# Understanding the Strong Interactions with Effective Theories

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# Outline

- QED, Effective Field Theory, Hydrogen
- Introduction to QCD,  $\alpha_s(\mu)$
- Energetic Particles & Soft-Collinear Effective Theory
- Weak Decays of B mesons
- Outlook

## Introduction to QED

(quantum electromagnetism)

QED {Special Relativity:spacetime, $v \leq c$ QED {Quantum Mechanics:quantization, $\Delta x \Delta p \geq \frac{\hbar}{2}$ 



antiparticles, spin, gauge-theory parameters: charge & masses

Interactions





two factors of the coupling



~~~

pair creation

# The Standard Model Interactions

|                      |                                     | (leave out       | eave out gravity and the higgs)               |  |
|----------------------|-------------------------------------|------------------|-----------------------------------------------|--|
|                      | Strong                              | Electromagnetism | Weak                                          |  |
|                      | QCD                                 | QED              |                                               |  |
| mediator:            | gluons                              | photons          | $W^{\pm}, Z^0$                                |  |
| typical<br>strength: | $\sim 1$                            | $\sim 10^{-2}$   | $\sim 10^{-6}$                                |  |
| range:               | $\sim 1  {\rm fm}$                  | $\infty$         | $\frac{1}{m_W} \to \sim 10^{-3}  \mathrm{fm}$ |  |
|                      |                                     |                  | $n  ightarrow pe \overline{ u}$ , radioactive |  |
|                      | proton                              | $\vec{B}$        | decay                                         |  |
| Other for<br>be de   | cces can (in prin<br>rived from the | nciple)<br>ese b | W Z <sup>2</sup><br>V<br>V<br>V<br>e          |  |

## Physics compartmentalized



But, one doesn't need nuclear physics to build a boat

Generality vs. Precision





Dynamics at long distance does not depend on the details of what happens at short distance In the quantum realm,  $\lambda \sim \frac{1}{p}$ , wavelength and momentum are related, so

> Low energy interactions do not depend on the details of high energy interactions

Bad:
 we have to work harder to probe the interesting physics at short distances

#### Good:

we can focus on the
relevant interactions &
degrees of freedom

• calculations are simpler

Newton didn't need quantum gravity for projectile motion





## Effective Field Theory Idea:



exact answer is irrelevant, work to the desired level of precision

Degrees of freedom can change:



## Effective Field Theory Idea:



exact answer is irrelevant, work to the desired level of precision

Symmetries of Theory 1 constrain the form of Theory 2:

Charge conjugation ( $e^+ \leftrightarrow e^-$ ) Parity ( $\vec{x} \rightarrow -\vec{x}$ ) Time-Reversal ( $t \rightarrow -t$ ) constrain the  $H_m$ 's Spin-Statistics Theorem





#### Effective Field Theory for Non-relativistic bound states

NRO

 $nL_J$  $F = J + S_p$ 



Compute the  $H_m$ :

**Relativity:** 
$$\frac{p^4}{8m_e^3} + .$$

$$H = H_0 + \sum_{m=1}^{\infty} \epsilon^m H_m$$

QED:  $\mu_e$ ,  $\vec{L} \cdot \vec{S}$ ,... (coefficients determined by  $\alpha$ ,  $m_e$ )

#### What about quarks?

When

u 
$$Q_u = +2/3$$
  
d u  $Q_d = -1/3$ 

size ~  $1 \,\mathrm{fm} \rightarrow 200 \,\mathrm{MeV} \gg p_{\gamma}$ 

low momentum photons do not resolve the quarks

matching

• •

couplings change too:  $Q_{u,d} \rightarrow Q_p$ 



This is just an application of the multipole expansion, familiar from electromagnetism:

$$\mathcal{V}(\vec{r}) = \frac{1}{r} \int \rho \, d^3 r' + \frac{1}{r^2} \int r' \cos\theta \rho \, d^3 r' + \dots$$
total
charge
$$200 \,\mathrm{MeV} \gg p_{\gamma} \Leftrightarrow r' \ll r$$
keV

Compute the  $H_m$ :

**Relativity:** 
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What about quarks?

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When  $\longrightarrow$  couplings change too:  $Q_{u,d} \rightarrow Q_p$ other parameters:  $m_p, \mu_p, \ldots$ in principle fixed by QCD, but it is more accurate to use experimental measurements measure a parameter in one place, then use it in others = universality

## Vacuum Polarization



 $e^+$   $\alpha$  coupling is

renormalized

 $\frac{e^2}{4\pi}$ 

resolution  $\mu = E$   $\mu \frac{d}{d\mu} \alpha(\mu) = \frac{2}{3\pi} \alpha^2(\mu)$ 

at larger energy E, we probe shorter distances and see a larger charge

 $\alpha(E) = \frac{\alpha(0)}{1 - \frac{\alpha(0)}{3\pi} \ln\left(\frac{E^2}{m^2}\right)}$ 



# Lamb Shift in NRQED

Two parts:



i) effective potentials
 ii) radiation in the bound
 (short distance)
 ii) radiation in the bound
 state (long distance)

p

$$\delta E_n = \left[\frac{4\alpha^2}{3m_e^2}|\psi_n(0)|^2\ln\left(\frac{\mu}{m_e}\right) + \dots\right] + \left[\frac{1}{m_e^2}\sum_{k\neq n}|\langle n|\hat{p}|k\rangle|^2(E_k - E_n)\ln\left(\frac{\mu}{|E_n - E_k|}\right) + \dots\right]$$

 $\mu$  dependence cancels, but allows us to give separate meaning to the two pieces

History:

• 1947 Bethe computed ii), with  $\mu = m_e$ 

large log:  $\sim \ln\left(\frac{m_e}{m_e\alpha^2}\right) = -2\ln(\alpha)$ 

 IP49 French & Weisskopf Lamb & Kroll (Feynman, Schwinger)



 $1040\,\mathrm{MHz}$ 

the

swer

 $\Delta E(2S -$ 

1058

# The structure of QED logs can be derived from a non-relativistic renormalization group Manohar, I.S.

 $\mu_E \sim \frac{\mu_p^2}{m}$ 

 $E = \frac{p^2}{2m}$ 

energy resolution  $\mu_E$ momentum resolution  $\mu_p$ 

|          | Comparison | System                                     | Observable                             | Correction              |
|----------|------------|--------------------------------------------|----------------------------------------|-------------------------|
|          | agrees*    | Н                                          | Lamb shift                             | $lpha^8 \ln^3 lpha$     |
|          | new        | $\mu^+ e^-, \ e^+ e^-$                     |                                        |                         |
| 11 0     |            |                                            | (no h.f.s., no $\Delta\Gamma/\Gamma$ ) |                         |
| all from | acrees     | $H = \mu^{+} \rho^{-} \rho^{+} \rho^{-}$   | hfs                                    | $\alpha^7 \ln^2 \alpha$ |
| 000      | agrees     | $H, \mu c, c c$<br>$H, \mu^+ e^-, e^+ e^-$ | I amh shift                            | a ma                    |
| one      | agrees     | $\mu$ e, e e                               |                                        | 0.0                     |
|          | agrees     | $e^+e^-$ or tho and para                   | $\Delta\Gamma/\Gamma$                  | $\alpha^3 \ln^2 \alpha$ |
| equation | agrees     | $H, u^+e^-, e^+e^-$                        | Lamb shift                             | $\alpha^6 \ln \alpha$   |
|          | a graad    | $H_{\mu}^{+} a^{-} a^{+} a^{-}$            | h f c                                  |                         |
|          | agrees     | $\mu$ , $\mu$ , $e$ , $e$ , $e$            | 11.1.5.                                |                         |
|          | agrees     | $e^+e^-$ or tho and para                   | $\Delta\Gamma/\Gamma$                  | $\alpha^2 \ln \alpha$   |
|          | agrees     | $H, \mu^+ e^-, e^+ e^-$                    | Lamb shift                             | $\alpha^5 \ln lpha$     |

LO anomalous dimension:  $\alpha^4 (\alpha \ln \alpha)^k$  stops at k = 1NLO anomalous dimension:  $\alpha^5 (\alpha \ln \alpha)^k$  stops at k = 3

# The structure of QED logs can be derived from a non-relativistic renormalization group Manohar, I.S.

 $E = \frac{p^2}{2m} \qquad \begin{array}{c} \text{energy resolution} & \mu_E \\ \text{momentum resolution} & \mu_p \end{array} \qquad \mu_E \sim \frac{\mu_p^2}{m}$ 

NRQED methods are also used for the non-logarithmic terms

|             |                     | Expt.(MHz)                | Theory(MHz)               | Agree?      |
|-------------|---------------------|---------------------------|---------------------------|-------------|
| Η           | Lamb                | 1057.845(9)               | 1057.85(1)                | $< r_p^2 >$ |
|             | h.f.s               | 1420.405751768(1)         | 1420.399(2)               | $G_E,G_M$   |
| $\mu^+ e^-$ | h.f.s               | 4463.30278(5)             | 4463.30288(55)            | $m_e/m_\mu$ |
| $e^+e^-$    | Lamb                | 13012.4(1)                | 13012.41(8)               | agree       |
|             | h.f.s               | 203389.10(74)             | 203391.70(50)             | $3\sigma$   |
|             | $\Gamma_{ m para}$  | 7990.9(1.7) $\mu s^{-1}$  | 7989.62(4) $\mu s^{-1}$   | agree       |
|             | $\Gamma_{ m ortho}$ | $7.0404(13) \ \mu s^{-1}$ | $7.03996(2) \ \mu s^{-1}$ | agree       |

The ideas we've discussed in QED:

• resolution  $\mu$ 

- changes in degrees of freedom & couplings
- expansions, multiple scales
- universality

become even more crucial for QCD

#### QCD Interactions are more complicated than QED:

#### strong coupling: $g(\mu)$



q q q g g g



these interactions involve the same coupling (gauge symmetry)

#### Vacuum response?





gluons have spin, carry color charge behave like a permanent magnet anti-screen the charge

$$\beta(g) = \mu \frac{d}{d\mu} g(\mu) = -\frac{g(\mu)^3}{16\pi^2} \left(11 - \frac{2}{3}n_f\right) < 0$$

In QCD, the coupling ,  $g(\mu)$  , behaves in the opposite way to QED, it gets weaker at short distances

slope is negative



large change in the value

 $\alpha_{s}(\mu) = \frac{g(\mu)^{2}}{4\pi} \qquad \beta(g) = \mu \frac{d}{d\mu} g(\mu) < 0$ Gross, Politzer, Wilczek







Nobel Prize, 2004 Asymptotic freedom large  $\mu = Q$ , small  $\alpha_s$ , free quarks Infrared slavery

as  $\mu = Q$  approaches a few 100 MeV ( $r \rightarrow 1 \text{ fm}$ ), the coupling gets large an expansion in  $\alpha_s(\mu < 1 \,\text{GeV})$  is no good

coupling gets so strong that quarks never escape unless they form a color singlet (bound) state with other quarks, ie. they are confined

Mesons  $\pi, K, \rho, \dots$  Baryons  $p, n, \Sigma, \Delta, \dots$ 

degrees of freedom change













 $m_b$ 

 $m_{c}$ 

 $\Lambda_{
m QCD}$ 

 $m_s$ 

 $m_{u,d}$ 

 $m_b \gg \Lambda_{\rm QCD}$ 

heavy quark symmetry Isgur & Wise

Decay by weak interactions; long lived

 $B \to X_u \ell \bar{\nu} \qquad B \to D\pi \qquad B \to K^* \gamma$  $B \to \pi \ell \bar{\nu} \qquad B \to X_s \gamma \qquad B \to \rho \gamma$  $B \to D^* \eta' \qquad B \to \rho \rho \quad B \to \pi \pi$  $B \to \gamma \ell \bar{\nu}$ 

Precision studies are sensitive to scales >  $m_W$ The B is heavy, so many of its decay products are energetic, E

eg.  $B \to D \ e \ \bar{\nu}$ ,  $M_W^2 \gg m_h^2 \gg \Lambda^2$ 



 $\mu$ 



$$\mathcal{L}_1 \to \mathcal{L}_2 \to \mathcal{L}_3 \to \mathcal{L}_4$$

expansion parameters

$$\frac{m_b^2}{m_W^2} \simeq \frac{1}{250}, \quad \alpha_s(m_b) \simeq 0.2, \quad \frac{\Lambda}{m_b} \simeq 0.1$$

# Soft - Collinear Effective Theory Bauer, Pirjol, I.S. Fleming, Luke

An effective field theory for energetic hadrons & jets

 $E \gg \Lambda_{\rm QCD}$ 



#### SCET is a field theory which:

- explains how these degrees of freedom communicate with each other, and with hard interactions
- organizes the interactions in a series expansion in
- provides a simple operator language to derive factorization theorems in fairly general circumstances

eg. unifies the treatment of factorization for exclusive and inclusive QCD processes

- new symmetry constraints
- scale separation & decoupling



 $\Lambda_{\rm QCD}$ 

#### How is SCET used?

- cleanly separate short and long distance effects in QCD
  - → derive new factorization theorems
  - → find universal hadronic functions, exploit symmetries & relate different processes
- model independent, systematic expansion
  - → study power corrections
- keep track of  $\mu$  dependence

→ sum logarithms, reduce uncertainties



 $\langle \pi | O_c(x) | 0 \rangle = f_\pi \phi_\pi(x)$ 

corrections will be  $\Lambda/m_c \sim 30\%$ 

#### **Color Suppressed Decays** Mantry, Pirjol, I.S. $\bar{B}^0 \to D^0 \pi^0$ Intractable without SCET subleading interaction (b) (a) $D^0$ $\overline{u}(\overline{s})$ $\bar{B}^0$ u (s) d $A_{00}^{D^{(*)}} = N_0^{(*)} \int dx \, dz \, dk_1^+ dk_2^+$ $J^{(i)}(z, x, k_1^+, k_2^+) S^{(i)}(k_1^+, k_2^+) \phi_M(x)$ $\Lambda^2$ $\gg$ $\gg$

 $Q = m_b, E_\pi, m_c$ 

prove S is same for D and D\*

#### Comparison to Data

(Cleo, Belle, Babar)



Extension to isosinglets: Blechman, Mantry, I.S.

Extension to baryons  $(\Lambda_b)$ : Leibovich, Ligeti, I.S., Wise



Not yet tested:

•  $Br(D^*\rho_{\parallel}^0) \gg Br(D^*\rho_{\perp}^0)$ ,  $Br(D^{*0}K_{\parallel}^{*0}) \sim Br(D^{*0}K_{\perp}^{*0})$ 

• equal ratios  $D^{(*)}K^*$ ,  $D_s^{(*)}K$ ,  $D_s^{(*)}K^*$ ; triangles for  $D^{(*)}\rho$ ,  $D^{(*)}K$ 

## $B \rightarrow \pi \pi$ Decays & Weak Interactions



CKM Matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

#### Violate

C: exchange of particles & antiparticles P: parity  $\vec{x} \rightarrow -\vec{x}$ 

Can use CP-violating observables in  $B \rightarrow \pi \pi$ to measure  $\gamma$ , but need to control QCD interactions



![](_page_35_Figure_8.jpeg)

#### $p^2 \sim \Lambda^2$ Factorization with SCET Bauer, Pirjol, Rothstein, I.S.; $p^2 \sim Q^2$ Beneke, Buchalla, Resolution $\mu = m_b$ Neubert, Sachrajda $B \to M_1 M_2 \quad (\sim 120 \text{ channels})$ Nonleptonic $p^2 \sim \Lambda^2$ $p^2 \sim \Lambda^2$ $p^2 \sim O\Lambda$ $A(B \to M_1 M_2) = A^{c\bar{c}} + N \left\{ f_{M_2} \zeta^{BM_1} \int du T_{2\zeta}(u) \phi^{M_2}(u) + f_{M_2} \int du dz \right\}$ $\left|\zeta_J^{BM_1}(z)\phi^{M_2}(u)+(1\leftrightarrow 2)\right|$ Form Factors $f(E) = \int dz \, T(z, E) \, \zeta_J^{BM}(z, E)$ universality at $+ C(E) \zeta^{BM}(E)$ $B \to \pi \ell \bar{\nu}$ , $E\Lambda$ $B \to K^* \ell^+ \ell^ B \to \rho \gamma$ , ... Resolution $\mu = \sqrt{E\Lambda}$ , expansion in $\alpha_s(\sqrt{E\Lambda})$ Beneke, Feldmann $\zeta_J^{BM}(z) = f_M f_B \int_0^1 dx \int_0^\infty dk^+ J(z, x, k^+, E) \phi_M(x) \phi_B(k^+)$

 $\zeta^{BM} = ?$  (left as a form factor)

Beneke, Feldmann Bauer, Pirjol, I.S. Becher, Hill, Lange, Neubert

$$\begin{array}{ccc} \bar{B}^0 \to \pi^+ \pi^-, & B^- \to \pi^0 \pi^-, & \bar{B}^0 \to \pi^0 \pi^0 \\ & B^0 \to \pi^+ \pi^-, & B^0 \to \pi^0 \pi^0 \end{array}$$

•  $C_{\pi^0\pi^0} = -0.28 \pm 0.39$ , uncertainty precludes measuring  $\gamma$  here (Belle & Babar)

 $\frac{\Lambda_{\text{QCD}}}{E_{\pi}} \ll 1 \quad \text{Factorization predicts a small relative} \\ \text{phase for two amplitudes } \epsilon \sim 0, \ \tau^{(t)} \sim 0 \end{cases}$ 

$$\epsilon = \operatorname{Im}\left(\frac{C}{T}\right) = \mathcal{O}\left(\alpha_s(m_b), \frac{\Lambda}{E}\right) \lesssim 0.2$$

![](_page_37_Figure_4.jpeg)

Bauer, Rothstein, I.S.

![](_page_37_Figure_5.jpeg)

Use this to get lpha without  $C_{\pi^0\pi^0}$  .

![](_page_37_Figure_7.jpeg)

![](_page_38_Figure_0.jpeg)

![](_page_38_Figure_1.jpeg)

![](_page_38_Figure_2.jpeg)

![](_page_39_Figure_0.jpeg)

#### SCET has been applied to many processes

| Process                                        | Non-Pert. functions                        | Utility                             |
|------------------------------------------------|--------------------------------------------|-------------------------------------|
| $\bar{B}^0 \to D^+ \pi^-, \dots$               | $\xi(w),  \phi_{\pi}$                      | study QCD                           |
| $\bar{B}^0 \to D^0 \pi^0, \dots$               | $S(k_i^+), \phi_{\pi}$                     | study QCD                           |
| $B \to X_s^{endpt} \gamma$                     | $f(k^+)$                                   | new physics, measure $f$            |
| $B \to X_u^{endpt} \ell \nu$                   | $f(k^+)$                                   | measure $ V_{ub} $                  |
| $B 	o \pi \ell  u, \dots$                      | $\phi_B(k^+),  \phi_\pi(x),  \zeta_\pi(E)$ | measure $ V_{ub} $ , study QCD      |
| $B \to \gamma \ell \nu,  \gamma \ell^+ \ell^-$ | $\phi_B$                                   | measure $\phi_B$ , new physics      |
| $B \to \pi \pi, K \pi, \ldots$                 | $\phi_B,  \phi_\pi,  \zeta_\pi(E)$         | new physics, CP violation, $\gamma$ |
|                                                | $\phi_{\bar{K}},\zeta_K(E)$                | study QCD                           |
| $B \to K^* \gamma, \ \rho \gamma$              | $\phi_B,  \phi_K,  \zeta_{K^*}^{\perp}(E)$ | measure $ V_{td}/V_{ts} $ ,         |
|                                                | $\phi_{\rho},\zeta_{\rho}^{\perp}(E)$      | new physics                         |
| $B \to X_s \ell^+ \ell^-$                      | $f(k^{+})$                                 | new physics                         |
| $e^-p \to e^-X$                                | $f_{i/p}(\xi), f_{g/p}(\xi)$               | study QCD, measure p.d.f's          |
| $p\bar{p} \to X\ell^+\ell^-$                   | $f_{i/p}(\xi), f_{g/p}(\xi)$               | study QCD                           |
| $e^-\gamma \to e^-\pi^0$                       | $\phi_{\pi}$                               | measure $\phi_{\pi}$                |
| $\gamma^* M \to M'$                            | $\phi_M,\phi_{M'}$                         | study QCD                           |
| $e^+e^- \rightarrow j_1 + \text{jets}$         | $\tilde{S}(k^+)$                           | event shapes & universality         |
| $e^+e^- \to J/\Psi X$                          | $S^{(8,n)}(k^+)$                           | study QCD                           |
| $\Upsilon 	o X\gamma$                          | $S^{(8,n)}(k^+)$                           | study QCD                           |
|                                                |                                            |                                     |

At MIT: C.Arnesen, D.Pirjol, A.Jain, B.Lange, K.Lee, S.Mantry

![](_page_41_Picture_0.jpeg)

# Who needs to understand QCD?

![](_page_42_Picture_1.jpeg)

## Immediate future:

Babar, Belle • For many channels, control of hadronic uncertainties is crucial to test standard model & look for new physics.

 $B \to X_s \ell^+ \ell^-, B \to \pi \pi, B \to K \pi, B \to \rho \pi, \dots$  $B \to \rho \gamma, B \to K^* \gamma, B \to \phi K_s, B \to \eta' K_s$ 

CDF, DØ • Test standard model / new physics in  $B_s, \Lambda_b, \ldots$ • Heavy quark production, jets, ...

## LHC era:

pp collider with  $E_{cm} = 14 \text{ TeV}$ scales:  $m_W, m_t, E_T^{\text{jet}}$ 

Energetic QCD (SCET)

![](_page_44_Picture_3.jpeg)

Effective theory concepts will be helpful whether we're: • exploring QCD,

- computing precision standard model cross sections (resolution scales or resummation of logs),
- or puzzling out signals of unexplored particle physics

# **Concluding Remarks**

- QED fundamental parameters & precision quantum field theory
- QCD today is as rich & diverse as ever many subfields which focus on different degrees of freedom and different relevant interactions
- SCET a new approach to derive factorization theorems and treat power corrections for energetic hadrons & jets
   Nonleptonic B-decays
  - ⇒ predictions for the size of amplitudes
     ⇒ universal hadronic parameters, strong phases
     ⇒ γ (or α) from individual B → M<sub>1</sub>M<sub>2</sub> channels
     A lot of theory and phenomenology left to study ...