\phi_B, \phi_\pi, \zeta_{\pi}(E)$</td>
</tr>
<tr>
<td>$B \rightarrow \gamma \ell\nu, \gamma\gamma$</td>
<td>$c (\Lambda^2), s (\Lambda^2)$</td>
<td>$\phi_B, \phi_K, \zeta_{K^*}(E)$</td>
</tr>
<tr>
<td>$B \rightarrow \pi\pi$</td>
<td>$c (\Lambda^2), s (\Lambda^2), c (\Lambda^2)$</td>
<td>$f_{i/p}(\xi), f_{g/p}(\xi)$</td>
</tr>
<tr>
<td>$B \rightarrow K^*\gamma$</td>
<td>$c (\Lambda^2)$</td>
<td>$\phi_\pi$</td>
</tr>
<tr>
<td>$e^- p \rightarrow e^- X$</td>
<td>$c (\Lambda^2), s (\Lambda^2)$</td>
<td>$\phi_M, \phi_{M'}$</td>
</tr>
<tr>
<td>$e^- \gamma \rightarrow e^- \pi^0$</td>
<td>$c (\Lambda^2)$</td>
<td></td>
</tr>
<tr>
<td>$\gamma^* M \rightarrow M'$</td>
<td>$c (\Lambda^2), s (\Lambda^2)$</td>
<td></td>
</tr>
</tbody>
</table>
$B \rightarrow D\pi$

"Tree" 

$B^0 \rightarrow D^+\pi^-$  
$B^- \rightarrow D^0\pi^-$

"Color suppressed" 

$B^- \rightarrow D^0\pi^-$  
$\bar{B}^0 \rightarrow D^0\pi^0$

"Exchange" 

$B^0 \rightarrow D^+\pi^-$  
$\bar{B}^0 \rightarrow D^0\pi^0$

Naive $N_c$-counting: very predictive

$O^8 = \left( \frac{\pi^0}{\bar{B}} \right)^{A \cdot A} \left( \bar{D}_u \right) \left( \bar{T} \right) \frac{1}{N_c} D^0 |(\bar{c}u)|0 \rangle$

QCDF - $D^0\pi^0$ is nonfactorizable channel

pQCD - predicted with expansion in $m_c/m_b$
• $D = D^*$
• $\text{Br}(D^+ M^-)$ agree with factorization
• $\text{Br}(D^0 M^0)$ small as expected
• but 20-30% power corrections for $\text{Br}(D^0 M^-) / \text{Br}(D^+ M^-)$
• significant strong phase $\delta \sim 30^\circ$

<table>
<thead>
<tr>
<th>Type</th>
<th>Decay</th>
<th>$\text{Br}(10^{-3})$</th>
<th>Decays</th>
<th>$\text{Br}(10^{-3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$B^0 \to D^+ \pi^-$</td>
<td>2.68 ± 0.29 $^a$</td>
<td>$B^0 \to D^{*+} \pi^-$</td>
<td>2.76 ± 0.21</td>
</tr>
<tr>
<td>III</td>
<td>$B^- \to D^0 \pi^-$</td>
<td>4.97 ± 0.38 $^a$</td>
<td>$B^- \to D^{*0} \pi^-$</td>
<td>4.6 ± 0.4</td>
</tr>
<tr>
<td>II</td>
<td>$\bar{B}^0 \to D^0 \pi^0$</td>
<td>0.292 ± 0.045 $^b$</td>
<td>$\bar{B}^0 \to D^{*0} \pi^0$ $^b$</td>
<td>0.25 ± 0.07</td>
</tr>
<tr>
<td>I</td>
<td>$B^0 \to D^+ \rho^-$</td>
<td>7.8 ± 1.4</td>
<td>$B^0 \to D^{*+} \rho^-$</td>
<td>6.8 ± 1.0 $^c$</td>
</tr>
<tr>
<td>III</td>
<td>$B^- \to D^0 \rho^-$</td>
<td>13.4 ± 1.8</td>
<td>$B^- \to D^{*0} \rho^-$</td>
<td>9.8 ± 1.8 $^c$</td>
</tr>
<tr>
<td>II</td>
<td>$\bar{B}^0 \to D^0 \rho^0$</td>
<td>0.29 ± 0.11 $^d$</td>
<td>$\bar{B}^0 \to D^{*0} \rho^0$</td>
<td>$&lt; 0.56$</td>
</tr>
</tbody>
</table>

$\delta \sim 30^\circ$
Color Suppressed Decays

Factorization with SCET

Single class of power suppressed SCET\(_1\) operators \(T\{O^{(0)}, L_{\xi q}^{(1)}, L_{\xi q}^{(1)}\}\)

We find \((M = \pi, \rho)\)

\[
A^{D(\ast)}_{00} = N^{(\ast)}_0 \int dx \ dz \ dk_1^+ \ dk_2^+ \ T^{(i)}(z) \ J^{(i)}(z, x, k_1^+, k_2^+) \ S^{(i)}(k_1^+, k_2^+) \ \phi_M(x)
\]

\[
Q^2 \gg Q \Lambda \gg \Lambda^2
\]

new soft function \(S^{(i)}(k_1^+, k_2^+)\) - like generalized parton distributions

Mantry, Pirjol, I.S. ‘03
i) \( \langle D^{(*)0} | O_s^{(0,8)} | \bar{B}^0 \rangle \rightarrow S^{(0,8)}(k_1^+, k_2^+) \) same for \( D \) and \( D^* \)

ii) \( S^{(i)}(k_1^+, k_2^+) \) is complex, new mechanism for rescattering

\[
O^{(0,8)} = \left[ \langle h_v^{(c)} S | \Gamma, q^{(1)}, \bar{T} \rangle^{n} \right] (S^\dagger h_v^{(b)}) (\bar{d} S)_{k_1^+} \Gamma_s \{1, T^a\} (S^\dagger u)_{k_2^+}
\]

**Predict**

equal strong phases \( \delta^D = \delta^{D^*} \)
equal amplitudes \( A_{00}^D = A_{00}^{D^*} \)
corrections to this are \( \alpha_s(m_b), \Lambda/Q \)
Tests and Predictions

Expt Average (CLEO, Belle, summer 2003):

\[
\begin{array}{cccc}
\text{Expt Average (CLEO, Belle, summer 2003)}: \\
0 & 0.2 & 0.4 & 0.6 \\
0.8 & 0.6 & 0.4 & 0.2 \\
\end{array}
\]

New BaBar results, hep-ph/0310028:

\[
\begin{align*}
Br(\bar{B}^0 \to D^0\pi^0) &= (0.29 \pm 0.04) \times 10^{-3}, \\
Br(\bar{B}^0 \to D^{*0}\pi^0) &= (0.29 \pm 0.06) \times 10^{-3}, \\
\delta^{D\pi} &= 30^\circ \pm 5^\circ, \\
\delta^{D^{*}\pi} &= 33^\circ \pm 5^\circ
\end{align*}
\]

isospin gives triangle:

\[
A_{0-} = \sqrt{2}A_{00} + A_{+-}
\]

rearrange:

\[
1 = RI + \frac{3A_{00}}{\sqrt{2}A_{0-}}
\]

\[
R_I = \frac{A_{1/2}}{\sqrt{2}A_{3/2}}
\]

\[
\delta = \arg(A_{1/2}A_{3/2}^*)
\]
Tests and Predictions

Also predict (not post-dict):

\[ r_{00}^\rho = \frac{A(\bar{B}^0 \rightarrow D^*0 \rho^0)}{A(\bar{B}^0 \rightarrow D^0 \rho^0)} = 1, \]

\[ r_{00}^{K^-} = \frac{A(\bar{B}^0 \rightarrow D_s^* K^-)}{A(\bar{B}^0 \rightarrow D_s K^-)} = 1, \]

\[ r_{00}^{K^0} = \frac{A(\bar{B}^0 \rightarrow D_s^{*0} \bar{K}^0)}{A(\bar{B}^0 \rightarrow D_s^{*0} \bar{K}^0)} = 1, \]

ie. same Br and same strong phases

All predictions so far are independent of the form of \( J^{(i)}(z, x, k_1^+, k_2^+) \)

and \( S^{(i)}(k_1^+, k_2^+), \phi_M(x) \)
More Predictions

If we expand $J(z, x, k_1^+, k_2^+)$ in $\alpha_s(Q\Lambda)$, we can make more predictions

Relate $\pi$ and $\rho$

- $\langle x^{-1} \rangle_\pi \simeq \langle x^{-1} \rangle_\rho$ implies $|r^{D\pi}| = |r^{D\rho}|$. Data gives

$$|r^{D\pi}| = \frac{|A(\bar{B}^0 \to D^+\pi^-)|}{|A(B^- \to D^0\pi^-)|} = 0.77 \pm 0.05, \quad |r^{D\rho}| = 0.80 \pm 0.09$$

- also predict that $\phi^{D\rho} = \phi^{D\pi}$, not yet tested
naive factorization for color suppressed decays
Similar Color Allowed Decays

\[ B_s \rightarrow D_s \pi \quad \text{from CDF} \]

\[ Br = (4.2 \pm 1.6) \times 10^{-3} \]

- pure “Tree” topology gives interesting information
- can handle SU(3) violation with

\[ \left. \frac{d\Gamma}{dq^2} (B_s \rightarrow D_s \ell^- \bar{\nu}_\ell) \right|_{q^2=m^2_\pi} \]
Add a soft quark

\[ \Lambda_b \rightarrow \Lambda_c \pi \]
Add three soft quarks

\[ \Theta_b \rightarrow \Theta_c \pi \]

Diquark model

Jaffe, Wilczek ‘03

- \( \Theta_b, \Theta_c \) may decay weakly
- \( \Theta_s \rightarrow K_S p \) penalty for breaking diquark correlation

Look for:

\[ \Theta_b^+ \rightarrow \Theta_c^0 \pi^+ \]
\[ \rightarrow \Theta_c^+ \pi^- \pi^+ \]
\[ \rightarrow (K_S p) \pi^- \pi^+ \]
\[ \rightarrow \pi^+ \pi^- p \pi^- \pi^+ \]

- all Cabbibo allowed
- all charged
- only pay diquark penalty once
- try to calculate it ...
Before we tackle the nonleptonic we should consider the semileptonic:

\[ B \rightarrow M_1 M_2 \]

\[ B \rightarrow \pi \pi \quad B \rightarrow \pi K \quad B \rightarrow \rho K^* \]
\[ B \rightarrow \pi K^* \quad B \rightarrow \rho \rho \quad B \rightarrow \pi \rho \quad B \rightarrow KK \]
\[ B_s \rightarrow \pi^0 \eta \quad B_s \rightarrow K^+ K^{*-} \]

PP = 21 + 13 decays
PV = 40 + 23 decays
VV = 21 + 13 decays

\[ B_s \rightarrow \pi^0 \eta \quad B_s \rightarrow K^+ K^{*-} \]

\[ \text{eg. SU(3) analysis} \quad \text{eg. QCDF analysis} \quad \text{Chiang et al.} \quad \text{Beneke et al.} \]

Before we tackle the nonleptonic we should consider the semileptonic:

\[ B \rightarrow \pi \ell \bar{\nu}_\ell \quad B \rightarrow K^* \gamma \quad B \rightarrow \rho \gamma \]
\[ B \rightarrow \rho \ell \bar{\nu}_\ell \quad B \rightarrow K e^+ e^- \quad B \rightarrow K^* \ell^+ \ell^- \]
Heavy-to-Light Decays

- Large $q^2$ accessible on the Lattice ($B \to \pi \ell \nu$, $q^2 > 17 \text{ GeV}^2$)
- For small $q^2$, $E \gg \Lambda_{\text{QCD}}$ and large energy factorization applies

$B \to \rho \ell \bar{\nu}_\ell$

Form factors

- Pseudoscalar: $f^+, f_0, f_T$
- Vector: $V, A_0, A_1, A_2, T_1, T_2, T_3$
\[
f^F(Q) = \frac{f_B f_M m_B}{4E^2} \int_0^1 dz \int_0^1 dx \int_\infty^0 dr_+ T(z, E, m_b) \\
\times J(z, x, r_+, E) \phi_M(x) \phi^+_B(r_+)
\]

\[
f^{NF}(Q) = C_k(E, m_b) \zeta_k(Q \Lambda, \Lambda^2)
\]

result at \text{LO} in \lambda, all orders in \alpha_s, where \(Q = \{m_b, E_M\}\)

\[p^2 \sim \Lambda^2\]  \[p^2 \sim Q \Lambda\]  \[p^2 \sim \Lambda^2\]
Some Controversy:

- Can $\zeta(Q\Lambda, \Lambda^2)$ be factored further without singularities?
- Does $f^{NF}$ dominate over $f^F$? (QCDF assumes this)
- How large are the Sudakov double logarithms?

Can be addressed with SCET
A few comments on the already established pattern from charmless 2-body decays

\[ B \rightarrow K\pi, \pi\pi, KK \]

Connected via Isospin symmetry (Gronau & London) (Isospin engineering)

Constructing these triangles is a major goal of the experiments

With Tree alone, expect:

- \[ B \rightarrow K\pi \approx 5\% \]
- \[ B \rightarrow \pi\pi \approx 4\% \]

\( \Delta \alpha \)

\( \tilde{A}(B^o \rightarrow \varphi^0\varphi^0) \)

\( \tilde{A}(B^- \rightarrow \varphi^-\varphi^0) = A(B^+ \rightarrow \varphi^0\varphi^0) \)

Penguins are significant players

With SU(3) connections to estimate P

Measurements:

Penguins are significant players
$B \to M_1 M_2$ Factorization

\[ F^{B\to M_1}, \phi_{M_2}(x) \]

\[ \phi_B(r^+), \phi_{M_1}(x), \phi_{M_2}(y) \]

- involves $\zeta_{M_1}, \phi_B(r^+), \phi_{M_i}(x)$
  - same as form factors
- same function $J(z, x, r_+, E)$
- one formula for PP, PV, VV
SU(3) Violation in $\phi_M(x)$

Factorization theorems usually do not try to untangle

$$m_{u,d,s} \text{ from } \Lambda_{\text{QCD}} \rightarrow \text{left in nonperturbative functions}$$

$$\phi_M(x) = \phi_0(x) + \sum_{P=\pi,K,\eta} \frac{m_P^2}{(4\pi f)^2} \left[ a_P^M(x) \ln \left( \frac{m_P^2}{\mu^2} \right) + b_P^M(x, \mu) \right]$$

Using chiral perturbation theory we find:

- Non-analytic terms vanish
- With all NLO operators, ie all the leading SU(3) violation:

$$\phi_\pi(x) + 3\phi_\eta(x) = 2 \left[ \phi_{K^+}(x) + \phi_{K^-}(x) \right]$$

“Gell-Mann Okubo”

J.Chen, I.S. ’03
Uncertainty in pure isospin analysis

Extrapolation to future

Future B-factories: LHCb, BTeV, $e^+e^-@10^{36}$ luminosity
D. del Re
(SuperB ‘03 workshop)

Today 2006 1yr @ $10^{36}$
Outlook

- The theory of nonleptonic $B$ decays is challenging, but progress is being made

- **The SCET can be applied to:**
  
  - Nonleptonic decays, Other $B$ decays
  - Jet physics, Exclusive form factors
  - Charmonium, Upsilon physics
  - ... others?

- Allows power corrections to be addressed in a model independent way

- For B’s, need to carefully examine expansion for each process and improve our understanding of power corrections to trust results beyond the 20% level

- Lots of data now, lots of data still to come

- A lot of theory and phenomenology left to study ...