Theoretical Introduction to B-decays

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QCD and High Energy Interactions
XXX VIIIth Rencontres de Moriond
March 22-29, 2003

Outline

- B-Physics Motivation

Heavy Quark Decays in 3 parts

- (1) $m_b/m_W \ll 1$, trees, penguins
- Applications: $\sin(2\beta) = \sin(2\phi_1)$, $\Delta m_{d,s}$
- (2) $\Lambda/m_b \ll 1$, Heavy Quark Effective Theory
- Applications: V_{cb}, V_{ub}
- (3) $\Lambda/Q \ll 1$ where $Q = \{E_H, m_b\}$, Factorization, Soft-Collinear Effective Theory
- Conclusion

Motivation

b-Hadrons:

- Heaviest bound states $\rightarrow$ Laboratory for EW, new physics, & QCD
- The lightest B^0, $\bar{B}^0$, $B^\pm$ decay weakly to many channnels

$$B \to D^{(*)} e\nu,\ B \to D_{1,2}^{(*)} e\nu,\ B \to \pi e\nu,\ B \to \rho e\nu,$$
$$B \to K^*\gamma,\ B \to K e^+ e^-,\ B \to \rho\gamma,$$
$$B \to \tau\nu,\ B \to \gamma e\nu,\ B \to e^+ e^- e\nu,$$
$$B \to D\pi,\ B \to \pi\pi,\ B \to K\pi,\ B \to J/\Psi K_S,$$
$$B \to X_u e\nu,\ B \to X_s\gamma,\ B \to X_s \nu\bar{\nu},$$
$$\ldots$$

(Repeat for B_s, Λ_b, $\ldots$)

$\longrightarrow$ To study high energy physics we need to elliminate hadronic uncertainties from QCD

$\longrightarrow$ Alternatively, B-decays provide a system where we can study hadronic structure and power corrections in QCD

Motivation

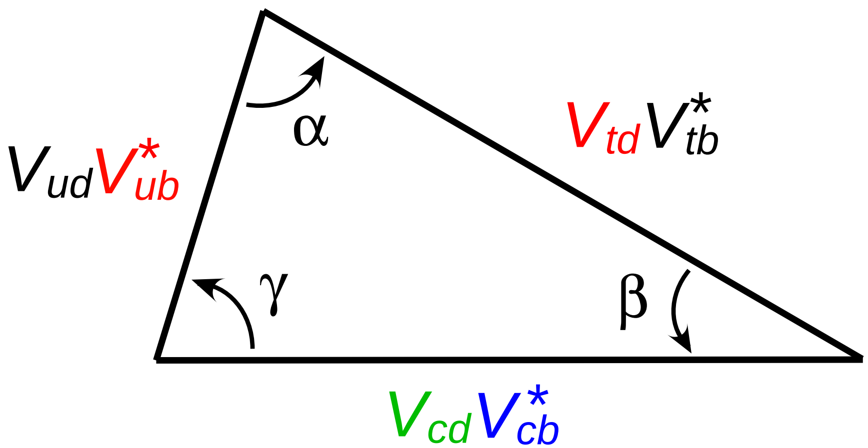
CP violation, SM $\rightarrow$ CKM matrix:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

3 angles + 1 phase which violates CP

Heirarchy: ~ 1, $\sim \lambda$, $\sim \lambda^2$, $\sim \lambda^3$
($\lambda \simeq .22$)

- unlike K and D systems CP violation is a large effect
- top quark loops not CKM suppressed (larger mixing, rare decays)



$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

Want to overconstrain SM with many (clean) measurements

Motivation

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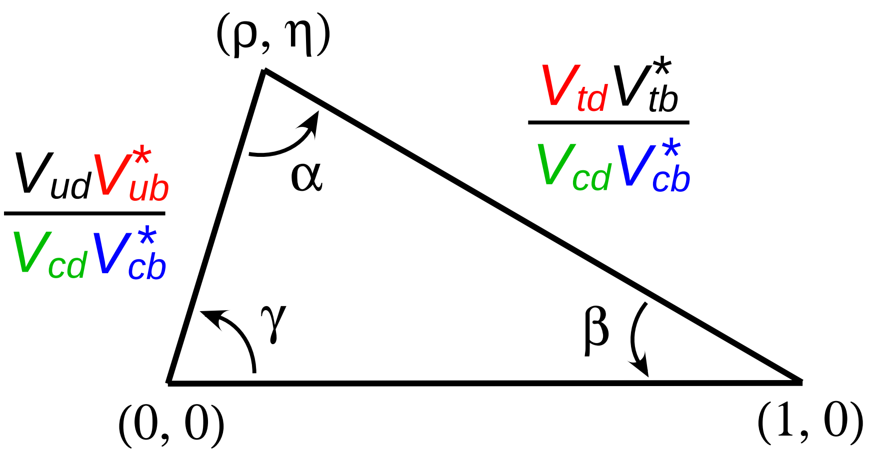

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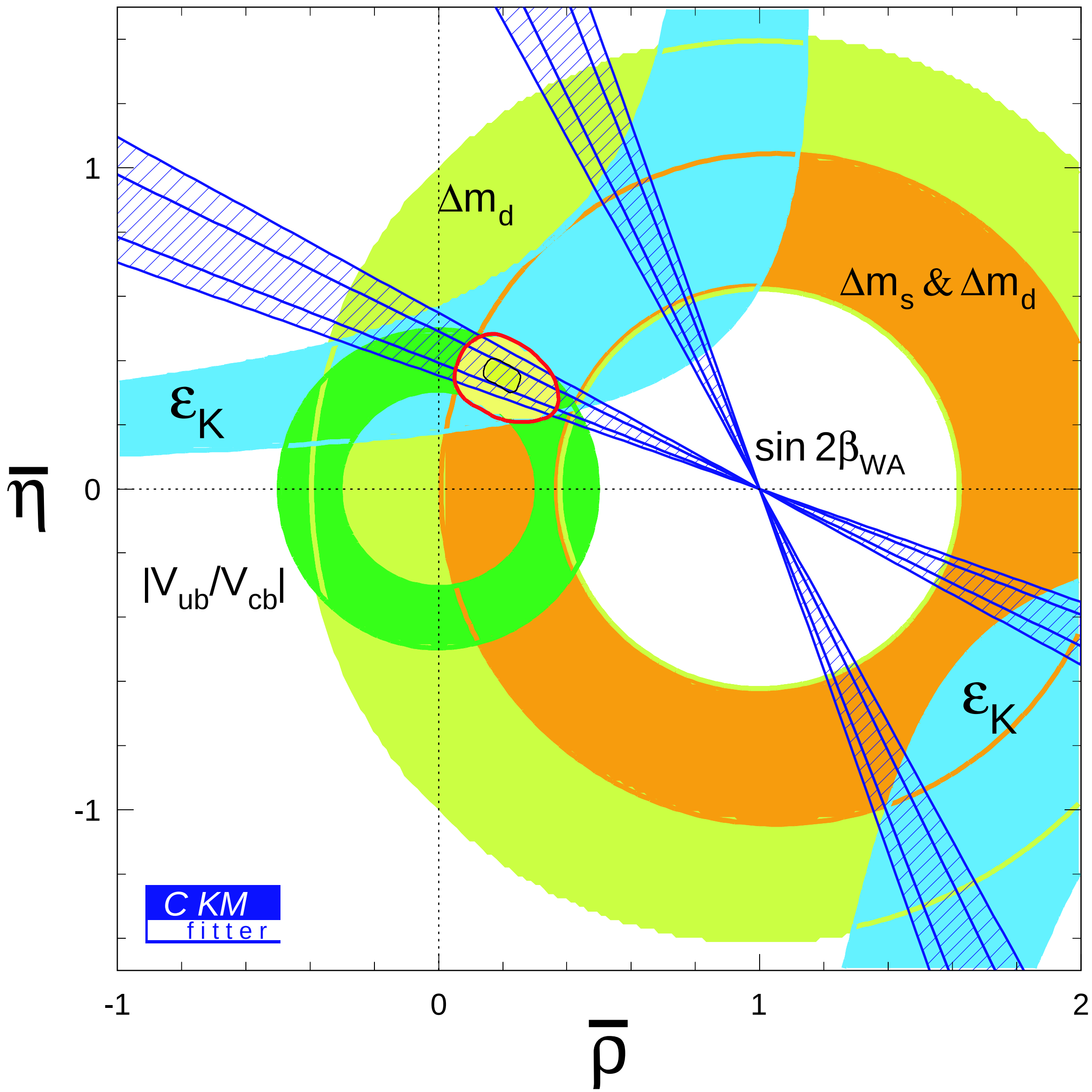
Unitarity Triangle



Success for SM!

Assume $VV^\dagger = 1$, check consistency

Used:

-V_{cd} through norm

Shown:

-ϵ_K from $K \to \pi\pi$

-$\Delta m_{d,s}$ from B^0-$\bar{B}^0$, B_s-$\bar{B}_s$

-V_{ub} from LEP, CLEO

-$\sin(2\beta)$(Belle,BaBar)

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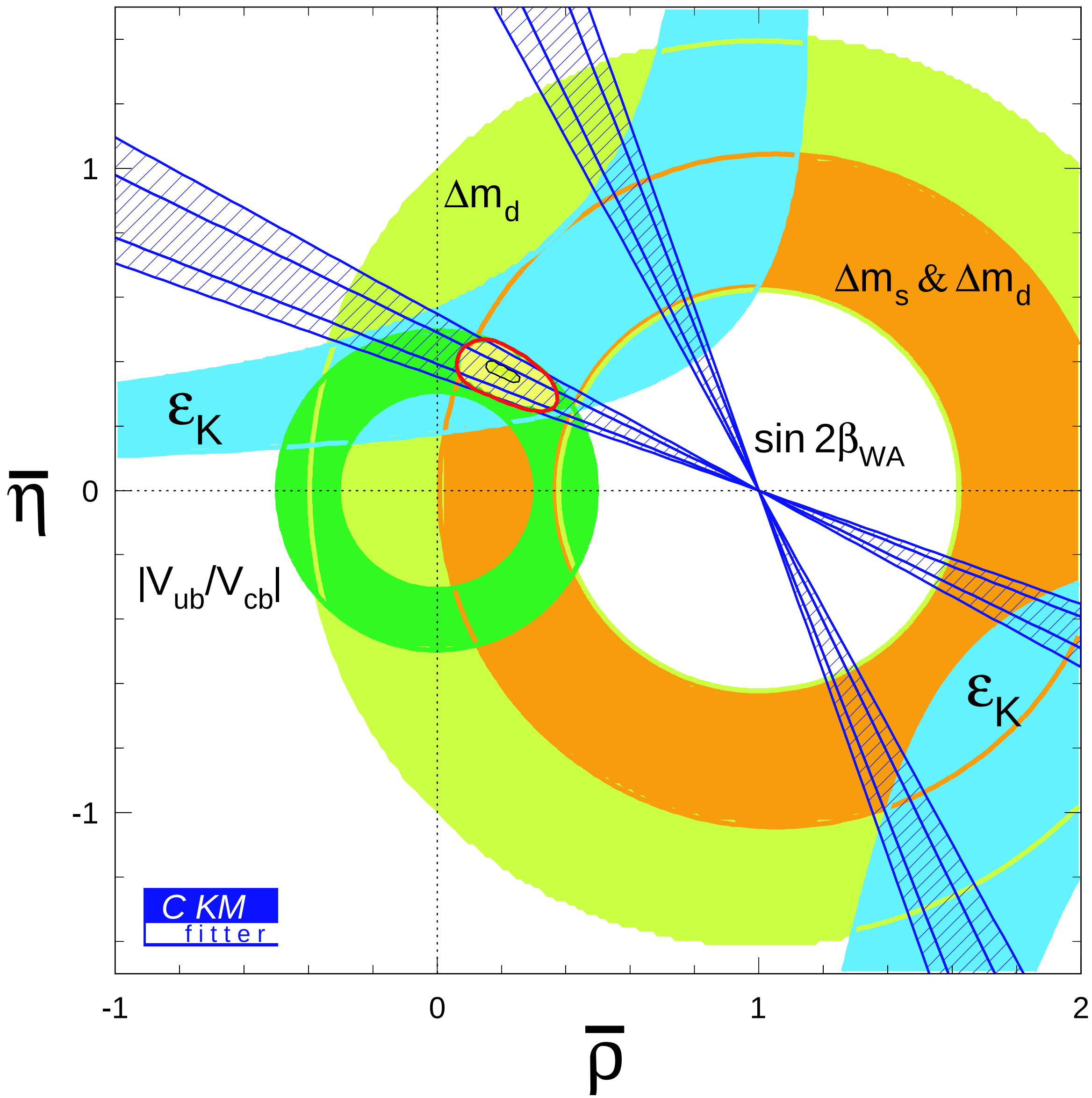

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Theory

Effective Field Theory: Useful for separating physics at different momentum scales

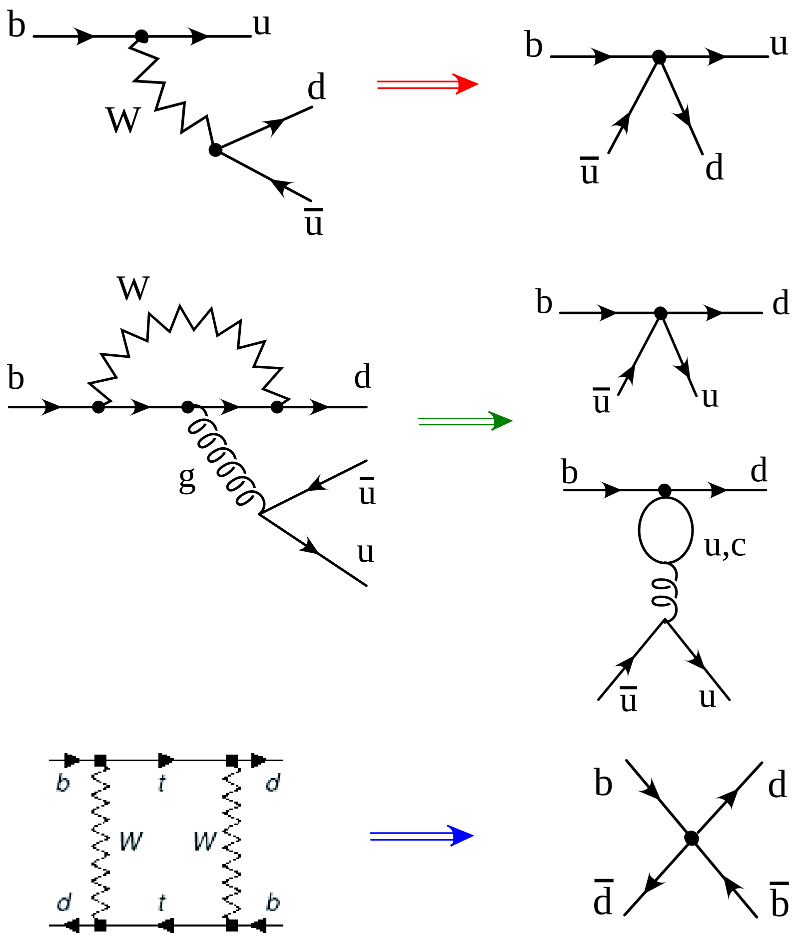
Factorization:

To understand hadronic uncertainties we need to separate short ($p \sim Q$) and long ($p \sim \Lambda_{\rm QCD}$) distance contributions

Some processes are cleaner than others!

Theory

1) Integrating out the W, t ($m_W, m_t \gg m_b$):



$$H_W = \frac{G_F}{\sqrt{2}} \sum_i \lambda^i C_i(\mu) O_i(\mu)$$

$$O_1 = (\bar{u}b)_{V-A}(\bar{d}u)_{V-A}$$

$$O_2 = (\bar{u}_i b_j)_{V-A}(\bar{d}_j u_i)_{V-A}$$

$$O_3 = (\bar{d}b)_{V-A} \sum_q (\bar{q}q)_{V-A}$$

$$O_{4,5,6} = \ldots$$

$$O_{7\gamma,8G} = \ldots$$

$$O^{ew}_{7,\ldots,10} = \ldots$$

$$O_{\Delta B=2} = (\bar{d}b)_{V-A}(\bar{d}b)_{V-A}$$

$$\ldots$$

Operators come with different CKM elements, $\lambda^1 = V_{ub}V^*_{ud}, \ldots$

$B \to \psi K_s$

$$B^0 \to \bar{B}^0(\, b\bar{d} \to c\bar{c}s\bar{d}\,) \to \psi K_S$$
$$B^0 \to B^0(\, \bar{b}d \to \bar{c}c\bar{s}d\,) \to \psi K_S$$

$$a_{CP} = \frac{\Gamma[\bar{B}^0(t) \to f] - \Gamma[B^0(t) \to f]}{\Gamma[\bar{B}^0(t) \to f] + \Gamma[B^0(t) \to f]}$$

Has a dominant weak phase $\to$ theoretically clean

Tree: $\bar{A}_T = V_{cb} V_{cs}^* A_{c\bar{c}s}$

Penguin: $\bar{A}_P = V_{tb} V_{ts}^* P_t + V_{cb} V_{cs}^* P_c + V_{ub} V_{us}^* P_u$

Use Unitarity, $\sum_i V_{ib} V_{is}^* = 0$, to remove $V_{tb} V_{ts}^*$:

$$\bar{A} = \underbrace{V_{cb} V_{cs}^*}_{\mathcal{O}(\lambda^2)} [A_{c\bar{c}s} + P_c - P_t] + \underbrace{V_{ub} V_{us}^*}_{\mathcal{O}(\lambda^4)} [P_u - P_t]$$

Strong phase cancels in $\bar{A}/A$ since QCD preserves CP

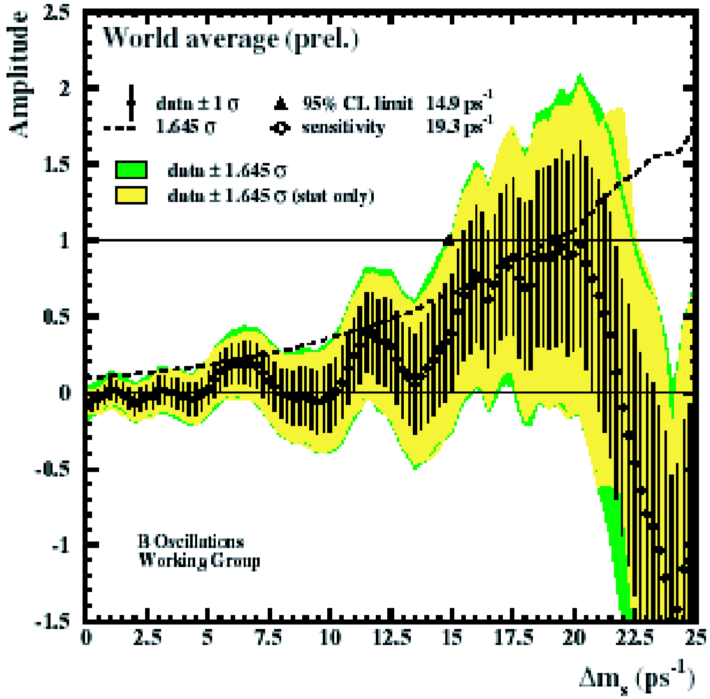
Asymmetry: $a_{CP} = Im(\lambda_{\psi K_S}) \sin(\Delta m\, t) = sin(2\beta) sin(\Delta m\, t)$

$\Delta m_{d,s}$

B^0-$\bar{B}^0$, B_s-$\bar{B}_s$ mixing dominated by top quarks

$$\Delta m_q = \frac{1}{6\pi^2} G_F^2 m_W^2 m_{B_q} |V_{tb} V_{tq}^*|^2 \, \eta_B S(x_t) f(\mu) f_{B_q}^2 B_{B_q}(\mu)$$

perturbative corrections



Lattice: ($\%\sigma_{rel}$ in CKM Fitter)

$\Delta m_d \to f_B^2 B_B$ (36%)

$\frac{\Delta m_s}{\Delta m_d} \to f_{B_s}^2 B_{B_s}/(f_B^2 B_B)$ (10%)

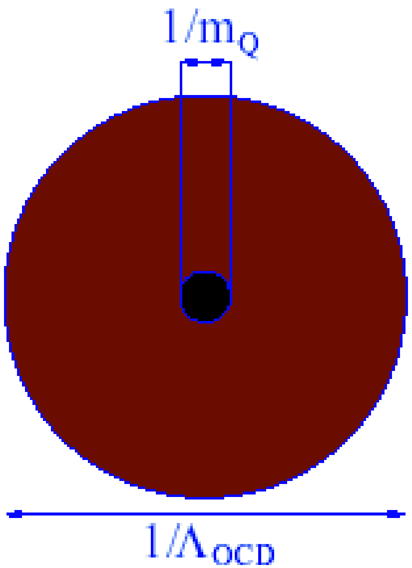
(HQET or NRQCD actions for b)

Future: improvements in Lattice, help from CLEO$_c$ and f_B/f_D, $f_{B_s} f_D/(f_B f_{D_s})$

$\Delta m_d = 0.489 \pm .008\, ps^{-1}$

$\Delta m_s > 14.4\, ps^{-1}$
(measurement $\to$ Run-II)

Heavy Quark Effective Theory



Expand in $\Lambda_{\rm QCD}/m_b$

$$\mathcal{L}^b_{QCD} = \bar{b}(i\not{D} - m_b)b \quad\Longrightarrow\quad \sum_{n=0} \mathcal{L}^{(n)}_{\rm HQET}$$

$$\mathcal{L}^{(0)}_{\rm HQET} = \bar{b}_v\, iv\cdot D\, b_v$$

$$\mathcal{L}^{(1)}_{\rm HQET} = \bar{b}_v \frac{(iD_\perp)^2}{2m_Q} b_v + c_F(\mu)\, \bar{b}_v \frac{\sigma_{\mu\nu} g G^{\mu\nu}}{4m_Q} b_v$$

Benefits:

- Rigorous power counting in Λ/m_b,
- Separation of perturbative and non-perturbative contributions: $\alpha_s(m_b)$ corrections & universal hadronic parameters $\lambda_{1,2}$
- heavy-quark spin-flavor symmetry in $\mathcal{L}^{(0)}_{\rm HQET}$

$$m_{B^{(*)}} = m_b + \bar{\Lambda} - \frac{\lambda_1}{2m_b} + d_h \frac{\lambda_2}{2m_Q} + \ldots$$

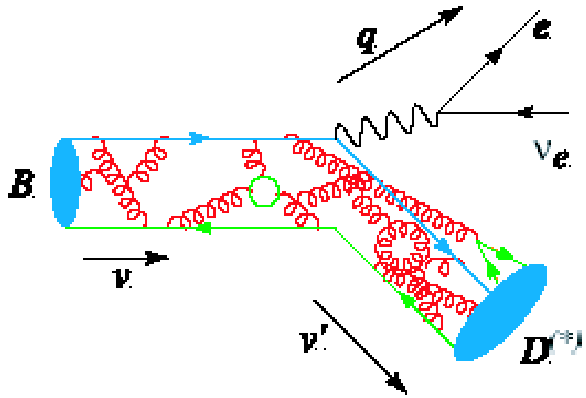
Application: $B \to D^{(*)} e\bar{\nu}$

Exclusive Decays:

Brown muck only feels $v \to v'$

six form factors $\Longrightarrow$ $\xi^{IW}(w)$

where $w = v\cdot v'$, $\xi^{IW}(1) = 1$



$$\frac{d\Gamma(B \to D^{(*)}\ell\bar{\nu})}{dw} = \text{(phase space)}|V_{cb}|^2 \mathcal{F}^2_{(*)}(w)$$

$$\mathcal{F}(1) = 1 + f(\alpha_s) + \frac{(lattice)}{m_{c,b}} + \ldots$$

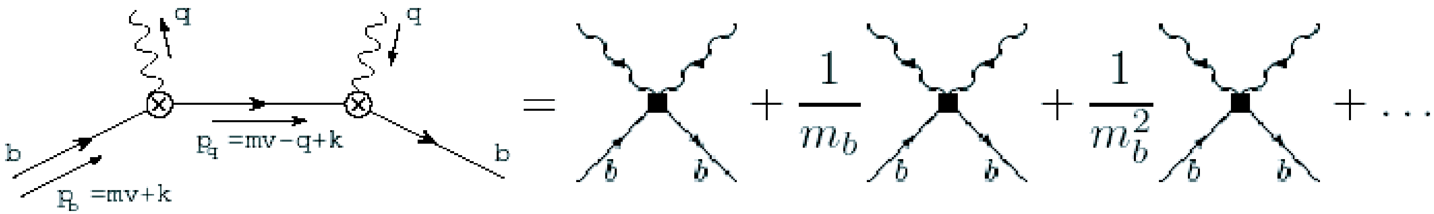
$$\mathcal{F}_*(1) = 1 - g(\alpha_s) + \frac{0}{m_{c,b}} + \frac{(lattice)}{m^2_{c,b}} + \ldots$$

$|V_{cb}|^2$ from extrapolation of $B \to D^*\ell\nu$ data to zero recoil, $w = 1$

Application: $B \to X_c \ell \bar{\nu}$

Inclusive Decay: OPE in Λ/m_b

$$\mathrm{Im} \int d^4x\, e^{-iq\cdot x} \langle B | T\{ J^{\mu\dagger}(x) J^{\nu}(0) \} | B \rangle$$



- $m_b \to \infty$ is free quark decay, $\alpha_s(m_b)$ corrections computable
- No Λ/m_b corrections $\to$ OPE gives 0 at this order
- At Λ^2/m_b^2 have dependence on λ_1, λ_2
- OPE results need to be smeared (duality assumed)

Method: fit E_ℓ, m_X^2 moments of spectra to m_b ($\bar{\Lambda}$), λ_1, $|V_{cb}|$

CLEO: $|V_{cb}| = (40.4 \pm 1.3) \times 10^{-3}$, $\bar{\Lambda} = 0.35 \pm 0.12\,\mathrm{GeV}$,

$$\lambda_1 = -.24 \pm .11\,\mathrm{GeV}^2$$

V_{ub}

$B \to X_u \ell \nu$:

Cuts to exclude $b \to c$ background make observables less inclusive

1) $m_X^2 \gg E_X \Lambda \gg \Lambda^2$, OPE in Λ/m_B
2) $m_X^2 \sim E_X \Lambda \gg \Lambda^2$, non-perturbative shape function — Neubert
3) $m_X^2 \sim \Lambda^2$, resonance region, $B \to \pi \ell \nu$, $B \to \rho \ell \nu$

Inclusive

- Can cut on q^2 (& m_X^2) and still remain in 1) — Bauer,Ligeti,Luke
- Can attempt to reconstruct the total rate — LEP Working Group
- Make cuts on E_ℓ (or m_X) which put us in 2), and use shape function extracted from $B \to X_s \gamma$ — CLEO

V_{ub}

$B \to X_u \ell \nu$:

Cuts to exclude $b \to c$ background make observables less inclusive

1) $m_X^2 \gg E_X \Lambda \gg \Lambda^2$, OPE in Λ / m_B
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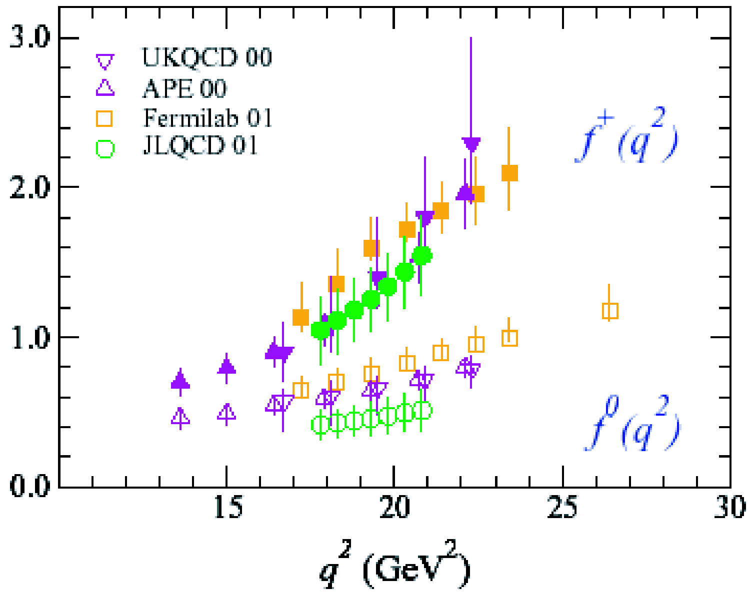
3) $m_X^2 \sim \Lambda^2$, resonance region, $B \to \pi \ell \nu$, $B \to \rho \ell \nu$

Exclusive

- Use exclusive channels plus form factor models (eventually Lattice)



$B \to \pi$ form factors

Yamada (Lattice '02)

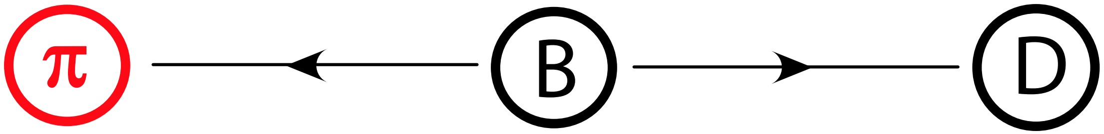
- consistent, $\sim 20\%$ errors
- quenched, only large q^2 so far

Factorization

Theory so far dealt with soft hadrons (B, $D^{(*)}$) and fully inclusive quantities (Λ/m_b expansion)

For processes with energetic hadrons or collimated jets we need collinear degrees of freedom too.

eg1.



Pion has: $p_\pi^\mu \;=\; (2.310\,\mathrm{GeV}, 0, 0, -2.306\,\mathrm{GeV}) = Q\,n^\mu\,, \quad n^2 = 0$

$Q \gg \Lambda_{QCD}$

collinear constituents:

$$(p^+, p^-, p^\perp) \;\sim\; \Big(\frac{\Lambda_{QCD}^2}{Q}, Q, \Lambda_{QCD}\Big) \sim Q(\lambda^2, 1, \lambda) \qquad \lambda \ll 1$$

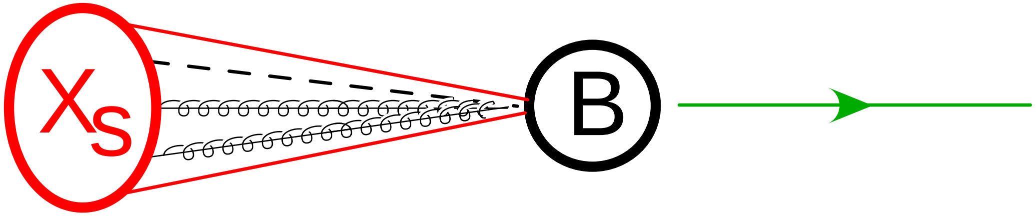
Factorization Theorem:

$$\langle D\pi|\bar{d}u\bar{c}b|B\rangle = N F^{B\to D}(0) \int_0^1 d\xi\; T(\xi, \tfrac{m_c}{m_b}, Q, \mu)\; \phi_\pi(\xi, \mu) + \mathcal{O}(\Lambda/Q)$$

Politzer-Wise
BBNS (two-loops)

Factorization

eg2.



γ endpoint

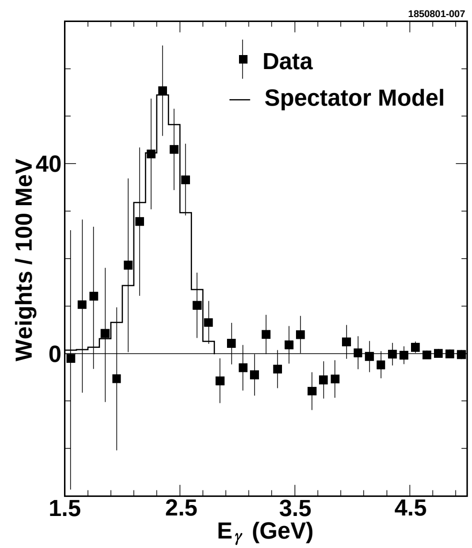
jet constituents:

$$(p^+, p^-, p^\perp) \quad \sim \quad \left(\Lambda_{QCD}, Q, \sqrt{Q\Lambda_{QCD}}\right) \sim Q(\lambda^2, 1, \lambda) \qquad\qquad \lambda \ll 1$$

Decay rate is given by factorized form — Korchemsky, Sterman

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = H(m_b, \mu) \int_{2E_\gamma - m_b}^{\bar\Lambda} dk^+ \, S(k^+, \mu) \, J(k^+ + m_b - 2E_\gamma, \mu)$$
$$+\mathcal{O}(\Lambda/Q)$$

$S(k^+, \mu)$ is shape function



Soft-Collinear Effective Theory

Bauer, Fleming, Luke, Pirjol, I.S.

Introduce fields for infrared degrees of freedom

modes	$p^\mu = (+, -, \perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_{n,p}$, $A^\mu_{n,q}$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_{s,p}$, $A^\mu_{s,q}$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	q_{us}, A^μ_{us}

Offshell modes with $p^2 \gg Q^2\lambda^2$ are integrated out into coefficients

Goals of EFT:

- simplify power counting in $\lambda = \Lambda_{\text{QCD}}/Q$,
  $\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \ldots, \quad O = O^{(0)} + O^{(1)} + \ldots$
- make symmetries explicit (eg. gauge, spin, rpi)
- sum IR logarithms
- understand universal parts of factorization
- model independent description of power corrections

Other people:
Rothstein, Chay
Kim, Manohar
Mehen, Leibovich
Neubert, Hill
Beneke, Feldmann
Chapovsky, Diehl
Wyler, Lunghi
...

Properties

Note: understanding power corrections will be crucial for B-decays since $\lambda \sim 1/5 - 1/3$ is not that small

Gauge Symmetry:

• have separate collinear, soft, usoft gauge invariance

• integrating out offshell fluctuations builds up Wilson line $W[\bar{n}\cdot A_n]$

• $C(i\bar{n}\cdot D_c) = WC(\bar{\mathcal{P}})W^\dagger$ is hard-collinear factorization

• collinear fields can be chosen so that gauge transformations do not connect different orders in λ

Spinor reduction:

• $\not{n}\xi_n = 0$, for each n only have "good" components

Reparameterization Invariance:

• invariance under small changes in n, $\bar{n}$ restore Lorentz invariance order by order in λ

• constrains form of subleading operators, and uniquely fixes many of their Wilson coefficients

Properties

Matching:

- EFT reproduces IR divergences of full theory order by order in λ
- Matching for hard coefficients is IR-safe and independent of choice of IR regulator

Power Counting:

- SCET operators and diagrams are homogeneous in λ
- formula $\delta = 4 + \sum_k (V_c^k - 4) + \ldots$ counts powers of λ solely through vertices in any graph, just like Chiral perturbation theory

Ultrasoft-collinear factorization

decoupling of ultrasoft gluons in $\mathcal{L}_c^{(0)}$ by field redefinitions

$$\xi_{n,p} \quad = \quad Y_n \, \xi_{n,p}^{(0)} \, , \qquad A_{n,q} = Y_n \, A_{n,q}^{(0)} \, Y_n^\dagger$$

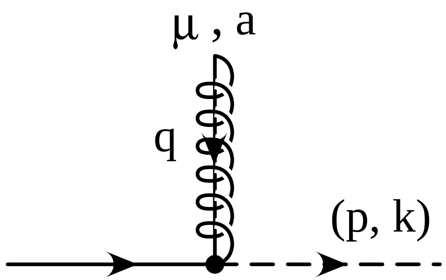
where $Y_n = \mathrm{P} \exp \left[ig \int_{-\infty}^{x} ds \, n \cdot A^{us}(ns) \right]$ (color transparency)

Properties

eg. usoft-collinear Lagrangian

$$\mathcal{L}_c = \mathcal{L}^{(0)}_{\xi\xi} + \mathcal{L}^{(0)}_{cg} + \mathcal{L}^{(0)}_{u} + \mathcal{L}^{(1)}_{\xi\xi} + \mathcal{L}^{(1)}_{\xi q} + \mathcal{L}^{(1)}_{cg} + \ldots$$

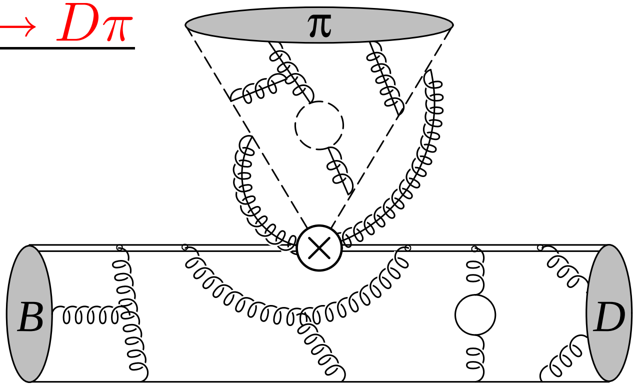
$$\mathcal{L}^{(1)}_{\xi q} = \bar{\xi}_n \frac{1}{i\bar{n}\cdot D_c}\, ig\, \not{B}^{\perp}_c W q_{us} + \text{h.c.}$$



$$ig\not{B}^{\perp}_c = [i\bar{n}\cdot D_c, i\not{D}^{\perp}_c],$$

Results

$\underline{B \to D\pi}$



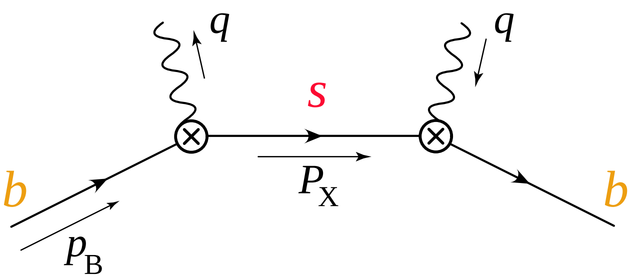
$$Q = m_b, m_c, E_\pi \gg \Lambda_{QCD}$$

B, D are soft, π collinear

All orders proof

Bauer, Pirjol, I.S.

$\underline{B \to X_s\gamma}$



For endpoint region:

$$E_\gamma \gtrsim \frac{m_B}{2} - \Lambda_{\rm QCD} \simeq 2.2\,{\rm GeV},$$

X_s collinear, B usoft, $\lambda = \sqrt{\frac{\Lambda_{QCD}}{m_B}}$

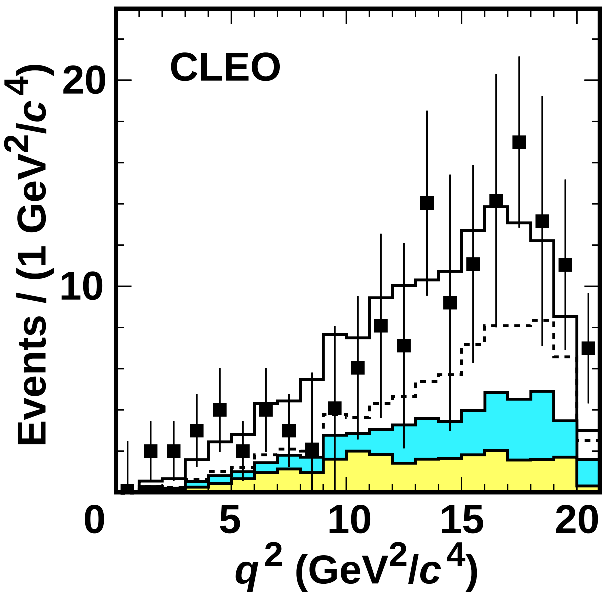
reproduces Korchemsky, Sterman

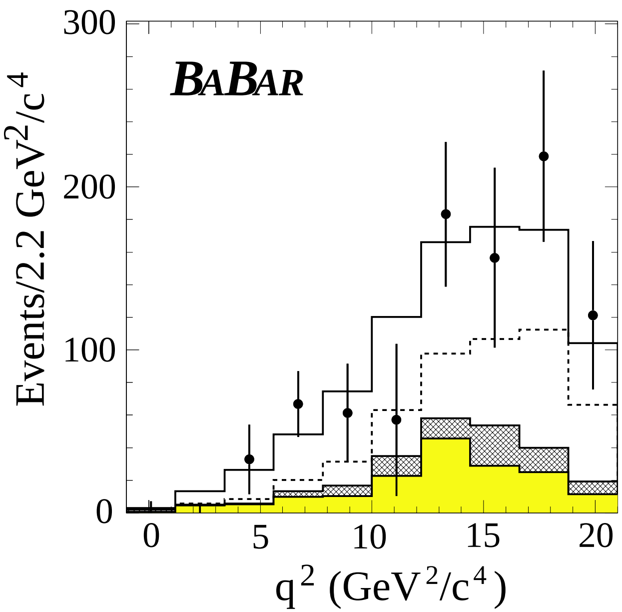
Power corrections tractable

Heavy-to-Light Decays

- Large q^2 accessible on the Lattice ($B \to \pi\ell\nu$, $q^2 \gtrsim 17\,\mathrm{GeV}^2$)
- For small q^2, $E \gg \Lambda_{\mathrm{QCD}}$ and large energy factorization applies

$B \to \rho\ell\nu$





Why is it interesting?

- Important ingredient for $B \to \pi\pi$ (PQCD vs. QCDF)
- Phenomenology: $|V_{ub}|$, $B \to \rho\gamma$, $B \to K^* e^+ e^-$, ...
- Interesting Features: Endpoint singularities, Sudakov suppression

Heavy-to-Light Form Factor

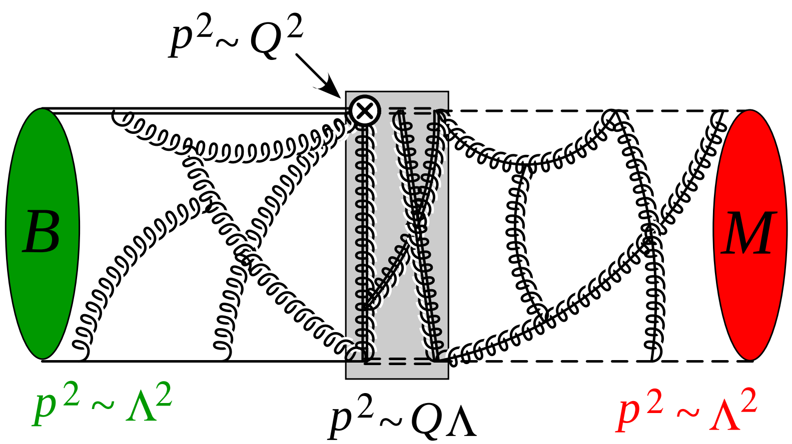
- For small q^2, $E \gg \Lambda_{\rm QCD}$ and large energy factorization applies

SCET result $f(Q) = f^F(Q) + f^{NF}(Q)$

Bauer,Pirjol,I.S. (hep-ph/0211069)

$$f^F(Q) = \frac{f_B f_M m_B}{4E^2} \int_0^1 dz \int_0^1 dx \int_0^\infty dr_+ \, T(z,Q,\mu_0) \times J(z,x,r_+,Q,\mu_0,\mu) \phi_M(x,\mu) \phi_B^+(r_+,\mu)$$

$$f^{\rm NF}(Q) = C_k(Q,\mu)\, \zeta_k^M(Q,\Lambda,\mu)\,.$$



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&\quad \times J(z,x,r_+,Q,\mu_0,\mu)\phi_M(x,\mu)\phi_B^+(r_+,\mu) \\
f^{\rm NF}(Q) &= C_k(Q,\mu)\, \zeta_k^M(Q,\Lambda,\mu)\,.
\end{aligned}
$$

- result at LO in $1/Q$, all orders in α_s, where $Q = \{m_b, E_M\}$
- no double counting, no missing pieces
- no assumption about tail of wavefunctions in power counting
- "endpoint singularities" occur in f^{NF}, regulated by non-perturbative gluons in $Y^\dagger D_\perp^{us} Y$
- f^{NF} obey "symmetry" relations of Charles et al. (one ζ for pseudoscalar mesons, two for vector mesons)

$B \to \pi^+\pi^-$

CP Asymmetry for $B^0(t) \to \pi^+\pi^-$, $\bar{B}^0(t) \to \pi^+\pi^-$

$$\mathcal{A}_{CP}(t) = -S_{\pi\pi}\sin(\Delta m_B t) + C_{\pi\pi}\cos(\Delta m_B t)$$

$$S_{\pi\pi} = \frac{2\,\mathrm{Im}\lambda}{1+|\lambda|^2}\,, \quad C_{\pi\pi} = \frac{1-|\lambda|^2}{1+|\lambda|^2}\,, \quad \lambda = e^{2i\alpha}\frac{1+e^{i\gamma}P/T}{1+e^{-i\gamma}P/T}$$

Unlike $B \to \psi K_S$ both weak phases give sizeable contributions

T = tree **P** = penguin

,

remove Penguin pollution with $B^0, \bar{B}^0 \to \pi^0\pi^0$, $B^+ \to \pi^+\pi^0$ plus isospin analysis

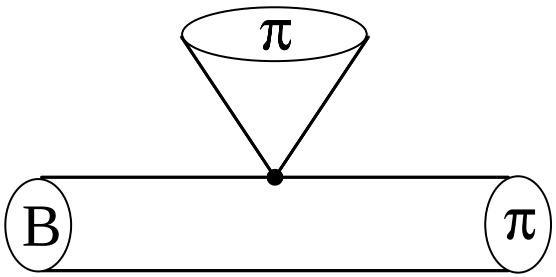
Gronau and London

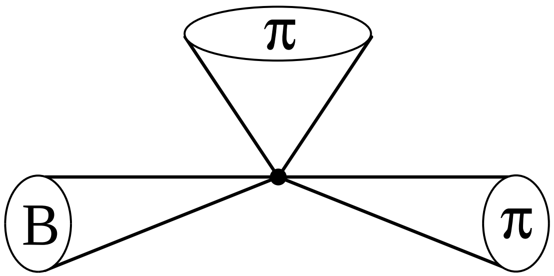
$B \to \pi^+\pi^-$ Factorization

Two proposals for computing P/T:

1) "QCD Factorization" Beneke, Buchalla, Neubert, Sachrajda

$$\langle O_i \rangle = \quad F^{B\to\pi}\, T(x) \otimes \phi_\pi(x) \quad + \quad T(\xi, x, y) \otimes \phi_B(\xi) \otimes \phi_\pi(x) \otimes \phi_\pi(y)$$





- same size in $1/m_b$
- second term suppressed by $\alpha_s(\sqrt{m_b\Lambda})$, first dominates
- predicts small strong phases

2) "k_T Factorization" Keum, Li, Sanda

(see next talk)

$B \to \pi^+\pi^-$ Factorization

Recent $B \to \pi\pi$ SCET result

Chay, Kim

- Claims agreement with BBNS, . . .

Current Experimental Results:

Babar: $S_{\pi\pi} = .02 \pm .34 \pm .05, \qquad C_{\pi\pi} = -.30 \pm .25 \pm .04$

Belle: $S_{\pi\pi} = -1.23 \pm .41\, ^{+.08}_{-.07}, \qquad C_{\pi\pi} = -.77 \pm .27 \pm .08$

Conclusions

- B-physics is a fundamental testing ground for electroweak physics, new physics, and QCD
- CKM picture passed its first test, and is probably the dominant source of CP violation in flavor changing processes
- Uncertainties from QCD need to be handled in a controlled fashion (with convergent power expansions and/or Lattice).

SCET-Conclusions

- Factorization emerges from an effective theory, operator formulation
- SCET allows us to address power corrections in model independent way
- Can also be applied to Inclusive/Exclusive Hard Scattering Processes (not discussed here)