Factorization in B-Decays & the Soft-Collinear Effective Theory

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Outline

power expansion of QCD

- Motivation
- Factorization & the Soft-Collinear Effective Theory (SCET)
- Focus on B decays:

  i) charm (test factorization):

  $B \to D\pi \qquad B \to D\rho \qquad \Lambda_b \to \Sigma_c^{(*)}\pi$

  ii) ~~CP~~: $B \to \pi\pi \qquad B \to K\pi \qquad B \to \rho\pi$

  relation to $B \to \pi\ell\bar{\nu}$

  iii) inclusive decays (Vub, shape functions):

  $B \to X_u \ell\bar{\nu} \qquad B \to X_s\gamma$

- Outlook

B decays - Motivation

- Heavy Stable Hadrons → lots of decays

BOTTOM MESONS
$(B = \pm 1)$

$B^+ = u\overline{b}$, $B^0 = d\overline{b}$, $\overline{B}^0 = \overline{d}\,b$, $B^- = \overline{u}\,b$, similarly for B^*'s

B-particle organization

Many measurements of B decays involve admixtures of B hadrons. Previously we arbitrarily included such admixtures in the $B^\pm$ section, but because of their importance we have created two new sections: "$B^\pm/B^0$ Admixture" for $\Upsilon(4S)$ results and "$B^\pm/B^0/B^0_s/b$-baryon Admixture" for results at higher energies. Most inclusive decay branching fractions and χ_b at high energy are found in the Admixture sections. B^0-$\overline{B}^0$ mixing data are found in the B^0 section, while B^0_s-$\overline{B}^0_s$ mixing data and B-$\overline{B}$ mixing data for a B^0/B^0_s admixture are found in the B^0_s section. CP-violation data are found in the $B^\pm$, B^0, and $B^\pm$ B^0 Admixture sections. b-baryons are found near the end of the Baryon section.

The organization of the B sections is now as follows, where bullets indicate particle sections and brackets indicate reviews.

- $B^\pm$
  mass, mean life, branching fractions CP violation
- B^0
  mass, mean life, branching fractions
  polarization in B^0 decay, B^0-$\overline{B}^0$ mixing, CP violation
- $B^\pm$ B^0 Admixtures
  branching fractions, CP violation
- $B^\pm/B^0/B^0_s/b$-baryon Admixtures
  mean life, production fractions, branching fractions
  χ_b at high energy, V_{cb} measurements
  - B^*
    mass
  - B^0_s
    mass, mean life, branching fractions
    polarization in B^0_s decay, B^0_s-$\overline{B}^0_s$ mixing
  - $B^\pm_c$
    mass, mean life, branching fractions

At end of Baryon Listings:

- Λ_b
  mass, mean life, branching fractions
- b-baryon Admixture
  mean life, branching fractions

$B^\pm$

$I(J^P) = \frac{1}{2}(0^-)$

I, J, P need confirmation. Quantum numbers shown are quark-model predictions.

Mass $m_{B^\pm} = 5279.0 \pm 0.5$ MeV
Mean life $\tau_{B^\pm} = (1.671 \pm 0.018) \times 10^{-12}$ s
$c\tau = 501$ μm

CP violation

$A_{CP}(B^+ \to J/\psi(1S)K^+) = -0.007 \pm 0.019$
$A_{CP}(B^+ \to J/\psi(1S)\pi^+) = -0.01 \pm 0.13$
$A_{CP}(B^+ \to \psi(2S)K^+) = -0.037 \pm 0.025$
$A_{CP}(B^+ \to \overline{D}^0 K^+) = 0.04 \pm 0.07$
$A_{CP}(B^+ \to D_{CP(+1)}K^+) = 0.06 \pm 0.19$
$A_{CP}(B^+ \to D_{CP(-1)}K^+) = -0.19 \pm 0.18$
$A_{CP}(B^+ \to \pi^+\pi^0) = 0.05 \pm 0.15$
$A_{CP}(B^+ \to K^+\pi^0) = -0.10 \pm 0.08$
$A_{CP}(B^+ \to K^0_S\pi^+) = 0.03 \pm 0.08$ (S = 1.1)
$A_{CP}(B^+ \to \pi^+\pi^-\pi^+) = -0.39 \pm 0.35$
$A_{CP}(B^+ \to \rho^+\rho^0) = -0.09 \pm 0.16$
$A_{CP}(B^+ \to K^+\pi^-\pi^+) = 0.01 \pm 0.08$
$A_{CP}(B^+ \to K^+K^-K^+) = 0.02 \pm 0.08$
$A_{CP}(B^+ \to K^+\eta') = 0.009 \pm 0.035$
$A_{CP}(B^+ \to \omega\pi^+) = -0.21 \pm 0.19$
$A_{CP}(B^+ \to \omega K^+) = -0.21 \pm 0.28$
$A_{CP}(B^+ \to \phi K^+) = 0.03 \pm 0.07$
$A_{CP}(B^+ \to \phi K^*(892)^+) = 0.09 \pm 0.15$
$A_{CP}(B^+ \to \rho^0 K^*(892)^+) = 0.20 \pm 0.31$

B^- modes are charge conjugates of the modes below. Modes which do not identify the charge state of the B are listed in the $B^\pm/B^0$ ADMIXTURE section.

The branching fractions listed below assume 50% $B^0\overline{B}^0$ and 50% B^+B^- production at the $\Upsilon(4S)$. We have attempted to bring older measurements up to date by rescaling their assumed $\Upsilon(4S)$ production ratio to 50:50 and their assumed D, D_s, D^*, and ψ branching ratios to current values whenever this would affect our averages and best limits significantly.

Indentation is used to indicate a subchannel of a previous reaction. All resonant subchannels have been corrected for resonance branching fractions to the final state so the sum of the subchannel branching fractions can exceed that of the final state.

For inclusive branching fractions, *e.g.*, $B \to D^{\pm}$ anything, the values usually are multiplicities, not branching fractions. They can be greater than one.

B^+ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Semileptonic and leptonic modes			
$\ell^+ \nu_\ell$ anything	[a] (10.2 ±0.9) %		–
$\overline{D}^0 \ell^+ \nu_\ell$	[a] (2.15±0.22) %		2310
$\overline{D}^*(2007)^0 \ell^+ \nu_\ell$	[a] (6.5 ±0.5) %		2258
$\overline{D}_1(2420)^0 \ell^+ \nu_\ell$	(5.6 ±1.6) $\times 10^{-3}$		2084
$\overline{D}^*_2(2460)^0 \ell^+ \nu_\ell$	< 8 $\times 10^{-3}$	CL=90%	2067
$\pi^0 e^+ \nu_e$	(9.0 ±2.8) $\times 10^{-5}$		2638
$\eta \ell^+ \nu_\ell$	(8 ±4) $\times 10^{-5}$		2611
$\omega \ell^+ \nu_\ell$	[a] < 2.1 $\times 10^{-4}$	CL=90%	2582
$\rho^0 \ell^+ \nu_\ell$	[a] ($1.34^{+0.32}_{-0.35}$) $\times 10^{-4}$		2583
$p \overline{p} e^+ \nu_e$	< 5.2 $\times 10^{-3}$	CL=90%	2467
$e^+ \nu_e$	< 1.5 $\times 10^{-5}$	CL=90%	2640
$\mu^+ \nu_\mu$	< 2.1 $\times 10^{-5}$	CL=90%	2638
$\tau^+ \nu_\tau$	< 5.7 $\times 10^{-4}$	CL=90%	2340
$e^+ \nu_e \gamma$	< 2.0 $\times 10^{-4}$	CL=90%	2640
$\mu^+ \nu_\mu \gamma$	< 5.2 $\times 10^{-5}$	CL=90%	2638
***D*, *D**, or *D*$_s$ modes**			
$\overline{D}^0 \pi^+$	(4.98±0.29) $\times 10^{-3}$		2308
$\overline{D}^0 \rho^+$	(1.34±0.18) %		2236
$\overline{D}^0 K^+$	(3.7 ±0.6) $\times 10^{-4}$	S=1.1	2280
$\overline{D}^0 K^*(892)^+$	(6.1 ±2.3) $\times 10^{-4}$		2213
$\overline{D}^0 K^+ \overline{K}^0$	(5.5 ±1.6) $\times 10^{-4}$		2189
$\overline{D}^0 K^+ \overline{K}^*(892)^0$	(7.5 ±1.7) $\times 10^{-4}$		2071
$\overline{D}^0 \pi^+ \pi^+ \pi^-$	(1.1 ±0.4) %		2289
$\overline{D}^0 \pi^+ \pi^+ \pi^-$ nonresonant	(5 ±4) $\times 10^{-3}$		2289
$\overline{D}^0 \pi^+ \rho^0$	(4.2 ±3.0) $\times 10^{-3}$		2207
$\overline{D}^0 a_1(1260)^+$	(5 ±4) $\times 10^{-3}$		2123
$\overline{D}^0 \omega \pi^+$	(4.1 ±0.9) $\times 10^{-3}$		2206
$D^*(2010)^- \pi^+ \pi^+$	(2.1 ±0.6) $\times 10^{-3}$		2247
$D^- \pi^+ \pi^+$	< 1.4 $\times 10^{-3}$	CL=90%	2299
$\overline{D}^*(2007)^0 \pi^+$	(4.6 ±0.4) $\times 10^{-3}$		2256
$\overline{D}^*(2007)^0 \omega \pi^+$	(4.5 ±1.2) $\times 10^{-3}$		2149
$\overline{D}^*(2007)^0 \rho^+$	(9.8 ±1.7) $\times 10^{-3}$		2181
$\overline{D}^*(2007)^0 K^+$	(3.6 ±1.0) $\times 10^{-4}$		2227
$\overline{D}^*(2007)^0 K^*(892)^+$	(7.2 ±3.4) $\times 10^{-4}$		2156
$\overline{D}^*(2007)^0 K^+ \overline{K}^0$	< 1.06 $\times 10^{-3}$	CL=90%	2132
$\overline{D}^*(2007)^0 K^+ K^*(892)^0$	(1.5 ±0.4) $\times 10^{-3}$		2008

$\overline{D}^*(2007)^0 \pi^+ \pi^+ \pi^-$	(9.4 ±2.6) $\times 10^{-3}$		2236
$\overline{D}^*(2007)^0 a_1(1260)^+$	(1.9 ±0.5) %		2062
$\overline{D}^*(2007)^0 \pi^- \pi^+ \pi^+ \pi^0$	(1.8 ±0.4) %		2219
$D^*(2010)^+ \pi^0$	< 1.7 $\times 10^{-4}$	CL=90%	2255
$\overline{D}^*(2010)^+ K^0$	< 9.5 $\times 10^{-5}$	CL=90%	2225
$D^*(2010)^- \pi^+ \pi^+ \pi^0$	(1.5 ±0.7) %		2235
$D^*(2010)^- \pi^+ \pi^+ \pi^+ \pi^-$	< 1 %	CL=90%	2217
$\overline{D}^*_1(2420)^0 \pi^+$	(1.5 ±0.6) $\times 10^{-3}$	S=1.3	2081
$\overline{D}^*_1(2420)^0 \rho^+$	< 1.4 $\times 10^{-3}$	CL=90%	1995
$\overline{D}^*_2(2460)^0 \pi^+$	< 1.3 $\times 10^{-3}$	CL=90%	2064
$\overline{D}^*_2(2460)^0 \rho^+$	< 4.7 $\times 10^{-3}$	CL=90%	1977
$\overline{D}^0 D_s^+$	(1.3 ±0.4) %		1815
$\overline{D}^0 D_{sJ}(2317)^+$	seen		1605
$\overline{D}^0 D_{sJ}(2457)^+$	seen		–
$\overline{D}^0 D_{sJ}(2536)^+$	not seen		1447
$\overline{D}^*(2007)^0 D_{sJ}(2536)^+$	not seen		1338
$\overline{D}^0 D_{sJ}(2573)^+$	not seen		1417
$\overline{D}^*(2007)^0 D_{sJ}(2573)^+$	not seen		1306
$\overline{D}^0 D_s^{*+}$	(9 ±4) $\times 10^{-3}$		1734
$\overline{D}^*(2007)^0 D_s^+$	(1.2 ±0.5) %		1737
$\overline{D}^*(2007)^0 D_s^{*+}$	(2.7 ±1.0) %		1651
$D_s^{(*)+} \overline{D}^{**0}$	(2.7 ±1.2) %		–
$\overline{D}^*(2007)^0 D^*(2010)^+$	< 1.1 %	CL=90%	1713
$\overline{D}^0 D^*(2010)^+ + \overline{D}^*(2007)^0 D^+$	< 1.3 %	CL=90%	1792
$\overline{D}^0 D^+$	< 6.7 $\times 10^{-3}$	CL=90%	1866
$\overline{D}^0 D^+ K^0$	< 2.8 $\times 10^{-3}$	CL=90%	1571
$\overline{D}^*(2007)^0 D^+ K^0$	< 6.1 $\times 10^{-3}$	CL=90%	1475
$\overline{D}^0 \overline{D}^*(2010)^+ K^0$	(5.2 ±1.2) $\times 10^{-3}$		1476
$\overline{D}^*(2007)^0 D^*(2010)^+ K^0$	(7.8 ±2.6) $\times 10^{-3}$		1362
$\overline{D}^0 D^0 K^+$	(1.9 ±0.4) $\times 10^{-3}$		1577
$\overline{D}^*(2010)^0 D^0 K^+$	< 3.8 $\times 10^{-3}$	CL=90%	–
$\overline{D}^0 D^*(2007)^0 K^+$	(4.7 ±1.0) $\times 10^{-3}$		1481
$\overline{D}^*(2007)^0 D^*(2007)^0 K^+$	(5.3 ±1.6) $\times 10^{-3}$		1368
$D^- D^+ K^+$	< 4 $\times 10^{-4}$	CL=90%	1571
$D^- D^*(2010)^+ K^+$	< 7 $\times 10^{-4}$	CL=90%	1475
$D^*(2010)^- D^+ K^+$	(1.5 ±0.4) $\times 10^{-3}$		1475
$D^*(2010)^- D^*(2010)^+ K^+$	< 1.8 $\times 10^{-3}$	CL=90%	1363
$(\overline{D}+\overline{D}^*)(D+D^*)K$	(3.5 ±0.6) %		–
$D_s^+ \pi^0$	< 2.0 $\times 10^{-4}$	CL=90%	2270
$D_s^{*+} \pi^0$	< 3.3 $\times 10^{-4}$	CL=90%	2215
$D_s^+ \eta$	< 5 $\times 10^{-4}$	CL=90%	2235
$D_s^{*+} \eta$	< 8 $\times 10^{-4}$	CL=90%	2178

$D_s^+ \rho^0$	$< 4 \times 10^{-4}$	CL=90%	2197
$D_s^{*+} \rho^0$	$< 5 \times 10^{-4}$	CL=90%	2138
$D_s^+ \omega$	$< 5 \times 10^{-4}$	CL=90%	2195
$D_s^{*+} \omega$	$< 7 \times 10^{-4}$	CL=90%	2136
$D_s^+ a_1(1260)^0$	$< 2.2 \times 10^{-3}$	CL=90%	2079
$D_s^{*+} a_1(1260)^0$	$< 1.6 \times 10^{-3}$	CL=90%	2014
$D_s^+ \phi$	$< 3.2 \times 10^{-4}$	CL=90%	2141
$D_s^{*+} \phi$	$< 4 \times 10^{-4}$	CL=90%	2079
$D_s^+ \overline{K}^0$	$< 1.1 \times 10^{-3}$	CL=90%	2241
$D_s^{*+} \overline{K}^0$	$< 1.1 \times 10^{-3}$	CL=90%	2184
$D_s^+ \overline{K}^*(892)^0$	$< 5 \times 10^{-4}$	CL=90%	2172
$D_s^{*+} \overline{K}^*(892)^0$	$< 4 \times 10^{-4}$	CL=90%	2112
$D_s^- \pi^+ K^+$	$< 8 \times 10^{-4}$	CL=90%	2222
$D_s^{*-} \pi^+ K^+$	$< 1.2 \times 10^{-3}$	CL=90%	2164
$D_s^- \pi^+ K^*(892)^+$	$< 6 \times 10^{-3}$	CL=90%	2138
$D_s^{*-} \pi^+ K^*(892)^+$	$< 8 \times 10^{-3}$	CL=90%	2076

Charmonium modes

$\eta_c K^+$	$(9.0 \pm 2.7) \times 10^{-4}$		1754
$J/\psi(1S) K^+$	$(1.00 \pm 0.04) \times 10^{-3}$		1683
$J/\psi(1S) K^+ \pi^+ \pi^-$	$(7.7 \pm 2.0) \times 10^{-4}$		1612
$X(3872) K^+$	seen		–
$J/\psi(1S) K^*(892)^+$	$(1.35 \pm 0.10) \times 10^{-3}$		1571
$J/\psi(1S) K(1270)^+$	$(1.8 \pm 0.5) \times 10^{-3}$		1390
$J/\psi(1S) K(1400)^+$	$< 5 \times 10^{-4}$	CL=90%	1308
$J/\psi(1S) \phi K^+$	$(5.2 \pm 1.7) \times 10^{-5}$	S=1.2	1227
$J/\psi(1S) \pi^+$	$(4.0 \pm 0.5) \times 10^{-5}$		1727
$J/\psi(1S) \rho^+$	$< 7.7 \times 10^{-4}$	CL=90%	1611
$J/\psi(1S) a_1(1260)^+$	$< 1.2 \times 10^{-3}$	CL=90%	1414
$J/\psi(1S) p \overline{\Lambda}$	$(1.2 {}^{+0.9}_{-0.6}) \times 10^{-5}$		567
$\psi(2S) K^+$	$(6.8 \pm 0.4) \times 10^{-4}$		1284
$\psi(2S) K^*(892)^+$	$(9.2 \pm 2.2) \times 10^{-4}$		1115
$\psi(2S) K^+ \pi^+ \pi^-$	$(1.9 \pm 1.2) \times 10^{-3}$		1178
$\chi_{c0}(1P) K^+$	$(6.0 {}^{+2.4}_{-2.1}) \times 10^{-4}$		1478
$\chi_{c1}(1P) K^+$	$(6.8 \pm 1.2) \times 10^{-4}$		1411
$\chi_{c1}(1P) K^*(892)^+$	$< 2.1 \times 10^{-3}$	CL=90%	1265

***K* or *K** modes**

$K^0 \pi^+$	$(1.88 \pm 0.21) \times 10^{-5}$		2614
$K^+ \pi^0$	$(1.29 \pm 0.12) \times 10^{-5}$		2615
$\eta' K^+$	$(7.8 \pm 0.5) \times 10^{-5}$		2528
$\eta' K^*(892)^+$	$< 3.5 \times 10^{-5}$	CL=90%	2472

ηK^+	$< 6.9 \times 10^{-6}$	CL=90%	2588
$\eta K^*(892)^+$	$(2.6 {}^{+1.0}_{-0.9}) \times 10^{-5}$		2534
ωK^+	$(9.2 {}^{+2.8}_{-2.5}) \times 10^{-6}$		2557
$\omega K^*(892)^+$	$< 8.7 \times 10^{-5}$	CL=90%	2503
$K^*(892)^0 \pi^+$	$(1.9 {}^{+0.6}_{-0.8}) \times 10^{-5}$		2562
$K^*(892)^+ \pi^0$	$< 3.1 \times 10^{-5}$	CL=90%	2562
$K^+ \pi^- \pi^+$	$(5.7 \pm 0.4) \times 10^{-5}$		2609
$K^+ \pi^- \pi^+$ nonresonant	$< 2.8 \times 10^{-5}$	CL=90%	2609
$K^+ \rho^0$	$< 1.2 \times 10^{-5}$	CL=90%	2558
$K_2^*(1430)^0 \pi^+$	$< 6.8 \times 10^{-4}$	CL=90%	2445
$K^- \pi^+ \pi^+$	$< 1.8 \times 10^{-6}$	CL=90%	2609
$K^- \pi^+ \pi^+$ nonresonant	$< 5.6 \times 10^{-5}$	CL=90%	2609
$K_1(1400)^0 \pi^+$	$< 2.6 \times 10^{-3}$	CL=90%	2451
$K^0 \pi^+ \pi^0$	$< 6.6 \times 10^{-5}$	CL=90%	2609
$K^0 \rho^+$	$< 4.8 \times 10^{-5}$	CL=90%	2558
$K^*(892)^+ \pi^+ \pi^-$	$< 1.1 \times 10^{-3}$	CL=90%	2556
$K^*(892)^+ \rho^0$	$(1.1 \pm 0.4) \times 10^{-5}$		2504
$K^*(892)^+ K^*(892)^0$	$< 7.1 \times 10^{-5}$	CL=90%	2484
$K_1(1400)^+ \rho^0$	$< 7.8 \times 10^{-4}$	CL=90%	2387
$K_2^*(1430)^+ \rho^0$	$< 1.5 \times 10^{-3}$	CL=90%	2381
$K^+ \overline{K}^0$	$< 2.0 \times 10^{-6}$	CL=90%	2593
$\overline{K}^0 K^+ \pi^0$	$< 2.4 \times 10^{-5}$	CL=90%	2578
$K^+ K_S^0 K_S^0$	$(1.34 \pm 0.24) \times 10^{-5}$		2521
$K_S^0 K_S^0 \pi^+$	$< 3.2 \times 10^{-6}$	CL=90%	2577
$K^+ K^- \pi^+$	$< 6.3 \times 10^{-6}$	CL=90%	2578
$K^+ K^- \pi^+$ nonresonant	$< 7.5 \times 10^{-5}$	CL=90%	2578
$K^+ K^+ \pi^-$	$< 1.3 \times 10^{-6}$	CL=90%	2578
$K^+ K^+ \pi^-$ nonresonant	$< 8.79 \times 10^{-5}$	CL=90%	2578
$K^+ K^*(892)^0$	$< 5.3 \times 10^{-6}$	CL=90%	2540
$K^+ K^- K^+$	$(3.08 \pm 0.21) \times 10^{-5}$		2522
$K^+ \phi$	$(9.3 \pm 1.0) \times 10^{-6}$	S=1.3	2516
$K^+ K^- K^+$ nonresonant	$< 3.8 \times 10^{-5}$	CL=90%	2522
$K^*(892)^+ K^+ K^-$	$< 1.6 \times 10^{-3}$	CL=90%	2466
$K^*(892)^+ \phi$	$(9.6 \pm 3.0) \times 10^{-6}$	S=1.9	2460
$K_1(1400)^+ \phi$	$< 1.1 \times 10^{-3}$	CL=90%	2339
$K_2^*(1430)^+ \phi$	$< 3.4 \times 10^{-3}$	CL=90%	2332
$K^+ \phi \phi$	$(2.6 {}^{+1.1}_{-0.9}) \times 10^{-6}$		2306
$K^*(892)^+ \gamma$	$(3.8 \pm 0.5) \times 10^{-5}$		2564
$K_1(1270)^+ \gamma$	$< 9.9 \times 10^{-5}$	CL=90%	2486
$\phi K^+ \gamma$	$(3.4 \pm 1.0) \times 10^{-6}$		2516
$K^+ \pi^- \pi^+ \gamma$	$(2.4 {}^{+0.6}_{-0.5}) \times 10^{-5}$		2609

$K^*(892)^0\pi^+\gamma$	$(2.0\ {}^{+0.7}_{-0.6}) \times 10^{-5}$		2562
$K^+\rho^0\gamma$	$< 2.0 \times 10^{-5}$	CL=90%	2558
$K^+\pi^-\pi^+\gamma$ nonresonant	$< 9.2 \times 10^{-6}$	CL=90%	2609
$K_1(1400)^+\gamma$	$< 5.0 \times 10^{-5}$	CL=90%	2453
$K_2^*(1430)^+\gamma$	$< 1.4 \times 10^{-3}$	CL=90%	2447
$K^*(1680)^+\gamma$	$< 1.9 \times 10^{-3}$	CL=90%	2360
$K_3^*(1780)^+\gamma$	$< 5.5 \times 10^{-3}$	CL=90%	2341
$K_4^*(2045)^+\gamma$	$< 9.9 \times 10^{-3}$	CL=90%	2243

Light unflavored meson modes

$\rho^+\gamma$	$< 2.1 \times 10^{-6}$	CL=90%	2583
$\pi^+\pi^0$	$(5.6\ {}^{+0.9}_{-1.1}) \times 10^{-6}$		2636
$\pi^+\pi^+\pi^-$	$(1.1 \pm 0.4) \times 10^{-5}$		2630
$\rho^0\pi^+$	$(8.6 \pm 2.0) \times 10^{-6}$		2581
$\pi^+ f_0(980)$	$< 1.4 \times 10^{-4}$	CL=90%	2547
$\pi^+ f_2(1270)$	$< 2.4 \times 10^{-4}$	CL=90%	2483
$\pi^+\pi^-\pi^+$ nonresonant	$< 4.1 \times 10^{-5}$	CL=90%	2630
$\pi^+\pi^0\pi^0$	$< 8.9 \times 10^{-4}$	CL=90%	2631
$\rho^+\pi^0$	$< 4.3 \times 10^{-5}$	CL=90%	2581
$\pi^+\pi^-\pi^+\pi^0$	$< 4.0 \times 10^{-3}$	CL=90%	2621
$\rho^+\rho^0$	$(2.6 \pm 0.6) \times 10^{-5}$		2523
$a_1(1260)^+\pi^0$	$< 1.7 \times 10^{-3}$	CL=90%	2494
$a_1(1260)^0\pi^+$	$< 9.0 \times 10^{-4}$	CL=90%	2494
$\omega\pi^+$	$(6.4\ {}^{+1.8}_{-1.6}) \times 10^{-6}$	S=1.3	2580
$\omega\rho^+$	$< 6.1 \times 10^{-5}$	CL=90%	2522
$\eta\pi^+$	$< 5.7 \times 10^{-6}$	CL=90%	2609
$\eta'\pi^+$	$< 7.0 \times 10^{-6}$	CL=90%	2551
$\eta'\rho^+$	$< 3.3 \times 10^{-5}$	CL=90%	2492
$\eta\rho^+$	$< 1.5 \times 10^{-5}$	CL=90%	2553
$\phi\pi^+$	$< 4.1 \times 10^{-7}$	CL=90%	2539
$\phi\rho^+$	$< 1.6 \times 10^{-5}$		2480
$\pi^+\pi^+\pi^+\pi^-\pi^-$	$< 8.6 \times 10^{-4}$	CL=90%	2608
$\rho^0 a_1(1260)^+$	$< 6.2 \times 10^{-4}$	CL=90%	2433
$\rho^0 a_2(1320)^+$	$< 7.2 \times 10^{-4}$	CL=90%	2410
$\pi^+\pi^+\pi^+\pi^-\pi^-\pi^0$	$< 6.3 \times 10^{-3}$	CL=90%	2592
$a_1(1260)^+ a_1(1260)^0$	< 1.3 %	CL=90%	2335

Charged particle ($h^\pm$) modes

$h^\pm = K^\pm$ or $\pi^\pm$

$h^+\pi^0$	$(1.6\ {}^{+0.7}_{-0.6}) \times 10^{-5}$		2636
ωh^+	$(1.38\ {}^{+0.27}_{-0.24}) \times 10^{-5}$		2580
$h^+ X^0$ (Familon)	$< 4.9 \times 10^{-5}$	CL=90%	–

Baryon modes

$p\overline{p}\pi^+$	$< 3.7 \times 10^{-6}$	CL=90%	2439
$p\overline{p}\pi^+$ nonresonant	$< 5.3 \times 10^{-5}$	CL=90%	2439
$p\overline{p}\pi^+\pi^+\pi^-$	$< 5.2 \times 10^{-4}$	CL=90%	2369
$p\overline{p}K^+$	$(4.3\ {}^{+1.2}_{-1.0}) \times 10^{-6}$		2348
$p\overline{p}K^+$ nonresonant	$< 8.9 \times 10^{-5}$	CL=90%	2348
$p\overline{\Lambda}$	$< 1.5 \times 10^{-6}$	CL=90%	2430
$p\overline{\Lambda}\pi^+\pi^-$	$< 2.0 \times 10^{-4}$	CL=90%	2367
$\overline{\Delta}^0 p$	$< 3.8 \times 10^{-4}$	CL=90%	2402
$\Delta^{++}\overline{p}$	$< 1.5 \times 10^{-4}$	CL=90%	2402
$D^+ p\overline{p}$	$< 1.5 \times 10^{-5}$	CL=90%	1860
$D^*(2010)^+ p\overline{p}$	$< 1.5 \times 10^{-5}$	CL=90%	1786
$\overline{\Lambda}_c^- p\pi^+$	$(2.1 \pm 0.7) \times 10^{-4}$		1981
$\overline{\Lambda}_c^- p\pi^+\pi^0$	$(1.8 \pm 0.6) \times 10^{-3}$		1936
$\overline{\Lambda}_c^- p\pi^+\pi^+\pi^-$	$(2.3 \pm 0.7) \times 10^{-3}$		1881
$\overline{\Lambda}_c^- p\pi^+\pi^+\pi^-\pi^0$	< 1.34 %	CL=90%	1823
$\overline{\Sigma}_c(2455)^0 p$	$< 8 \times 10^{-5}$	CL=90%	1939
$\overline{\Sigma}_c(2520)^0 p$	$< 4.6 \times 10^{-5}$	CL=90%	1905
$\overline{\Sigma}_c(2455)^0 p\pi^0$	$(4.4 \pm 1.8) \times 10^{-4}$		1897
$\overline{\Sigma}_c(2455)^0 p\pi^-\pi^+$	$(4.4 \pm 1.7) \times 10^{-4}$		1845
$\overline{\Sigma}_c(2455)^{--} p\pi^+\pi^+$	$(2.8 \pm 1.2) \times 10^{-4}$		1845
$\overline{\Lambda}_c(2593)^- / \overline{\Lambda}_c(2625)^- p\pi^+$	$< 1.9 \times 10^{-4}$	CL=90%	–

Lepton Family number (LF) or Lepton number (L) violating modes, or $\Delta B = 1$ weak neutral current ($B1$) modes

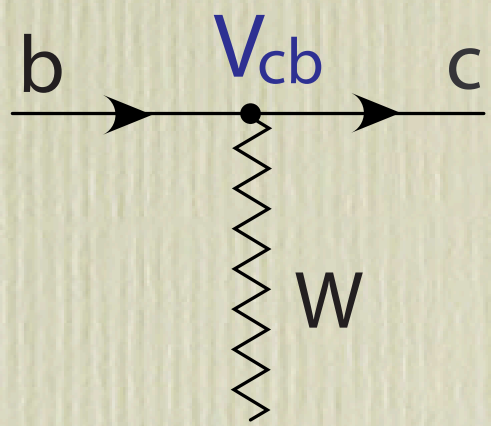
$\pi^+ e^+ e^-$	*B1*		$< 3.9 \times 10^{-3}$	CL=90%	2638
$\pi^+\mu^+\mu^-$	*B1*		$< 9.1 \times 10^{-3}$	CL=90%	2633
$K^+ e^+ e^-$	*B1*		$(6.3\ {}^{+1.9}_{-1.7}) \times 10^{-7}$		2616
$K^+\mu^+\mu^-$	*B1*		$(4.5\ {}^{+1.4}_{-1.2}) \times 10^{-7}$		2612
$K^+\ell^+\ell^-$	*B1*	[a]	$(5.3 \pm 1.1) \times 10^{-7}$		2616
$K^+\overline{\nu}\nu$	*B1*		$< 2.4 \times 10^{-4}$	CL=90%	2616
$K^*(892)^+ e^+ e^-$	*B1*		$< 4.6 \times 10^{-6}$	CL=90%	2564
$K^*(892)^+\mu^+\mu^-$	*B1*		$< 2.2 \times 10^{-6}$	CL=90%	2560
$K^*(892)^+\ell^+\ell$	*B1*	[a]	$< 2.2 \times 10^{-6}$	CL=90%	2564
$\pi^+ e^+\mu^-$	*LF*		$< 6.4 \times 10^{-3}$	CL=90%	2637
$\pi^+ e^-\mu^+$	*LF*		$< 6.4 \times 10^{-3}$	CL=90%	2637
$K^+ e^+\mu^-$	*LF*		$< 8 \times 10^{-7}$	CL=90%	2615
$K^+ e^-\mu^+$	*LF*		$< 6.4 \times 10^{-3}$	CL=90%	2615
$K^*(892)^+ e^\pm\mu^\mp$	*LF*		$< 7.9 \times 10^{-6}$	CL=90%	2563
$\pi^- e^+ e^+$	*L*		$< 1.6 \times 10^{-6}$	CL=90%	2638
$\pi^-\mu^+\mu^+$	*L*		$< 1.4 \times 10^{-6}$	CL=90%	2633

B decays - Motivation

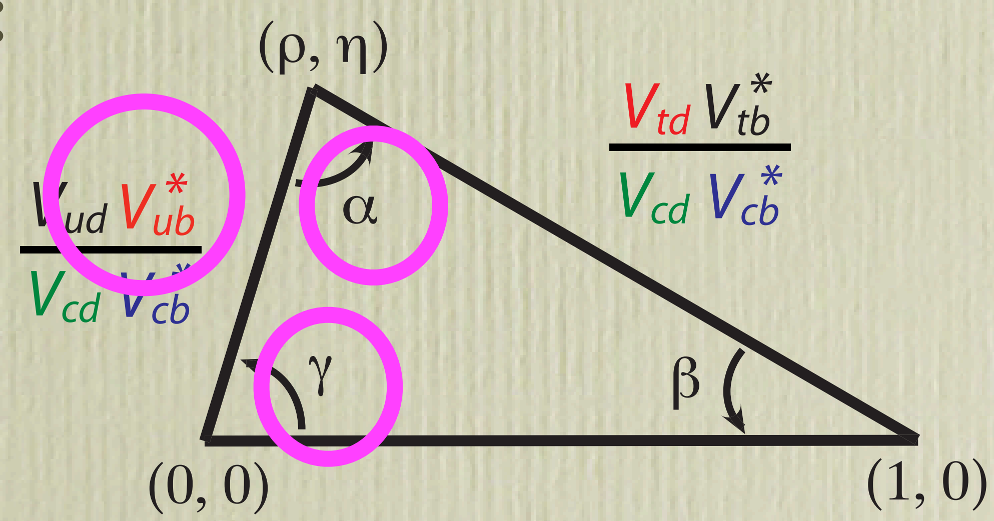
- Heavy Stable Hadrons → lots of decays
- Probe the flavor sector of the SM

CKM matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$\not{CP}$:

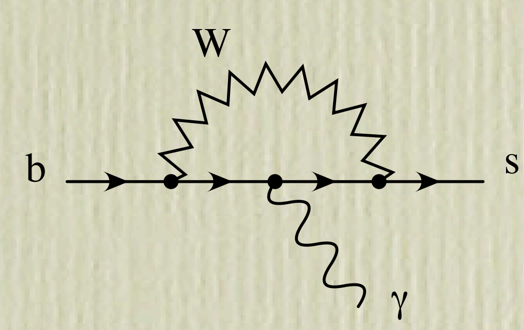


B decays - Motivation

- Heavy Stable Hadrons → lots of decays
- Probe the flavor sector of the SM; CKM matrix
- Look for new physics: ~~CP~~

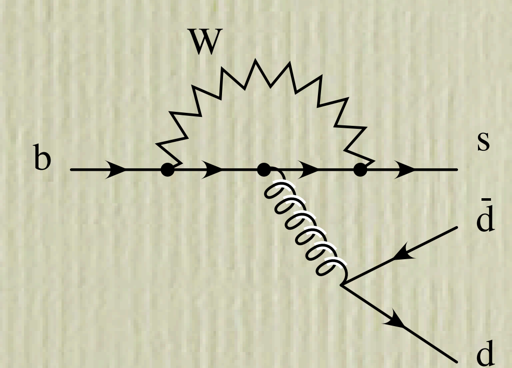
  redundant measurements,

  precision measurements,



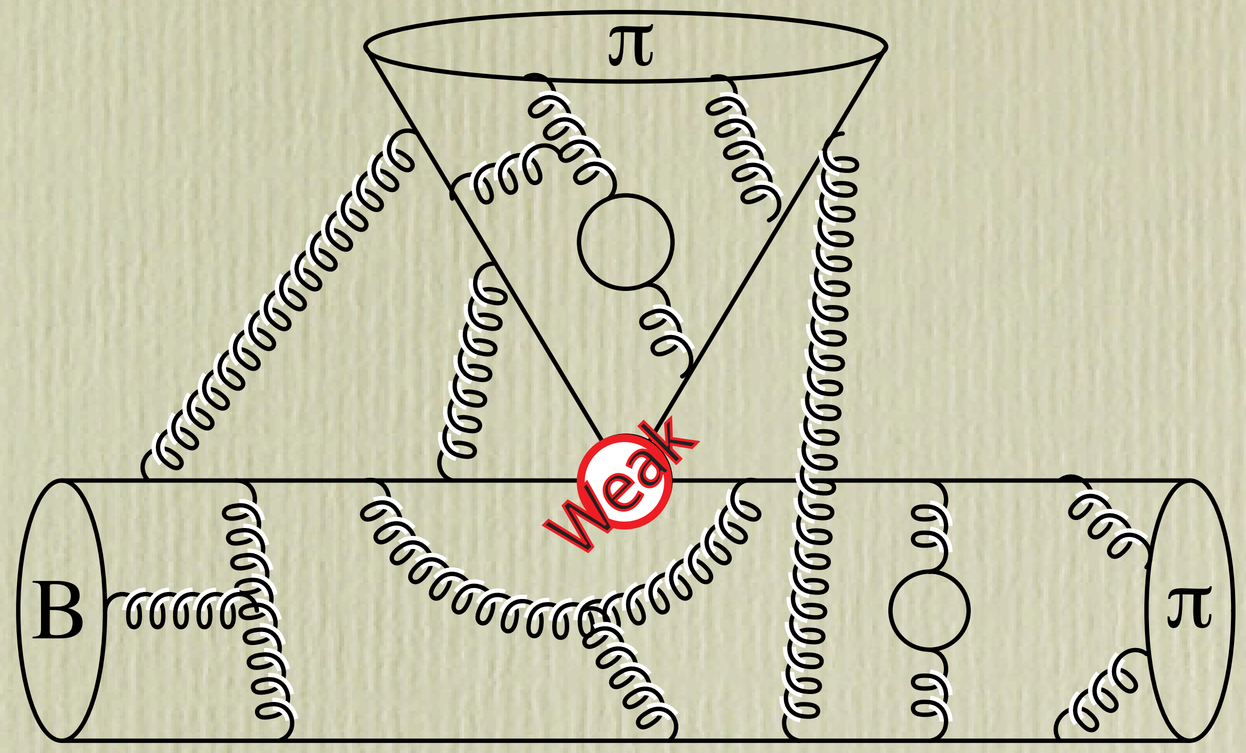
$B \to X_s \gamma$

  rare decays



$B \to K\pi$

- Measure fundamental hadronic parameters & improve our understanding of QCD


π
Weak
B
π

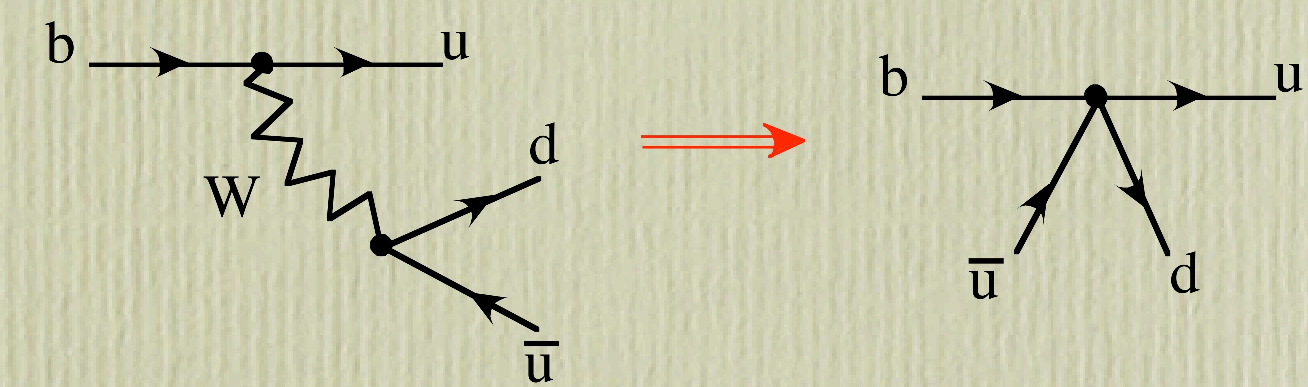
Electroweak Hamiltonian

$m_W, m_t \gg m_b$

λ^i = CKM factors

$\lambda^1 = V_{ub}V_{ud}^* \quad \lambda^3 = V_{tb}V_{td}^*$

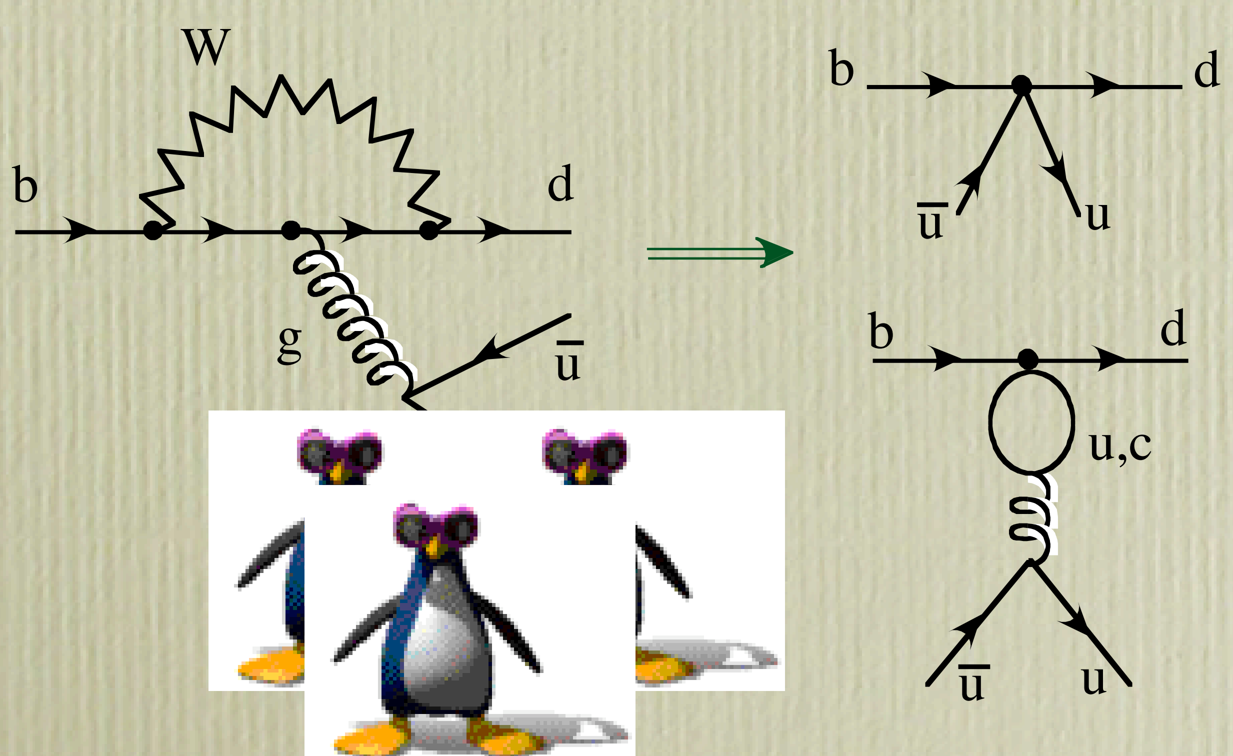
$$H_{\text{weak}} = \frac{G_F}{\sqrt{2}} \sum_i \lambda^i C_i(\mu) O_i(\mu)$$



trees

$$O_1 = (\bar{u}b)_{V-A}(\bar{d}u)_{V-A}$$

$$O_2 = (\bar{u}_i b_j)_{V-A}(\bar{d}_j u_i)_{V-A}$$



penguins

$$O_3 = (\bar{d}b)_{V-A} \sum_q (\bar{q}q)_{V-A}$$

$$O_{4,5,6} = \ldots$$

$$O_{7\gamma,8G} = \ldots$$

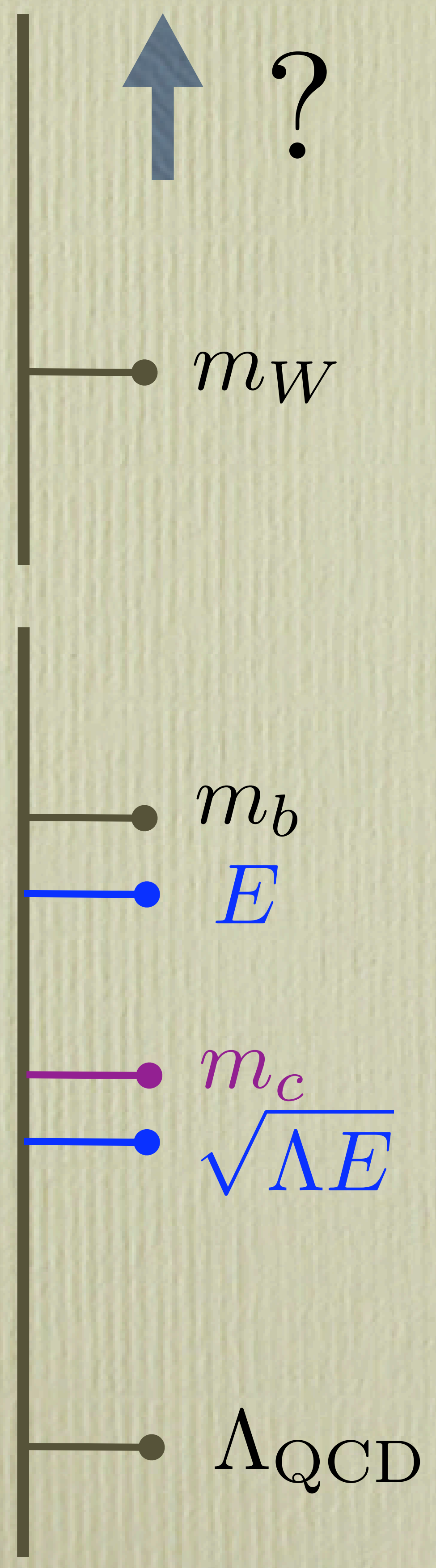
$$O^{ew}_{7,\ldots,10} = \ldots$$

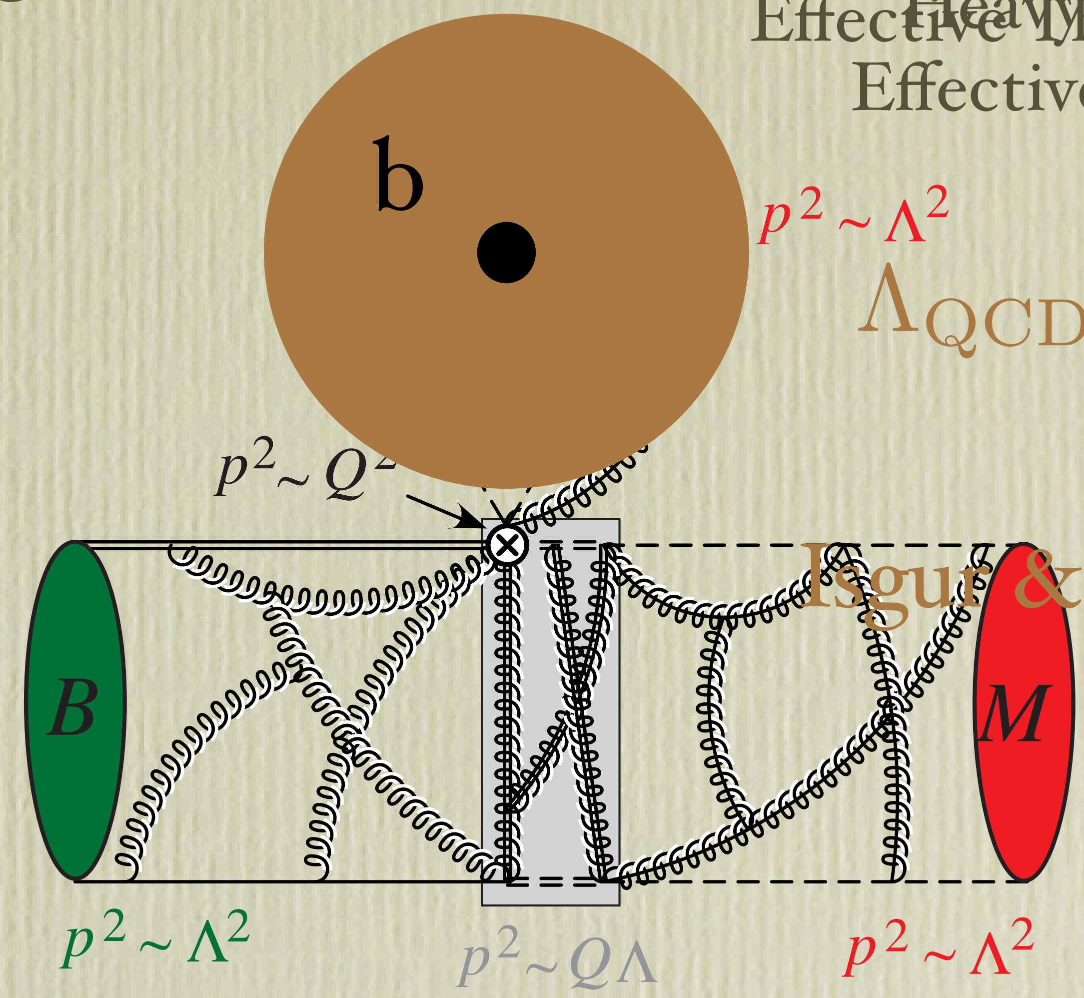
B-meson

Energetic Hadrons

Soft-Collinear Effective Theory

Heavy Quark Effective Theory





$\Lambda_{QCD} \ll m_b$

Isgur & Wise

Need expansion parameters

$$\alpha_s(m_b) \simeq 0.2 \qquad \frac{\Lambda}{m_b} \simeq 0.1 \qquad \frac{\Lambda}{E_M} \simeq 0.2$$

Soft - Collinear Effective Theory

Bauer, Pirjol, Fleming, Stewart

An effective field theory for energetic hadrons & jets

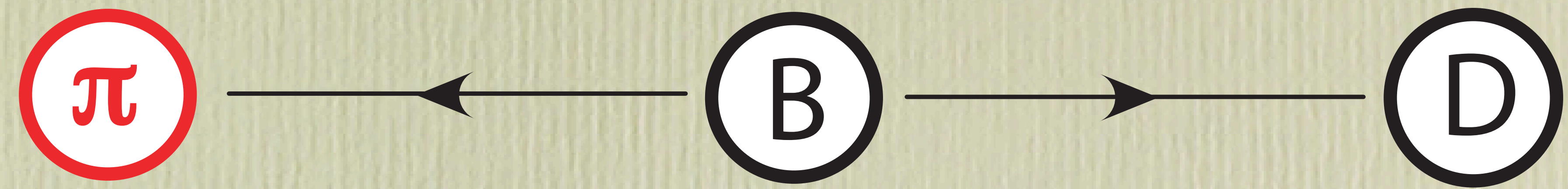
$E \gg \Lambda_{\rm QCD}$

- Separate physics at different momentum scales
- Model independent, systematically improvable
- Exploit symmetries
- power expansion, explore factorization beyond LO
- Resum Sudakov logarithms

egs. H_W, HQET, ChPT

Soft Collinear Effective Theory

eg.



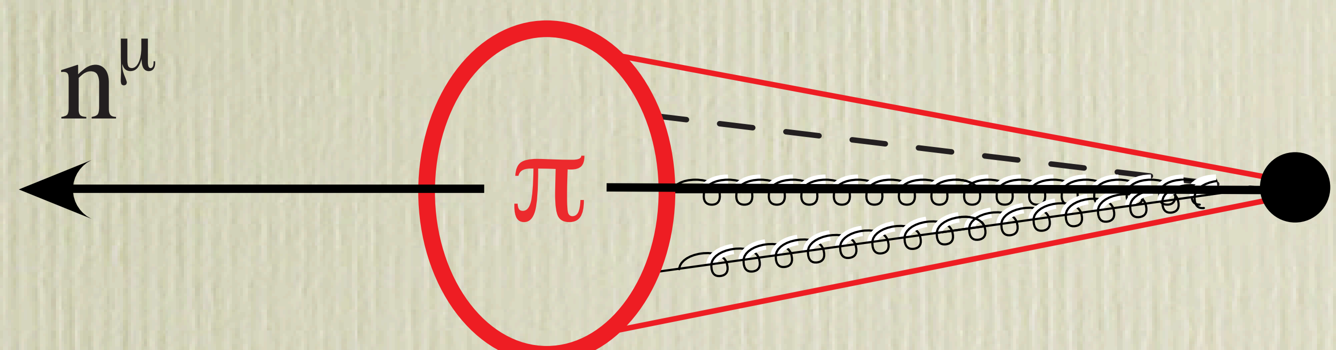
Pion has: $p_\pi^\mu \;=\; (2.3\,\mathrm{GeV})n^\mu = Q\,n^\mu \qquad n^2 = \bar{n}^2 = 0,\ \ (\bar{n}\cdot p = p^-)$

Soft constituents:

$$p_s^\mu = (p^+, p^-, p^\perp) \sim (\Lambda, \Lambda, \Lambda)$$

Collinear constituents:

$$p_c^\mu = (p^+, p^-, p^\perp) \sim \left(\frac{\Lambda^2}{Q}, Q, \Lambda\right) \sim Q(\lambda^2, 1, \lambda) \qquad \lambda = \frac{\Lambda}{Q}$$



Degrees of freedom in SCET

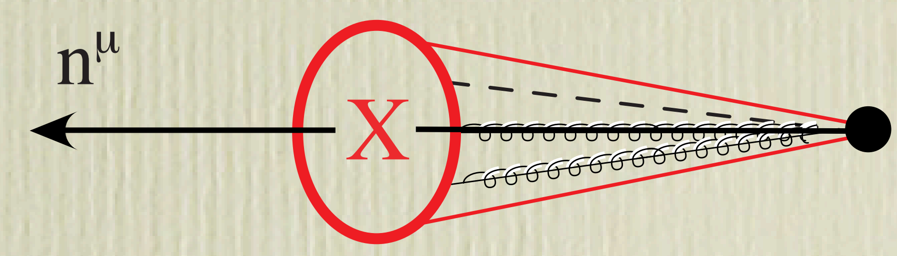
Introduce fields for infrared degrees of freedom (in operators)

modes	$p^\mu = (+, -, \perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us},\ A_{us}^\mu$

$\mathrm{SCET_I}$ → Energetic jets $\Lambda^2 \ll Q\Lambda \ll Q^2$

usoft $p^\mu \sim \Lambda$

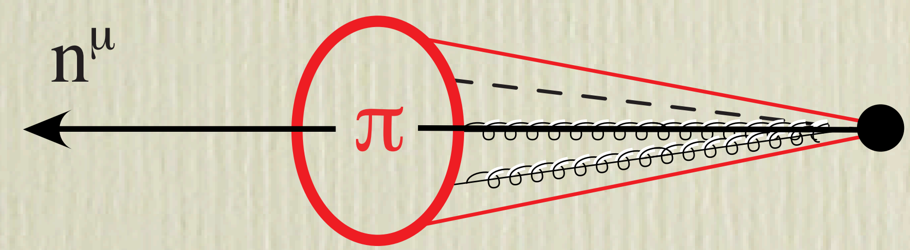
collinear $p_c^2 \sim Q\Lambda, \quad \lambda = \sqrt{\Lambda/Q}$



$\mathrm{SCET_{II}}$ → Energetic hadrons

soft $p^\mu \sim \Lambda$

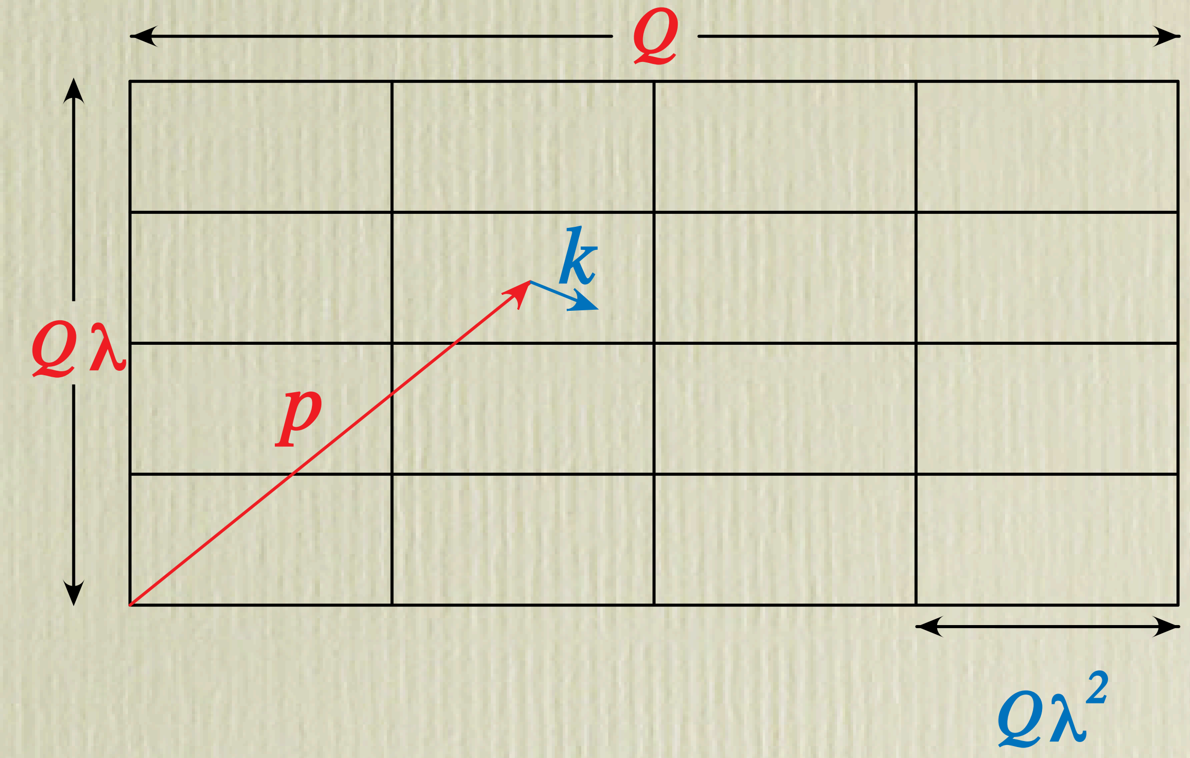
collinear $p_c^2 \sim \Lambda^2, \quad \lambda = \Lambda/Q$



Separate Momenta (multipole expansion)

		label	residual	
HQET	$P^\mu =$	$m_b v^\mu$	$+\ k^\mu$	$h_v(x)$
SCET	$P^\mu =$	p^μ	$+\ k^\mu$	$\xi_{n,p}(x)$

$(1, \lambda)$



Collinear Quarks

- $\psi(x) \to \sum_p e^{-ip\cdot x} \xi_{n,p}(x)$
- $\not{n}\, \xi_{n,p} = 0$
- $\partial^\mu\, \xi_{n,p} \sim (Q\lambda^2)\, \xi_{n,p}$

Introduce Label Operator

$$\mathcal{P}^\mu (\phi^\dagger_{q_1} \cdots \phi_{p_1} \cdots) = (p_1^\mu + \ldots - q_1^\mu - \ldots)(\phi^\dagger_{q_1} \cdots \phi_{p_1} \cdots)$$

$$i\partial^\mu e^{-ip\cdot x}\, \phi_p(x) = e^{-ip\cdot x}(\mathcal{P}^\mu + i\partial^\mu)\phi_p(x)$$

Power Counting

Type	$(p^+, p^-, p^\perp)$	Fields	Field Scaling
collinear	$(\lambda^2, 1, \lambda)$	$\xi_{n,p}$	λ
		$(A^+_{n,p}, A^-_{n,p}, A^\perp_{n,p})$	$(\lambda^2, 1, \lambda)$
soft	$(\lambda, \lambda, \lambda)$	$q_{s,p}$	$\lambda^{3/2}$
		$A^\mu_{s,p}$	λ
usoft	$(\lambda^2, \lambda^2, \lambda^2)$	q_{us}	λ^3
		A^μ_{us}	λ^2

Make kinetic terms order λ^0 $\quad \int d^4X \quad \bar{\xi}_{n,p'} \, \frac{\bar{n}\!\!\!/}{2} \Big(in\cdot\partial + \ldots \Big) \, \xi_{n,p}$

$$\lambda^0 = \quad \lambda^{-4} \quad \lambda \quad \lambda^2 \quad \lambda$$

- At leading power only λ^0 interactions are required

$$\text{LO:} \quad \mathcal{O}^{(0)} \text{ with } \mathcal{L}^{(0)}$$

$$\text{NLO:} \quad \mathcal{O}^{(1)} \text{ with } \mathcal{L}^{(0)}, \quad \& \quad T\{\mathcal{O}^{(0)}, \mathcal{L}^{(1)}\} \text{ with } \mathcal{L}^{(0)}$$

LO SCET Lagrangian

$$\mathcal{L}_c^{(0)} = \bar{\xi}_n \Big\{ n \cdot iD_{us} + gn \cdot A_n + i\not{D}_\perp^c \frac{1}{i\bar{n} \cdot D_c} i\not{D}_\perp^c \Big\} \frac{\not{\bar{n}}}{2} \xi_n$$

- most general order λ^0 gauge invariant action
- propagator $\frac{i\not{n}}{2}\, \bar{n}\cdot p / [\, n\cdot(k+p)\, \bar{n}\cdot p + p_\perp^2 + i\epsilon]$
- eikonal for usoft gluons interacting with collinear quark
- $\mathcal{L}_{cg}^{(0)} = \mathcal{L}_{cg}^{(0)}(A_n^\mu, n\cdot A_{us})$, $\mathcal{L}_{us}^{(0)} = \bar{q}\, i\not{D}\, q$

Consider the following field redefinitions in SCET

$\xi_n \to Y \xi_n$, $A_n \to Y A_n Y^\dagger$ $\qquad Y(x) = P \exp\Big(ig \int_{-\infty}^0 ds\, n\cdot A_{us}(x+ns) \Big)$

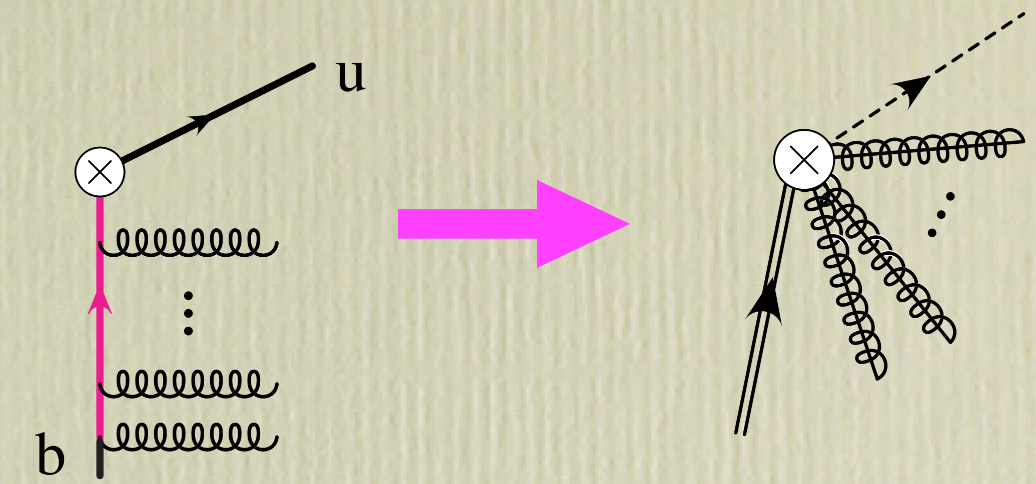
$$n\cdot D_{us} Y = 0,\ Y^\dagger Y = 1$$

gives: $\mathcal{L} = \bar{\xi}_n [in\cdot D_{us} + \ldots] \xi_n \Longrightarrow \bar{\xi}_n [in\cdot\partial + \ldots] \xi_n$

Moves all usoft gluons to operators, simplifies cancellations

Factorization

- Separation of scales
  and Decoupling



eg. $\bar{u}\,\Gamma\,b$

➡ $\bar{\xi}_n W\ \Gamma\ h_v$ integrate out offshell quarks

➡ $(\bar{\xi}_n W)\,\Gamma\,(Y^\dagger h_v)$ usoft-collinear factorization (field redefn.)

➡ $\int d\omega\, C(\omega) (\bar{\xi}_n W)_\omega\,\Gamma\,(Y^\dagger h_v)$ hard-collinear factorization

$$\omega \sim p_c^- \sim Q$$

- operators are gauge invariant,
  so factorization is too

$$W = P \exp\left(ig \int_{-\infty}^{y} ds\, \bar{n}\cdot A_n(s\bar{n}^\mu)\right)$$

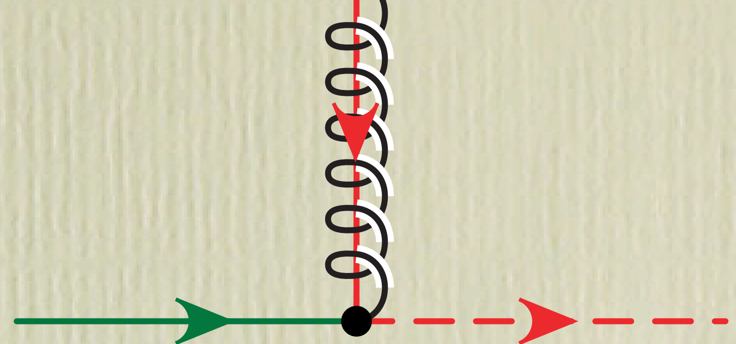
$$Y = P \exp\left(ig \int_{-\infty}^{0} ds\, n\cdot A_{us}(x+ns)\right)$$

$$S = P \exp\left(ig \int_{-\infty}^{0} ds\, n\cdot A_{s}(x+ns)\right)$$

Subleading Lagrangians and Currents

examples

$$\mathcal{L}_{\xi q}^{(1)} = (\bar{\xi}_n W)\Big(\frac{1}{\bar{\mathcal{P}}} W^\dagger i g \not{B}_c^\perp W\Big)(Y^\dagger q_{us}) + \text{h.c.}$$



$$\mathcal{L}_{\xi\xi}^{(2)} = (\bar{\xi}_n W)\Big(Y^\dagger i \not{D}_{us}^\perp i \not{D}_{us}^\perp Y\Big)\, \frac{\not{\bar{n}}}{2} \frac{1}{\bar{\mathcal{P}}} (W \xi_n)$$

$$J_i^{(1)}(\omega_1, \omega_2) = \frac{1}{m_b}\, (\bar{\xi}_n W)_{\omega_1} \Theta_i^\alpha \Big(\frac{1}{\bar{\mathcal{P}}} W^\dagger i g B_{c\,\alpha}^\perp W\Big)_{\omega_2} (Y^\dagger h_v)$$

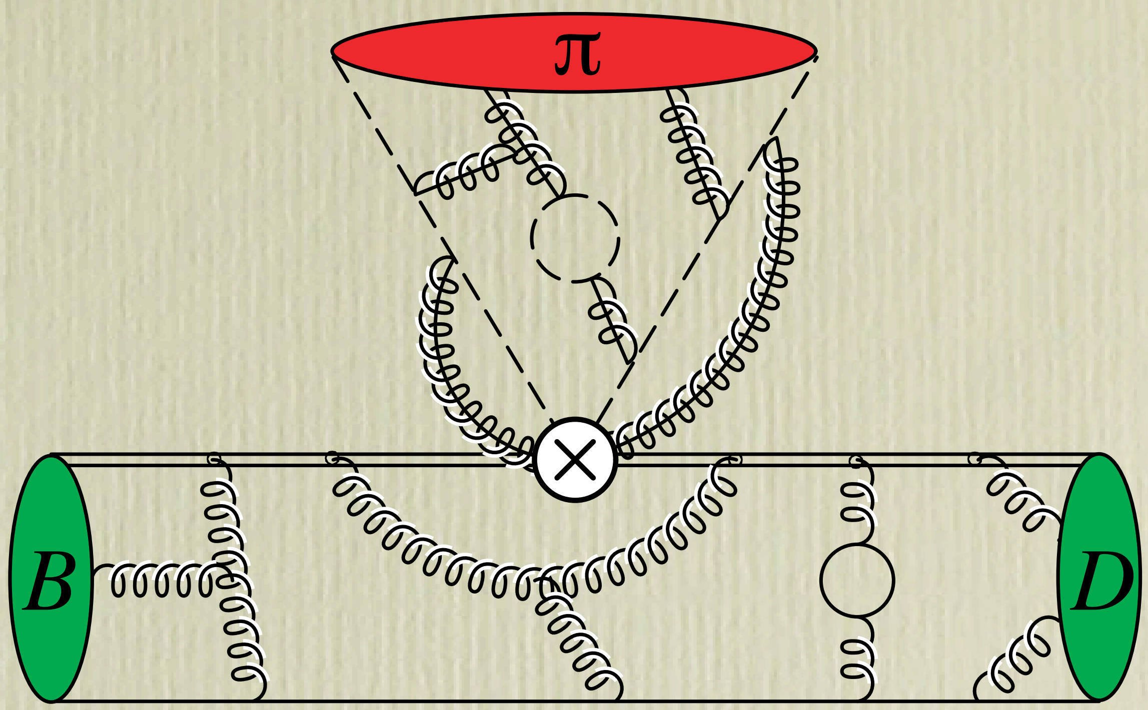
- reparameterization invariance for $n,\ \bar{n}$ relates Wilson coefficients of some leading leading & subleading operators

Chay & Kim
Manohar et al.
Beneke et al.

Factorization Example

Bauer, Pirjol, I.S.

$\bar{B}^0 \to D^+\pi^- \ , \ B^- \to D^0\pi^-$



B, D are soft, π collinear

$$\mathcal{L}_{\rm SCET} = \mathcal{L}_s^{(0)} + \mathcal{L}_c^{(0)}$$

Factorization if $\mathcal{O} = O_c \times O_s$

$$\mathcal{O} = \big[\bar{h}_{v'}^{(c)}\Gamma_h h_v^{(b)}\big]\big[(\bar{\xi}_n^{(d)} W)_{\omega_1}\Gamma_n (W^\dagger \xi_n^{(u)})\big]$$

$$\langle D\pi|(\bar{c}b)(\bar{u}d)|B\rangle = N\, \xi(v\cdot v')\int_0^1 dx\, T(x,\mu)\, \phi_\pi(x,\mu)$$

Universal functions:

$$\langle D^{(*)}|O_s|B\rangle = \xi(v\cdot v')$$

$$\langle \pi|O_c(x)|0\rangle = f_\pi \phi_\pi(x)$$

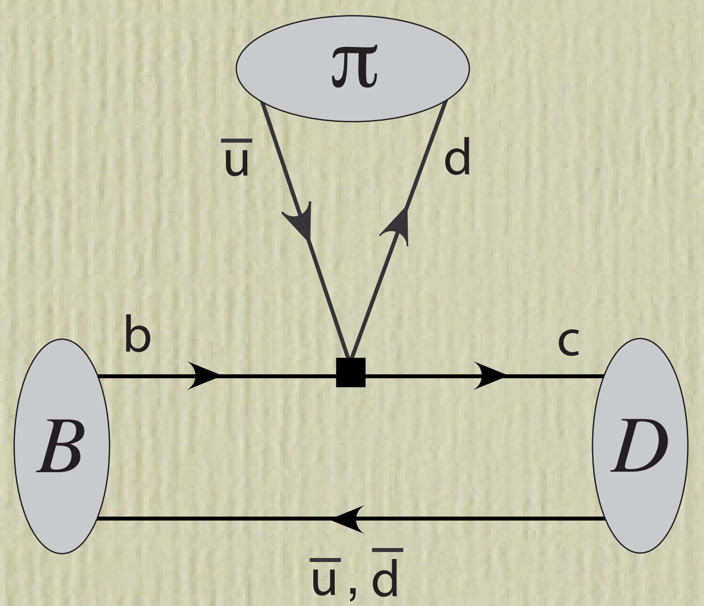
Calculate T, $\alpha_s(Q)$

$$Q = E_\pi, m_b, m_c$$

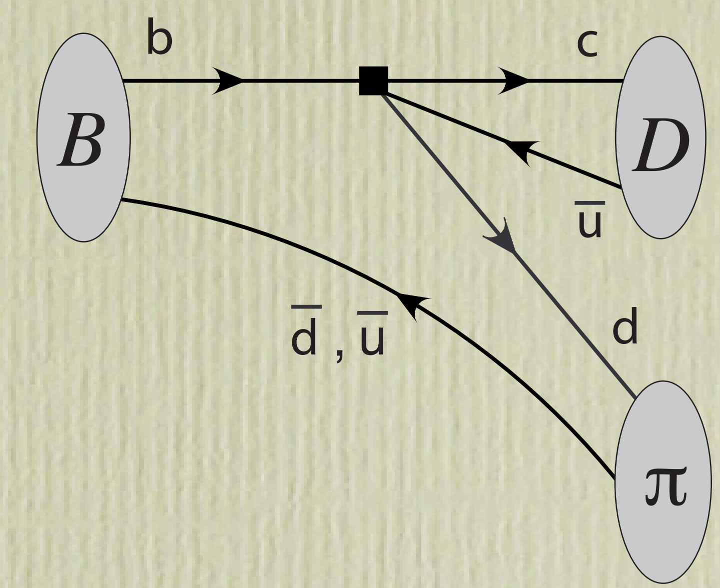
corrections will be $\Lambda/m_c \sim 30\%$

Color Suppressed Decays

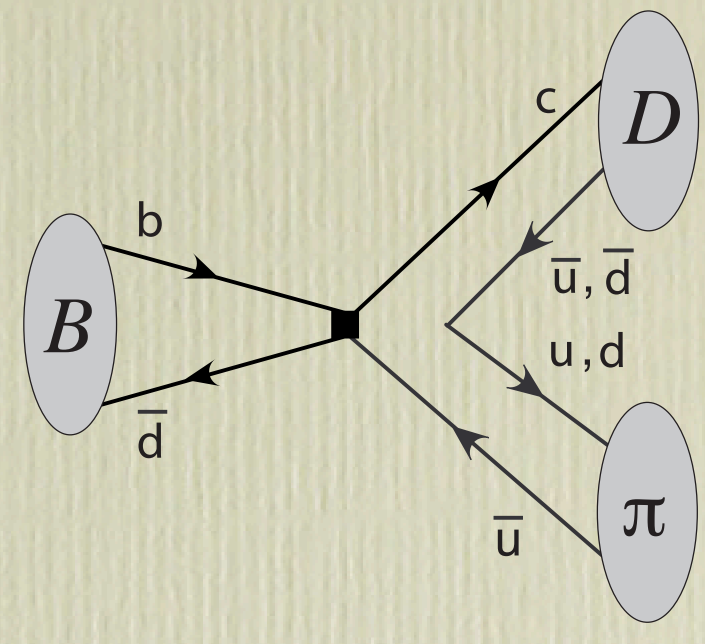
"Tree"



"Color suppressed"



"Exchange"



$$\bar{B}^0 \to D^+\pi^- \qquad B^- \to D^0\pi^- \qquad \bar{B}^0 \to D^+\pi^-$$

$$B^- \to D^0\pi^- \qquad \bar{B}^0 \to D^0\pi^0 \qquad \bar{B}^0 \to D^0\pi^0$$

Naive Factorization too small

Large N_c not very predictive

$$A(\bar{B}^0 \to D^0\pi^0) \sim a_2 \langle \pi^0 | (\bar{d}b) | \bar{B}^0 \rangle \langle D^0 | (\bar{c}u) | 0 \rangle$$

$(N_c)^0$ $\quad 1/N_c$ $\quad 1/N_c$

$$\bar{B}^0 \to D^{(*)0}\pi^0 , D^{(*)0}\rho^0, D^{(*)0}K^0, D^{(*)0}K^{*0}, D_s^{(*)}K^-, D_s^{(*)}K^{*-},$$

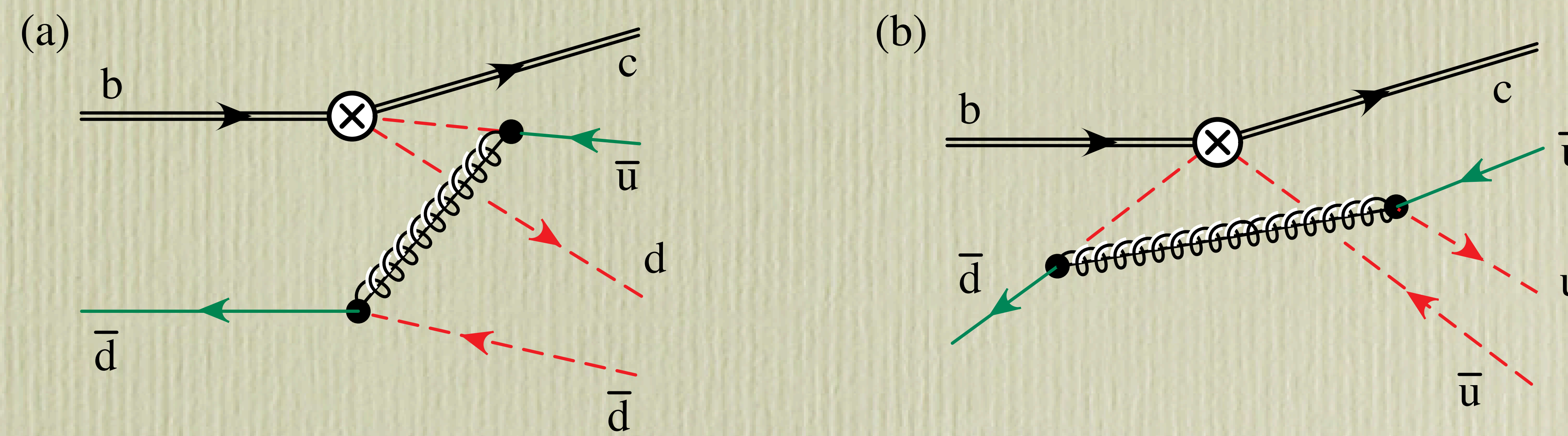
$\ldots$

Color Suppressed Decays

Factorization with SCET

Mantry, Pirjol, I.S.

Single class of power suppressed $\mathrm{SCET_I}$ operators $T\{\mathcal{O}^{(0)}, \mathcal{L}^{(1)}_{\xi q}, \mathcal{L}^{(1)}_{\xi q}\}$



Order $\lambda^2 = \left(\sqrt{\Lambda/E}\right)^2 = \Lambda/E$

$$A_{00}^{D^{(*)}\pi} = N_0^{(*)} \int dx\, dz\, dk_1^+ dk_2^+\, T^{(i)}(z)\, J^{(i)}(z, x, k_1^+, k_2^+)\, S^{(i)}(k_1^+, k_2^+)\, \phi_\pi(x)$$

$$+A_{\rm long}^{D^{(*)}\pi}$$

$Q^2 \gg Q\Lambda \gg \Lambda^2$

QCD → $\mathrm{SCET_I}$ → $\mathrm{SCET_{II}}$

1) $\langle D^{(*)0}|O_s^{(0,8)}|\bar{B}^0\rangle \rightarrow S^{(0,8)}(k_1^+, k_2^+)$ same for D and D^*

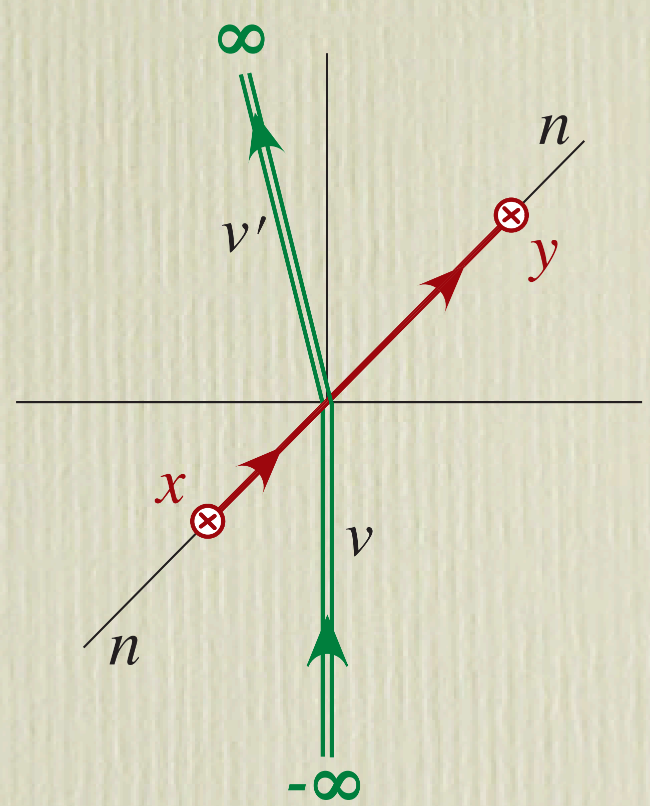
with HQET for $\langle D^{(*)0}\pi|(\bar{c}\,b)(\bar{d}\,u)|\bar{B}^0\rangle$ get $\frac{p_\pi^\mu}{m_c} \longrightarrow \frac{E_\pi}{m_c} = 1.5$

not a convergent expansion

2) $S^{(i)}(k_1^+, k_2^+)$ is complex, new mechanism for rescattering

$$O_s^{(0,8)} = \left[(\bar{h}_{v'}^{(c)} S)\Gamma^h\{1, T^a\}\,(S^\dagger h_v^{(b)})\,(\bar{d}\,S)_{k_1^+}\Gamma_s\{1, T^a\}(S^\dagger u)_{k_2^+}\right]$$

$$= O^{(0,8)}[v, v', n]$$



Predict

equal strong phases $\delta^D = \delta^{D^*}$

equal amplitudes $A_{00}^D = A_{00}^{D*}$

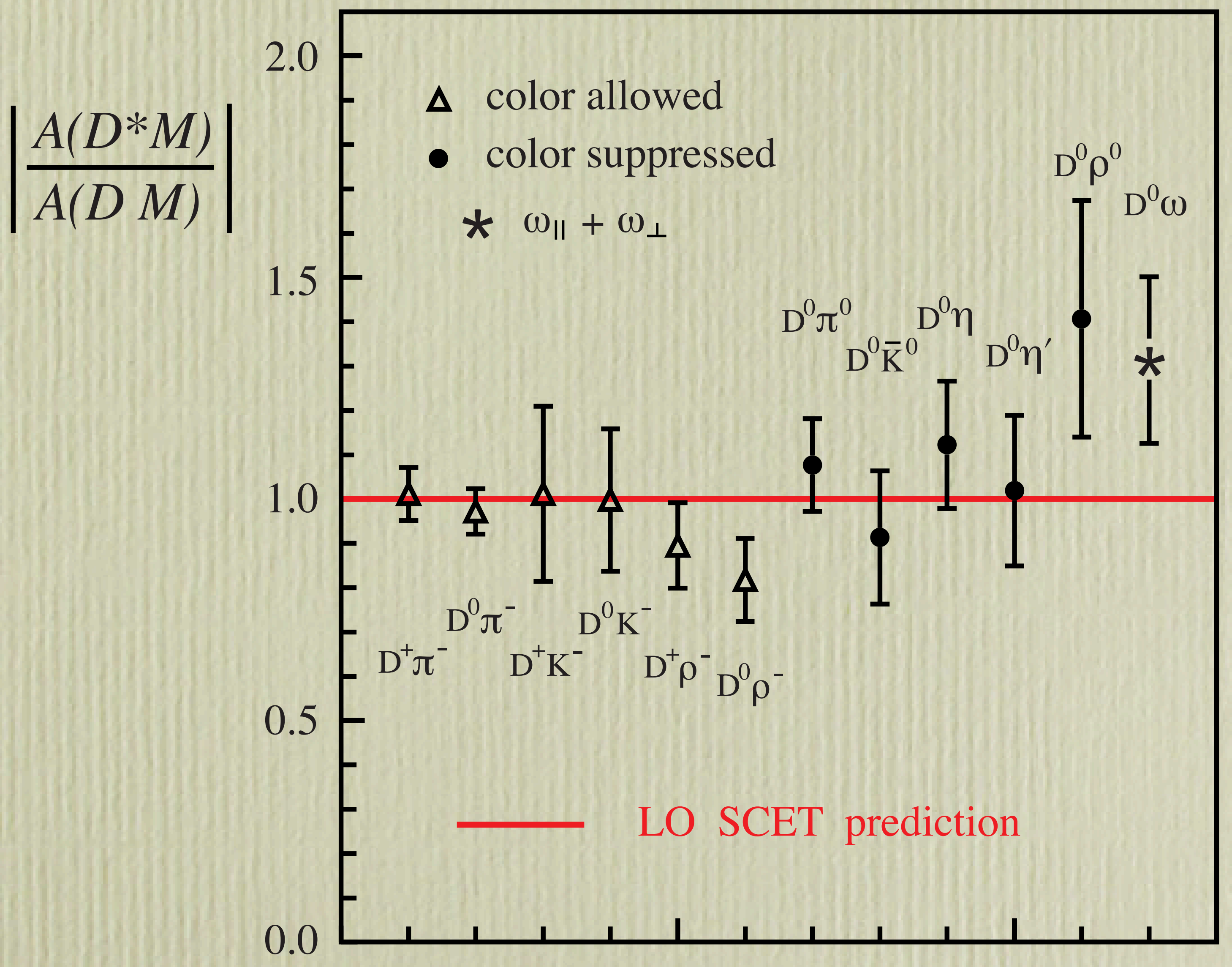
corrections to this are $\alpha_s(m_b)$, Λ/Q

Note: independent of the form of $J^{(i)}(z, x, k_1^+, k_2^+)$

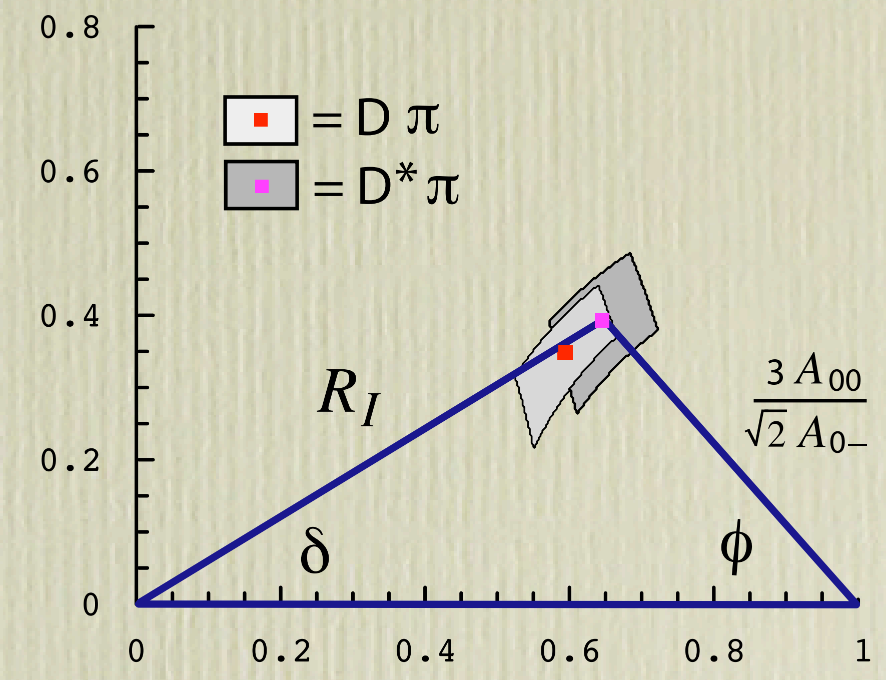
and $S^{(i)}(k_1^+, k_2^+)$, $\phi_M(x)$

Tests and Predictions

Expt Average (Cleo, Belle, Babar):



isospin triangle



$$\delta(D\pi) = 30.4 \pm 4.8^\circ$$
$$\delta(D^*\pi) = 31.0 \pm 5.0^\circ$$

Extension to isosinglets:

Blechman, Mantry, I.S.

More Predictions

If we expand $J(z, x, k_1^+, k_2^+)$ in $\alpha_s(E\Lambda)$, we can make more predictions

<u>Relate π and ρ</u>

- predict that $\phi^{D\rho} = \phi^{D\pi}$, not yet tested
- Recall data gives

$$|r^{D\pi}| = 0.77 \pm 0.05 \qquad |r^{D\rho}| = 0.80 \pm 0.09$$

if $\langle x^{-1}\rangle_\pi \simeq \langle x^{-1}\rangle_\rho$ then this implies $\delta^{D\pi} \simeq \delta^{D\rho}$

FKS mixing angle

- $$\frac{Br(\bar B \to D^{(*)}\eta')}{Br(\bar B \to D^{(*)}\eta)} = \tan^2(\theta) = 0.67 + \mathcal{O}(\alpha_s(\sqrt{E\Lambda}))$$

$$\frac{|A(\bar B^0 \to D^+\pi^-)|}{|A(B^- \to D^0\pi^-)|}$$

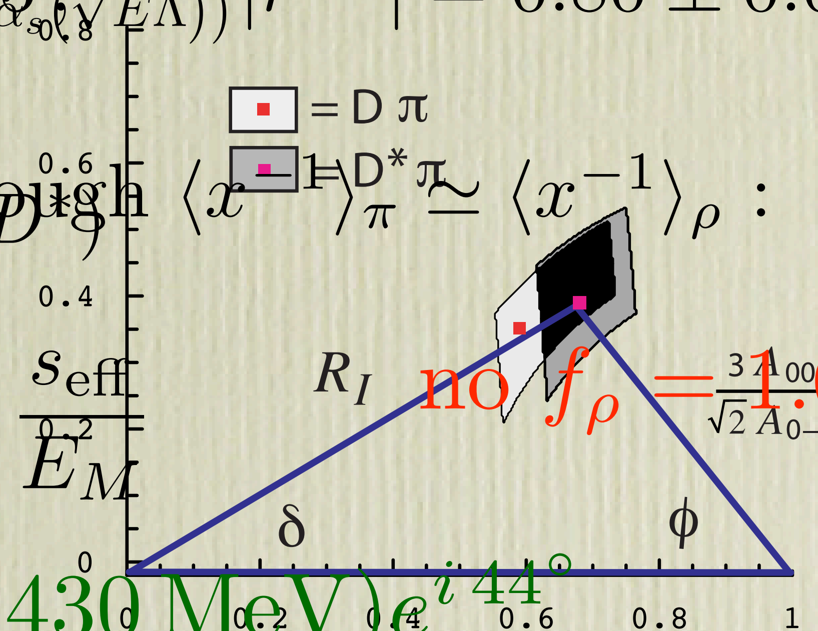
data $= 0.61 \pm 0.12 (D), \quad 0.51 \pm 0.18 (D^*)$

SCET predicts weak dependence on M through $\langle x^{-1}\rangle_\pi \simeq \langle x^{-1}\rangle_\rho$:

$$r^{DM} = 1 - \frac{16\pi\alpha_s m_D}{9(m_B + m_D)} \frac{\langle x^{-1}\rangle_M}{\xi(w_{max})} \frac{s_{\rm eff}}{E_M}$$

no $f_\rho = 1.6\, f_\pi$

natural parameters fit data, $s_{\rm eff} \simeq (430\ \text{MeV}) e^{i\,44^\circ}$



isospin triangle

$B \to M_1 M_2$

$B \to \pi\pi$ $B \to \pi K$ $B \to \rho K^*$

$B \to \rho\rho$ $B \to KK$

$B \to \pi K^*$ $B \to \pi\rho$

$B_s \to \pi^0\eta$ $B_s \to K^+ K^{*-}$

PP = 21 + 13 decays
PV = 40 + 23 decays → many of them observed
VV = 21 + 13 decays

First we need to look at semileptonic decays

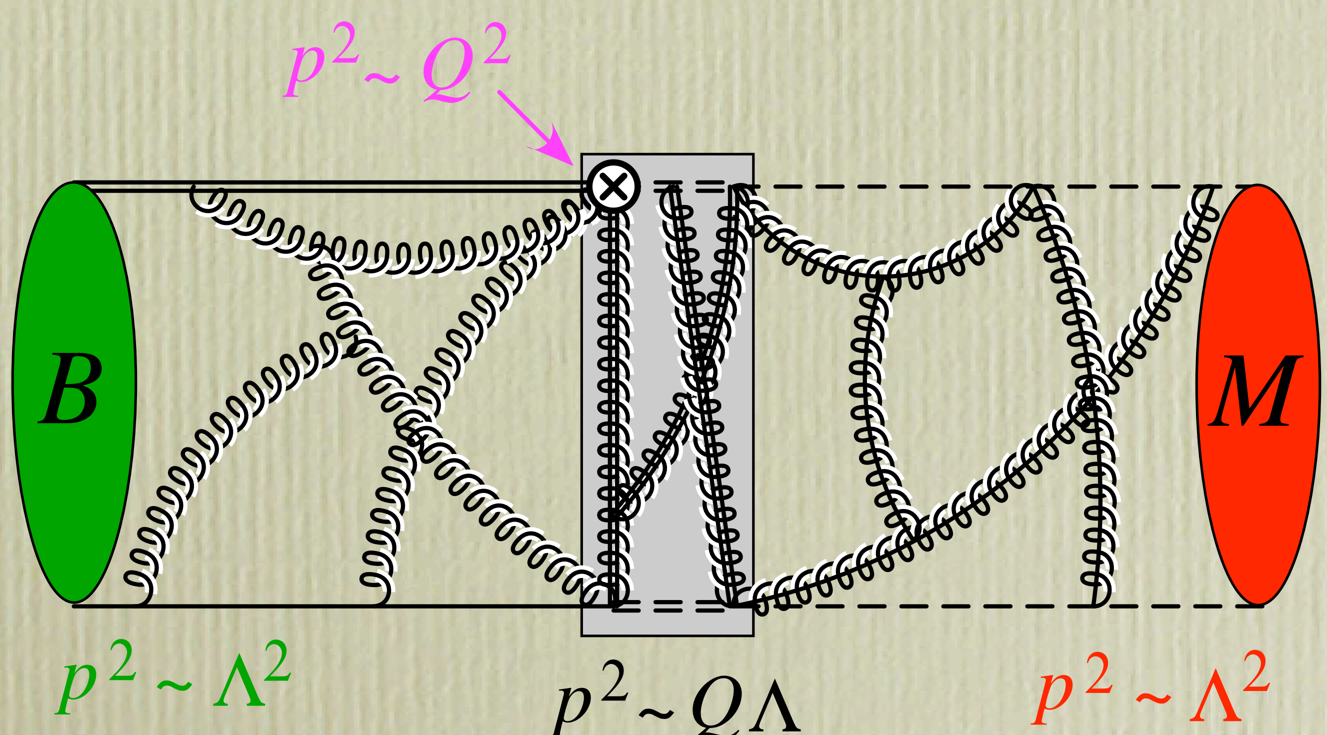
Form Factors in SCET

$B \to$ pseudoscalar: $f_+,\ f_0,\ f_T$
$B \to$ vector: $V,\ A_0,\ A_1,\ A_2,\ T_1,\ T_2,\ T_3$

$$f(E) = \int_0^1 dz\, T(z, E, m_b)\, \zeta_J^{BM}(z, E)$$
} "hard spectator", "factorizable"

$$+ \quad C(E, m_b)\, \zeta^{BM}(Q\Lambda, \Lambda^2)$$
} "soft form factor", "non-factorizable"

$$\zeta_J^{BM}(z) = f_M f_B \int_0^1 dx \int_0^\infty dk^+ J(z, x, k^+, E)\phi_M(x)\phi_B(k^+)$$



result at LO in λ, all orders in α_s, where $Q = \{m_b, E_M\}$

$$\Lambda/Q \ll 1$$

power corrections are $\sim 20\%$

Form Factors in SCET

One Loop Matching Known:

$C_k(E, m_b)$ Bauer, Fleming, Pirjol, I.S.

$T_i(z, E, m_b)$ Beneke, Kiyo, Yang

$J(z, x, r_+, E)$ Becher, Hill, Lee, Neubert

Lange, Neubert

Log Resummation:

Sudakov suppression of "soft" relative to "hard" form factors

→ small for physical b-quark mass

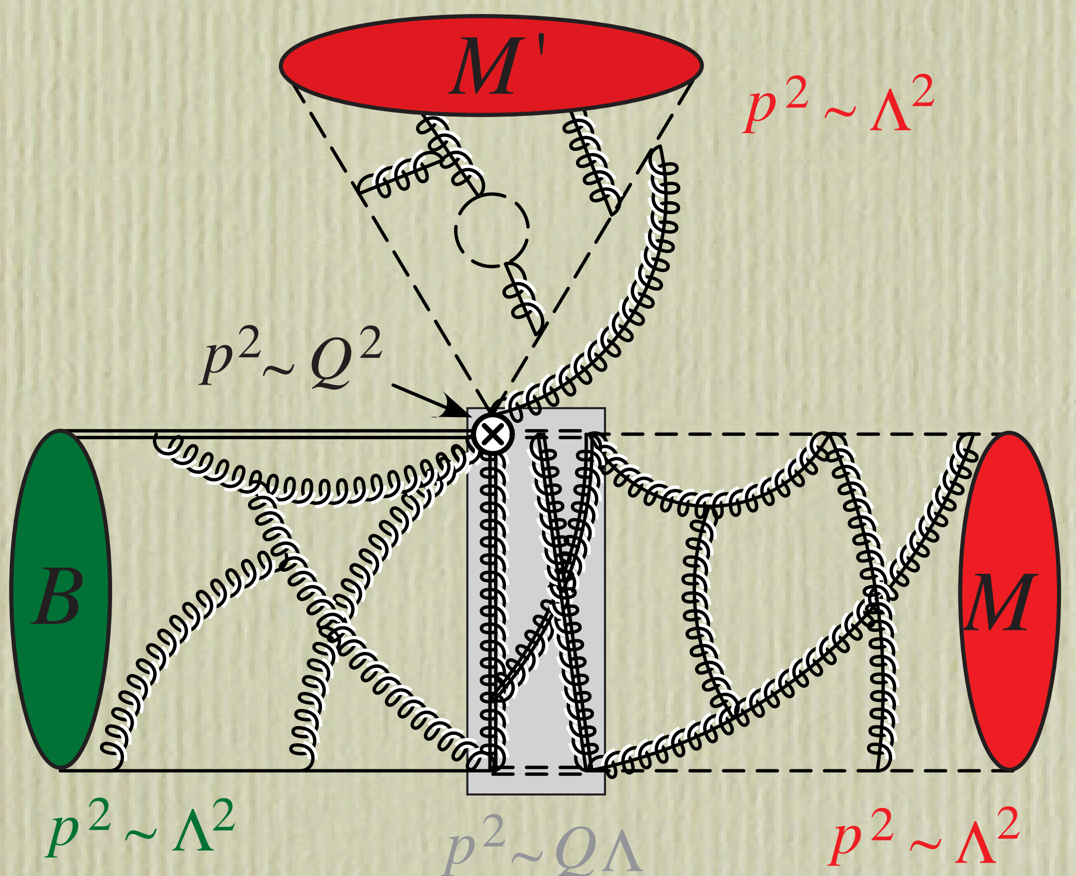
Which of ζ^{BM}, ζ_J^{BM} is bigger?

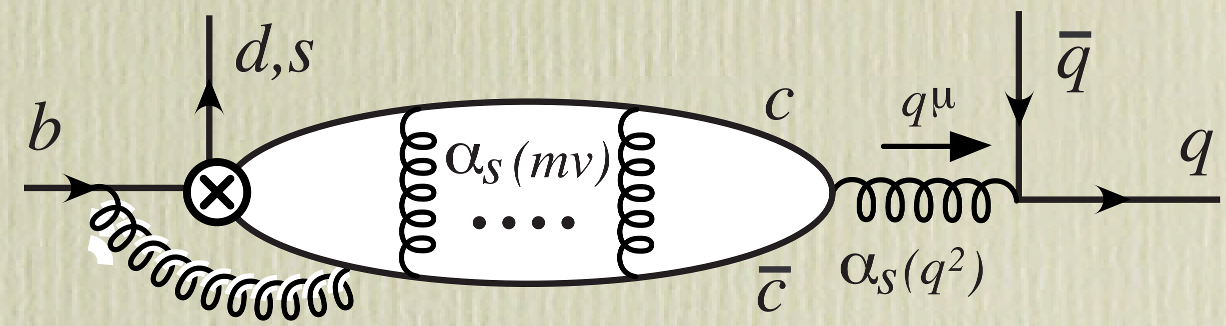
$$f(E) = \int_0^1 dz\, T(z, E, m_b)\, \zeta_J^{BM}(z, E) + C(E, m_b)\, \zeta^{BM}(Q\Lambda, \Lambda^2)$$

$B \to M_1 M_2$ Factorization in SCET

$\Lambda^2 \ll E\Lambda \ll E^2, m_b^2$

Bauer, Pirjol, Rothstein, I.S.
Chay, Kim
(earlier work by B.B.N.S.)





Ciuchini et al,
Colangelo et al

- hard spectator & form factor terms → same

  → Same Jet function as $B \to M$ form factors

- long distance charming penguin amplitude $= A_{c\bar{c}}$

Operators

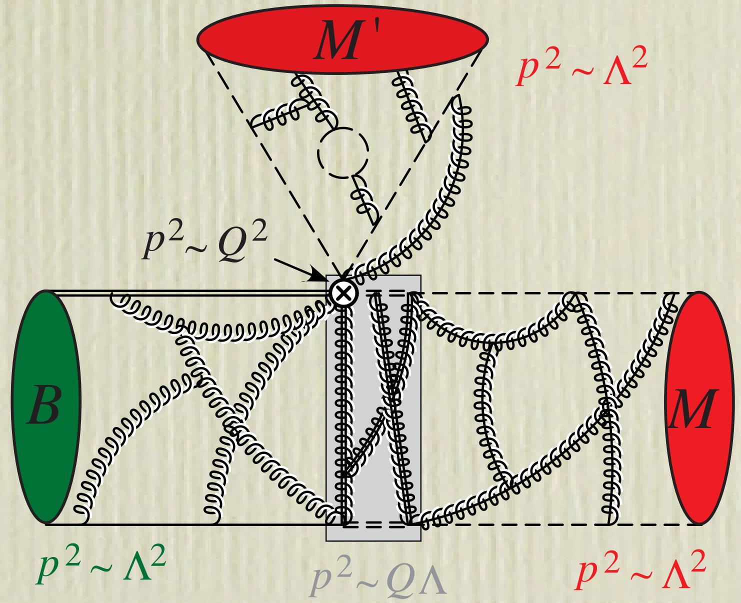
QCD $$H_W = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \Big(C_1 O_1^p + C_2 O_2^p + \sum_{i=3}^{10,8g} C_i O_i \Big)$$

$\mathrm{SCET_I}$ Integrate out $\sim m_b$ fluctuations

$$H_W = \frac{2G_F}{\sqrt{2}} \Big\{ \sum_{i=1}^{6} \int d\omega_j c_i^{(f)}(\omega_j) Q_{if}^{(0)}(\omega_j) + \sum_{i=1}^{8} \int d\omega_j b_i^{(f)}(\omega_j) Q_{if}^{(1)}(\omega_j) + \mathcal{Q}_{c\bar{c}} + \dots \Big\}$$

$$Q_{1d}^{(0)} = \big[\bar{u}_{n,\omega_1} \bar{n}\!\!\!/ P_L b_v\big]\big[\bar{d}_{\bar{n},\omega_2} n\!\!\!/ P_L u_{\bar{n},\omega_3}\big], \quad \dots$$

$$Q_{1d}^{(1)} = \frac{-2}{m_b} \big[\bar{u}_{n,\omega_1}\, ig \mathcal{B}\!\!\!/_{n,\omega_4}^{\perp} P_L b_v\big]\big[\bar{d}_{\bar{n},\omega_2} n\!\!\!/ P_L u_{\bar{n},\omega_3}\big], \quad \dots$$



New Nonperturbative Result in $\alpha_s(\sqrt{E\Lambda})$:

$$A(B \to M_1 M_2) = A^{c\bar{c}} + N\Big\{ f_{M_2}\, \zeta^{BM_1} \int_0^1 du\, T_{2\zeta}(u)\phi^{M_2}(u) + f_{M_1}\zeta^{BM_2} \int_0^1 du\, T_{1\zeta}(u)\phi^{M_1}(u)$$
$$+ f_{M_2} \int_0^1 du \int_0^1 dz\, T_{2J}(u,z)\zeta_J^{BM_1}(z)\phi^{M_2}(u) + f_{M_1} \int_0^1 du \int_0^1 dz\, T_{1J}(u,z)\zeta_J^{BM_2}(z)\phi^{M_1}(u)\Big\}$$

where $\zeta^{BM} \sim \zeta_J^{BM}(z) \sim (\Lambda/Q)^{3/2}$ and appear in $B \to M$

Focus on model independent results at LO:

- fit ζ's, calculate T's
- strong phase only in $A_{c\bar{c}}$ and small pert. corrections

BBNS: Factorization similar, but does not separate $E\Lambda \ll E^2, m_b^2$
Phenomenological inputs gave $\zeta_J^{BM} \ll \zeta^{BM}$

Hard Coefficients

M_1M_2	$T_{1\zeta}(u)$	$T_{2\zeta}(u)$	M_1M_2	$T_{1\zeta}(u)$	$T_{2\zeta}(u)$
$\pi^-\pi^+$, $\rho^-\pi^+$, $\pi^-\rho^+$, $\rho_\parallel^-\rho_\parallel^+$	$c_1^{(d)}+c_4^{(d)}$	0	$\pi^+K^{(*)-}$, ρ^+K^-, $\rho_\parallel^+K_\parallel^{*-}$	0	$c_1^{(s)}+c_4^{(s)}$
$\pi^-\pi^0$, $\rho^-\pi^0$	$\frac{1}{\sqrt{2}}(c_1^{(d)}+c_4^{(d)})$	$\frac{1}{\sqrt{2}}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\pi^0K^{(*)-}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}-c_3^{(s)})$	$\frac{1}{\sqrt{2}}(c_1^{(s)}+c_4^{(s)})$
$\pi^-\rho^0$, $\rho_\parallel^-\rho_\parallel^0$	$\frac{1}{\sqrt{2}}(c_1^{(d)}+c_4^{(d)})$	$\frac{1}{\sqrt{2}}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	ρ^0K^-, $\rho_\parallel^0K_\parallel^{*-}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}+c_3^{(s)})$	$\frac{1}{\sqrt{2}}(c_1^{(s)}+c_4^{(s)})$
$\pi^0\pi^0$	$\frac{1}{2}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\frac{1}{2}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\pi^-\bar{K}^{(*)0}$, $\rho^-\bar{K}^0$, $\rho_\parallel^-\bar{K}_\parallel^{*0}$	0	$-c_4^{(s)}$
$\rho^0\pi^0$	$\frac{1}{2}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\frac{1}{2}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\pi^0\bar{K}^{(*)0}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}-c_3^{(s)})$	$-\frac{1}{\sqrt{2}}c_4^{(s)}$
$\rho_\parallel^0\rho_\parallel^0$	$\frac{1}{2}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\frac{1}{2}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\rho^0\bar{K}^0$, $\rho_\parallel^0\bar{K}_\parallel^{*0}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}+c_3^{(s)})$	$-\frac{1}{\sqrt{2}}c_4^{(s)}$
$K^{(*)0}K^{(*)-}$, $K^{(*)0}\bar{K}^{(*)0}$	$-c_4^{(d)}$	0	$K^{(*)-}K^{(*)+}$	0	0

similar for T_J's in terms of $b_i^{(f)}$'s

Note: have not used isospin yet

Matching

$$c_1^{(f)} = \lambda_u^{(f)}\Big(C_1+\frac{C_2}{N_c}\Big) - \lambda_t^{(f)}\frac{3}{2}\Big(C_{10}+\frac{C_9}{N_c}\Big) + \Delta c_1^{(f)}\,,$$

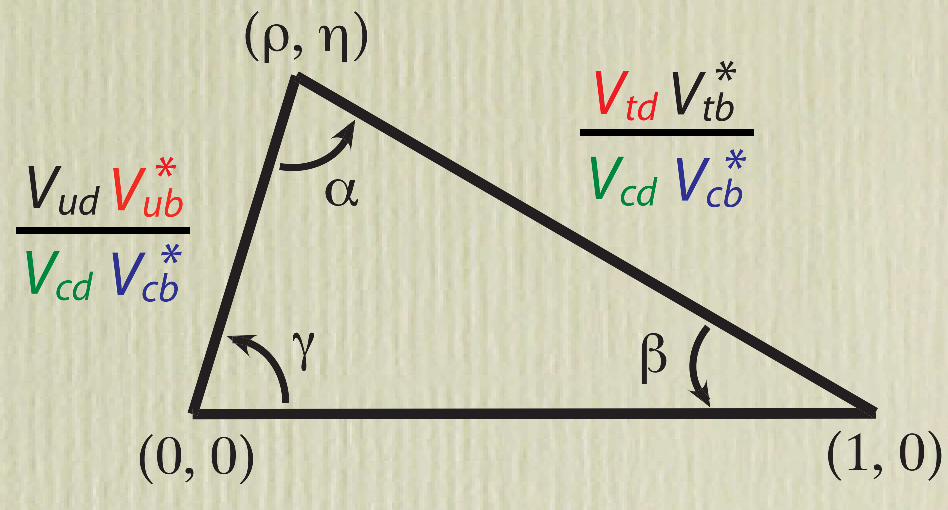
$$b_1^{(f)} = \lambda_u^{(f)}\Big[C_1 + \Big(1-\frac{m_b}{\omega_3}\Big)\frac{C_2}{N_c}\Big] - \lambda_t^{(f)}\Big[\frac{3}{2}C_{10} + \Big(1-\frac{m_b}{\omega_3}\Big)\frac{3C_9}{2N_c}\Big] + \Delta b_1^{(f)}\,,$$

Phenomenology for $B \to \pi\pi$

CP Asymmetries

Test
CP violation

$$\mathcal{A}_{\rm CP}(t) = -S_{\pi\pi}\sin(\Delta m_B t) + C_{\pi\pi} cos(\Delta m_B t)$$



World Averages (BABAR, BELLE)

	$\overline{\rm Br}\times 10^6$	$C_{\pi\pi}$	$S_{\pi\pi}$
$\pi^+\pi^-$	4.6 ± 0.4	-0.37 ± 0.11	-0.61 ± 0.13
$\pi^0\pi^0$	1.51 ± 0.28	-0.28 ± 0.39	
$\pi^+\pi^0$	5.61 ± 0.63		

Warning: The BaBar and Belle asymmetries do not agree.

ICHEP '04

	$C_{\pi^+\pi^-}$	$S_{\pi^+\pi^-}$
Babar	-0.09 ± 0.15	-0.30 ± 0.17
Belle	-0.58 ± 0.17	-1.00 ± 0.22

Aspen'05 (Belle)

-0.56 ± 0.13 $\quad - 0.67 \pm 0.17$

Pure Isospin Analysis

ala Gronau, London

$$A(\bar{B}^0 \to \pi^+\pi^-) = e^{-i\gamma}\,|\lambda_u|\,T - |\lambda_c|\,P$$
$$A(\bar{B}^0 \to \pi^0\pi^0) = e^{-i\gamma}\,|\lambda_u|\,C + |\lambda_c|P$$
$$\sqrt{2}A(B^- \to \pi^0\pi^-) = e^{-i\gamma}\,|\lambda_u|\,(T + C)$$

$|\lambda_{c,u}| =$ CKM factors

Parameters: β known

isospin: γ +5 hadronic

one, say T, just sets Br scale

$$p_c \equiv -\frac{|\lambda_c|}{|\lambda_u|}\,\mathrm{Re}\Big(\frac{P}{T}\Big), \qquad p_s \equiv -\frac{|\lambda_c|}{|\lambda_u|}\,\mathrm{Im}\Big(\frac{P}{T}\Big),$$

$$t_c \equiv \frac{|T|}{|T+C|}, \qquad \epsilon \equiv \mathrm{Im}\Big(\frac{C}{T}\Big).$$

(SCET: $\epsilon = 0$

$Re(A_{c\bar{c}})$, $Im(A_{c\bar{c}})$, $\zeta^{B\pi}$, $\zeta_J^{B\pi}$)

Data:

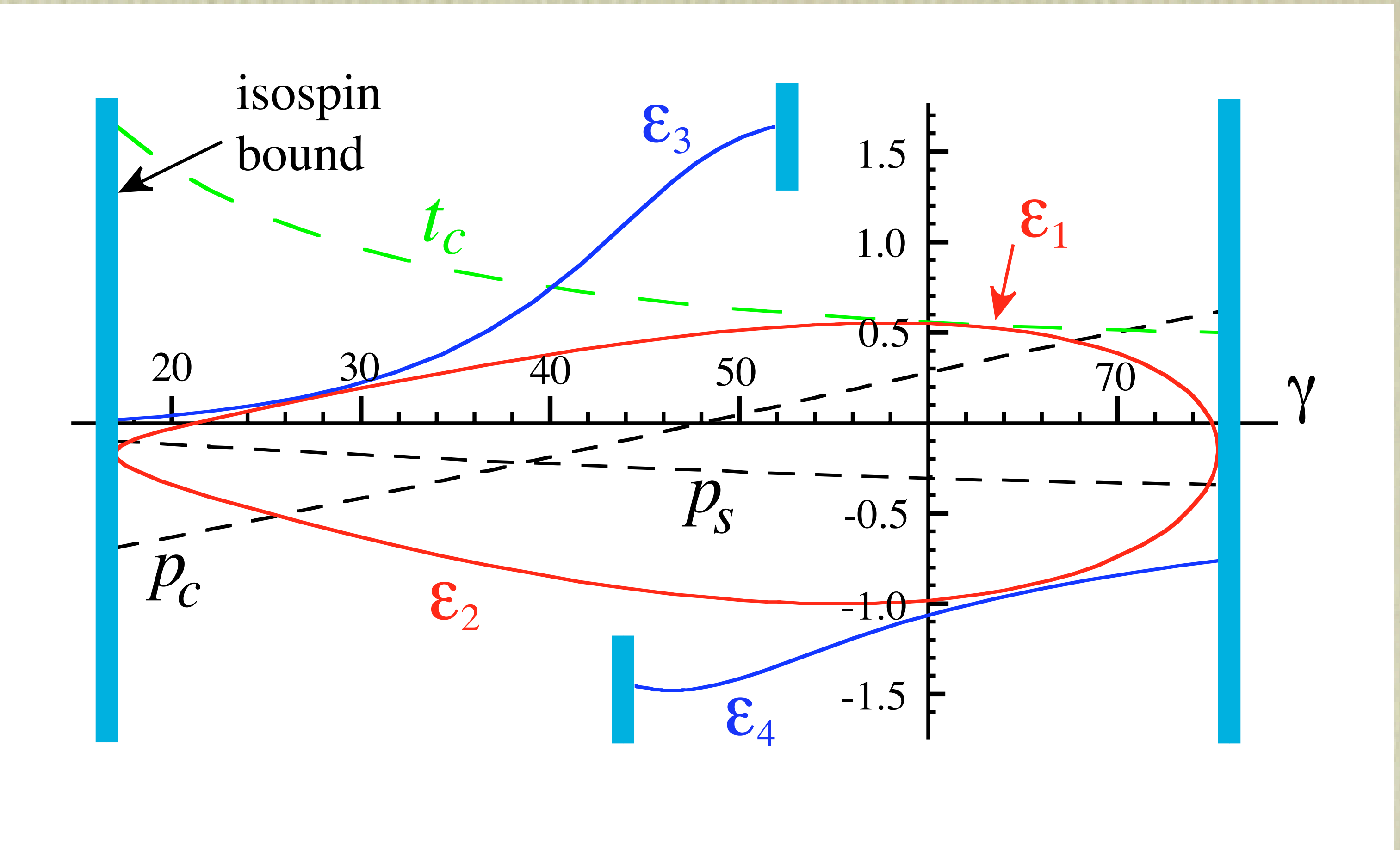
$$S_{\pi^+\pi^-}, C_{\pi^+\pi^-} \Rightarrow p_c, p_s$$

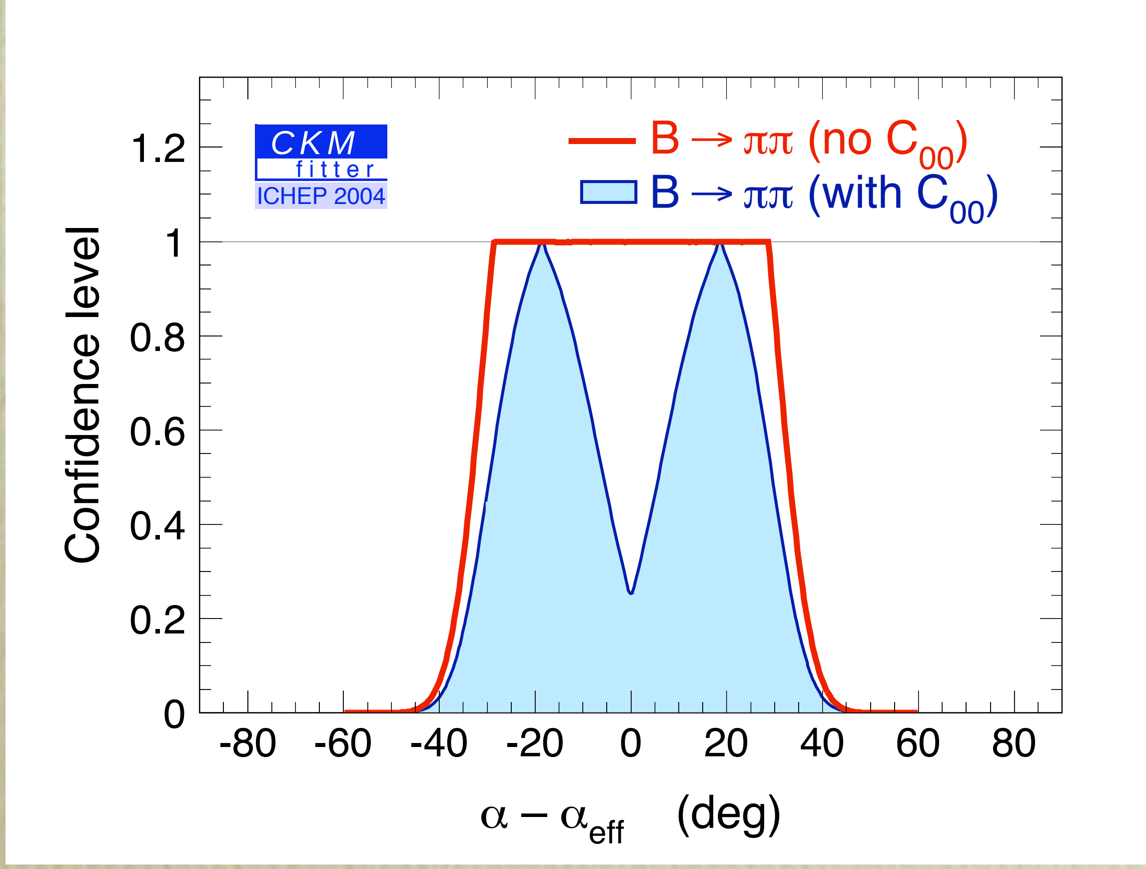
$$\frac{Br(\pi^+\pi^-)}{Br(\pi^0\pi^-)} \Rightarrow t_c$$

$$\frac{Br(\pi^0\pi^0)}{Br(\pi^0\pi^-)} \Rightarrow \epsilon_{1,2}$$

$$C_{\pi^0\pi^0} \Rightarrow \epsilon_{3,4}$$

determined as functions of γ





large C amplitude
large penguin

SCET:

- an extra term $\frac{C_1}{N_c}\langle \bar{u}^{-1}\rangle_\pi \, \zeta_J^{B\pi}$ ruins color suppression
- size of penguin consistent with $A_{c\bar{c}} \sim v\ \alpha_s(2m_c)$

just isospin:

Problem is that $C_{\pi^0\pi^0}$ will remain uncertain for quite some time

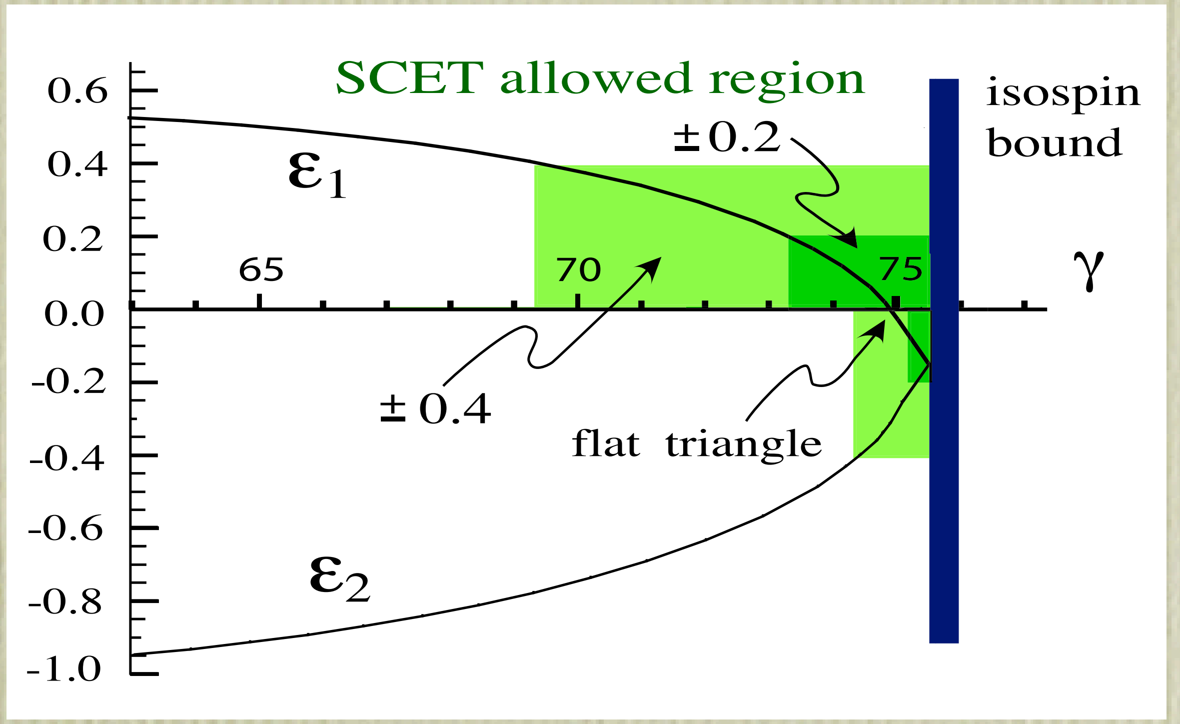
A New Method for Determining γ

Bauer, Rothstein, I.S.

Isospin + bare minimum from Λ/m_b expansion

Factorization from SCET: $\epsilon \sim \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{m_b}\,, \alpha_s(m_b)\Big)\,.$

This gives



2nd solution $\gamma = 21.5^{\circ}{}^{+8.7^\circ}_{-4.4^\circ}{}^{+11.1^\circ}_{-7.9^\circ}$

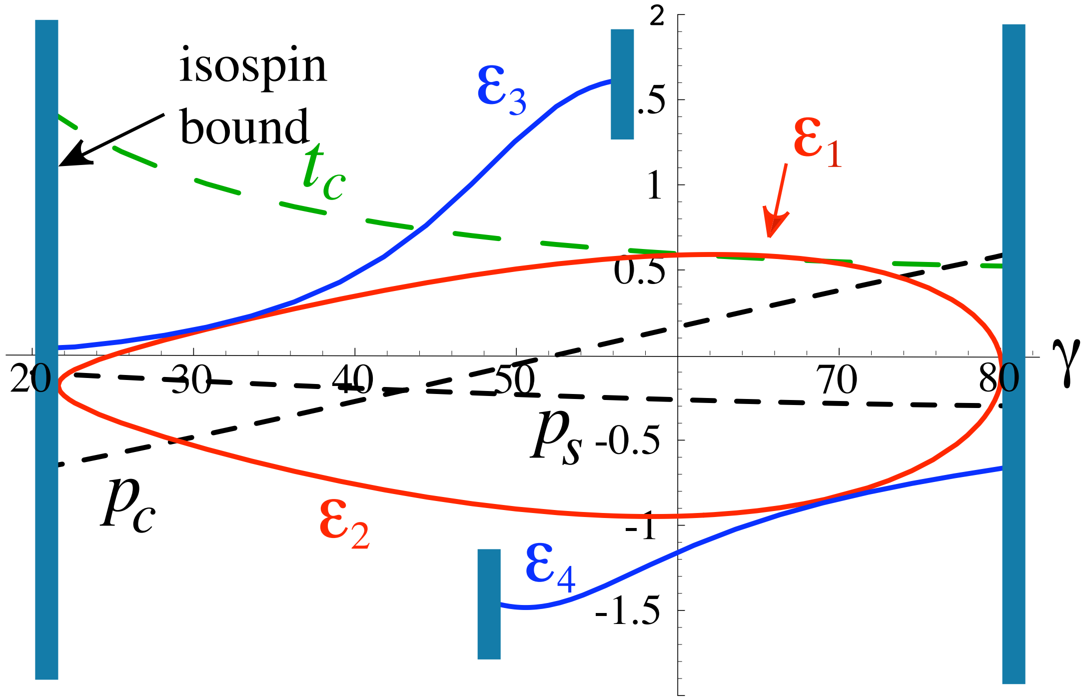
global fits give

$$\gamma \simeq 62^\circ \pm 12^\circ$$

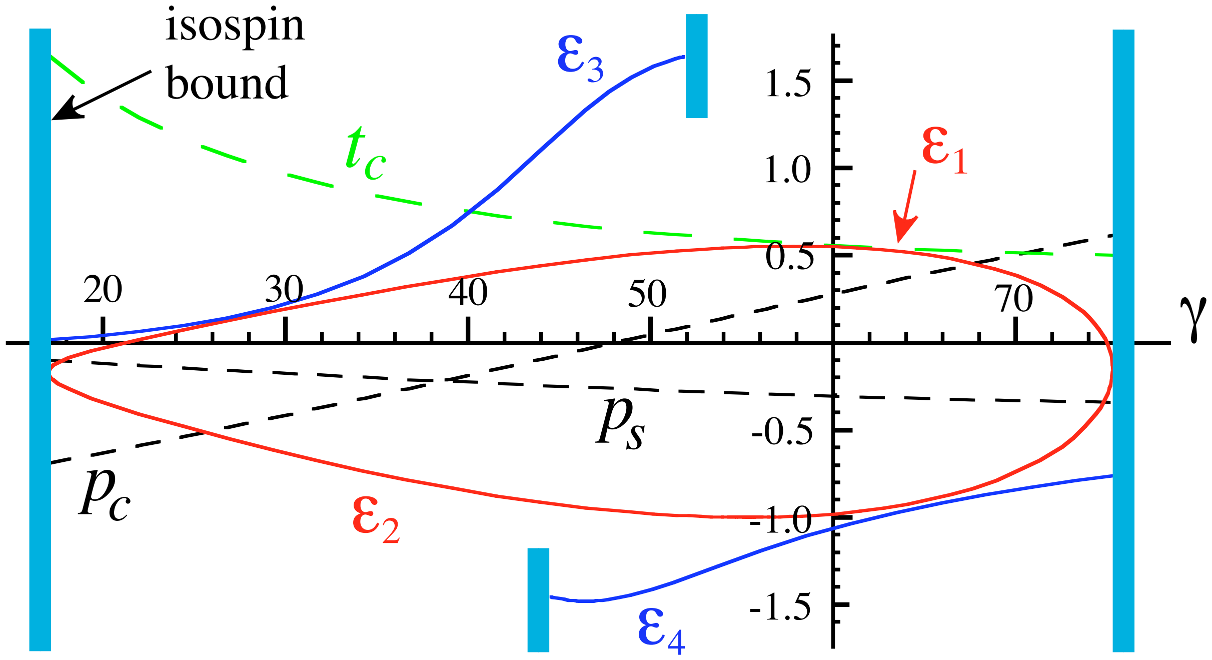
$$\gamma = 74.9^\circ \pm 2^\circ{}^{+9.4^\circ}_{-13.3^\circ}\,.$$

$$(\text{or}{}^{+2^\circ}_{-5.2^\circ})$$

Theory uncertainty is small



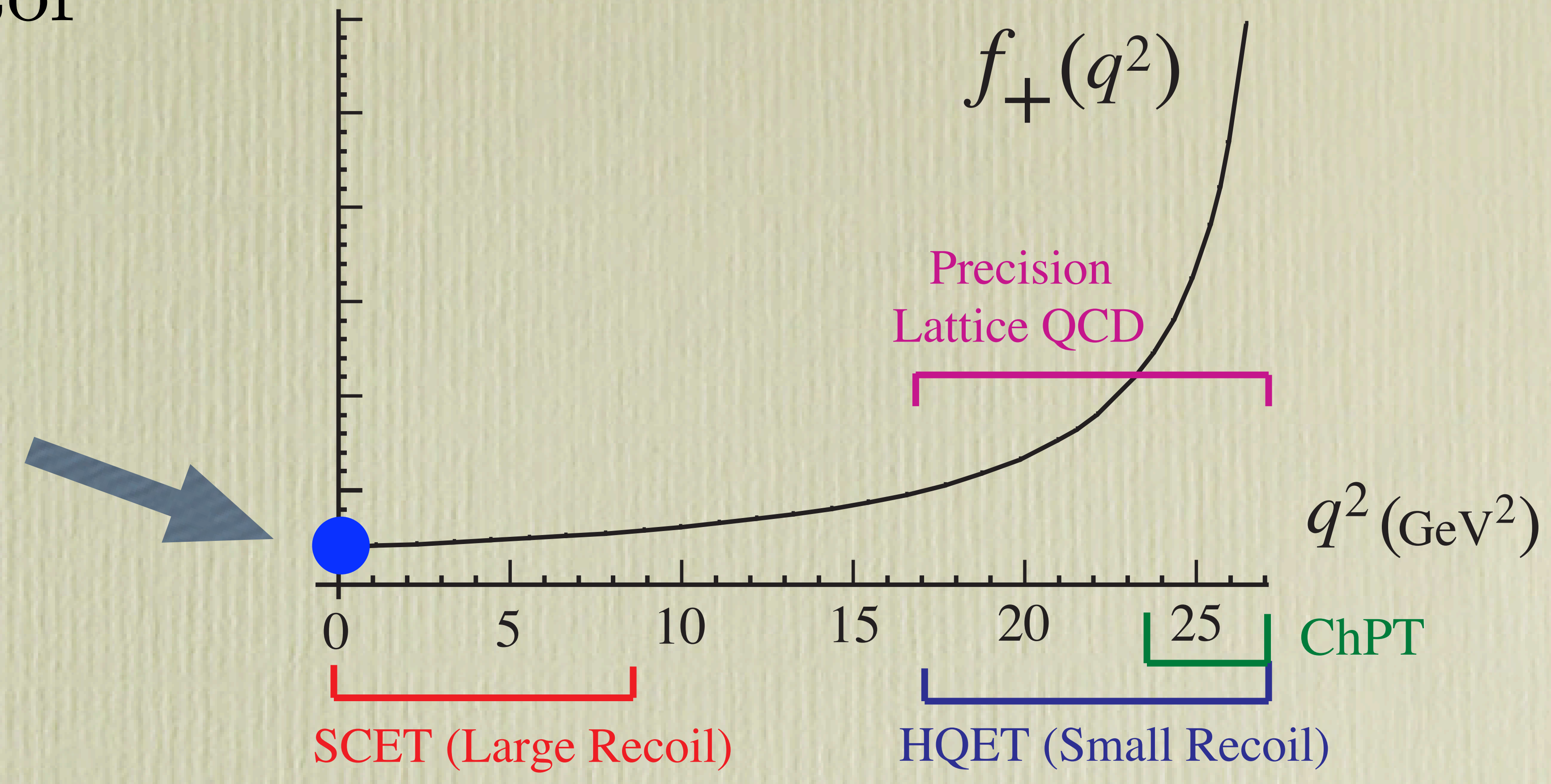
averages with new Belle data



ICHEP averages

$B \to \pi \ell \bar{\nu}$ form factor

q^2 relevant for nonleptonic



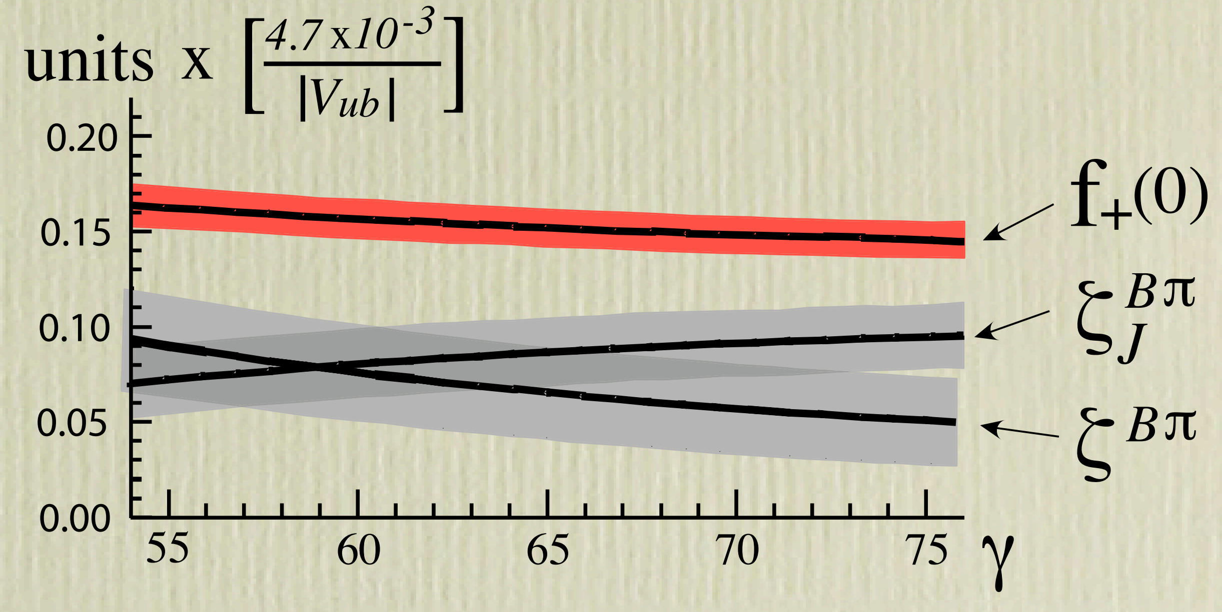
Use nonleptonic data: $B \to \pi\pi$ determines the parameters

$$\zeta^{B\pi}\,|_{\gamma=75^\circ} = (0.052 \pm 0.023)\left(\frac{4.7\times10^{-3}}{|V_{ub}|}\right)$$

$$\zeta_J^{B\pi}\,|_{\gamma=75^\circ} = (0.095 \pm 0.017)\left(\frac{4.7\times10^{-3}}{|V_{ub}|}\right)$$

$$\int dx \frac{\phi_\pi(x)}{x} = 3$$

hard scattering bigger than soft form factor



$$f_+(0) = \zeta^{B\pi} + \zeta_J^{B\pi}$$

$$f_+(0) = (0.15 \pm 0.01 \pm 0.04)\left(\frac{4.7\times10^{-3}}{|V_{ub}|}\right)$$

expt. theory estimate

smaller than models

$$f_+(0) \sim 0.25$$

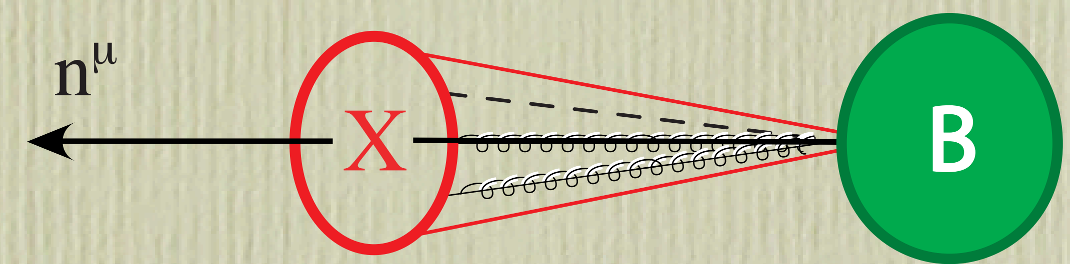
$$\int dx \frac{\phi_\pi(x)}{x} = 2.25 \qquad \zeta^{B\pi}|_{\gamma=75^\circ} = 0.02 \qquad \zeta_J^{B\pi}|_{\gamma=75^\circ} = 0.13$$

$$\int dx \frac{\phi_\pi(x)}{x} = 3.75 \qquad \zeta^{B\pi}|_{\gamma=75^\circ} = 0.07 \qquad \zeta_J^{B\pi}|_{\gamma=75^\circ} = 0.08$$

Inclusive Decays

$B \to X_u \ell \bar{\nu}$ $\quad$ $B \to X_s \gamma$

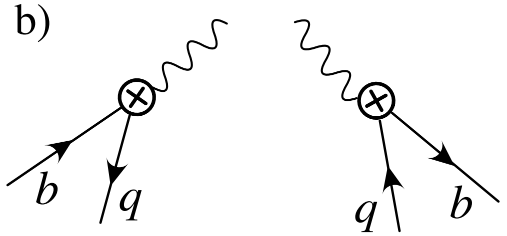
- With enough phase space can use local OPE, known to $\frac{1}{m_b^3}$
- But some cuts put us in endpoint region:

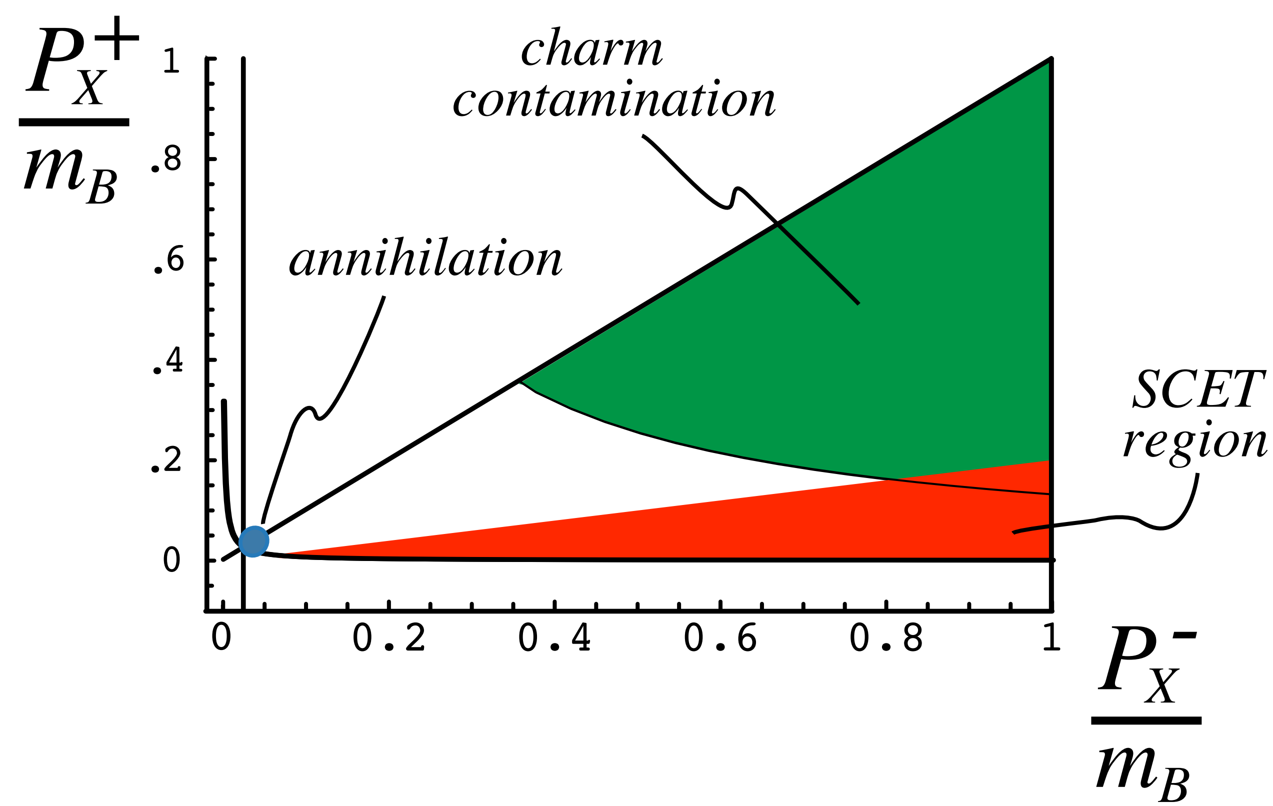


$$m_X^2 \sim m_b \Lambda \qquad P_X^- \gg P_X^+$$

$$\frac{P_X^+}{P_X^-} \le 0.2$$

b)



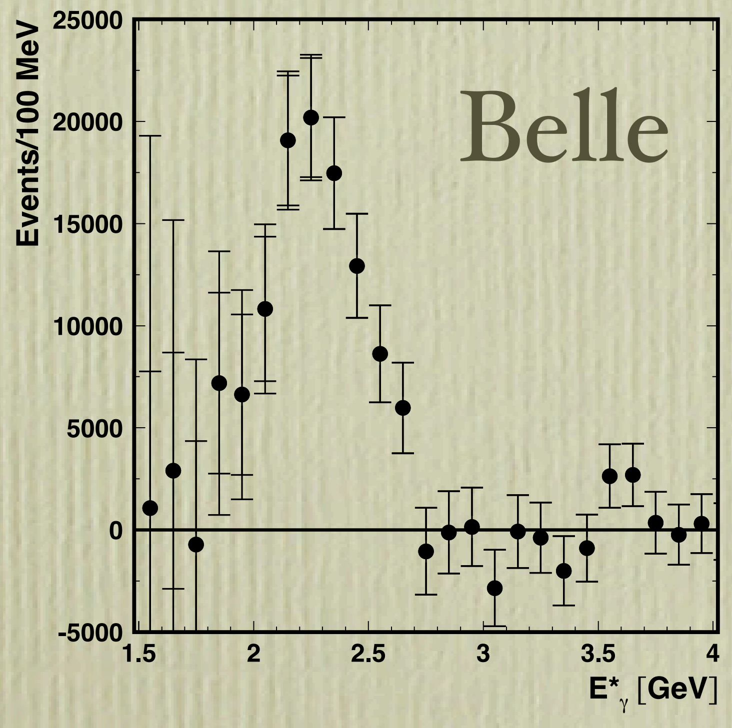


$$d\Gamma = H(m_b, p_X^-) \int dk^+ \, J(p_X^- k^+) \, f(k^+ + \overline{\Lambda} - p_X^+)$$

Korchemsky, Sterman

SCET gives systematic expansion in this region

$$\lambda^2 = \frac{\Lambda}{m_b}$$

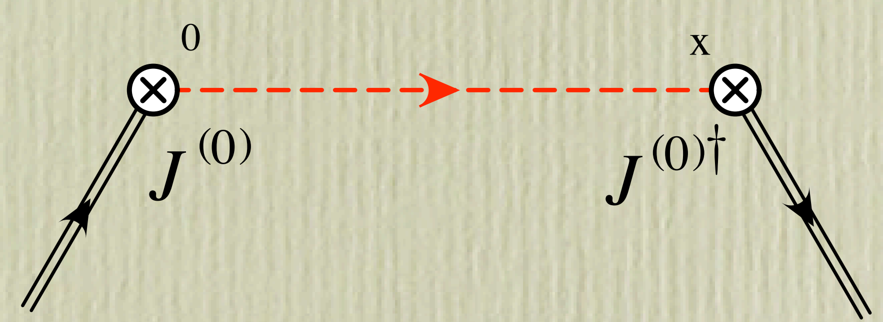


$B \to X_s \gamma$

shape function f can be measured in $B \to X_s \gamma$, then used to measure Vub with $B \to X_u \ell \bar{\nu}$

Leading Order Factorization in SCET

$$J^{(0)} = \int d\omega\, C(\omega)(\bar{\xi}_n W)_\omega\, \Gamma\, (Y^\dagger h_v)$$

T-product	Example Diagram	Hard, Jet, and Shape Functions	Usoft Operator
$T^{(0)}$		$h^{[0]}\,\mathcal{J}^{(0)}\,f^{(0)}$	$\bar{h}_v(x)h_v(0)$

$$d\Gamma = H(m_b, p_X^-) \int dk^+\, J(p_X^- k^+)\, f(k^+ + \overline{\Lambda} - p_X^+)$$

Factorization at NLO

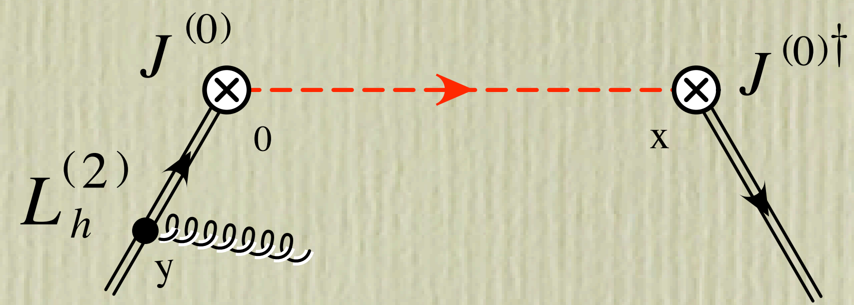
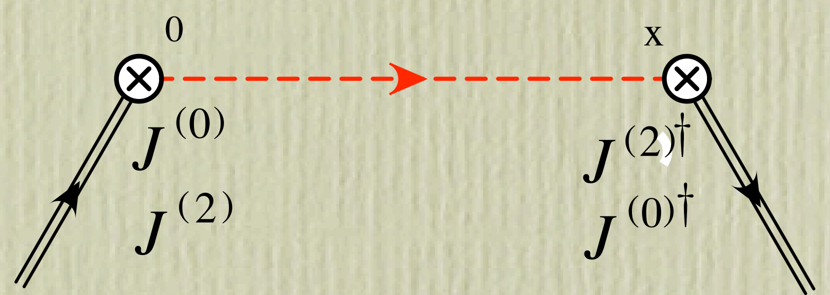
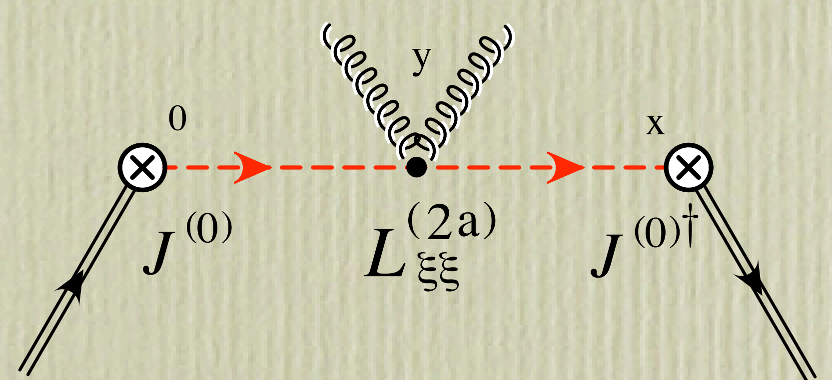
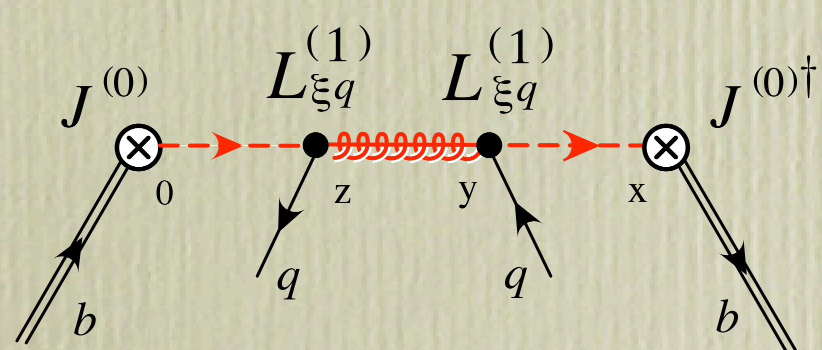
K. Lee, I.S. hep-ph/0409045

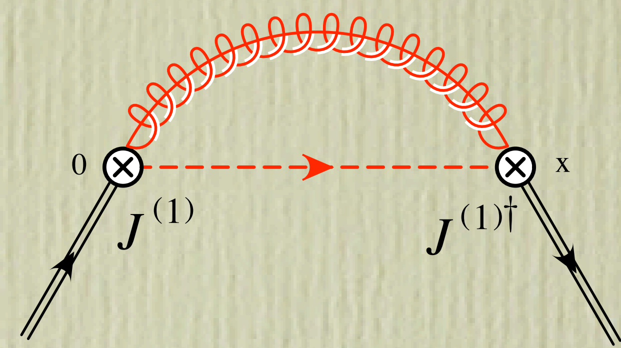
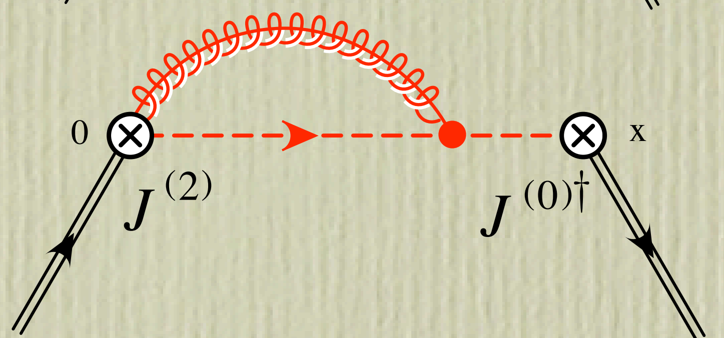
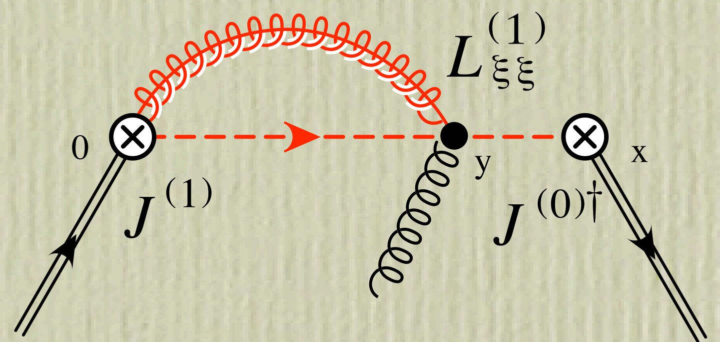
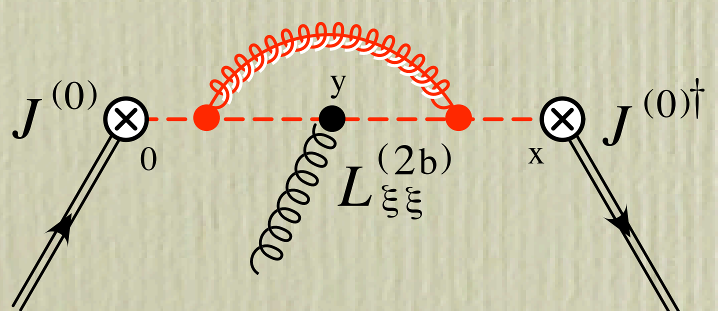
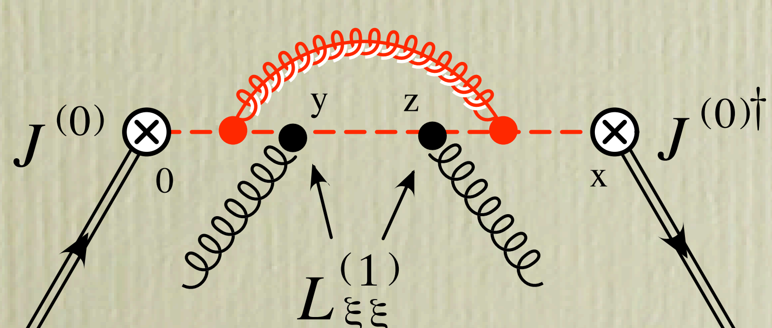
- derive factorization theorems at subleading order
- complete categorization of all terms at $\dfrac{\Lambda}{m_b}$
- all orders in α_s

$$J = J^{(0)} + J^{(1)} + J^{(2)} + \ldots$$
$$\mathcal{L} = \mathcal{L}_c^{(0)} + \mathcal{L}_{us}^{(0)} + \mathcal{L}_{\xi q}^{(1)} + \mathcal{L}_j^{(1)} + \mathcal{L}_j^{(2)} + \ldots$$

Bosch et al. hep-ph/0409115

Beneke et al. hep-ph/0411395

T-product	Example Diagram	Hard, Jet, and Shape Functions	Usoft Operator
$\hat{T}^{(2H)}$		$h^0 \mathcal{J}^{(0)}\, f_0^{(2)}$	$\bar{h}_v(x)h_v(0)i\mathcal{L}_{\rm h}^{(2)}(y)$
$\hat{T}^{(2a)}$		$h^{1,2}\mathcal{J}^{(0)}\, f_{1,2}^{(2)}$	$\bar{h}_v(x)(D_{T,\perp}h_v)(0)$ $(\bar{h}_v D_{T,\perp})(x)h_v(0)$
$\hat{T}^{(2L)}$		$h^{3,4}\, \mathcal{J}_{1,2}^{(-2)}\, f_{3,4}^{(4)}$ $h^{3,4}\, \mathcal{J}_{3,4}^{(-2)}\, g_{3,4}^{(4)}$	$\bar{h}_v(x)(D_\perp D_\perp)(y)h_v(0)$
$\hat{T}^{(2q)}$		$h^{5,6}\, \mathcal{J}_1^{(-4)}\, f_{5,6}^{(6)}$ $h^{5-8}\, \mathcal{J}_{2-4}^{(-4)}\, g_{5-10}^{(6)}$	$\bar{h}_v(x)q(y)\bar{q}(z)h_v(0)$

<table>
<tr><th>T-product</th><th>Example Diagram</th><th>Hard, Jet, and Shape Functions</th><th>Usoft Operator</th></tr>
<tr><td>$\hat{T}^{(2b)}$</td><td>
</td><td>$h^{[2b]}\,\mathcal{J}^{(2)}_{1,2}\,f^{(0)}$</td><td>$\bar{h}_v(x)h_v(0)$</td></tr>
<tr><td>$\hat{T}^{(2c)}$</td><td>
</td><td>$h^{[2c]}\,\mathcal{J}^{(2)}_{3-10}\,f^{(0)}$</td><td>$\bar{h}_v(x)h_v(0)$</td></tr>
<tr><td>$\hat{T}^{(2La)}$</td><td>
</td><td>$h^{[2La]}\,\mathcal{J}^{(0)}_{j'}\,g^{(2)}_{11,12}$</td><td>$\bar{h}_v(x)D_\perp(y)h_v(0)$</td></tr>
<tr><td>$\hat{T}^{(2Lb)}$</td><td>
</td><td>$h^{[2Lb]}\,\mathcal{J}^{(0)}_{j'}\,g^{(2)}_{13,14}$</td><td>$\bar{h}_v(x)\bar{n}\cdot D(y)h_v(0)$</td></tr>
<tr><td>$\hat{T}^{(2LL)}$</td><td>
</td><td>$h^{[2LL]}\,\mathcal{J}^{(-2)}_{j'}\,g^{(4)}_{15-26}$</td><td>$\bar{h}_v(x)D_\perp(y)D_\perp(z)h_v(0)$</td></tr>
</table>

$$
\begin{aligned}
&\frac{h_i^{0f}(\bar n\cdot p)}{2m_b}\int_0^{p_X^+} dk^+\, \mathcal{J}^{(0)}(\bar n\cdot p\, k^+,\mu)\, f_0^{(2)}(k^+ + r^+,\mu)\\
&+\sum_{r=1}^{2}\frac{h_i^{rf}(\bar n\cdot p)}{m_b}\int_0^{p_X^+} dk^+\, \mathcal{J}^{(0)}(\bar n\cdot p\, k^+,\mu)\, f_r^{(2)}(k^+ + r^+,\mu)\\
&+\sum_{r=3}^{4}\frac{h_i^{rf}(\bar n\cdot p)}{m_b}\int dk_1^+\, dk_2^+\, \mathcal{J}_{1\pm 2}^{(-2)}(\bar n\cdot p\, k_j^+,\mu)\, f_r^{(4)}(k_j^+ + r^+,\mu)\\
&+\sum_{r=5}^{6}\frac{h_i^{rf}(\bar n\cdot p)}{\bar n\cdot p}\int dk_1^+ dk_2^+ dk_3^+\, \mathcal{J}_1^{(-4)}(\bar n\cdot p\, k_{j'}^+,\mu)\, f_r^{(6)}(k_{j'}^+ + r^+,\mu)\\
&+\frac{h_i^{00f}(\bar n\cdot p)}{m_b}\int_0^{p_X^+} dk^+\, \mathcal{J}^{(0)}(\bar n\cdot p\, k^+,\mu)\, g_0^{(2)}(k^+ + r^+,\mu)\\
&+\sum_{r=3}^{4}\frac{h_i^{rf}(\bar n\cdot p)}{m_b}\int dk_1^+\, dk_2^+\, \mathcal{J}_{3\pm 4}^{(-2)}(\bar n\cdot p\, k_j^+,\mu)\, g_r^{(4)}(k_j^+ + r^+,\mu)\\
&+\sum_{r=5}^{6}\frac{h_i^{rf}(\bar n\cdot p)}{\bar n\cdot p}\int dk_1^+ dk_2^+ dk_3^+\, \mathcal{J}_2^{(-4)}(\bar n\cdot p\, k_{j'}^+,\mu)\, g_r^{(6)}(k_{j'}^+ + r^+,\mu)\\
&+\sum_{r=7}^{8}\frac{h_i^{rf}(\bar n\cdot p)}{\bar n\cdot p}\int dk_1^+ dk_2^+ dk_3^+\, \big[\mathcal{J}_3^{(-4)}(\bar n\cdot p\, k_{j'}^+,\mu)\, g_r^{(6)}(k_{j'}^+ + r^+,\mu)\\
&\qquad\qquad + \mathcal{J}_4^{(-4)}(\bar n\cdot p\, k_{j'}^+,\mu)\, g_{r+2}^{(6)}(k_{j'}^+ + r^+,\mu)\big]\\
&+\sum_{m=1,2}\int dz_1 dz_2\, \frac{h_i^{[2b]m+8}(z_1,z_2,\bar n\cdot p)}{m_b}\int_0^{p_X^+} dk^+\, \mathcal{J}_m^{(2)}(z_1,z_2,p_X^- k^+)\, f^{(0)}(k^+ + \bar\Lambda - p_X^+)\\
&+\sum_{m=3,4}\frac{h_i^{[2c]m+8}(\bar n\cdot p)}{m_b}\int_0^{p_X^+} dk^+\, \mathcal{J}_m^{(2)}(p_X^- k^+)\, f^{(0)}(k^+ + \bar\Lambda - p_X^+)\\
&+\sum_{m=5}^{10}\int dz_1\, \frac{h_i^{[2c]m+8}(z_1,\bar n\cdot p)}{m_b}\int_0^{p_X^+} dk^+\, \mathcal{J}_m^{(2)}(z_1,p_X^- k^+)\, f^{(0)}(k^+ + \bar\Lambda - p_X^+)\\
&+ W_i^{[2La]f}[g_{11,12}^{(2)}] + W_i^{[2Lb]f}[g_{13,14}^{(2)}] + W_i^{[2LL]f}[g_{15-26}^{(4)}] + W_i^{[2Ga]f}[f_{3,4}^{(4)}]
\end{aligned}
$$

+ phase space & kinematic corrections

$h_i(\bar n\cdot p)$: $\alpha_s(m_b^2)$

$\mathcal{J}(\bar n\cdot p k_j^+)$:

$\alpha_s(m_X^2) \sim \alpha_s(m_b\Lambda)$

- drop $\alpha_s \frac{\Lambda}{m_b}$
- keep $\frac{\Lambda}{m_b}$ and $4\pi\alpha_s \frac{\Lambda}{m_b}$

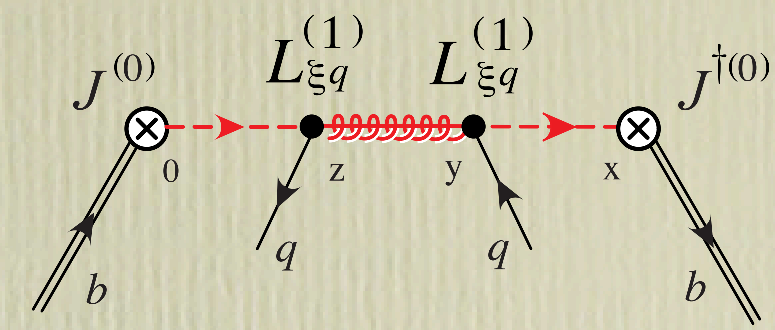
$$\frac{h_i^{0f}(\bar{n}\cdot p)}{2m_b}\int_0^{p_X^+} dk^+\ \mathcal{J}^{(0)}(\bar{n}\cdot p\, k^+,\mu)\ f_0^{(2)}(k^+ + r^+,\mu)$$

$$+\sum_{r=1}^{2}\frac{h_i^{rf}(\bar{n}\cdot p)}{m_b}\int_0^{p_X^+} dk^+\ \mathcal{J}^{(0)}(\bar{n}\cdot p\, k^+,\mu)\ f_r^{(2)}(k^+ + r^+,\mu)$$

$$+\sum_{r=3}^{4}\frac{h_i^{rf}(\bar{n}\cdot p)}{m_b}\int dk_1^+\, dk_2^+\ \mathcal{J}_{1\pm 2}^{(-2)}(\bar{n}\cdot p\, k_j^+,\mu)\ f_r^{(4)}(k_j^+ + r^+,\mu)$$

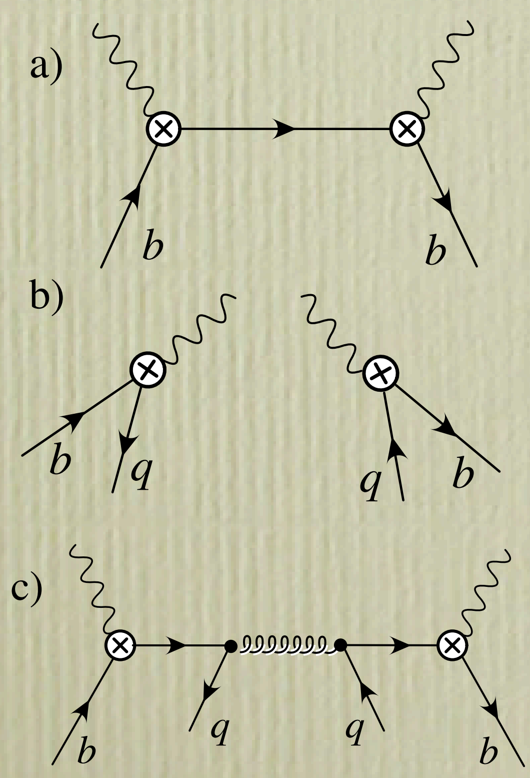
$$+\sum_{r=5}^{6}\frac{h_i^{rf}(\bar{n}\cdot p)}{\bar{n}\cdot p}\int dk_1^+ dk_2^+ dk_3^+\ \mathcal{J}_1^{(-4)}(\bar{n}\cdot p\, k_{j'}^+,\mu)\ f_r^{(6)}(k_{j'}^+ + r^+,\mu)$$

\+ phase space & kinematic corrections



$$\left(\text{dim } 6 = \frac{\Lambda^3}{m_b^3}\right)$$

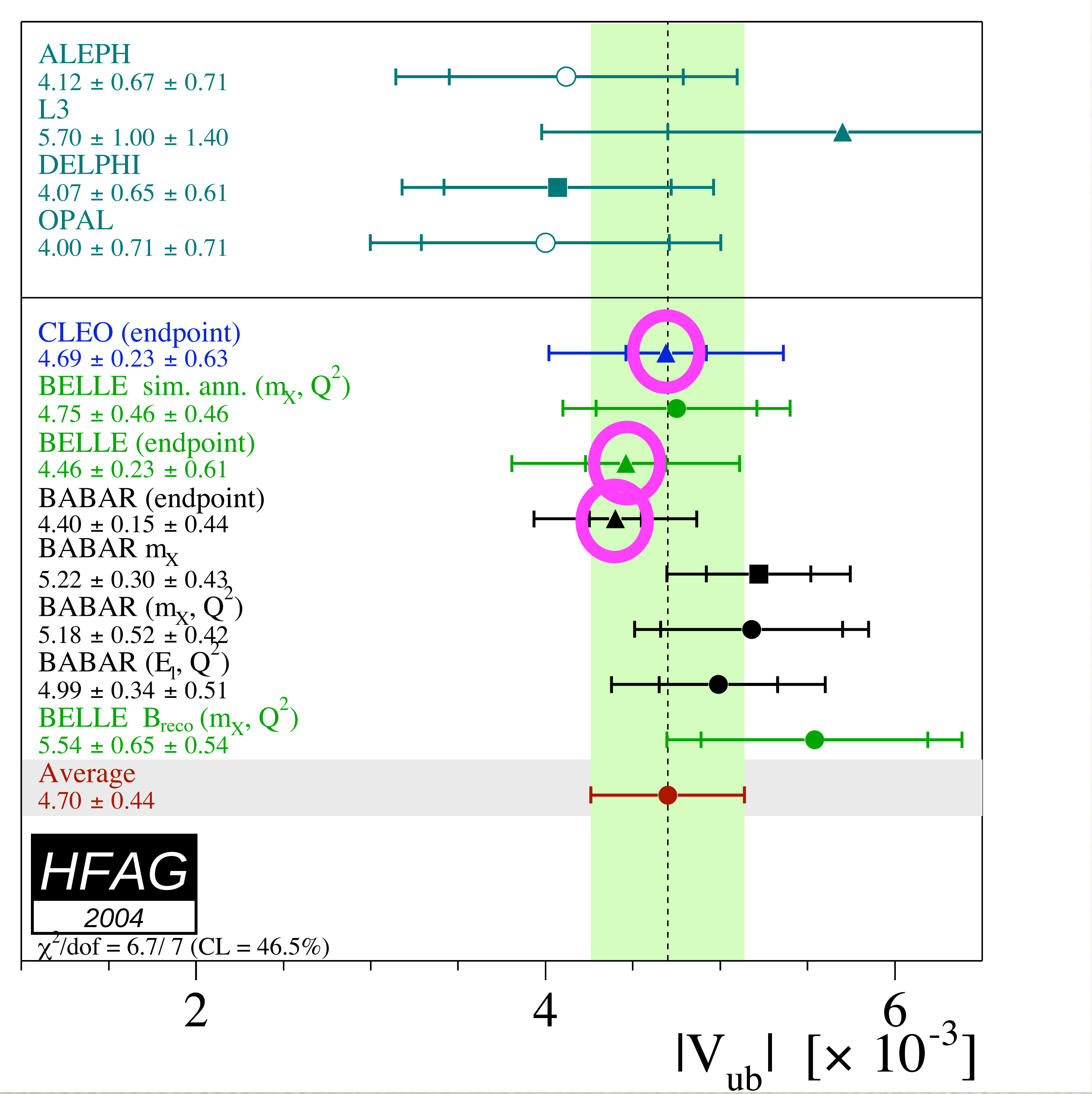
4-quark operators enhanced by $\dfrac{m_b^2}{\Lambda^2}$



	local OPE	endpoint
a)	~ 1	~ 1
b)	$\sim 16\pi^2 \frac{\Lambda^3}{m_b^3}\Delta B \sim 0.02$	$\sim 16\pi^2 \frac{\Lambda^2}{m_b^2}\Delta B \sim 0.2$
c)	$\sim 4\pi\alpha_s(m_b)\frac{\Lambda^3}{m_b^3} \sim 0.003$	$\sim 4\pi\alpha_s(1.4\,\text{GeV})\frac{\Lambda}{m_b} \sim 0.6$

times additional dynamical suppression

Inclusive Vub Results:



○ endpoint analysis

$$|V_{ub}|_{\text{endpoint}} = 4.5 \times 10^{-3}$$

$$|V_{ub}|_{\text{other incl.}} = 5.1 \times 10^{-3}$$

- a detailed study of the subleading shape functions is needed to reduce the theoretical uncertainty in the endpoint region

Outlook

- There is a theory for B-decays with energetic hadrons
  - ➡ predictions for the size of amplitudes
  - ➡ universal hadronic parameters, strong phases
  - ➡ γ (or α) from individual $B \to M_1 M_2$ channels

- We now have the tools to systematically study power corrections
  - ➡ color suppressed decays, inclusive decays

- The SCET can be applied to:
  Nonleptonic decays, Other B decays
  Jet physics, Exclusive form factors
  Charmonium, Upsilon physics
  ... others ?

- A <u>lot</u> of theory and phenomenology left to study ...