QCD Effects in
B & Λ_b Decays

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Outline

- Expansions and the Soft-Collinear EFT
- 1) Lessons from $\bar{B}^0 \to D^0\pi^0$ $(1/N_c$ & $\Lambda/E_\pi)$
    - $\Lambda_b \to \Lambda_c\pi \quad \Lambda_b \to \Sigma_c^{(*)}\rho$
    - $\bar{B} \to D^{**}\pi$
- 2) Results for $B \to P$ and $B \to V$ form factors
- 3) Factorization for $B \to M_1 M_2$ in SCET

  ie. $B \to PP, PV, VV$

    - $B \to M_1 M_2$ Factorization Theorem
-
    - $B \to \pi\pi$ Phenomenology at LO in $1/m_b$
- Outlook and Open Issues

$B \to M_1 M_2$

$B \to \pi\pi$ $B \to \pi K$ $B \to \rho K^*$

$B \to \rho\rho$ $B \to KK$

$B \to \pi K^*$ $B \to \pi\rho$

$B_s \to \pi^0\eta$ $B_s \to K^+ K^{*-}$

PP = 21 + 13 decays
PV = 40 + 23 decays
VV = 21 + 13 decays

$E_M \sim 2.3\,\mathrm{GeV}$ energetic

$B \to \rho^+\rho^-$ Babar $\alpha_{\mathrm{eff}} = 102^\circ\ {}^{+16}_{-12}(stat)\ {}^{+5}_{-4}(syst)$

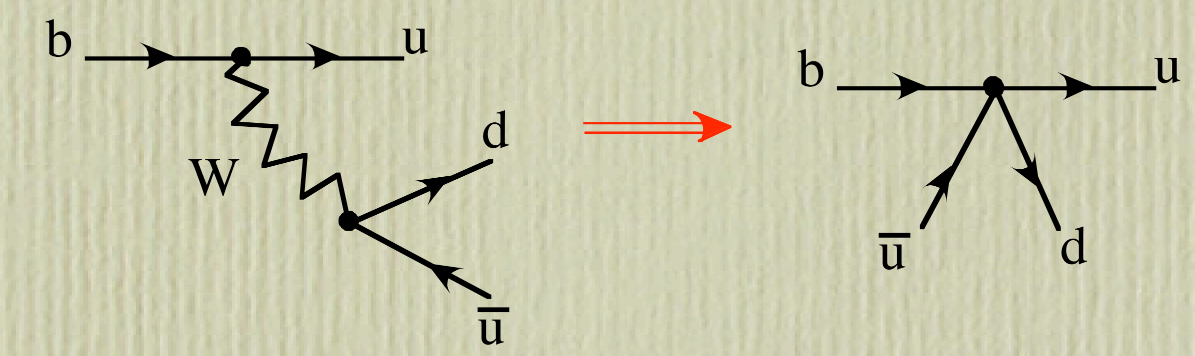
QCD contamination $|\alpha_{\mathrm{eff}} - \alpha| < 17^\circ$

Of course we want as many α's & γ's as we can get

Electroweak Hamiltonian

$$m_W, m_t \gg m_b$$

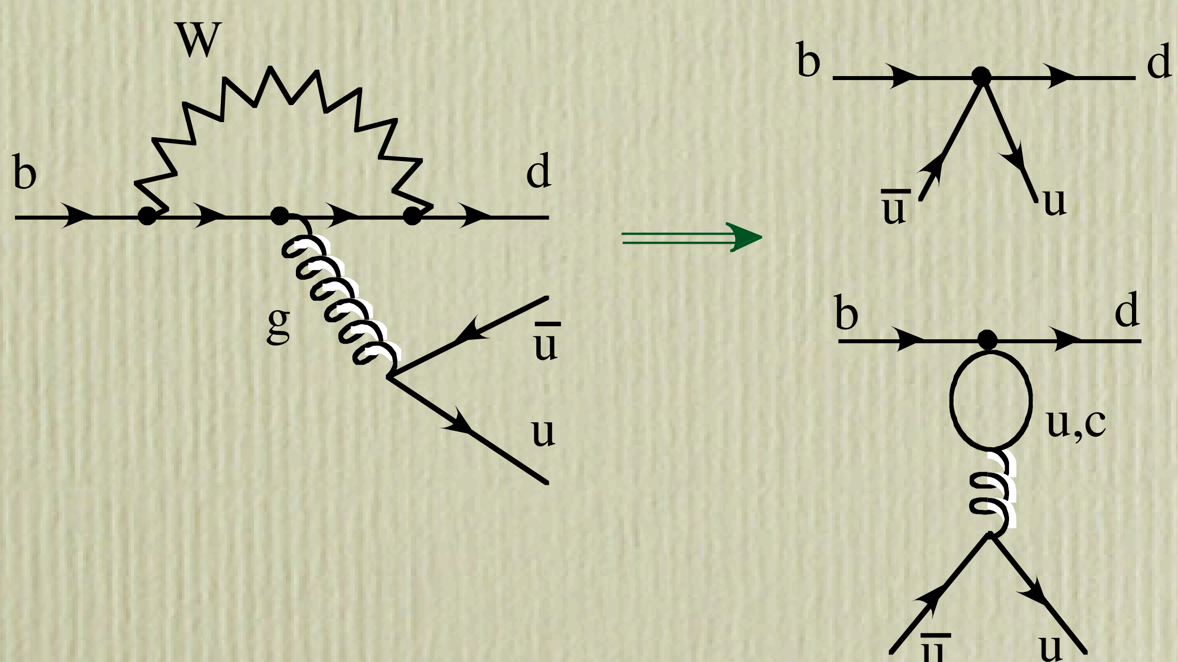
$$H_{\text{weak}} = \frac{G_F}{\sqrt{2}} \sum_i \lambda^i C_i(\mu) O_i(\mu)$$



trees

$$O_1 = (\bar{u}b)_{V-A}(\bar{d}u)_{V-A}$$

$$O_2 = (\bar{u}_i b_j)_{V-A}(\bar{d}_j u_i)_{V-A}$$



penguins

$$O_3 = (\bar{d}b)_{V-A} \sum_q (\bar{q}q)_{V-A}$$

$$O_{4,5,6} = \ldots$$

$$O_{7\gamma,8G} = \ldots$$

$$O^{ew}_{7,\ldots,10} = \ldots$$

λ^i = CKM factors

$\lambda^1 = V_{ub}V^*_{ud}$ $\quad$ $\lambda^3 = V_{tb}V^*_{td}$

Need expansion parameters to make model independent predictions

$$\alpha_s(m_b) \simeq 0.2 \qquad \frac{\Lambda}{m_b} \simeq 0.1 \qquad \frac{\Lambda}{E_M} \simeq 0.2$$

$$\frac{m_s}{\Lambda} \simeq 0.3$$

Effective Field Theory

- Separate physics at different momentum scales
- Model independent, systematically improvable
- Power expansion, can estimate uncertainty
- Exploit symmetries

Measuring CP violation in "unclean" decays:

1. Use SU(2) or SU(3) to relate amplitudes

   - Flavor symmetries of QCD, $m_u, m_d, m_s \ll \Lambda_{\mathrm{QCD}}$

2. Factorization from QCD to reduce the amplitudes to simple universal nonperturbative parameters.

   - Expand in $m_b, E_\pi \gg \Lambda_{\mathrm{QCD}}$

   Proof of Factorization means Known to be Model Independent once hadronic parameters are determined

These two possibilities are not exclusive.

Ask "What are the uncertainties?" and
"Is the expansion converging as expected?"

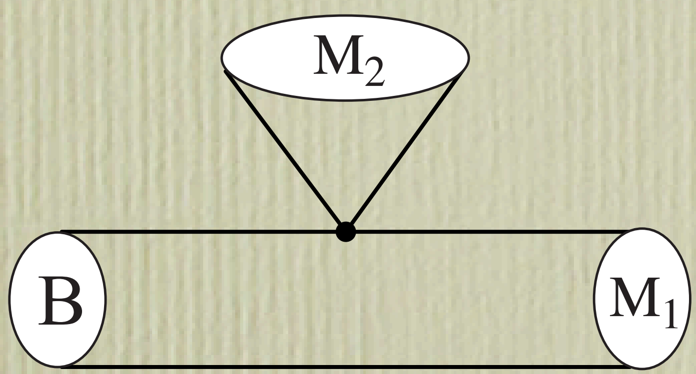
Factorization in QCD

- Beneke, Buchalla, Neubert, Sachrajda proposed a QCD factorization theorem for $B \to \pi\pi$, QCDF .
- Amplitude is reduced to simpler matrix elements

$$\langle \pi\pi | \cdots | B \rangle \longrightarrow \langle \pi | \cdots | B \rangle \ , \ \langle \pi | \cdots | 0 \rangle \ , \ \langle 0 | \cdots | B \rangle$$

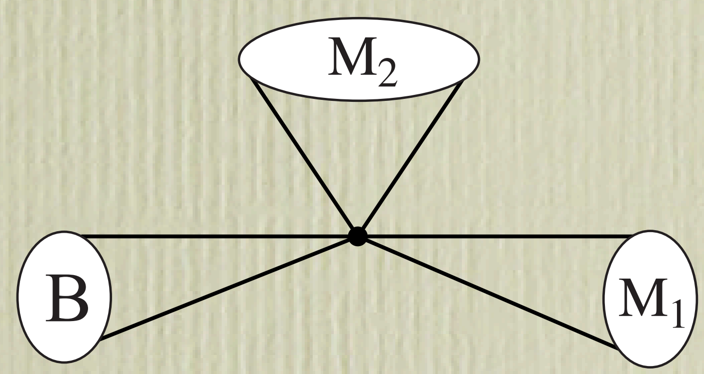
- At LO in $\frac{\Lambda_{\rm QCD}}{E_\pi}$ strong phases are perturbative, $i\alpha_s(m_b)$, and therefore small.

Keum, Li, Sanda: pQCD Factorization



$F^{B\to M_1}$, $\phi_{M_2}(x)$

form factor



$\phi_B(r^+), \phi_{M_1}(x), \phi_{M_2}(y)$

hard spectator

Ciuchini et al, Colangelo et al: charming penguins

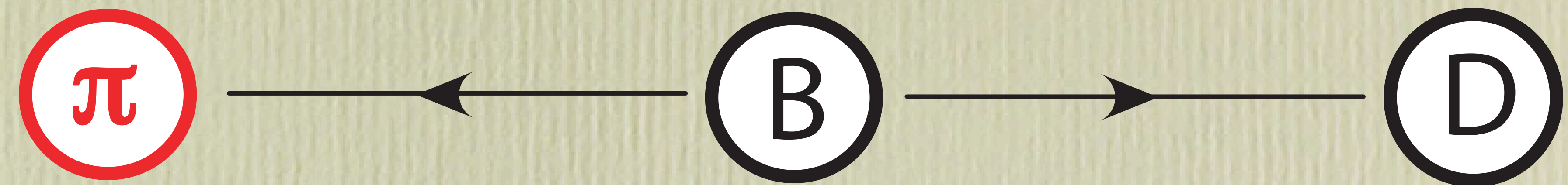
Soft - Collinear Effective Theory

Bauer, Pirjol, Stewart
Fleming, Luke

- An effective field theory for energetic hadrons, $E \gg \Lambda_{\rm QCD}$

Soft Collinear Effective Theory

eg.



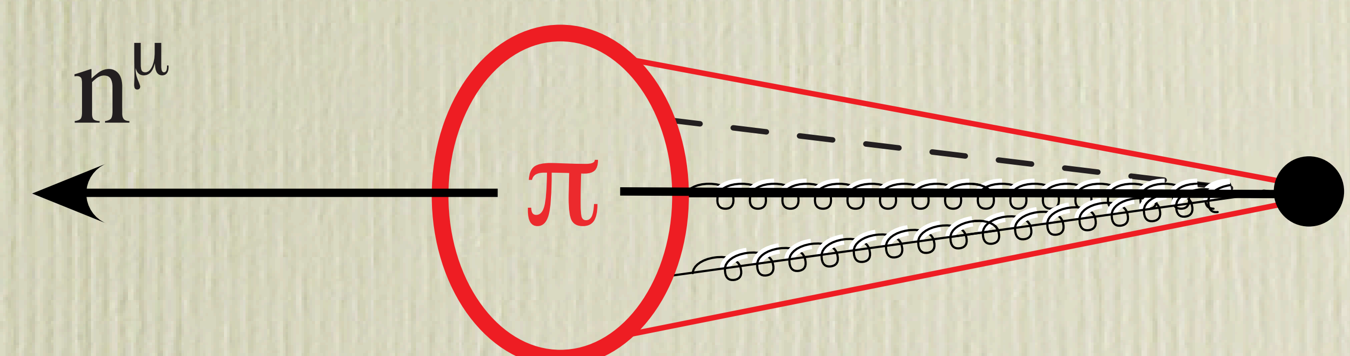
Pion has: $p_\pi^\mu \quad = \quad (2.3\,\mathrm{GeV})n^\mu = Q\,n^\mu \qquad n^2 = \bar{n}^2 = 0, \; (\bar{n}\cdot p = p^-)$

Soft brown muck:

$$p_s^\mu = (p^+, p^-, p^\perp) \sim (\Lambda, \Lambda, \Lambda)$$

Collinear constituents:

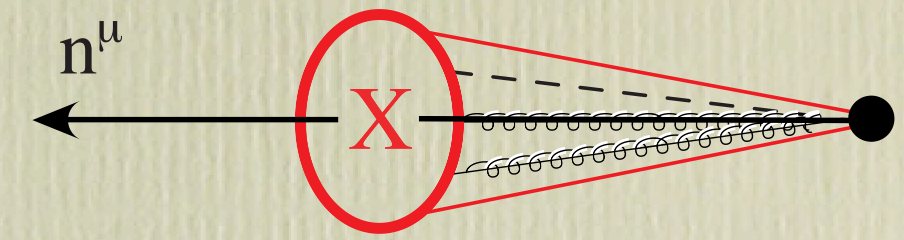
$$p_c^\mu = (p^+, p^-, p^\perp) \sim \left(\frac{\Lambda^2}{Q}, Q, \Lambda\right) \sim Q(\lambda^2, 1, \lambda) \qquad \lambda = \frac{\Lambda}{Q}$$



SCET$_{\mathrm{I}}$ → Energetic jets $\Lambda^2 \ll Q\Lambda \ll Q^2$

usoft $p^\mu \sim \Lambda$

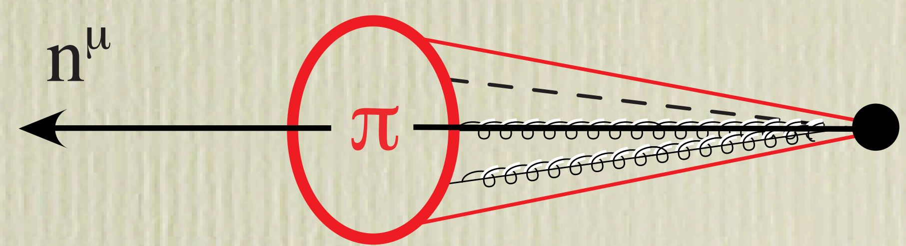
collinear $p_c^2 \sim Q\Lambda, \quad \lambda = \sqrt{\Lambda/Q}$



SCET$_{\mathrm{II}}$ → Energetic hadrons

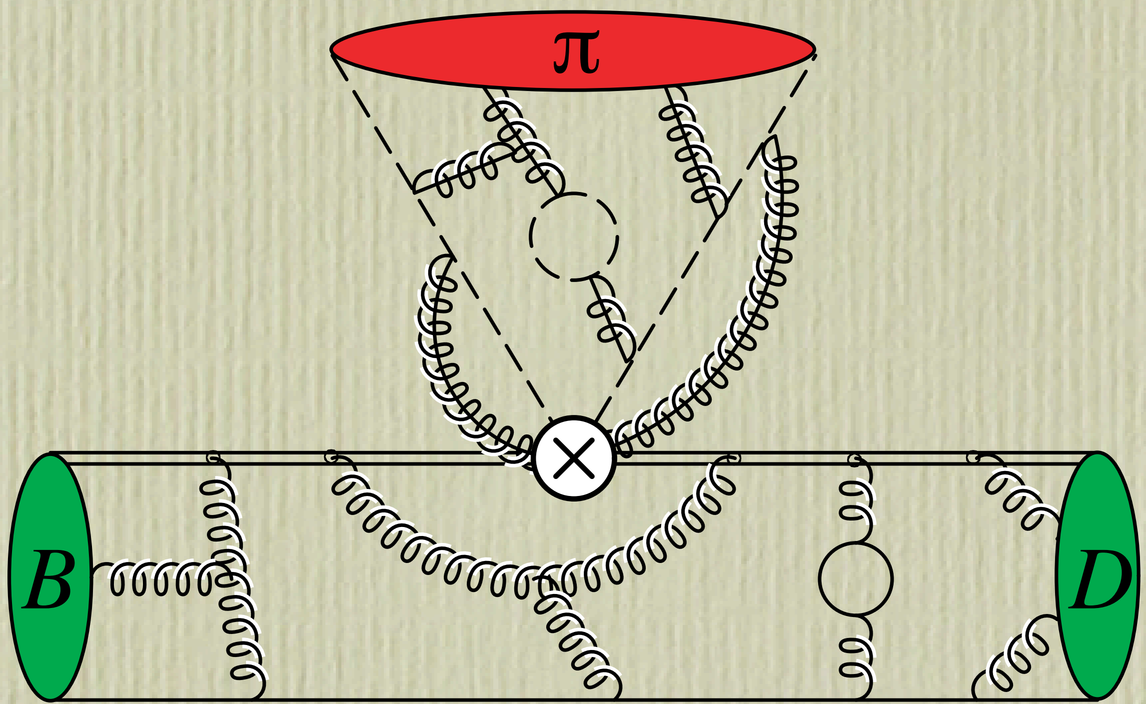
soft $p^\mu \sim \Lambda$

collinear $p_c^2 \sim \Lambda^2, \quad \lambda = \Lambda/Q$



Factorization

LO = λ^5 graphs



$\bar{B}^0 \to D^+\pi^-$, $B^- \to D^0\pi^-$

B, D are soft, π collinear

$$\mathcal{L}_{\rm SCET} = \mathcal{L}_s^{(0)} + \mathcal{L}_c^{(0)}$$

Factorization if $\mathcal{O} = O_c \times O_s$

Bauer, Pirjol, I.S.

$$\langle D\pi|(\bar{c}b)(\bar{u}d)|B\rangle = N\,\xi(v\cdot v')\int_0^1 dx\, T(x,\mu)\,\phi_\pi(x,\mu)$$

Universal functions:

$$\langle D^{(*)}|O_s|B\rangle = \xi(v\cdot v')$$

$$\langle \pi|O_c(x)|0\rangle = f_\pi \phi_\pi(x)$$

Calculate T, $\alpha_s(Q)$

$$Q = E_\pi, m_b, m_c$$

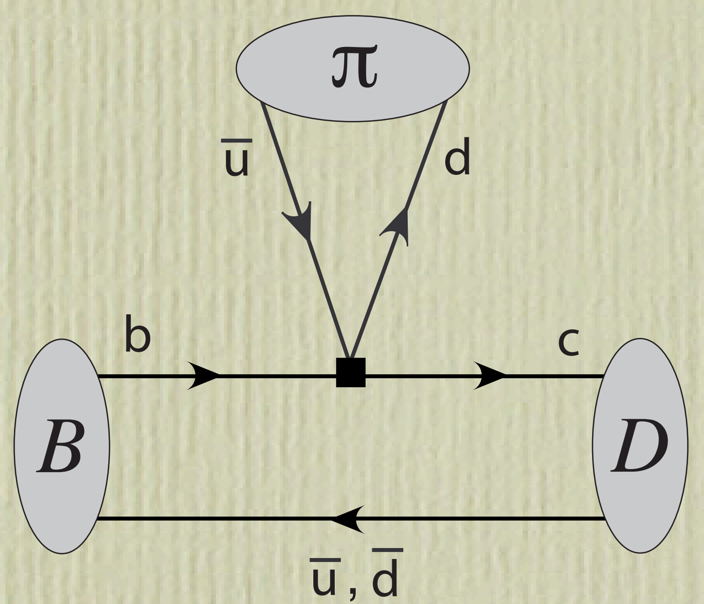
corrections will be $\Lambda/m_c \sim 30\%$

Universal hadronic parameters

Process	Degrees of Freedom (p^2)	Non-Pert. functions
$\bar{B}^0 \to D^+\pi^-, \ldots$	c (Λ^2), s (Λ^2)	$\xi(w)$, ϕ_π
$\bar{B}^0 \to D^0\pi^0, \ldots$	c (Λ^2), s (Λ^2), c $(Q\Lambda)$	$S(k_j^+)$, ϕ_π
$B \to X_s^{endpt}\gamma$, $B \to X_u^{endpt}\ell\nu$	c $(Q\Lambda)$, us (Λ^2)	$f(k^+)$
$B \to \pi\ell\nu, \ldots$	c $(Q\Lambda)$, s (Λ^2), c (Λ^2)	$\phi_B(k^+)$, $\phi_\pi(x)$, $\zeta_\pi(E)$
$B \to \gamma\ell\nu$, $\gamma\gamma$	c $(Q\Lambda)$, us (Λ^2)	ϕ_B
$B \to \pi\pi$	c (Λ^2), s (Λ^2), c $(Q\Lambda)$	ϕ_B, ϕ_π, $\zeta_\pi(E)$
$B \to K^*\gamma$	c $(Q\Lambda)$, s (Λ^2), c (Λ^2)	ϕ_B, ϕ_K, $\zeta_{K^*}^\perp(E)$
$e^-p \to e^-X$	c (Λ^2)	$f_{i/p}(\xi)$, $f_{g/p}(\xi)$
$e^-\gamma \to e^-\pi^0$	c (Λ^2), s (Λ^2)	ϕ_π
$\gamma^* M \to M'$	c (Λ^2), s (Λ^2)	ϕ_M, $\phi_{M'}$

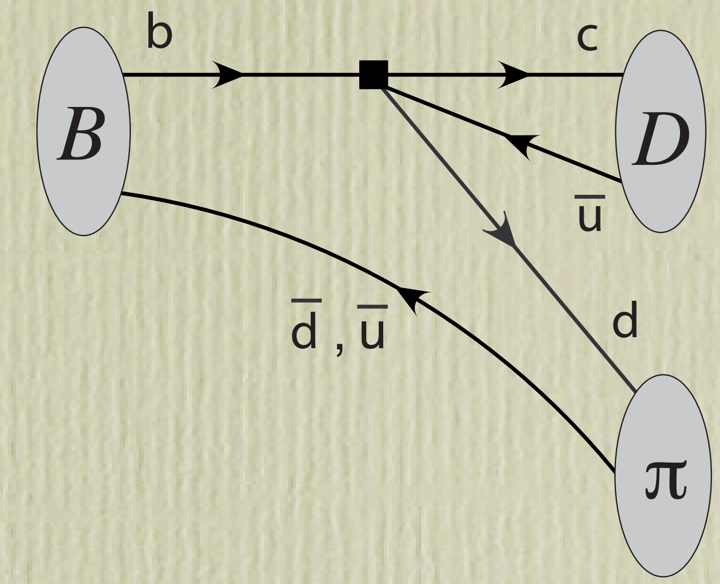
$B \to D\pi$

"Tree"



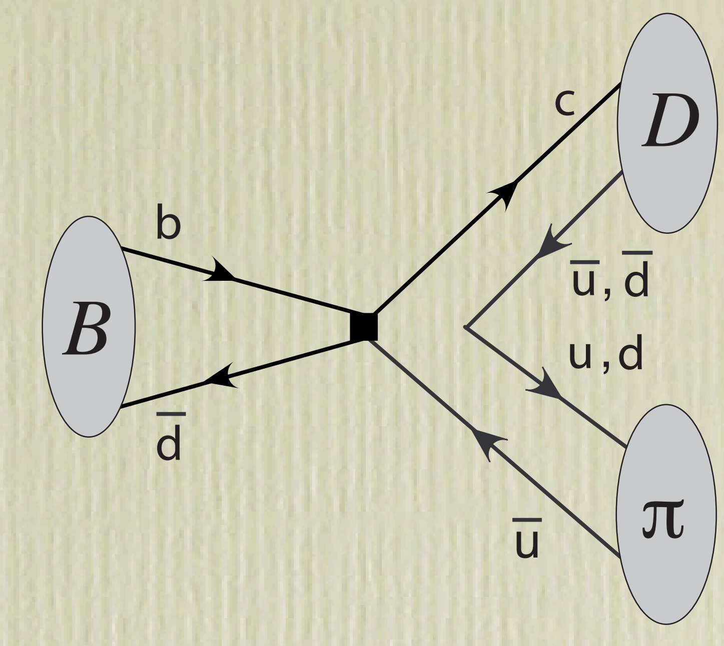
$$\bar{B}^0 \to D^+\pi^-$$
$$B^- \to D^0\pi^-$$

"Color suppressed"



$$B^- \to D^0\pi^-$$
$$\bar{B}^0 \to D^0\pi^0$$

"Exchange"



$$\bar{B}^0 \to D^+\pi^-$$
$$\bar{B}^0 \to D^0\pi^0$$

Large N_c - not very predictive

$(N_c)^0$ $\qquad$ $1/N_c$ $\qquad$ $1/N_c$

$$O^0 = (\bar{c}b)_{V-A}(\bar{d}u)_{V-A}$$
$$O^8 = (\bar{c}T^A b)_{V-A}(\bar{d}T^A u)_{V-A}$$

Data

(Cleo, Belle, Babar)

Type	Decay	Br(10^{-3})	Decay	Br(10^{-3})
I	$\bar{B}^0 \to D^+\pi^-$	2.68 ± 0.29	$\bar{B}^0 \to D^{*+}\pi^-$	2.76 ± 0.21
III	$B^- \to D^0\pi^-$	4.97 ± 0.38	$B^- \to D^{*0}\pi^-$	4.6 ± 0.4
II	$\bar{B}^0 \to D^0\pi^0$	0.29 ± 0.03	$\bar{B}^0 \to D^{*0}\pi^0$	0.26 ± 0.05
I	$\bar{B}^0 \to D^+\rho^-$	7.8 ± 1.4	$\bar{B}^0 \to D^{*+}\rho^-$	6.8 ± 1.0
III	$B^- \to D^0\rho^-$	13.4 ± 1.8	$B^- \to D^{*0}\rho^-$	9.8 ± 1.8
II	$\bar{B}^0 \to D^0\rho^0$	0.29 ± 0.11	$\bar{B}^0 \to D^{*0}\rho^0$	< 0.56

- size of $\mathrm{Br}(D^+M^-)$ agrees with factorization
- $\mathrm{Br}(D^0M^0)$ small as expected (power suppressed)
- color allowed Br are same for D and D^*
- significant strong phases $\delta \sim 30^\circ$

- but significant power corrections: $\mathrm{Br}(D^0M^-)/\mathrm{Br}(D^+M^-)$

$$\frac{|A_{0-}|}{|A_{+-}|} = \begin{cases} 0.77 \pm 0.05 & \text{for } D\pi \\ 0.81 \pm 0.05 & \text{for } D^*\pi \end{cases}$$

20-30% level

$$\frac{\mathrm{Br}(\bar{B}^0 \to D^{*+}\pi^-)}{\mathrm{Br}(\bar{B}^0 \to D^+\pi^-)} = 1.03 \pm 0.14 \qquad \frac{\mathrm{Br}(B^- \to D^{*0}\pi^-)}{\mathrm{Br}(B^- \to D^0\pi^-)} = 0.93 \pm 0.11$$

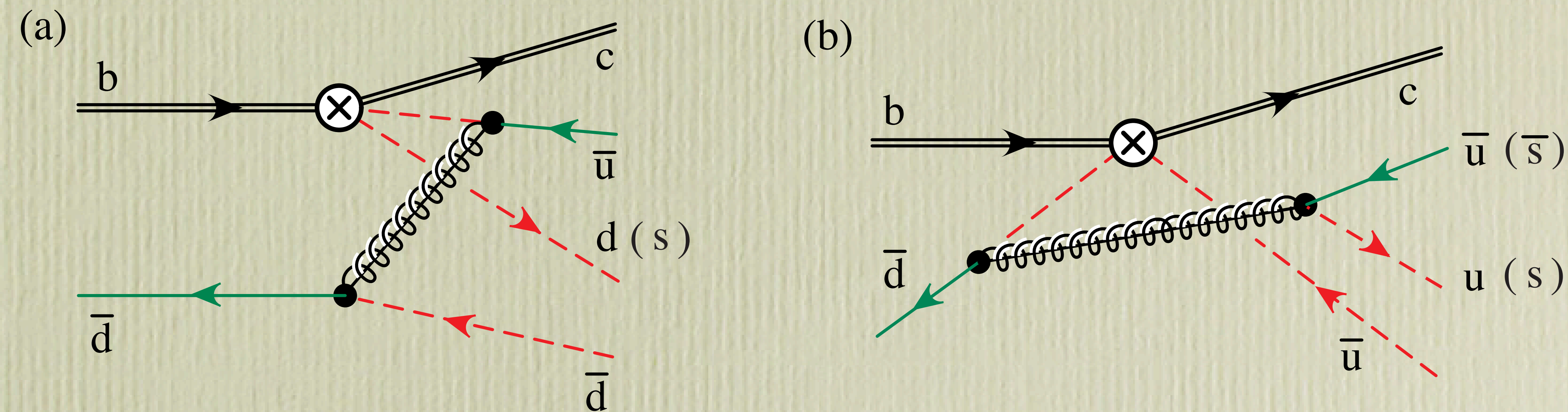
Color Suppressed Decays

$$\bar{B}^0 \to D^{(*)0}\pi^0 \ , D^{(*)0}\rho^0, D^{(*)0}K^0, D^{(*)0}K^{*0}, D_s^{(*)}K^-, D_s^{(*)}K^{*-}$$

Mantry, Pirjol, I.S.

Factorization with SCET

Single class of power suppressed $\mathrm{SCET_I}$ operators $T\{\mathcal{O}^{(0)}, \mathcal{L}_{\xi q}^{(1)}, \mathcal{L}_{\xi q}^{(1)}\}$



$$A_{00}^{D^{(*)}} = N_0^{(*)} \int dx\, dz\, dk_1^+ dk_2^+ \underbrace{T^{(i)}(z)}_{Q^2} \gg \underbrace{J^{(i)}(z,x,k_1^+,k_2^+)}_{Q\Lambda} \gg \underbrace{S^{(i)}(k_1^+,k_2^+)\, \phi_M(x)}_{\Lambda^2}$$

$$+A_{\rm long}^{D^{(*)}M}$$

new soft function $S^{(i)}(k_1^+, k_2^+)$ - like generalized parton distributions

Theory tidbits:

Implications

1) Long Distance Amplitude — polarization in D* V

2) Symmetry structure of $S^{(i)}$ — D versus D*

3) Complex nature of $S^{(i)}$ — universal strong phases

Phenomenology:

1) Predictions that are independent of form of $J^{(i)}$

2) Predictions with $J^{(i)}$ expanded in $\alpha_s(\mu^2 \sim E\Lambda)$

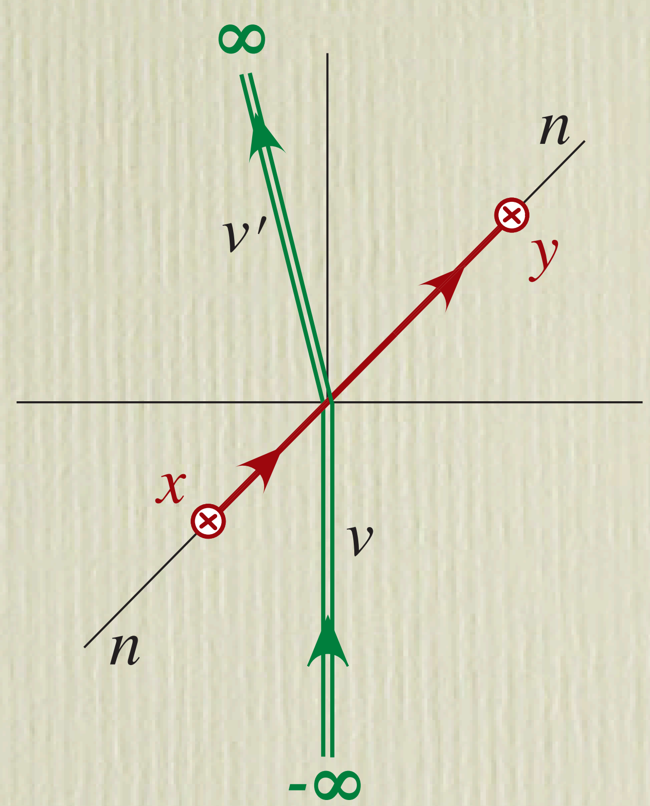
$\langle D^{(*)0}|O_s^{(0,8)}|\bar{B}^0\rangle \to S^{(0,8)}(k_1^+, k_2^+)$ same for D and D^*

with HQET for $\langle D^{(*)0}\pi|(\bar{c}\, b)(\bar{d}\, u)|\bar{B}^0\rangle$ get $\frac{p_\pi^\mu}{m_c} \longrightarrow \frac{E_\pi}{m_c} = 1.5$

not a convergent expansion

$S^{(i)}(k_1^+, k_2^+)$ is complex, new mechanism for rescattering

$$O^{(0,8)} = O^{(0,8)}[v, v', n]$$



Predict

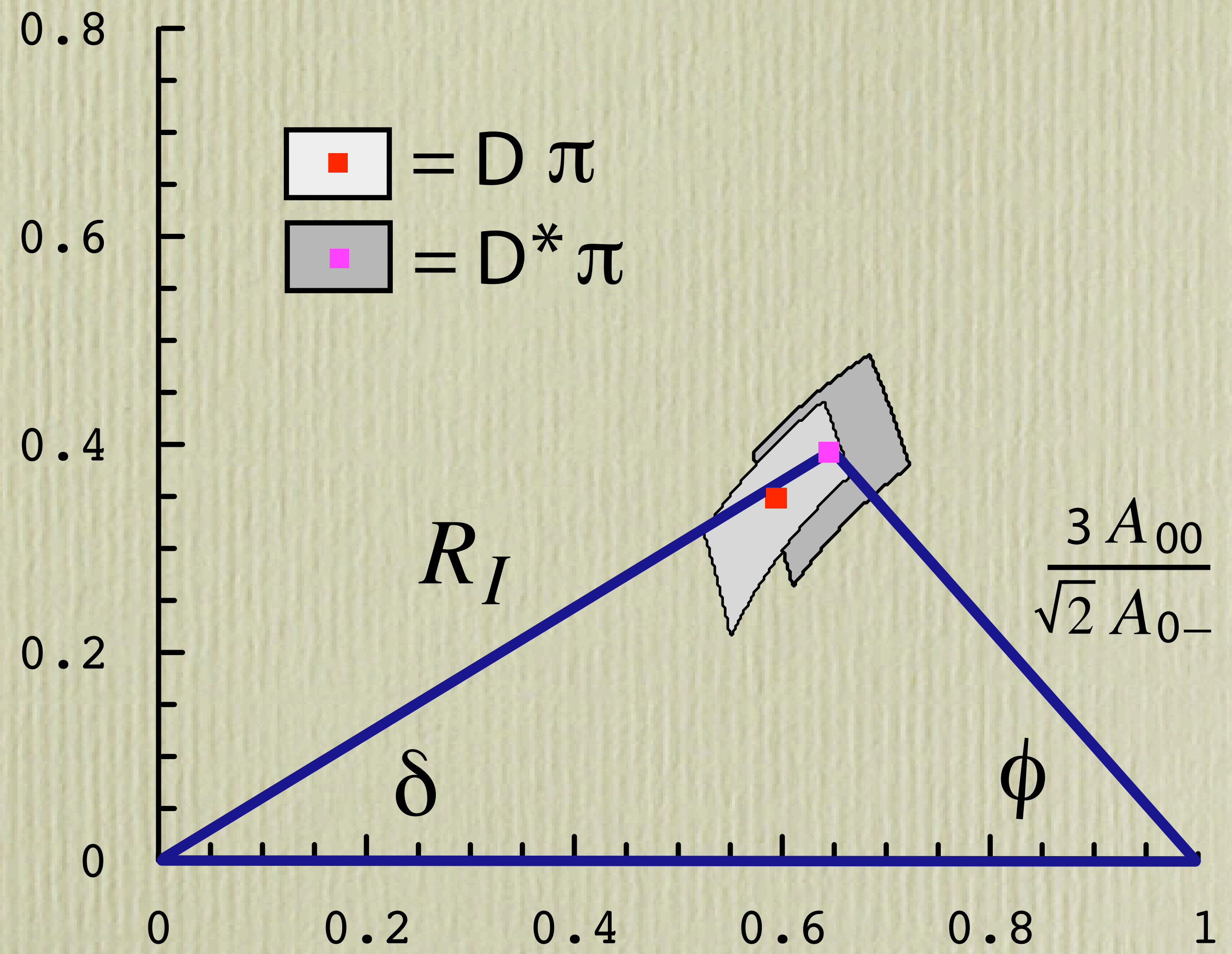
equal strong phases $\delta^D = \delta^{D^*}$

equal amplitudes $A_{00}^D = A_{00}^{D^*}$

corrections to this are $\alpha_s(m_b)$, Λ/Q

Tests and Predictions

Expt Average (Cleo, Belle, Babar):



isospin gives triangle:

$$A_{0-} = \sqrt{2} A_{00} + A_{+-}$$

rearrange:

$$\begin{aligned} 1 &= R_I + \frac{3A_{00}}{\sqrt{2}A_{0-}} \\ R_I &= \frac{A_{1/2}}{\sqrt{2}A_{3/2}} \\ \delta &= \arg(A_{1/2}A^*_{3/2}) \end{aligned}$$

$$Br(D^0\pi^0) = (0.29 \pm 0.03) \times 10^{-3}\,, \quad \delta(D\pi) = 30.4 \pm 4.8^\circ$$

$$Br(D^{*0}\pi^0) = (0.26 \pm 0.05) \times 10^{-3}\,, \; \delta(D^*\pi) = 31.0 \pm 5.0^\circ$$

Tests and Predictions

Also predict (not post-dict):

$$r_{00}^{\rho} = \frac{A(\bar{B}^0 \to D^{*0}\rho^0)}{A(\bar{B}^0 \to D^0\rho^0)} = 1\,,$$

$$r_{00}^{K^-} = \frac{A(\bar{B}^0 \to D_s^* K^-)}{A(\bar{B}^0 \to D_s K^-)} = 1\,, \qquad r_{00}^{K_\parallel^{*-}} = \frac{A(\bar{B}^0 \to D_s^* K_\parallel^{*-})}{A(\bar{B}^0 \to D_s K_\parallel^{*-})} = 1\,,$$

$$r_{00}^{K^0} = \frac{A(\bar{B}^0 \to D^{0*}\bar{K}^0)}{A(\bar{B}^0 \to D^0\bar{K}^0)} = 1\,, \qquad r_{00}^{K_\parallel^{*0}} = \frac{A(\bar{B}^0 \to D^{*0}\bar{K}_\parallel^{*0})}{A(\bar{B}^0 \to D^0\bar{K}_\parallel^{*0})} = 1$$

ie. same Br and same strong phases

All predictions so far are independent of the form of $J^{(i)}(z, x, k_1^+, k_2^+)$

and $S^{(i)}(k_1^+, k_2^+)\,,\quad \phi_M(x)$

More Predictions

If we expand $J(z, x, k_1^+, k_2^+)$ in $\alpha_s(E\Lambda)$, we can make more predictions

Relate π and ρ

- Recall data gives

predict that $\phi^{D\rho} = \phi^{D\pi}$, not yet tested

$$\frac{|A(\bar B^0 \to D^+\pi^-)|}{|A(B^- \to D^0\pi^-)|} = 0.77 \pm 0.05$$

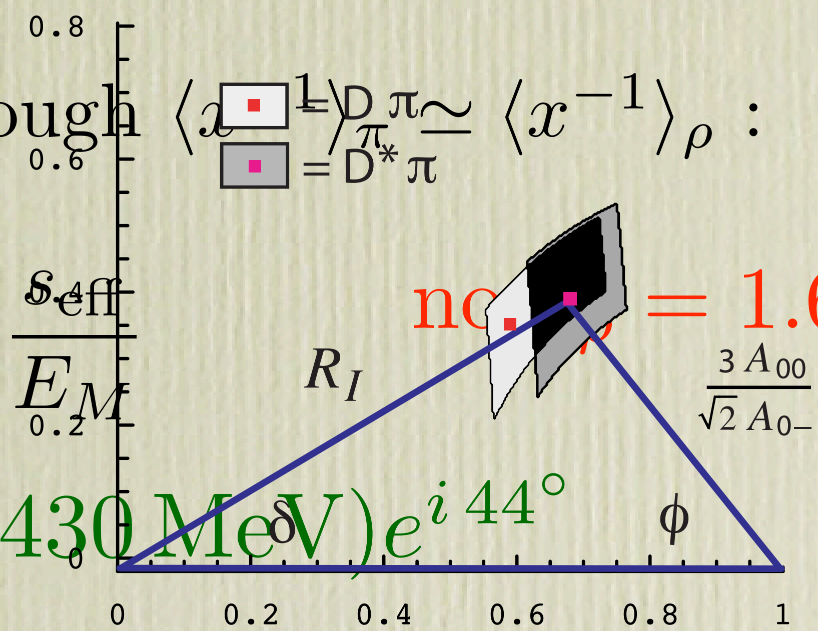
if $\langle x^{-1}\rangle_\pi \simeq \langle x^{-1}\rangle_\rho$ then this implies $\delta^{D\pi} \simeq \delta^{D\rho}$, $|r^{D\rho}| = 0.80 \pm 0.09$

SCET predicts weak dependence on M through $\langle x^{-1}\rangle_\pi \simeq \langle x^{-1}\rangle_\rho$:

$$r^{DM} = 1 - \frac{16\pi\alpha_s m_D}{9(m_B+m_D)} \frac{\langle x^{-1}\rangle_M}{\xi(w_{max})} \frac{s_{\rm eff}}{E_M}$$

no ... $= 1.6\, f_\pi$

natural parameters fit data, $s_{\rm eff} \simeq (430\text{ MeV})e^{i\,44^\circ}$

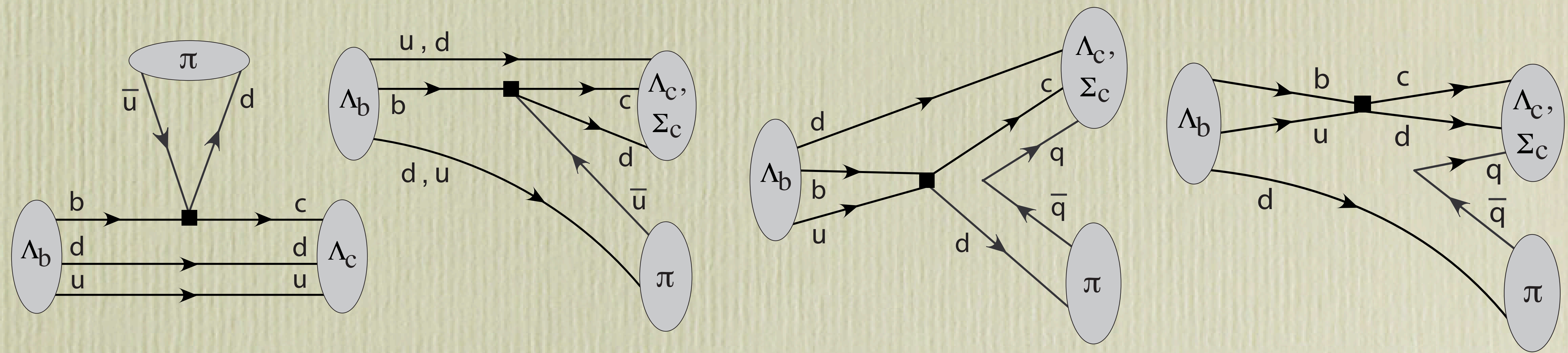


Baryon decays

Add a soft quark

Leibovich, Ligeti, I.S., Wise

$$\Lambda_b \to \Lambda_c \pi,\ \Lambda_c \rho,\ \Sigma_c^{(*)} \pi,\ \Sigma_c^{(*)} \rho$$



T=tree

C= color commensurate

E= exchange

B = bow-tie

Naive factorization, only makes sense for T

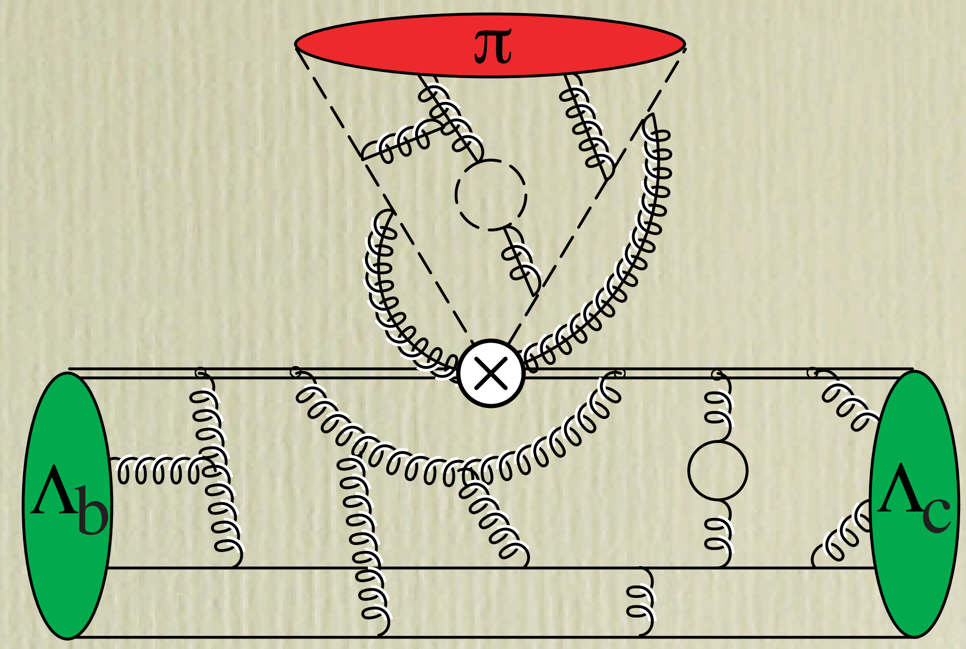
$\Lambda_b \to \Sigma_c^{(*)} \ell \bar{\nu}$ violates isospin and is $1/m_b$ suppressed

In SCET: $T \gg C \sim E \gg B$ similar factorization theorems

$\Lambda_b \to \Lambda_c \pi$

1.6

need semileptonic



$$\frac{\Gamma(\Lambda_b \to \Lambda_c \pi^-)}{\Gamma(\bar{B}^0 \to D^+ \pi^-)} = \frac{8 m_{\Lambda_b}^3 (1-r_\Lambda^2)^3\, r_D}{m_B^3 (1-r_D^2)^3 (1+r_D)^2} \left(\frac{\zeta(w_{\max}^\Lambda)}{\xi(w_{\max}^D)} \right)^2$$

$= 2$ in small velocity limit

CDF has 2.7 ± 0.8 for this ratio

$\Lambda_b \to \Sigma_c^{(*)} \pi$ similar SCET analysis to $\bar{B}^0 \to D^0 \pi^0$

$$\frac{Br(\Lambda_b \to \Sigma_c^* \pi)}{Br(\Lambda_b \to \Sigma_c \pi)} = 2\,, \qquad \frac{Br(\Lambda_b \to \Sigma_c^* \rho)}{Br(\Lambda_b \to \Sigma_c \rho)} = 2$$

$$\frac{Br(\Lambda_b \to \Xi_c^* K)}{Br(\Lambda_b \to \Xi_c' K)} = 2\,, \qquad \frac{Br(\Lambda_b \to \Xi_c^* K_\parallel^*)}{Br(\Lambda_b \to \Xi_c' K_\parallel^*)} = 2$$

Decays to Excited States

S. Mantry

$D_1: \quad J^\pi = 1^+, \quad m = 2420\,\mathrm{MeV}$

$D_2^*: \quad J^\pi = 2^+, \quad m = 2460\,\mathrm{MeV}$

Semileptonics:

LO and $1/m_{c,b}$ compete $\Rightarrow$ could spoil factorization in $B \to D^{**}\pi$

$$\langle D_{v'}^{**}|J|B_v\rangle \propto (v\cdot v' - 1) = 0.0 \text{ to } 0.3$$

Nonleptonics:

At max recoil find: $(v\cdot v' - 1)(v\cdot v' + 1) = \frac{E_\pi^2}{m_{D^{**}}^2} \sim 1$

can use SCET power counting & factorization

Predict: $$\frac{Br(B \to D_2^*\pi)}{Br(B \to D_1\pi)} = 1$$ color allowed & color suppressed

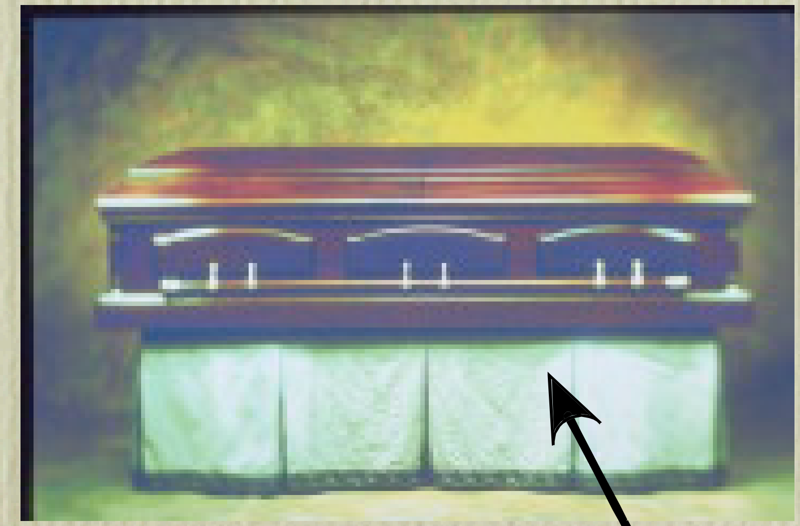
$$\phi_{D_2^*\pi} = \phi_{D_1\pi}$$ equal phases in isospin triangles

Belle: $$\frac{Br(B^- \to D_2^{*0}\pi^-)}{Br(B^- \to D_1^0\pi^-)} = 0.77 \pm 0.15$$ (prev. theory estimates uncertain: $= 0.3$ to 1.4)

Lessons

- nonperturbative strong phases $\delta \sim 30^\circ$ are natural

  from Λ/E !
- Nonperturbative J vs. Perturbative J
- With the entire amplitude power suppressed the polarization issue in B to VV is non-trivial



naive factorization for color suppressed decays

$B \to M$ Form Factors

Bauer, Pirjol, I.S.
Beneke, Feldmann,
Becker, Hill, Lange, Neubert

pseudoscalar: $f_+,\ f_0,\ f_T$

vector: $V,\ A_0,\ A_1,\ A_2,\ T_1,\ T_2,\ T_3$

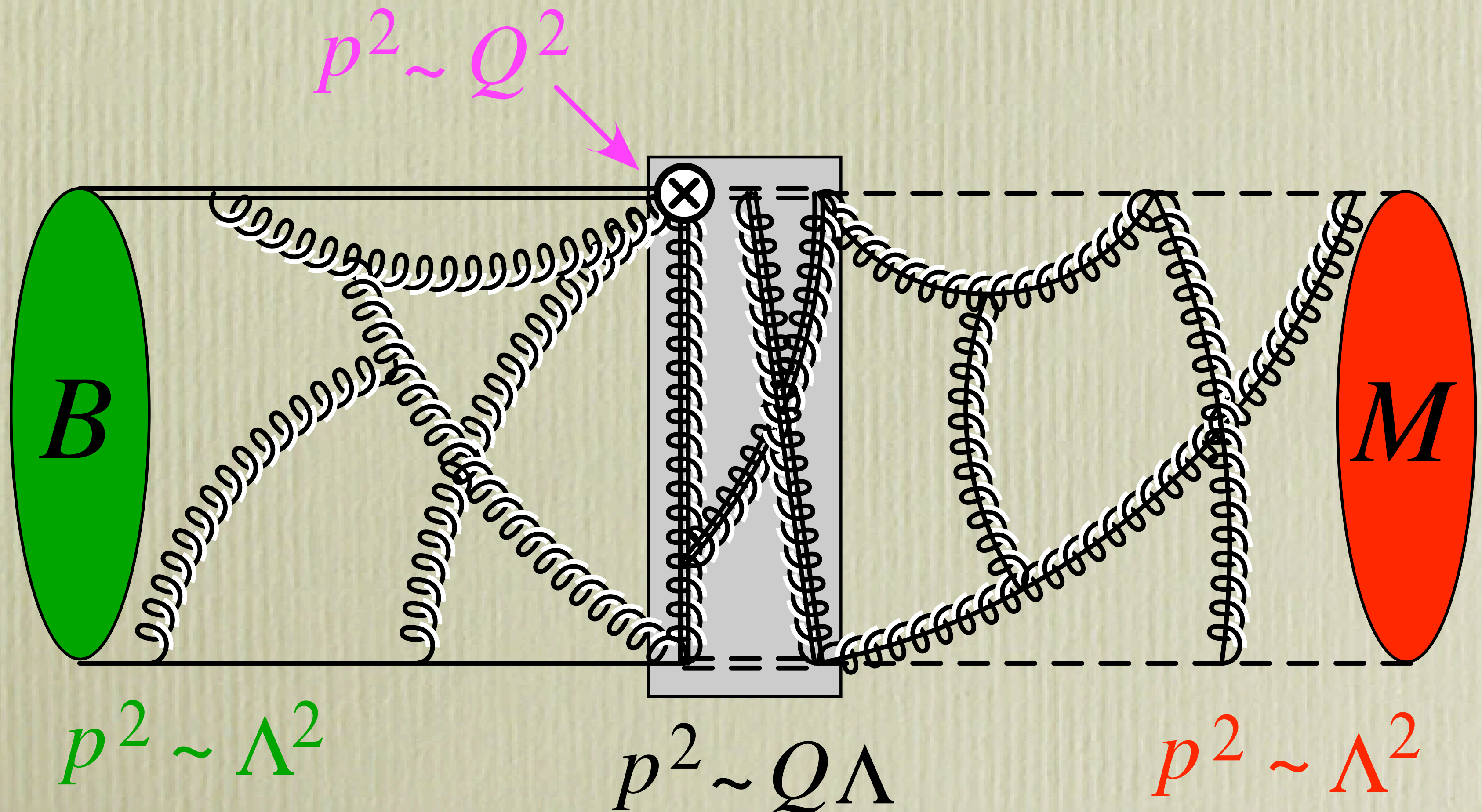
SCET Result

$$f^{F}(E) \equiv \frac{f_B f_M m_B}{4E^2} \int_0^1 dz \int_0^1 dx \int_0^\infty dr_+\, T(z,E,m_b) \times J(z,x,r_+,E)\, \phi_M(x)\, \phi_B^+(r_+)$$

$$f^{NF}(E) \equiv C(E,m_b)\, \zeta^{BM}(Q\Lambda,\Lambda^2)$$

$$f^{F}(E) \equiv \int_0^1 dz\, T(z,E,m_b)\, \zeta_J^{BM}(z,E)$$

$$f^{NF}(E) \equiv C(E,m_b)\, \zeta^{BM}(Q\Lambda,\Lambda^2)$$



$\Lambda/Q \ll 1$

result at LO in λ, all orders in α_s, where $Q = \{m_b, E_M\}$

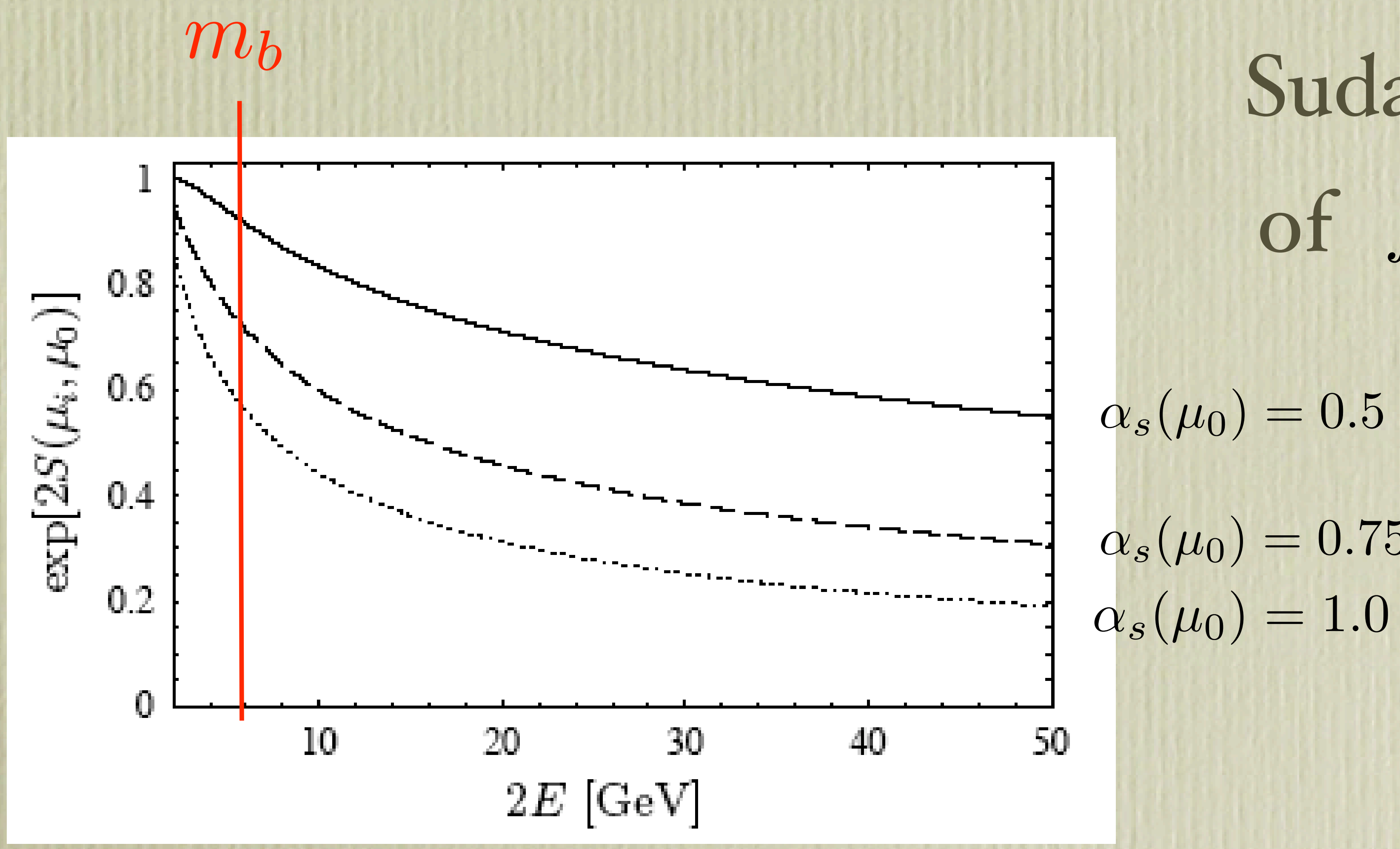
One Loop Matching: $C_k(E, m_b)$ Bauer, Fleming, Pirjol, I.S.

$T_i(z, E, m_b)$ Beneke, Kiyo, Yang

$J(z, x, r_+, E)$ Becher, Hill, Neubert

Log Resummation: Lange, Neubert; Becher, Hill, Neubert

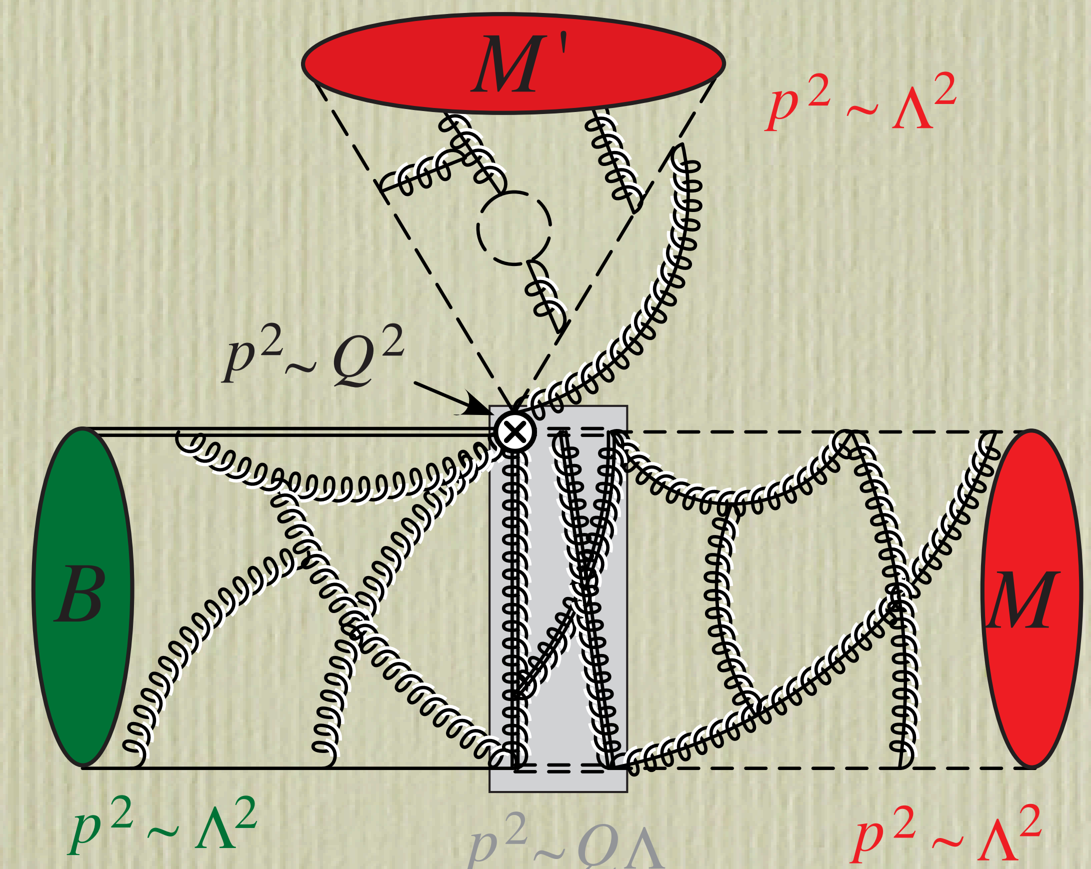
Sudakov suppression of f^{NF} relative to f^F

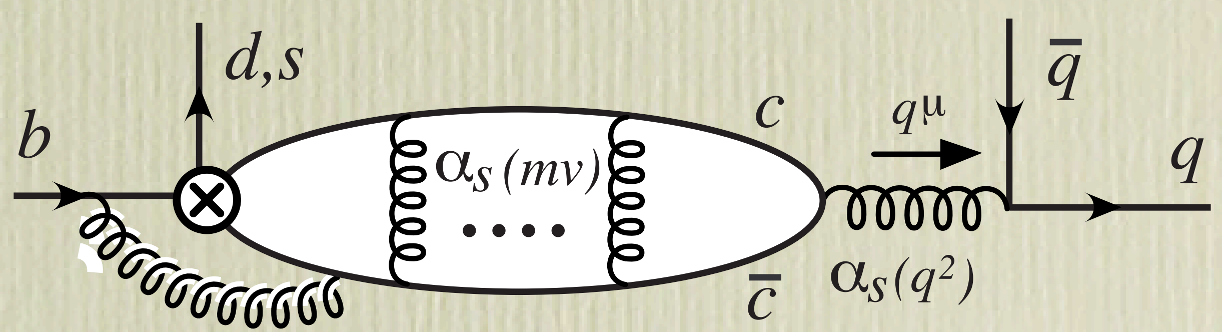


$B \to M_1 M_2$ Factorization in SCET

Chay, Kim

Bauer, Pirjol, Rothstein, I.S.





- hard spectator & form factor terms → same
- long distance charming penguin amplitude

$$\Lambda^2 \ll E\Lambda \ll E^2, m_b^2$$

Operators

QCD

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \Big(C_1 O_1^p + C_2 O_2^p + \sum_{i=3}^{10,8g} C_i O_i \Big)$$

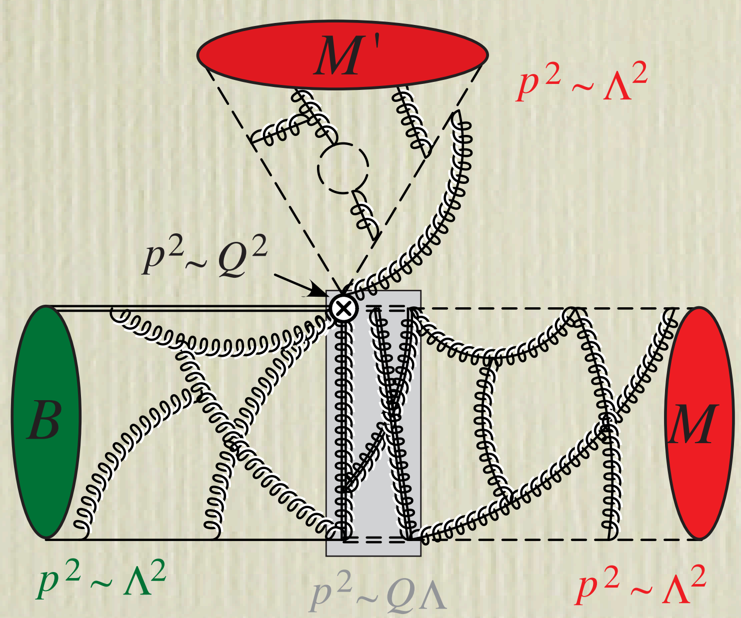
SCET$_\mathrm{I}$ Integrate out $\sim m_b$ fluctuations

$$H_W = \frac{2G_F}{\sqrt{2}} \Big\{ \sum_{i=1}^{6} \int d\omega_j c_i^{(f)}(\omega_j) Q_{if}^{(0)}(\omega_j) + \sum_{i=1}^{8} \int d\omega_j b_i^{(f)}(\omega_j) Q_{if}^{(1)}(\omega_j) + \mathcal{Q}_{c\bar{c}} + \ldots \Big\}$$

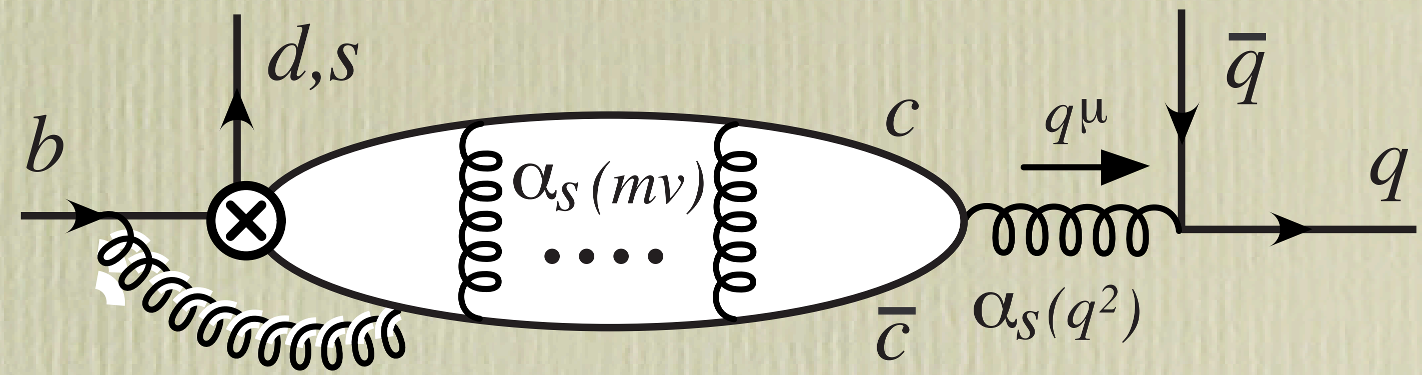
$$Q_{1d}^{(0)} = \big[\bar{u}_{n,\omega_1} \bar{n\!\!\!/} P_L b_v\big]\big[\bar{d}_{\bar{n},\omega_2} n\!\!\!/ P_L u_{\bar{n},\omega_3}\big] , \quad \ldots$$

$$Q_{1d}^{(1)} = \frac{-2}{m_b} \big[\bar{u}_{n,\omega_1}\, ig \mathcal{B\!\!\!/}_{n,\omega_4}^{\perp} P_L b_v\big]\big[\bar{d}_{\bar{n},\omega_2} n\!\!\!/ P_L u_{\bar{n},\omega_3}\big] , \quad \ldots$$

$$Q_{7d}^{(1)} = \frac{-2}{m_b} \big[\bar{u}_{n,\omega_1}\, ig\, \mathcal{B}_{n,\omega_4}^{\perp\,\mu} P_L b_v\big]\big[\bar{d}_{\bar{n},\omega_2} n\!\!\!/ \gamma_\mu^{\perp} P_R u_{\bar{n},\omega_3}\big] ,$$



Long Distance $c\bar{c}$



dangerous region near threshold

- $q^2 \simeq 4m_c^2 \quad , \quad x \simeq 4m_c^2/m_b^2 \sim 0.4$
- NRQCD $c\bar{c}$ couple to b, spectator

  suppression $\sim v = 0.5$ ie. none

These amplitudes appear to be LO ! (disagrees with QCDF)

If so:

- LO large strong phases (mechanism as before)
- LO transverse polarization in VV

→ Need to derive a Fact. Thm. to be sure

Polarization

VV channels

transverse vs. longitudinal

expect longitudinal
to be larger

$$\frac{R_T}{R_0} \sim \frac{1}{m_b^2}$$

Kagan

SCET factorization theorem agrees, except for $A_{c\bar{c}}$

Data:

	R_T/R_0
$\rho^+\rho^0$	0.04 ± 0.08
$\rho^+\rho^-$	0.01 ± 0.05
$K^{*0}\phi$	0.72 ± 0.30

Penguins are small in $B \to \rho\rho$

Penguin dominated

like ϕK_s also $b \to s\bar{s}s$

Charming penguins
might explain
polarization data at LO

Large power corrections
(eg. annihilation) are
another possibility

$\mathrm{SCET}_{\mathrm{II}}$ ➡ Same Jet function as $B \to M$

$$A(B \to M_1 M_2) \;=\; A^{c\bar{c}} + N\Big\{ f_{M_2}\, \zeta^{BM_1} \int_0^1 du\, T_{2\zeta}(u)\, \phi^{M_2}(u) + f_{M_1}\, \zeta^{BM_2} \int_0^1 du\, T_{1\zeta}(u)\, \phi^{M_1}(u)$$
$$+ \frac{f_B f_{M_1} f_{M_2}}{m_B} \int_0^1 du \int_0^1 dx \int_0^1 dz \int_0^\infty dk_+\, J(z,x,k_+) \left[T_{2J}(u,z)\phi^{M_1}(x)\phi^{M_2}(u) + T_{1J}(u,z)\phi^{M_2}(x)\phi^{M_1}(u)\right] \phi_B^+(k_+) \Big\}$$

New Nonperturbative Result in $\alpha_s(\sqrt{E\Lambda})$:

$$A(B \to M_1 M_2) \;=\; A^{c\bar{c}} + N\Big\{ f_{M_2}\, \zeta^{BM_1} \int_0^1 du\, T_{2\zeta}(u)\phi^{M_2}(u) + f_{M_1}\zeta^{BM_2} \int_0^1 du\, T_{1\zeta}(u)\phi^{M_1}(u)$$
$$+ f_{M_2} \int_0^1 du \int_0^1 dz\, T_{2J}(u,z)\zeta_J^{BM_1}(z)\phi^{M_2}(u) + f_{M_1} \int_0^1 du \int_0^1 dz\, T_{1J}(u,z)\zeta_J^{BM_2}(z)\phi^{M_1}(u) \Big\}$$

where $\zeta^{BM} \sim \zeta_J^{BM}(z) \sim (\Lambda/Q)^{3/2}$ and appear in $B \to M$

- fit ζ's, calculate T's

Hard Coefficients

M_1M_2	$T_{1\zeta}(u)$	$T_{2\zeta}(u)$	M_1M_2	$T_{1\zeta}(u)$	$T_{2\zeta}(u)$
$\pi^-\pi^+,\ \rho^-\pi^+,\ \pi^-\rho^+,\ \rho^-_\parallel\rho^+_\parallel$	$c_1^{(d)}+c_4^{(d)}$	0	$\pi^+K^{(*)-},\ \rho^+K^-,\ \rho^+_\parallel K^{*-}_\parallel$	0	$c_1^{(s)}+c_4^{(s)}$
$\pi^-\pi^0,\ \rho^-\pi^0$	$\frac{1}{\sqrt{2}}(c_1^{(d)}+c_4^{(d)})$	$\frac{1}{\sqrt{2}}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\pi^0K^{(*)-}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}-c_3^{(s)})$	$\frac{1}{\sqrt{2}}(c_1^{(s)}+c_4^{(s)})$
$\pi^-\rho^0,\ \rho^-_\parallel\rho^0_\parallel$	$\frac{1}{\sqrt{2}}(c_1^{(d)}+c_4^{(d)})$	$\frac{1}{\sqrt{2}}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\rho^0K^-,\ \rho^0_\parallel K^{*-}_\parallel$	$\frac{1}{\sqrt{2}}(c_2^{(s)}+c_3^{(s)})$	$\frac{1}{\sqrt{2}}(c_1^{(s)}+c_4^{(s)})$
$\pi^0\pi^0$	$\frac{1}{2}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\frac{1}{2}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\pi^-\bar{K}^{(*)0},\ \rho^-\bar{K}^0,\ \rho^-_\parallel\bar{K}^{*0}_\parallel$	0	$-c_4^{(s)}$
$\rho^0\pi^0$	$\frac{1}{2}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\frac{1}{2}(c_2^{(d)}-c_3^{(d)}-c_4^{(d)})$	$\pi^0\bar{K}^{(*)0}$	$\frac{1}{\sqrt{2}}(c_2^{(s)}-c_3^{(s)})$	$-\frac{1}{\sqrt{2}}c_4^{(s)}$
$\rho^0_\parallel\rho^0_\parallel$	$\frac{1}{2}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\frac{1}{2}(c_2^{(d)}+c_3^{(d)}-c_4^{(d)})$	$\rho^0\bar{K}^0,\ \rho^0_\parallel\bar{K}^{*0}_\parallel$	$\frac{1}{\sqrt{2}}(c_2^{(s)}+c_3^{(s)})$	$-\frac{1}{\sqrt{2}}c_4^{(s)}$
$K^{(*)0}K^{(*)-},\ K^{(*)0}\bar{K}^{(*)0}$	$-c_4^{(d)}$	0	$K^{(*)-}K^{(*)+}$	0	0

similar for T_J's in terms of $b_i^{(f)}$'s

Note: have not used isospin here

Matching

$$c_1^{(f)} = \lambda_u^{(f)}\Big(C_1+\frac{C_2}{N_c}\Big) - \lambda_t^{(f)}\frac{3}{2}\Big(C_{10}+\frac{C_9}{N_c}\Big) + \Delta c_1^{(f)}\,,$$

$$b_1^{(f)} = \lambda_u^{(f)}\Big[C_1 + \Big(1-\frac{m_b}{\omega_3}\Big)\frac{C_2}{N_c}\Big] - \lambda_t^{(f)}\Big[\frac{3}{2}C_{10} + \Big(1-\frac{m_b}{\omega_3}\Big)\frac{3C_9}{2N_c}\Big] + \Delta b_1^{(f)}\,,$$

Phenomenology for $B \to \pi\pi$

Beneke, Neubert - QCDF analysis

Buras, Fleischer, Recksiegel, Schwab - SU(2) analysis

Ali, Lunghi, Parkhomenko - SU(2) analysis

Chiang, Gronau, Rosner, Suprun - global $B \to PP$ analysis

Bauer, Pirjol, Rothstein, I.S. - LO analysis in SCET with A_{cc}

BABAR

$$S_{\pi\pi} = -0.40 \pm 0.22$$
$$C_{\pi\pi} = -0.19 \pm 0.20$$

BELLE

$$S_{\pi\pi} = -1.00 \pm 0.22$$
$$C_{\pi\pi} = -0.58 \pm 0.17$$

World Averages (HFAG, incl. CLEO)

$$S_{\pi\pi} = -0.74 \pm 0.16, \quad C_{\pi\pi} = -0.46 \pm 0.13\,,$$
$$Br(B^+ \to \pi^0\pi^+) = (5.2 \pm 0.8) \times 10^{-6}\,,$$
$$Br(B^0 \to \pi^+\pi^-) = (4.6 \pm 0.4) \times 10^{-6}\,,$$
$$Br(B^0 \to \pi^0\pi^0) = (1.9 \pm 0.5) \times 10^{-6}\,,$$

- a large Penguin, seems problematic for QCDF
- to test Λ/E expansion in a model independent way
  we should fit unknown hadronic parameters

Pure Isospin Analysis

$$A = \lambda_u^{(d)} T + \lambda_c^{(d)} P$$

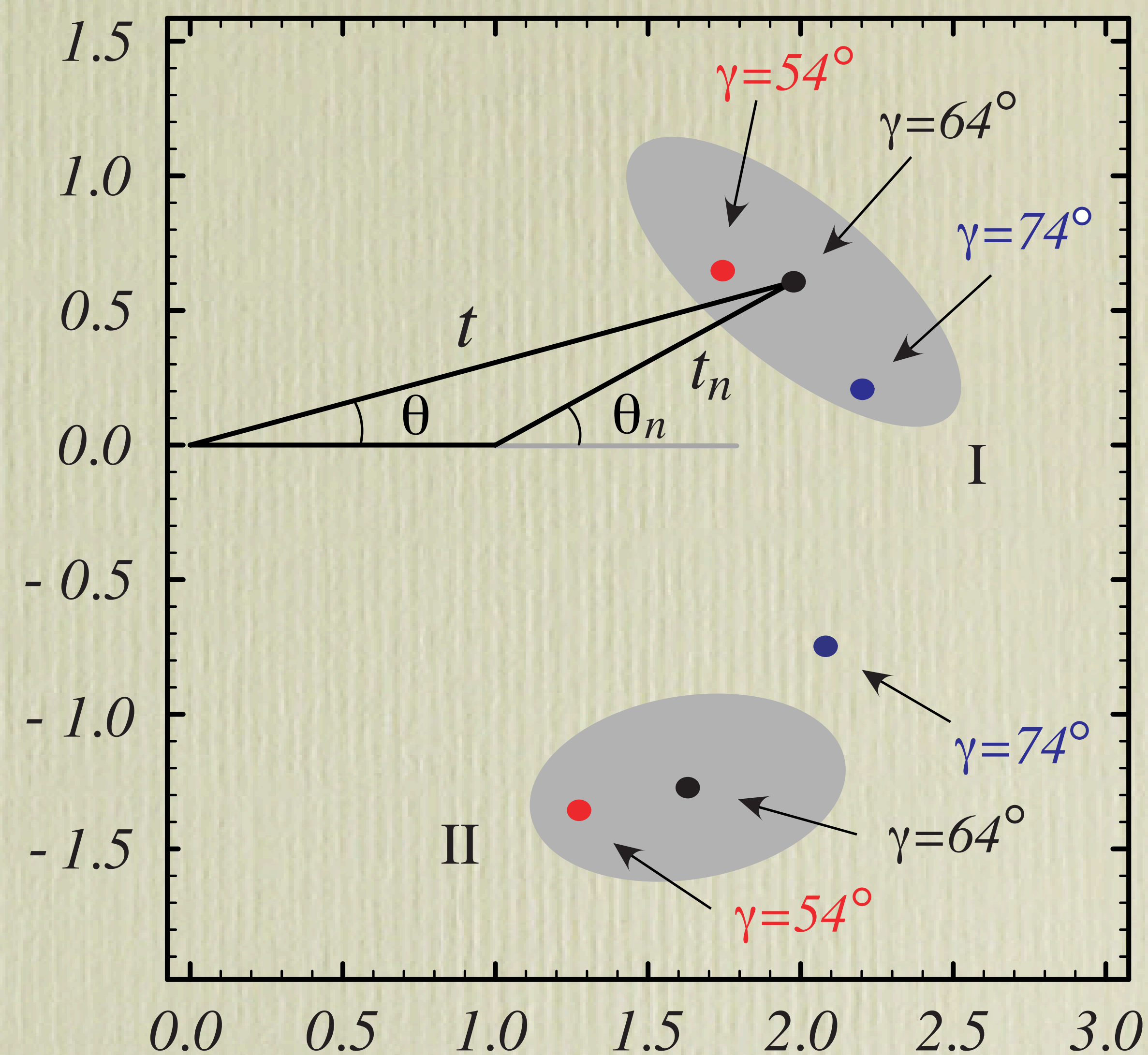
$$\begin{aligned} A(\bar{B}^0 \to \pi^+\pi^-) &= \lambda_u^{(d)}\, T_c (1 + r_c\, e^{i\delta_c} e^{i\gamma}), \\ A(\bar{B}^0 \to \pi^0\pi^0) &= \lambda_u^{(d)}\, T_n (1 + r_n\, e^{i\delta_n} e^{i\gamma}), \\ \sqrt{2} A(B^- \to \pi^0\pi^-) &= \lambda_u^{(d)}\, T, \end{aligned}$$

10 hadronic parameters

- 4 for isospin relation
- an overall phase

= 5 parameters

$$\left(T, r_c, \delta_c, |t| = \left|\frac{T}{T_c}\right|, |t_n| = \left|\frac{T_n}{T_c}\right| \right)$$



With $\gamma = 64^\circ$

$$r_c = 0.75 \pm 0.35, \quad \delta_c = -44^\circ \pm 12^\circ.$$

$$|t| = 2.07 \pm 0.42, \quad |t_n| = \begin{cases} 1.15 \pm 0.33 & \text{(I)} \\ 1.42 \pm 0.35 & \text{(II)} \end{cases}$$

large penguin

large C amplitude

SCET at LO

LO in Λ/E , LO in $\alpha_s(m_b)$ for T's

fit 4 parameters $\left(\zeta^{B\pi}, \zeta_J^{B\pi}, P \text{ (or } A_{c\bar{c}})\right)$ → do not need $|t_n|$ ie. $Br(B^0 \to \pi^0\pi^0)$

$T, \quad |t|$

$$\zeta^{B\pi}\big|_{\gamma=64^\circ} = (0.05 \pm 0.05)\Big(\frac{3.9\times10^{-3}}{|V_{ub}|}\Big),$$

$$\zeta_J^{B\pi}\big|_{\gamma=64^\circ} = (0.11 \pm 0.03)\Big(\frac{3.9\times10^{-3}}{|V_{ub}|}\Big),$$

← QCDF used $\zeta^{B\pi} \gg \zeta_J^{B\pi}$

$r_c, \quad \delta_c$

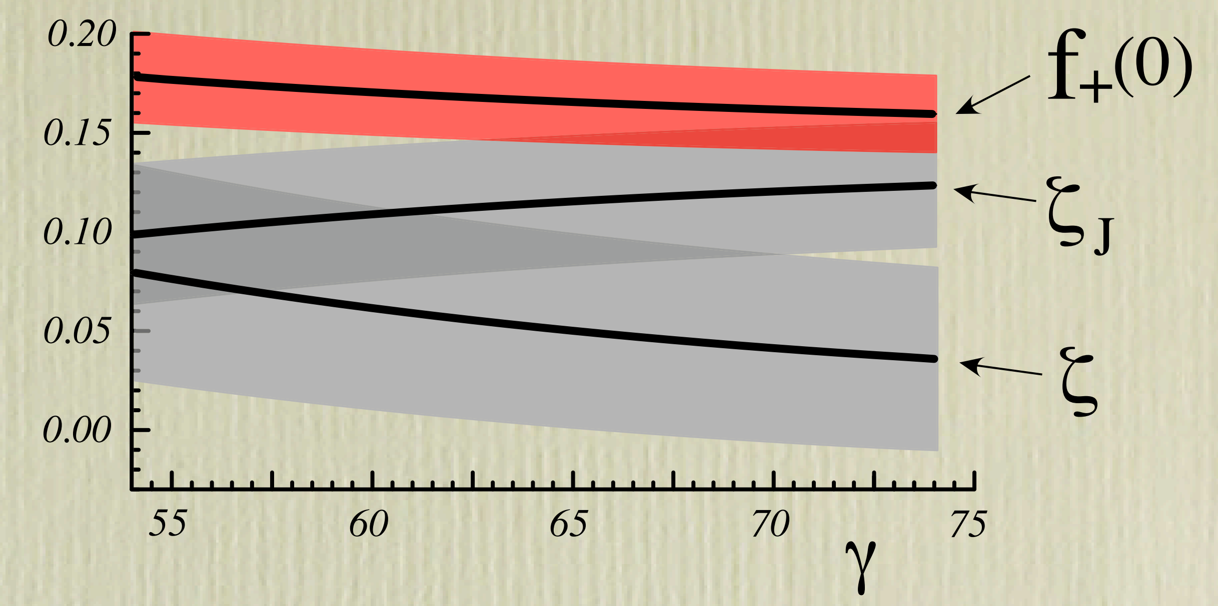
$$\frac{P}{N_\pi}\Big|_{\gamma=64^\circ} = (0.043 \pm 0.013)\, e^{i(136^\circ \pm 12^\circ)}$$

← compatible with large $A_{c\bar{c}}$

At this order the "Tree" isospin triangle is predicted to be flat

Predictions

units x $\left[\frac{3.9 \times 10^{-3}}{|V_{ub}|}\right]$



I) $f_+(0) = \zeta^{B\pi} + \zeta_J^{B\pi}$

- $A(B^- \to \pi^0\pi^-) \propto \zeta^{B\pi} + \Big(1 + \frac{\langle \bar{u}^{-1}\rangle_\pi}{4}\Big)\zeta_J^{B\pi}$

  naive factorization fails when $\zeta_J^{B\pi} \sim \zeta^{B\pi}$

- values are substantially smaller than model estimates

$$f_+(0)\,\big|_{\gamma=64^\circ} = (0.17 \pm 0.02)\Big(\frac{3.9\times 10^{-3}}{|V_{ub}|}\Big)$$

only expt.

II) Predict

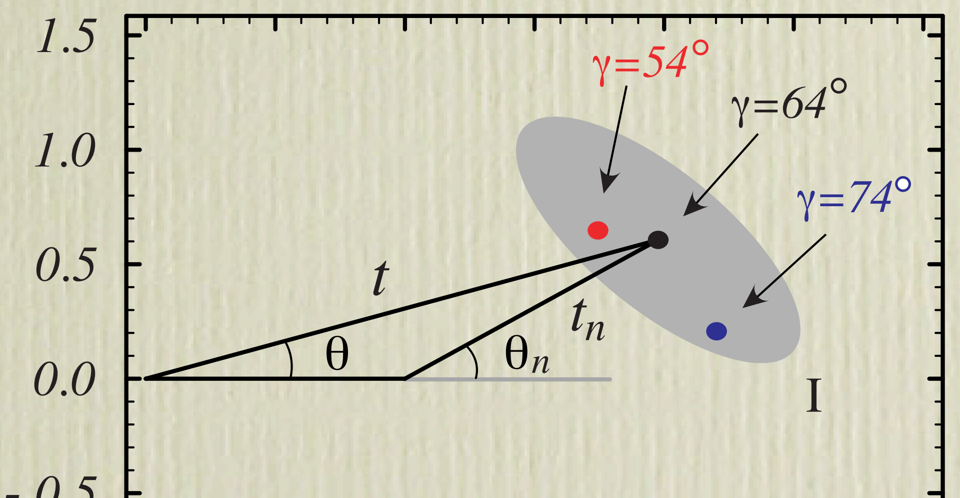
only expt.

$$Br(B^0 \to \pi^0\pi^0) = \begin{cases} (1.0 \pm 0.7)\times 10^{-6}, & \gamma = 54^\circ \\ (1.3 \pm 0.6)\times 10^{-6}, & \gamma = 64^\circ \\ (1.8 \pm 0.7)\times 10^{-6}, & \gamma = 74^\circ \end{cases}$$

an extra term $\frac{C_1}{N_c}\langle \bar{u}^{-1}\rangle_\pi\, \zeta_J^{B\pi}$ ruins color suppression

from flat “tree” triangle

or turn this around and predict γ



Open Issues in $B \to M_1 M_2$

- Factorization formula with charming penguins?
- Power Corrections:
  - expect nonperturbative phases $\delta \sim 30^\circ$
  - $C_1 \Lambda / E \sim C_2 \gg C_{j \geq 3}$
  - "chirally" enhanced terms, annihilation
- size of SU(3) breaking: not just f_M also $\phi_M(x)$

$\Rightarrow$ a lot of work left to do

Outlook

- The theory of nonleptonic B decays is challenging, but progress is being made

SCET

- Allows power corrections to be addressed in a model independent way
- For B's, need to carefully examine expansion for each process and improve our understanding of power corrections to trust results beyond the 20% level

- A lot of theory and phenomenology left to study ...

We have only seen
the tip of the iceberg

Comments on $K\pi$

Effective Field Theory

- Separate physics at different momentum scales
- Power expansion
- Make symmetries explicit
- Model independent, systematically improvable

Effective Theories	Expansion Parameter
(1) Electroweak (Fermi) Hamiltonian	$m_b/m_W \ll 1$
(2) Heavy Quark Effective Theory (HQET)	$\Lambda/m_b \ll 1$
(3) Chiral Perturbation Theory, SU(3)	$m_{u,d,s}/\Lambda \ll 1$

All designed to separate hard $p_h \sim Q$ and soft p_s momenta, $Q^2 \gg p_s^2$

Allow for energetic hadrons $\Longrightarrow$ collinear p_c, new class of processes

$$Q \gg \Lambda_{\rm QCD} \qquad\qquad Q = E_H$$

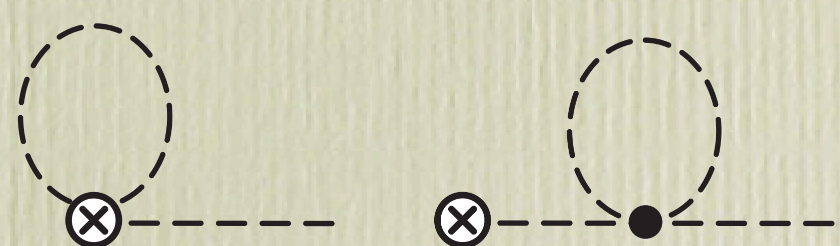
SU(3) Violation

$$\int_0^1 dx\, \phi_M(x) = 1 \qquad M = \pi, K, \eta$$

Using chiral perturbation theory:

J.Chen, I.S. '03

- Non-analytic terms vanish



all in f_M

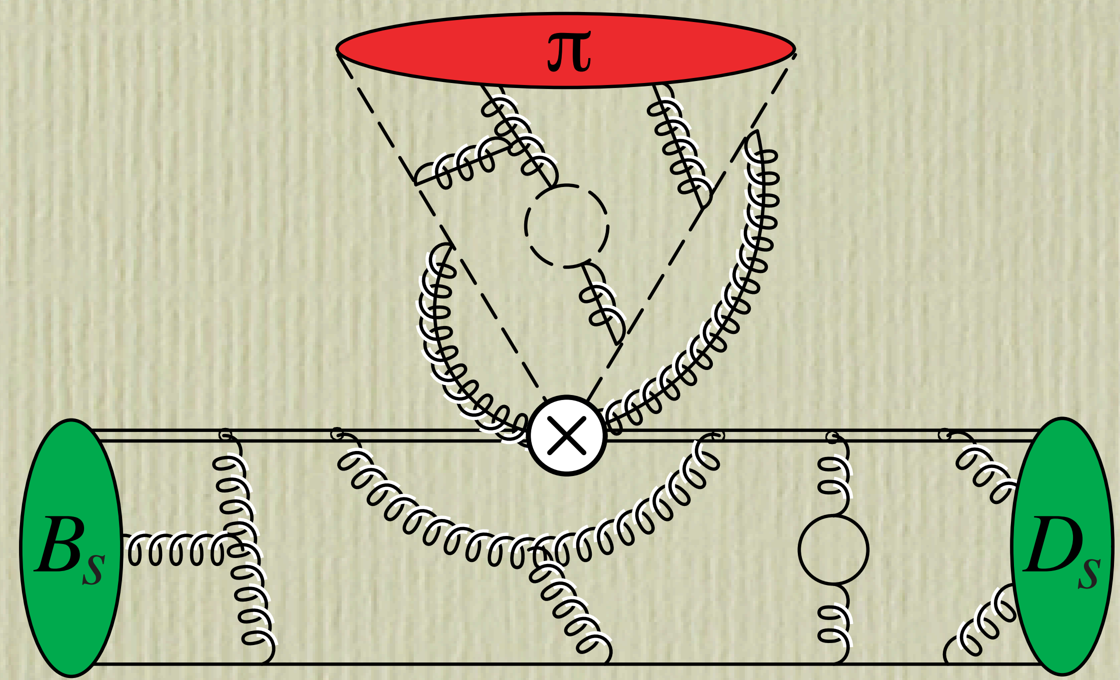
- At NLO, ie with all the leading SU(3) violation:

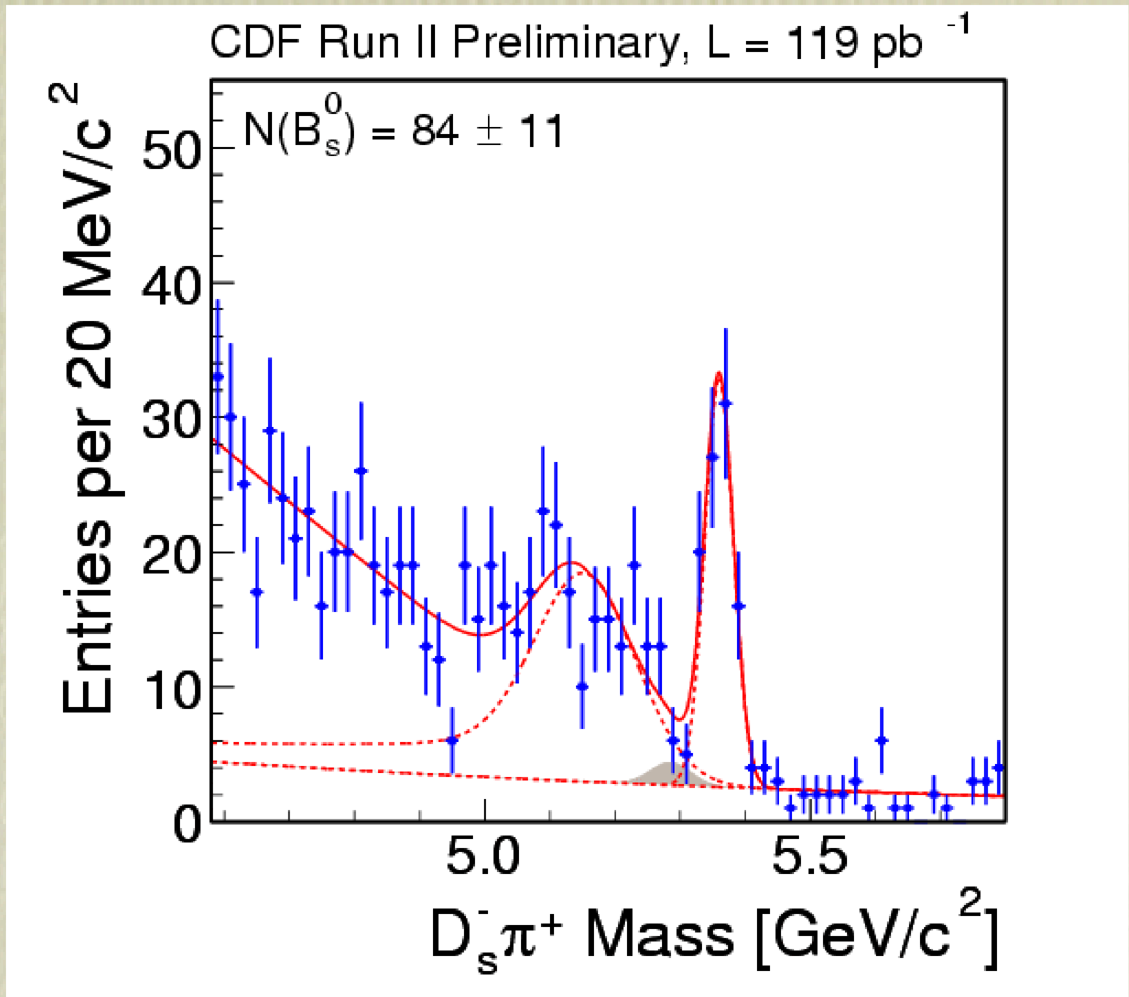
$$\phi_\pi(x) + 3\phi_\eta(x) = 2\left[\phi_{K^+}(x) + \phi_{K^-}(x)\right]$$

"Gell-Mann Okubo"

$B_s \to D_s \pi$

from CDF





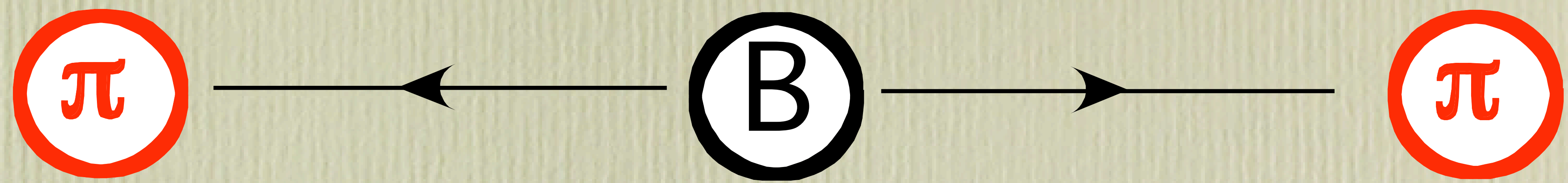
$$Br = (4.2 \pm 1.6) \times 10^{-3}$$

- pure "Tree" topology ➡ gives interesting information

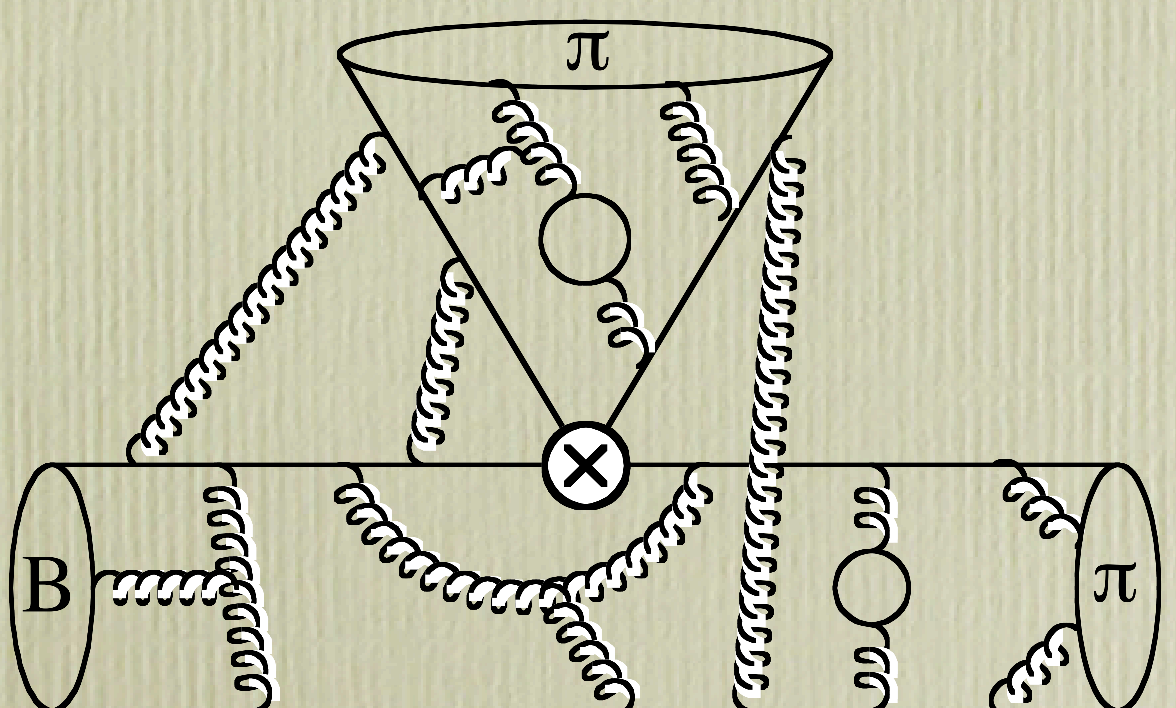
Using SU(3)

$$|T + E| = 5.9 \pm 0.3$$
$$|T + C| = 7.7 \pm 0.3$$
$$|T| = 7.3 \pm 1.5$$

Two body nonleptonic decays. Simple?



$$\Gamma = \frac{|\vec{p}_\pi|}{8\pi m_B^2}|A|^2 \qquad\qquad A = \langle \pi\pi | H_{\text{weak}} | B \rangle$$



Note: Nonleptonic B-decays are not Gold Plated Observables for Lattice QCD

SCET Expansion

LO: $\mathcal{O}^{(0)}$ with $\mathcal{L}^{(0)}$

NLO: $T\{O^{(0)},\mathcal{L}^{(1)}\} \sim O^{(1)}$ with $\mathcal{L}^{(0)}$

NNLO: $T\{O^{(0)},\mathcal{L}^{(2)}\} \sim T\{O^{(1)},\mathcal{L}^{(1)}\}$
$\sim T\{O^{(0)},\mathcal{L}^{(1)},\mathcal{L}^{(1)}\} \sim O^{(2)}$ with $\mathcal{L}^{(0)}$

$B \to M_1 M_2$ Factorization in SCET

Chay, Kim $\Lambda^2 \ll E\Lambda \ll E^2, m_b^2$

- operators, exponentiation of soft & collinear gluons
- involves ζ_{M_1}, $\phi_B(r^+)$, $\phi_{M_i}(x)$ same as form factors

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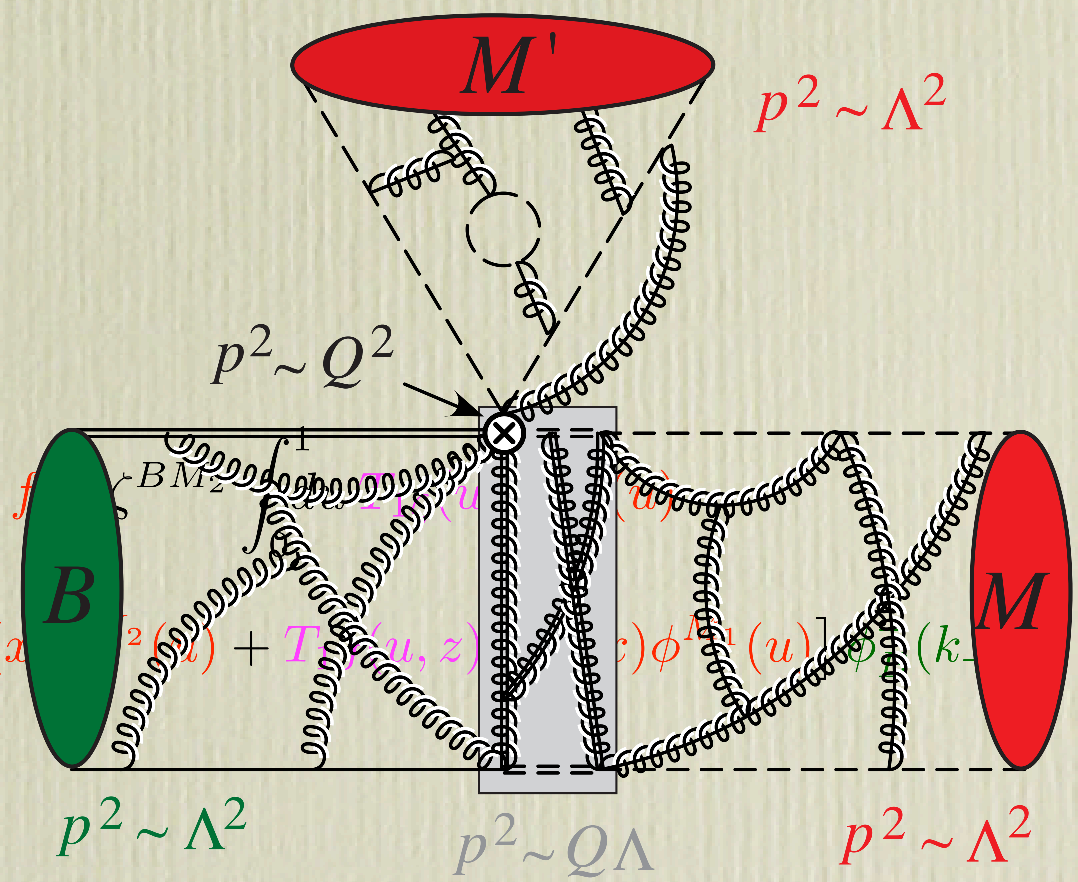
Bauer, Pirjol, Rothstein, I.S.

- hard spectator & form factor terms → same operators
- unique function $J(z, x, r_+, E)$ which is also in $B \to M$
- long distance charming penguins
- analysis for PP, PV, VV



$$A(B \to M_1 M_2) = A^{c\bar{c}} + N\Big\{ f_{M_2}\, \zeta^{BM_1} \int_0^1 du\, T_{2\zeta}(u)\, \phi^{M_2}(u) + f_{M_1}\, \zeta^{BM_2} \int_0^1 du\, T_{1\zeta}(u)\, \phi^{M_1}(u) \Big\}$$

$$+ \frac{f_B f_{M_1} f_{M_2}}{m_B} \int_0^1 du \int_0^1 dx \int_0^1 dz \int_0^\infty dk_+\, J(z, x, k_+) \left[T_{2J}(u,z)\phi^{M_1}(x)\phi^{M_2}(u) + T_{1J}(u,z)\phi^{M_2}(x)\phi^{M_1}(u) \right] \phi_B(k_+)$$