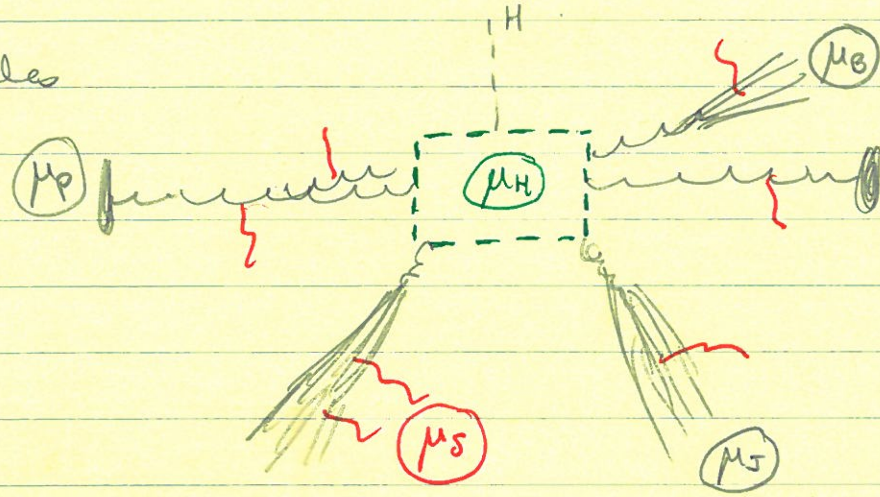
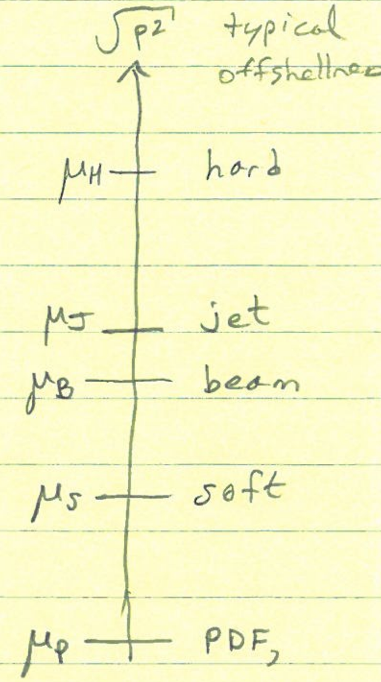
Use ideas of EFT to simplify collider physics calculations?

Give Operator definitions to various ingredients above (PDFs, fact. thms, scale separation & RGE)

Soft - Collinear Effective Theory

5 scales





Hard Scale

$\mu_H^2 \simeq p_{J1} \cdot p_{J2}\ ,\ p_H \cdot p_{J1}\ ,\ M_H^2$ hard partonic 4-vectors in $\hat{\sigma}(gg \to Hgg)$ calc

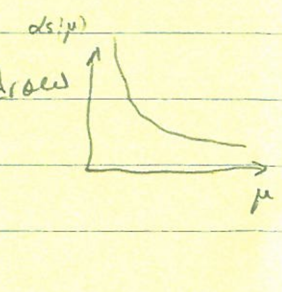
Jet Scale $\mu_J^2 \simeq M_{Jet}^2 = \left(\sum_{i \in J} p_i^\mu \right)^2$ $\mu_J^2 \ll \mu_H^2$ means collimated

Beam Scale $\mu_B^2 \simeq M_{T,beam}^2 = \left(\sum_{i \in B} \vec{p}_{iT} \right)^2$

Soft Scale $\mu_S \simeq \dfrac{\mu_J^2}{\mu_H}, \dfrac{\mu_B^2}{\mu_H}$ or could be $\simeq \mu_J, \mu_B$

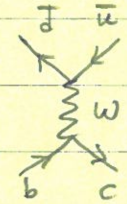
Proton Scale $\mu_p = \mu_0$ boundary condition $f_i(x, \mu_0)$ for PDF evolution

Redraw

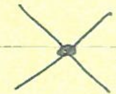


EFT Concepts

Decoupling eg.

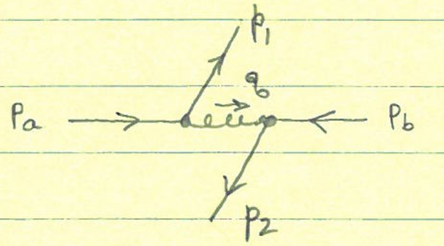


$\xrightarrow{p_i^2 \ll M_W^2}$

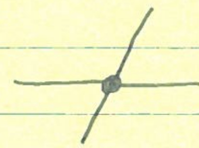


$\sim \dfrac{(\bar\psi \cdots \psi)(\bar\psi \cdots \psi)}{M_W^2}$

eg



$\xrightarrow{p_i^2 \ll |q^2|}$



$\sim \dfrac{1}{q^2} (\bar\psi \cdots \psi)(\bar\psi \cdots \psi)$

say $p_i^2 = 0$ on-shell , $q = p_a - p_1 = n_a E_a - n_1 E_1$

$n_a = (1, \hat z)$, $\bar n_a = (1, -\hat z)$

$n_1 = (1, \hat n)$ $\quad \bar n_1 = (1, -\hat n)$ $\qquad q^2 \sim Q^2$

← later use

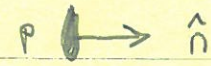
$q^2 = -2 E_a E_1\, n_1 \cdot n_a = -2 (E_a E_1)(1 - \hat z \cdot \hat n)$ $\qquad$ "hard"

large if energies big & deflection angle large

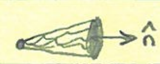
Construct $\mathcal{L}_{eff}$

- degrees of freedom? $\qquad$ low energy / on-shell modes
  → what fields
- symmetries → constrain interactions / operators
  [Lorentz, Gauge theory, Global, ...]
- expansions, leading order description → power counting

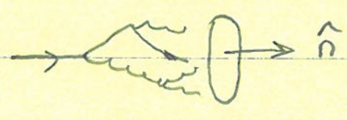
Collinear Modes $\qquad$ energetic hadron



$\langle 0 | O | p \rangle$



Jets



$p_\perp \sim \Lambda_{QCD} \ll |\vec p|$

energetic collimated radiation in direction $\hat n$

Let $n^\mu = (1, \hat{n})$ any $p^\mu = \underbrace{\bar{n}\cdot p}_{p^-} \frac{n^\mu}{2} + \underbrace{n\cdot p}_{p^+} \frac{\bar{n}^\mu}{2} + p_\perp^\mu$

$\bar{n}^\mu = (1, -\hat{n})$ $n\cdot p_\perp = \bar{n}\cdot p_\perp \equiv 0$

$n\cdot\bar{n} = 2$ light-cone coords

$n^2 = \bar{n}^2 = 0$ $p^2 = \bar{n}\cdot p \; n\cdot p + \underbrace{p_\perp^2}_{= -\vec{p}_\perp^{\,2}}$

1 massless particle: $p^\mu = \bar{n}\cdot p \frac{n^\mu}{2}$, $\bar{n}\cdot p \sim Q$

($E \doteq \bar{n}\cdot p/2$)

2 massless particles: 

$p_i^\mu = \bar{n}\cdot p_i \frac{n^\mu}{2} + n\cdot p_i \frac{\bar{n}^\mu}{2} + p_{i\perp}^\mu$

$i = 1, 2$

$\bar{n}\cdot p_i \sim Q$, $p_{i\perp}^\mu \ll Q$ collimated

say $p_{i\perp} \sim \lambda Q$

$\boxed{\lambda \ll 1 \text{ dimensionless Power Counting parameter}}$

on-shell $n\cdot p_i = \frac{-p_{\perp i}^2}{\bar{n}\cdot p_i} \sim \lambda^2 Q$

K-particles: same

n-Collinear: $\underline{p_i^\mu \sim Q\,(\overset{n\cdot p}{\lambda^2}, \overset{\bar{n}\cdot p}{1}, \overset{\perp}{\lambda})}$, $p_1 + p_2$ too!

Collinear Fields: quark ξ_n
gluon A_n^μ

$\boxed{\xi_n \quad A_n^\mu}$

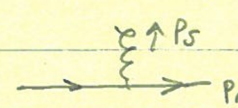
Soft all components small & homogeneous

$\underline{p_s^\mu \sim Q\,(\lambda^\alpha, \lambda^\alpha, \lambda^\alpha)}$ $\boxed{q_s, A_s^\mu}$

soft + soft = soft collinear + soft ?

soft + hard = hard

collinear + hard = hard



suppressed

$(p_s + p_n)^2 = 2\,p_n\cdot p_s = \bar{n}\cdot p_n \; n\cdot p_s + \cdots$

$\lambda^0 * \lambda^\alpha$

Value of α depends on what we measure

eg 1 Mass in (large enough) region a, $M_a^2 = \left(\sum_{i\in a} p_i^\mu\right)^2$

demand $M_a^2 \sim Q^2\lambda^2 \ll Q^2$

must add

collinear + collinear $\quad (p_{n1}+p_{n2})^2 = 2\, p_{n1}\cdot p_{n2} \sim Q^2\lambda^2$ ✓

$\quad\quad + \; - \quad - \; + \quad \perp \; \perp$

collinear + soft $\quad (p_n + p_s)^2 = Q^2\lambda^\alpha \quad \therefore \; \alpha = 2$ [to contrib]

[include all smaller α too]

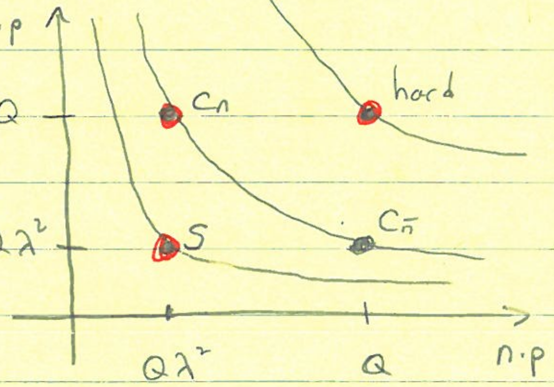
eg 2 $\quad B_\perp = \sum_{i\in a} |\vec{p}_i^{\,\perp}| \ll Q$

$\sim \lambda$

Σ collinear ✓

soft $\Rightarrow \alpha = 1$

① SCET$_{\mathrm{I}}$



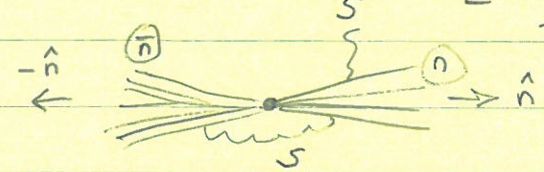
$\boxed{p^2 = p^+p^-}$

Q^2

$Q^2\lambda^2$

$Q^2\lambda^4$

[virtual too]



$e^+e^- \to 2$ jets

in CM frame

[s = ultrasoft]

② SCET$_{\mathrm{II}}$

Comments

- power counting requires multiple fields for some particle
- relative scaling is important
- 2 dims needed for classification

Field Power Counting

Use Free Kinetic term

ξ_n propogator

$$p^2 = n\cdot p\ \bar{n}\cdot p + p_\perp^2$$

$$\lambda^2 * \lambda^0 + (\lambda)^2 \qquad \text{same size}$$

$$\frac{i\not{p}}{p^2+i0} = \frac{i\not{n}}{2}\frac{\bar{n}\cdot p}{p^2+i0} + \dots = \frac{i\not{n}}{2}\frac{1}{n\cdot p + \frac{p_\perp^2}{\bar{n}\cdot p} + i0\, \mathrm{sign}(\bar{n}\cdot p)} + \dots$$

$$\underbrace{\int d^4x}_{\lambda^{-4}}\ \underbrace{e^{-ip\cdot x}}_{\lambda^0} \langle 0|T\,\xi_n(x)\,\bar{\xi}_n(0)|0\rangle = \frac{i\not{n}}{2}\underbrace{\frac{\bar{n}\cdot p}{p^2+i0}}_{\lambda^{-2}} \qquad (*)$$

$(d^4p \sim \lambda^4)$

need $\boxed{\xi_n \sim \lambda}$

Note: (*) implies $\not{n}\,\xi_n = 0$

take $u_n = \frac{\not{n}\bar{\not{n}}}{4} u$ for spinor.

$$\sum_s u_n^s \bar{u}_n^s = \frac{\not{n}\bar{\not{n}}}{4}\sum_s u^s\bar{u}^s \frac{\bar{\not{n}}\not{n}}{4} = \frac{\not{n}}{2}\bar{n}\cdot p \quad ✓$$

A_n^μ same as QCD.

$$p^\mu \sim (\lambda^2, 1, \lambda) \sim i\partial_n^\mu \quad , \quad iD_n^\mu = i\partial_n^\mu + gA_n^\mu$$

homogeneous

$$\therefore \quad \boxed{A_n^\mu \sim (\lambda^2, 1, \lambda)}$$

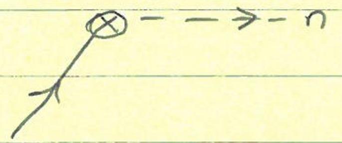
[or from propogator as for fermion]

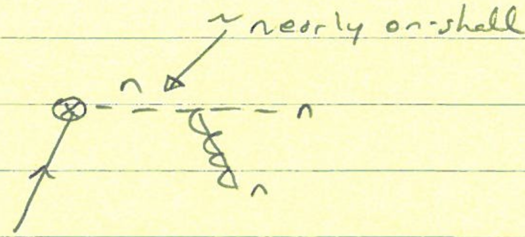
Similar for Soft: $A_s^\mu \sim p_s^\mu \sim \lambda^\alpha$, $p_s \sim \lambda^\alpha$

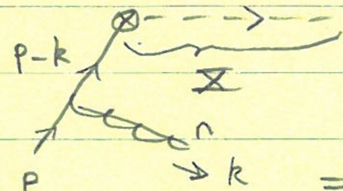
$\psi_s \sim \lambda^{3\alpha/2}$ $\qquad d^4x \sim \lambda^{-4\alpha}$

Operator are products of fields. $\bar{n}\cdot A_n \sim \lambda^0$?

Collinear Wilson lines







$$(p-k)^2 = \cancel{p^2} + \cancel{k^2} - 2p\cdot k = -\underbrace{\bar{n}\cdot k}_{\lambda^0}\,\underbrace{n\cdot q}_{\lambda^0} + \ldots \qquad \underline{\text{offshell}}$$

$$= X\ \frac{i(\not{p}-\not{k}+m)}{(p-k)^2-m^2+i0}\ (igT^a \not{\epsilon}_n^a)\ u(p)$$

$\frac{\not{n}}{2}\,\bar{n}\cdot\epsilon_n^a + \ldots$

since $\not{A} = \underbrace{\bar{n}\cdot A}_{\lambda^0}\frac{\not{n}}{2} + \ldots$

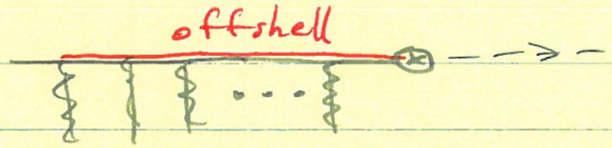
expand (Homework)

$$= X\ \frac{(-g)\,\bar{n}\cdot A_n^a\, T^a}{-\bar{n}\cdot k + i0}\ u(p)$$

- universal, indep of p, m, ...

$\Longrightarrow$ 

Keep Going



$+ \ldots \Longrightarrow$

Give Wilson line

$$W_n(y,-\infty) = P\exp\left(ig\int_{-\infty}^{0} ds\ \bar{n}\cdot A_n(s\bar{n}+y)\right)$$

(More Hmwk)

$W_n \sim \lambda^0$

SCET building block operator $\qquad \chi_n = W_n^\dagger \xi_n$

Gauge Invariance $\quad \bar{\xi}_n W_n \to \bar{\xi}_n \cancel{U_n^\dagger U_n} W_n \quad$ protects combination

Usually: couple to both quarks, here couple to one + W_n

$W_n^\dagger W_n = \mathbb{1} = W_n W_n^\dagger$

$i\bar{n}\cdot D_n W_n = 0$

singlet

$$\left[\begin{array}{l} i\bar{n}\cdot D_n W_n \Phi = W_n\, i\bar{n}\cdot\partial_n \Phi \\ W_n^\dagger\, i\bar{n}\cdot D_n W_n = i\bar{n}\cdot\partial_n \\ i\bar{n}\cdot D_n = W_n\, i\bar{n}\cdot\partial_n W_n^\dagger \end{array}\right.$$

as operator

<u>Hard Collinear Factorization</u> $\qquad \mathcal{L}^{\text{hard}} = C\, \mathcal{O}$

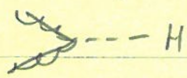
power counting, symmetry & matching calculations imply operators $\mathcal{O}$ are built from

$\chi_n = W_n^\dagger \xi_n$

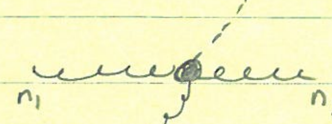
$\mathcal{B}_{n\perp}^\mu = \frac{1}{g}\left[W_n^\dagger\, iD_{n\perp}^\mu W_n\right]$ derivative only on W_n

$P_\perp^\mu$

& Soft Fields $\Big\}$ often suppressed

<u>Example</u>	<u>Operators</u>		
$e^+e^- \to 2$ jets	$\bar{\chi}_n \gamma_\perp^\mu \chi_{\bar{n}}$		(Amplitude)
$gg \to H$	$\mathcal{B}_{n\perp}^\mu \mathcal{B}_{\bar{n}\perp\mu} H$		(Ampl)
[quark PDF	$\bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \delta(\omega - i\bar{n}\cdot\partial_n)\chi_n$		Ampl^2]
gluon PDF	$\mathrm{tr}\left[\mathcal{B}_{n\perp}^\mu \delta(\omega - i\bar{n}\cdot\partial_n)\mathcal{B}_{n\perp\mu}\right]$		"

$pp \to H + 1\text{-jet}$, remove top



Ampl

- $\mathcal{B}_{\perp n_1}^{a_1\mu_1} \mathcal{B}_{\perp n_2}^{a_2\mu_2} \mathcal{B}_{\perp n_3}^{a_3\mu_3} H \, T_{\mu_1\mu_2\mu_3}$ $\quad (if^{a_1a_2a_3})$

  ← no $d^{a_1a_2a_3}$ by charge conjugation

- $\mathcal{B}_{\perp n_1}^{a\mu_1} \bar{\chi}_{n_2}^{\alpha} \Gamma^{\mu_2} \chi_{n_3}^{\beta} H \, T^{\Gamma}_{\mu_1\mu_2}$

how many operators?

Helicity Basis: $\mathcal{B}_{n\pm}^a \equiv -\epsilon_\mp^\mu(n,\bar{n})\, \mathcal{B}_{n\mu}^\perp$

$$\epsilon_\pm(n,\bar{n}) = \frac{1}{\sqrt{2}}(0, 1, \mp i, 0)$$

Similarly $\quad J_{n_1n_2\pm}^{\alpha\beta} \propto \epsilon_\mp^\mu(n_1,n_2)\, \bar{\chi}_{n_1\pm}^\alpha \gamma_\mu \underbrace{\chi_{n_2\pm}^\beta}_{\left(\frac{1\pm\gamma_5}{2}\right)\chi_n}$

Allowed	$\mathcal{B}\ \mathcal{B}\ \mathcal{B}$		$\mathcal{B}\ J$	
	$+\ +\ +$		$+\ +$	
	$+\ +\ -$		$-\ +$	
	$-\ -\ +$	Wilson Coeff fixed by	$+\ -$	fixed by
	$-\ -\ -$	Parity	$-\ -$	charge conj.

4 non-trivial Wilson Coefficients C for SCET

For more on Helicity based operators in SCET see
arXiv: 1508.02397