Soft-Collinear Effective Theory

- EFT treatment of soft & collinear IR physics for hard collisions in QCD (or decays with large E released)

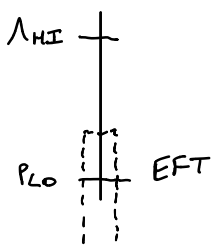
$\Rightarrow$ jets, energetic hadrons, soft partons / hadrons

eg. $e^+e^- \to$ 2-jets, $e^- p \to e^- X$ (DIS), $pp \to H + $ 1-jet, $B \to \pi\pi$, jet substructure, ... [many many more]

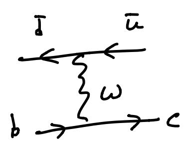
Concepts: Factorization, Wilson Lines, Sum Sudakov Double Logs, Power Corrections, ...

First, Review key EFT Concepts

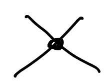
<u>Decoupling</u> Effects from heavy or <u>offshell</u> particles are suppressed / decouple $\quad p_{LO} \ll \Lambda_{HI}$



eg

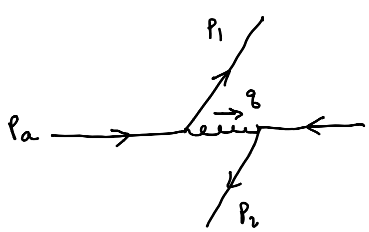


$\longrightarrow$



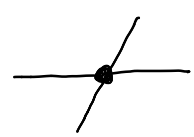
$\sim \dfrac{(\bar\psi\psi)(\bar\psi\psi)}{M_W^2} \qquad M_W^2 \gg p_i^2$

eg.



$p_i^2 \ll |q^2|$

$\longrightarrow$



$\sim \dfrac{1}{q^2} (\bar\psi \cdot\cdot \psi)(\bar\psi \cdot\cdot \psi)$

say $p_i^2 = 0$ on-shell, $\quad q = p_a - p_1 = n_a E_a - n_1 E_1$

$n_a = (1, \hat{z}) \qquad \bar{n}_a = (1, -\hat{z}) \qquad q^2 = -2E_aE_1\, n_a \cdot n_1$

$n_1 = (1, \hat{n}) \qquad \bar{n}_1 = (1, -\hat{n}_1) \qquad\quad = -2E_aE_1\, (1 - \hat{z}\cdot\hat{n})$

large if energies big & deflection angles large

$\boxed{q^2 \sim Q^2 \quad \text{"hard"}}$

Construct $\mathcal{L}_{eff}$

- degrees of freedom? low energy / nearly onshell modes
  → what fields
- symmetries → constrain interactions / operators
  [Lorentz, Gauge theory, Global, ...]
- expansions, leading order description
  → power counting

$$\mathcal{L}_{EFT} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots$$

∞ # operators, but only specific subset needed at given order
[often in mass dimension of operators, but not in SCET]

Matching

$$\mathcal{L}_{EFT}^{(k)} = \sum_i \underbrace{C_i(\mu)}_{\text{short. dist. (offshell)}} \underbrace{O_i^{(k)}(\mu)}_{\text{long dist. } (\sim \text{on-shell})}$$

- $\mathcal{L}_{HI}$ & $\mathcal{L}_{EFT}$ have same IR, differ in UV
- $C_i(\mu)$ does not depend on IR scales (masses in EFT, Λ_{QCD}, IR regulators, ...)

Λ_{HI}

μ splits HI from LO

p_{LO}

$\mathcal{L}_{HI} \to \mathcal{L}_{EFT}$ "top-down EFT" if we know $\mathcal{L}_{HI}(\Lambda_{HI}, p_{LO})$ we can (perturbatively) construct $\mathcal{L}_{EFT}$.

Calculate C, construct O [Hweak, HQET NRQCD, SCET, ...]

$\mathcal{L}_{HI}$?

$\mathcal{L}_{EFT}$ "bottom-up EFT" form $\sum_i C_i O_i$ complete basis

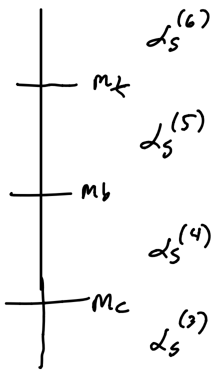
exploit symmetries

[eg. SM as EFT, Chiral Lagrangians ...]

Renormalization

- parameters g, C in QFT must be defined by a renormalization scheme, also $\mathcal{O}$ ($\overline{MS}$, Wilsonian Cutoff, ...)
- schemes depend on cutoff / renormalization scale "μ" $\quad g(\mu)$, $C(\mu)$

eg. $\alpha_s^{(n_f)}(\mu)$ in QCD $\qquad \mu \frac{d}{d\mu} \alpha_s^{(n_f)}(\mu) = -\frac{\beta_0^{(n_f)}}{2\pi} \left[\alpha_s^{(n_f)}(\mu)\right]^2 + \dots$

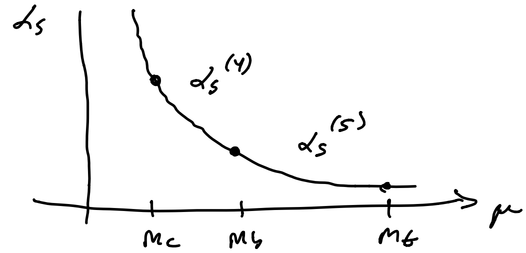


$$\beta_0^{(n_f)} = 11 - \frac{2}{3} n_f$$

Hmwk for anyone unfamiliar with this

Renormalization Group

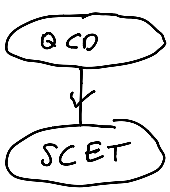
- sums logs between mass scales $\alpha_s^k \ln^k(m_b/m_t)$



[Typically]

- Power counting handles powers $\frac{p_{LO}}{\Lambda_{HI}} \ll 1$
- Renormalization group handles logs $\ln\left(\frac{p_{LO}}{\Lambda_{HI}}\right)$ which may be large $\alpha_s \ln(\cdot\cdot) \sim 1$

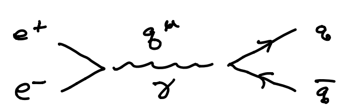
SCET



$\Lambda_{QCD}^2, p_{IR}^2 \ll Q^2$ hard collision

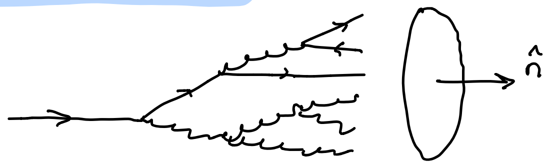
degrees of freedom

consider $e^+e^- \to 2$ jets



$Q^2 = q^2$

Jets / collinear



due to collinear (& soft) enhancements in QCD

- collimated radiation in direction $\hat{n}$
- $E_{Jet} \sim Q$

Let $n^\mu = (1, \hat{n})$
$\bar{n}^\mu = (1, -\hat{n})$

$n^2 = \bar{n}^2 = 0\,, \quad n\cdot\bar{n} = 2$

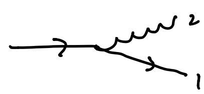
$$p^\mu = \underbrace{\bar{n}\cdot p}_{p^-} \frac{n^\mu}{2} + \underbrace{n\cdot p}_{p^+} \frac{\bar{n}^\mu}{2} + p_\perp^\mu$$

$$p^2 = n\cdot p\, \bar{n}\cdot p + \underbrace{p_\perp^2}_{-\vec{p}_\perp^{\,2}}$$

Collinear ?

1 massless particle : $\quad p^\mu = \bar{n}\cdot p \dfrac{n^\mu}{2}$

2 massless :



$$p_i^\mu = \bar{n}\cdot p_i \frac{n^\mu}{2} + p_{i\perp}^\mu + n\cdot p_i \frac{\bar{n}^\mu}{2}$$

$i = 1, 2$

$\bar{n}\cdot p_i \sim Q$ large $\qquad p_{i\perp}^\mu \ll Q \quad$ collimated

say $p_{i\perp} \sim \lambda Q \qquad \boxed{\lambda \ll 1}$ $\qquad$ dimensionless power counting parameter

on-shell $\quad n\cdot p_i = -\dfrac{p_{i\perp}^2}{\bar{n}\cdot p_i} \;\Rightarrow\; n\cdot p_i \sim \lambda^2 Q \quad$ nearly on-shell

n particles : same

$$\boxed{\text{n-Collinear}} : \quad p^\mu \sim Q(\overset{n\cdot p}{\lambda^2}, \overset{\bar{n}\cdot p}{1}, \overset{\perp}{\lambda})$$

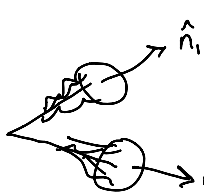
Collinear Fields :

quark $\quad \xi_n$
gluon $\quad A_n^\mu$

energetic hadron : $\quad p_\perp \sim \Lambda_{QCD} \Rightarrow \lambda \sim \dfrac{\Lambda_{QCD}}{Q}$ $\qquad$ energetic quarks & gluons confine into single hadron

jet of hadrons: $\quad 1 \gg \lambda \gg \dfrac{\Lambda_{QCD}}{Q}$

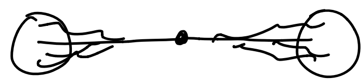
2-jets



$\hat{n}_1 \qquad n_1, \bar{n}_1 \qquad \xi_{n_1}, A_{n_1}^\mu$

$\hat{n}_2 \qquad n_2, \bar{n}_2 \qquad \xi_{n_2}, A_{n_2}^\mu$

back-to-back jets: $n_2 = \bar{n} = (1, -\hat{n})$, $\bar{n}_2 = n$



$n_1 = n = (1, \hat{n})$, $\bar{n}_1 = \hat{n}$

	$(+,-,\perp)$
$\xi_n,\ A_n$	$(\lambda^2, 1, \lambda)$
$\xi_{\bar{n}},\ A_{\bar{n}}$	$(1, \lambda^2, \lambda)$

Soft $\quad \underline{p_s^\mu \sim Q\lambda^\alpha}$ $\quad$ all components small & homogeneous

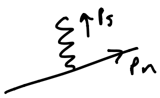
soft + soft = soft

soft + hard = hard

collinear + hard = hard

n_1-collinear + n_2-collinear = hard $\leftarrow$ hard interaction produces jets

collinear + soft ?



$$(p_n + p_s)^2 = 2p_n \cdot p_s = \underbrace{\bar{n}\cdot p_n\ n\cdot p_s}_{\lambda^0 * \lambda^\alpha} + \underbrace{\cdots}_{\text{suppressed}} \quad \sim Q^2\lambda^\alpha$$

Value of α depends on what we measure

eg 1 $\quad$ Mass in (large enough) region a, $\quad M_a^2 = \left(\sum_{i\in a} p_i^\mu\right)^2$

[mass of R = 1 jet, hemisphere mass, ...]

demand $\quad M_a^2 \sim Q^2\lambda^2 \ll Q^2$ $\quad$ [collimated jet has $E_J \gg M_J$]

Collinear + collinear $\quad (p_n + p_n')^2 = 2\, p_n \cdot p_n' \sim Q^2\lambda^2$

$$\begin{array}{cc} + & - \\ - & + \\ \perp & \perp \end{array}$$

collinear + soft $\quad (p_n + p_s)^2 \sim Q^2 \lambda^\alpha$

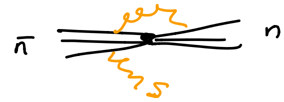
$\therefore\ \alpha = 2$ to contribute $\quad$ "ultrasoft"

eg 2 $\quad$ Transverse Momenta, $\quad$ broadening $B_\perp = \sum_{i\in a} \underbrace{|\vec{p}_i^{\,\perp}|}_{\sim\lambda} \ll Q$

$\sum$ collinear ✓

soft $\Rightarrow$ $\alpha = 1$ $\quad$ "soft"

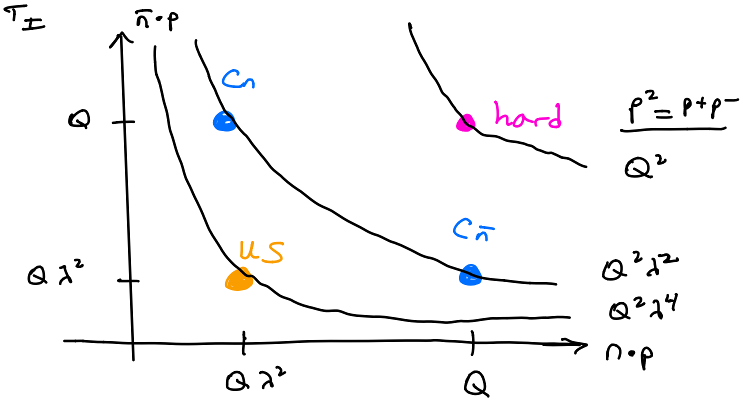
DOF Picture $\quad e^+e^- \to 2\,\text{jets}$ (cm frame)



[virtual too]

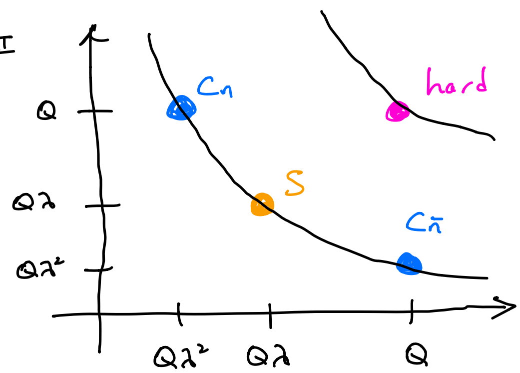
① $SCET_I$

$\alpha = 2$



- modes cover regions of momentum space, extend into IR

② $SCET_{II}$



- power counting requires multiple fields for same particle
- relative scaling of modes is important [boost invariant, unlike absolute scaling]
- modes not classified by p^2 alone

Study $SCET_I$, come back to $SCET_{II}$

Field Power Counting

Use free kinetic term

ξ_n propagator

$$p^2 = n\cdot p \;\bar{n}\cdot p + p_\perp^2$$
$$\lambda^2 * \lambda^0 + (\lambda)^2 \quad \text{same size}$$

$$\frac{i\not{p}}{p^2+i0} = \frac{i\not{n}}{2}\frac{\bar{n}\cdot p}{p^2+i0} + \dots = \frac{i\not{n}}{2}\frac{1}{n\cdot p + \frac{p_\perp^2}{\bar{n}\cdot p} + i0\,\mathrm{sign}(\bar{n}\cdot p)} + \dots$$

must have

$$\underbrace{\int d^4x}_{\lambda^{-4}} \; \underbrace{e^{-ip\cdot x}}_{\lambda^0} \langle 0 | T\, \xi_n(x)\, \bar{\xi}_n(0) | 0 \rangle = \frac{i \not{n}}{2} \underbrace{\frac{\bar{n}\cdot p}{p^2 + i0}}_{\lambda^{-2}} \quad (*)$$

$(d^4p \sim \lambda^4)$

thus $\boxed{\xi_n \sim \lambda}$ [differs from $\frac{3}{2}$ mass dimension]

Note: $(*)$ implies $\not{n}\, \xi_n = 0$ since $\not{n}^2 = n^2 = 0$

take $\xi_n = \underbrace{\frac{\not{n}\bar{\not{n}}}{4}}_{\text{projection op.}} \psi$ for spin

spinors $u_n = \frac{\not{n}\bar{\not{n}}}{4} u(p)$

$$\sum_s u_n^s \bar{u}_n^s = \frac{\not{n}\bar{\not{n}}}{4} \sum_s u^s \bar{u}^s \frac{\bar{\not{n}}\not{n}}{4} = \frac{\not{n}}{2} \bar{n}\cdot p \quad ✓$$

$$u_+(p) = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{p^-} \\ \cancel{\sqrt{p^+} e^{i\phi_p}} \\ \sqrt{p^-} \\ \cancel{\sqrt{p^+} e^{i\phi_p}} \end{pmatrix},$$

$\frac{\not{n}\bar{\not{n}}}{4} u(p)$ kills small terms

Dirac Rep: $\frac{\not{n}\bar{\not{n}}}{4} = \frac{1}{2}\begin{pmatrix} \mathbb{1} & \sigma^3 \\ \sigma^3 & \mathbb{1} \end{pmatrix}$

similar for $u_-(p)$ & antiquarks $v_+(p)$, $v_-(p)$

A_n^μ

same propagator as QCD

$p^\mu \sim (\lambda^2, 1, \lambda) \sim i\partial_n^\mu \qquad iD_n^\mu = i\partial_n^\mu + g A_n^\mu$

Want $i\partial_n^\mu \sim A_n^\mu$ so $\boxed{A_n^\mu \sim (\overset{+}{\lambda^2}, \overset{-}{1}, \overset{\perp}{\lambda})}$ true in gauge

[or derive from free propagator]

Soft

Similar analysis

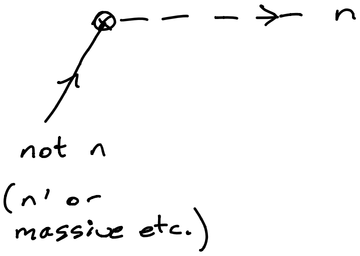
$p_s \sim \lambda^\alpha$

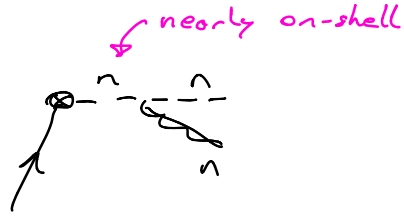
$$\boxed{\begin{aligned} A_s^\mu &\sim p_s^\mu \sim \lambda^\alpha \\ \psi_s &\sim \lambda^{3\alpha/2} \end{aligned}}$$

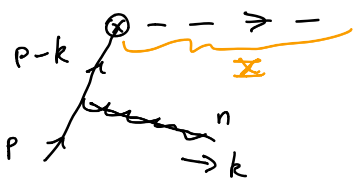
$$\underbrace{\int d^4x}_{\lambda^{-4\alpha}} \; \bar{\psi}_s \underbrace{i\not{\partial}}_{\lambda^\alpha} \psi_s$$

Collinear Wilson Lines

- $\bar{n}\cdot A_n \sim \lambda^0$? no suppression for building operators







$$(p-k)^2 = \cancel{p^2} + \cancel{k^2} - 2p\cdot k = -\underbrace{\bar{n}\cdot k}_{\lambda^0}\ \underbrace{n\cdot q}_{\lambda^0 \text{ since not } n\text{-collinear}} + \cdots$$

$\therefore$ offshell

integrate it out

$$= \overline{X}\ \frac{i(\not{p}-\not{k}+m)}{(p-k)^2 - m^2 + i0}\ (ig T^a \not{\epsilon}_n^a)\ u(p)$$

$$\not{\epsilon}_n^a \to \frac{\not{n}}{2}\,\bar{n}\cdot\epsilon_n^a + \cdots$$

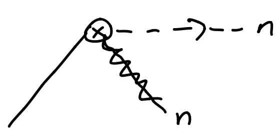
since $\not{A} = \underbrace{\bar{n}\cdot A}_{\lambda^0}\,\frac{\not{n}}{2} + \cdots$

expand Homework

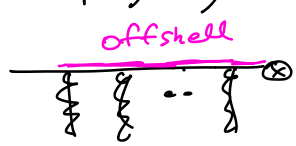
$$= \overline{X}\ \frac{(-g)\ \bar{n}\cdot A_n^a}{-\bar{n}\cdot k + i0}\ T^a\ u(p)$$

- universal, independent of $p, m, \ldots$

$\rightarrow$



keep going



+ perms $\Rightarrow$

Gives Wilson line

$$W_n(y,-\infty) = P\exp\left(ig\int_{-\infty}^{0} ds\ \bar{n}\cdot A_n(s\bar{n}+y)\right)$$

More Hmwk

$\boxed{W_n \sim \lambda^0}$

SCET operator $(\bar{\xi}_n W_n)\ (\Gamma \psi)$

generic, operator "building block"

quark $\underline{\chi_n \equiv W_n^\dagger \xi_n}$

gluon $\underline{\mathcal{B}_{n\perp}^\mu \equiv \frac{1}{g}[W_n^\dagger iD_{n\perp}^\mu W_n]} = \left[\frac{1}{g\, i\bar{n}\cdot\partial_n} W_n^\dagger [i\bar{n}\cdot D_n, iD_\perp^\mu] W_n\right]$

field strength + adjoint Wilson line

$$= A_{n\perp}^\mu - \frac{k_\perp^\mu}{\bar{n}\cdot k}\,\bar{n}\cdot A_n + \cdots$$

[vanishes if $A^\mu \to k^\mu$, g. inv.]

Gauge Symmetry — symmetry trnsfm. must leave us within the EFT

$$U(x) = e^{i\alpha^A(x)T^A}$$

$i\partial^\mu U_n(x) \sim P_n^\mu U_n(x)$ collinear

$i\partial^\mu U_{us}(x) \sim P_{us}^\mu U_{us}(x)$ ultrasoft

- $\xi_n \to U_n \xi_n$ $\quad iD_n^\mu \to U_n iD_n^\mu U_n^\dagger$ for A_n

  $q_{us} \to q_{us}$ [else not ultrasoft], $W_n \to U_n W_n$

- $q_{us} \to U_{us} q_{us}$ $\quad iD_{us} \to \cdots$

  $\xi_n \to U_{us}\xi_n$ $\quad A_n^\mu \to U_{us} A_n^\mu U_{us}^\dagger$, $W_n \to U_{us} W_n U_{us}^\dagger$

$\chi_n = W_n^\dagger \xi_n \to W_n^\dagger \cancel{U_n^\dagger}\cancel{U_n}\xi_n$ $\quad$ protected by g. inv. eg. stays together when we add loop corrections

build operators out of n-collinear gauge invariant building blocks χ_n, $\mathcal{B}_{n\perp}^\mu$

Wilson lines needed to ensure gauge invariance in presence of operators where gluons that only couple in on-shell manner to single colored field.

trades $\bar{n}\cdot A_n \to W_n$

$$W_n^\dagger W_n = \mathbb{1} = W_n W_n^\dagger$$

collinear gauge singlet

$$[i\bar{n}\cdot D_n \; W_n] = 0 \qquad \therefore \quad i\bar{n}\cdot D_n \, W_n \, \Phi = W_n \, i\bar{n}\cdot\partial_n \, \Phi$$

$$W_n^\dagger \, i\bar{n}\cdot D_n \, W_n = i\bar{n}\cdot\partial_n \quad \text{as operator}$$

$$i\bar{n}\cdot D_n = W_n \, i\bar{n}\cdot\partial_n \, W_n^\dagger$$

Hard - Collinear Factorization

$$\mathcal{L}^{hard} = C \otimes \mathcal{O}$$

What do Wilson Coefficients depend on ?

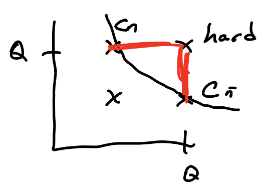
$i\bar{n}\cdot\partial_n \sim \lambda^0$

Allows $\underbrace{C(i\bar{n}\cdot\partial_n)\chi_n}_{\text{gauge inv.}} = \int d\omega \, C(\omega) \, \underbrace{\delta(\omega - i\bar{n}\cdot\partial_n)\chi_n}_{\text{operator} \equiv \chi_{n,\omega}}$

Hard & collinear modes communicate through $\sim \lambda^0$ momenta

constrained by gauge inv.

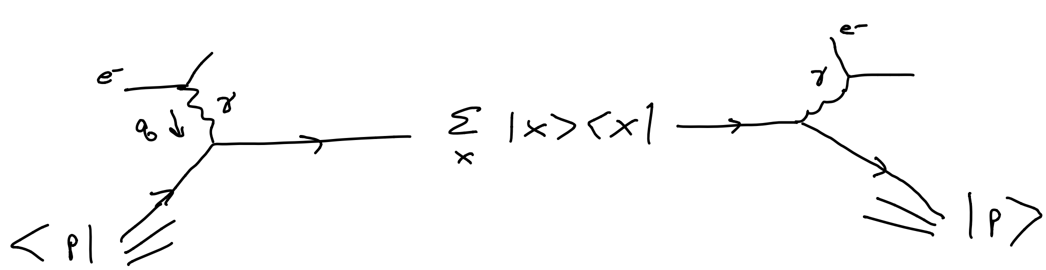
& momentum conservation



DIS

$e^- p \to e^- X$ Inclusive Factorization

[full analysis requires more knowledge, eg $\mathcal{L}$, cover few key parts]



$$q = (0,0,0,Q) = \frac{Q}{2}(\bar{n} - n) \qquad q^2 = -Q^2 \;\; \text{spacelike}$$

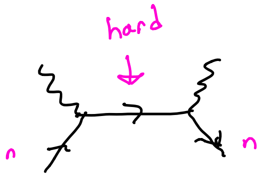
Bjorken $\quad x = \dfrac{Q^2}{2p\cdot q}$ $\qquad$ Breit frame, where proton is n-collinear

$p_X = p + q = \text{hard}$

Proton $p_p^\mu = \frac{n^\mu}{2}\bar{n}\cdot p_p + \frac{\bar{n}^\mu}{2}\underbrace{\frac{M_p^2}{\bar{n}\cdot p_p}}_{\text{small}}$, big $\bar{n}\cdot p_p = \frac{Q}{x} \sim \lambda^0$

$\lambda = \frac{\Lambda_{QCD}}{Q} \ll 1$

QCD



$\longrightarrow$

SCET

$$O_q = \underset{\lambda}{\bar{\chi}_n} \frac{\bar{n}\!\!\!/}{2} \underset{\lambda}{\chi_n}$$

$O \sim \lambda^2$ twist-2

Add arbitrary pert. α_s^k corrections:

also gluon $O_g = \mathcal{B}_{n\perp}^{\mu}\mathcal{B}_{n\perp\mu}$

$$\mathcal{L}_{hard} = \int d\omega\, \cancel{d\omega'}\, C(\omega,\cancel{\omega'},Q)\, \bar{\chi}_n \frac{\bar{n}\!\!\!/}{2}\, \cancel{\delta(\omega' + i\bar{n}\cdot\partial_n)}\, \delta(\omega - i\bar{n}\cdot\partial_n)\, \chi_n$$

forward $\langle p| \cdots |p\rangle$ matrix element fixes $\omega = \omega'$

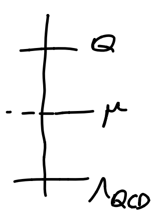
$$\sigma \sim \int d\omega\ \mathrm{Im}\, C(\omega,Q)\ \langle p|\ \bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \delta(\omega - i\bar{n}\cdot\partial_n)\chi_n\ |p\rangle$$

momentum of quark in proton

both dimensionless

$$\sim \int \frac{d\xi}{\xi}\ \underbrace{H\left(\frac{x}{\xi}, \frac{Q}{\mu}, \alpha_s(\mu)\right)}_{\text{Hard}}\ \underbrace{f_{q/p}\left(\xi, \frac{\mu}{\Lambda_{QCD}}\right)}_{\text{Collinear}}, \quad \xi = \frac{\omega}{\bar{n}\cdot p}$$

parton dist'n



$$\frac{Q}{\omega} = \frac{Q}{\xi\,\bar{n}\cdot p} = \frac{x}{\xi}$$

More Hard Operators

power counting, symmetry & matching calc imply O are built from

χ_n

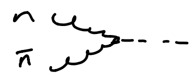
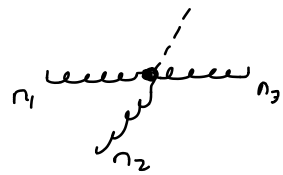
$\mathcal{B}_{n\perp}^{\mu}$

$P_\perp^\mu$

& Soft Fields

often suppressed

[Note: true at any order other collinear ops elliminated by operator identities & eqtns. of motion.]

Example	Operators		
$e^+e^- \to 2$ jets	$\bar\chi_n \, \gamma_\perp^\mu \, \chi_{\bar n}$		Amplitude
$gg \to H$	$\mathcal{B}_{n\perp}^\mu \, \mathcal{B}_{\bar n \perp \mu} \, H$		Ampl.
[quark PDF	$\bar\chi_n \frac{\bar{n}\!\!\!/}{2} \delta(\omega - i\bar n\cdot\partial) \chi_n$		Ampl.^{2}]
gluon PDF	$\mathrm{tr}\left[\mathcal{B}_{n\perp}^\mu \, \delta(\omega - i\bar n\cdot\partial) \mathcal{B}_{n\perp\mu}\right]$		Ampl^2
$pp \to H + 1$-jet	(remove top)		Ampl

$$\bullet \quad \mathcal{B}_{n_1\perp}^{a_1\mu_1} \; \mathcal{B}_{n_2\perp}^{a_2\mu_2} \; \mathcal{B}_{n_3\perp}^{a_3\mu_3} \; H \; T_{\mu_1\mu_2\mu_3} \; (i f^{a_1a_2a_3})$$

↑ no $d^{a_1a_2a_3}$ by charge conjugation

$$\bullet \quad \mathcal{B}_{n_1\perp}^{a\,\mu_1} \; \bar\chi_{n_2}^{\bar\alpha} \; \chi_{n_3}^{\beta} \; H \; T_{\mu_1}^{\alpha\bar\beta}$$

how many operators ?

Helicity basis: natural in SCET since we have direction to use $\hat n$

$$\mathcal{B}_{n\pm}^a \equiv - \epsilon_\mp^\mu(n,\bar n)\, \mathcal{B}_{n\mu}^\perp \,, \qquad \epsilon_\mp = \frac{1}{\sqrt 2}(0,1,\pm i,0)$$

$$J_{n_1n_2\pm}^{\bar\alpha\beta} \propto \epsilon_\mp^\mu(n_1,n_2)\, \bar\chi_{n_1\pm}^{\bar\alpha}\, \gamma_\mu\, \underbrace{\chi_{n_2\pm}^{\beta}}_{\left(\frac{1\pm\gamma_5}{2}\right)\chi_n}$$

Allowed

$\mathcal{B}$	$\mathcal{B}$	$\mathcal{B}$	
+	+	+	
+	+	−	
−	−	+	} Wilson Coeff fixed by Parity
−	−	−	}

$\mathcal{B}$	J	
+	+	
−	+	
+	−	} fixed by charge Conj.
−	−	}

4 non-trivial coefficients

[note: no evenescent operators in leading power SCET due to helicity conservation]

Easy to exploit modern spinor-helicity results.

[see 1508.02397 for more on helicity operators in SCET.]

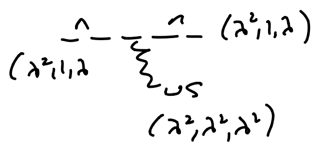
SCET $\mathcal{L}$ SCET$_I$ $(\alpha = 2)$

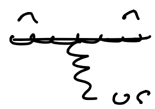
For interactions that are isolated and <u>purely</u> n-collinear or <u>purely</u> ultrasoft we just have full QCD $\mathcal{L}$ for each sector.

usoft: nothing to expand   n-collinear boost everything $\overset{+\ -\ \perp}{(\lambda^2, 1, \lambda)} \to (\lambda, \lambda, \lambda)$ same

<u>Key thing SCET describes is interactions between sectors</u>

For $\mathcal{L}^{(0)}$

- 



usoft leave collinear $\sim$ on-shell

- hard interactions produce collinear quarks with $\not{n}\,\xi_n = 0$
  [hard int. breaks boost arguement]

$$\psi = \left(\frac{\not{n}\bar{\not{n}}}{4} + \frac{\bar{\not{n}}\not{n}}{4}\right)\psi = \xi_n + \varphi_n$$

$$\mathcal{L}_{QCD} = \bar{\psi}\, i\not{D}\,\psi = \bar{\xi}_n \frac{\bar{\not{n}}}{2}\, in\cdot D\, \xi_n + \bar{\varphi}_n \frac{\not{n}}{2}\, i\bar{n}\cdot D\, \varphi_n + \bar{\xi}_n\, i\not{D}_\perp \varphi_n + \bar{\varphi}_n\, i\not{D}_\perp \xi_n$$

e.o.m. $\quad \delta / \delta\bar{\varphi}_n \quad \Rightarrow \quad \varphi_n = \dfrac{1}{i\bar{n}\cdot D}\, \underset{\lambda}{i\not{D}_\perp}\, \dfrac{\bar{\not{n}}}{2}\, \underset{\lambda}{\xi_n}$ smaller than ξ_n for hard production

$$\mathcal{L}_{QCD} = \bar{\xi}_n \left(in\cdot D + i\not{D}_\perp \frac{1}{i\bar{n}\cdot D}\, i\not{D}_\perp \right) \frac{\bar{\not{n}}}{2}\, \xi_n \qquad \underline{\text{still}}\ \ \underline{\text{QCD}}$$

Expand

- couple only to ξ_n in path integral $J\,\xi_n$
- $in\cdot D = \underset{\lambda^2}{in\cdot\partial} + \underset{\lambda^2}{g\,n\cdot A_n} + \underset{\lambda^2}{g\,n\cdot A_s}$ multipole expansion

label comment

$iD_\perp = \underset{\lambda}{i\partial_{n\perp}} + \underset{\lambda}{g A_{n\perp}} + \cdots$ $A_{us}^\perp \ll A_{n\perp}$

$\underline{i\partial_{us}^\perp} \ll \underline{i\partial_n^\perp}$

$i\bar{n}\cdot D = i\bar{n}\cdot\partial_n + g\,\bar{n}\cdot A_n + \cdots$ $\bar{n}\cdot A_{us} \ll \bar{n}\cdot A_n$

$i\bar{n}\cdot\partial_{us} \ll i\bar{n}\cdot\partial_n$

$$\mathcal{L}^{(0)}_{nq} = \bar\xi_n \left(i n\cdot D + i\slashed{D}_{n\perp} \frac{1}{i\bar n\cdot D_n} i\slashed{D}_{n\perp}\right)\frac{\bar{\slashed{n}}}{2}\,\xi_n$$

gives $\dfrac{(\bar{\slashed{n}}/2)}{n\cdot p + \frac{p_\perp^2}{\bar n\cdot p} + i0\,\mathrm{sign}(\bar n\cdot p)}$ ✓

bit more work for particle vs. antiparticle see EFTx

gluons

$$\mathcal{L}^{(0)}_{ng} = \mathcal{L}^{(0)}_{ng}\left[n\cdot D,\ D_{n\perp},\ \bar n\cdot D_n\right] \text{ too}$$

(+ gauge fixing & ghosts)

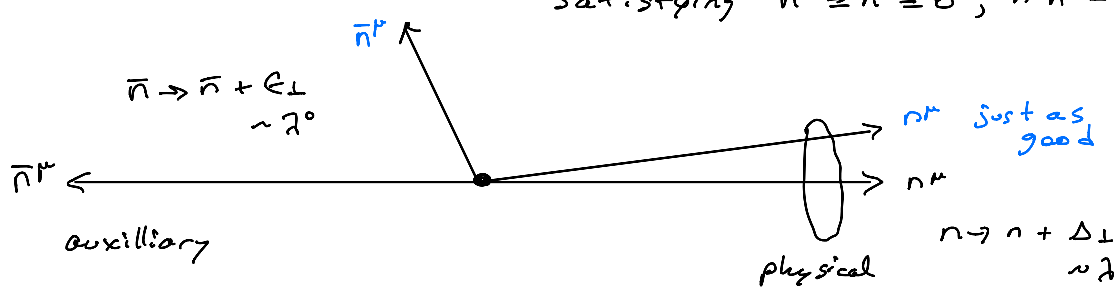
If we drop $n\cdot A_{us}$ these are QCD Lagrangians

<u>Higher Orders</u> eg. $\mathcal{L}^{(1)} = \underset{\lambda}{(\bar\xi_n W_n)}\, \underset{\lambda^2}{i\slashed{D}^{us}_\perp} \left(W_n^\dagger \frac{1}{i\bar n\cdot D_n} \underset{\lambda}{i\slashed{D}_{n\perp}}\frac{\bar{\slashed{n}}}{2}\underset{\lambda}{\xi_n}\right) \quad = \lambda^5$

eg. $\mathcal{L}^{(1)} = \underset{\lambda}{(\bar\xi_n W_n)}\ g\, \underset{\lambda}{\slashed{B}_{n\perp}}\ \underset{\lambda^3}{q_{us}} + \text{h.c.} \quad = \lambda^5$

<u>Gauge Inv</u> ✓

<u>Reparameterization Inv</u> (RPI) freedom to choose n & $\bar n$ satisfying $n^2 = \bar n^2 = 0$, $n\cdot\bar n = 2$



RPI_{III} $\quad n \to \kappa n$, $\quad \bar n \to \dfrac{\bar n}{\kappa}$

Numerator (# n's − # $\bar n$'s) = Denominator (# n's − # $\bar n$'s)

Each collinear sector has its own RPI symmetry

[protects $\mathcal{L}^{(k)}$ coeff from loop corrections, relates operator coeffs.]

$$\mathcal{L}^{(0)}_{SCET_I} = \mathcal{L}^{(0)}_{us} + \sum_n \left(\mathcal{L}^{(0)}_{nq} + \mathcal{L}^{(0)}_{ng}\right) + \mathcal{L}^{(0)}_{Glauber}$$

just full QCD q_{us}, A^μ_{us}

sum over distinct RPI equivalence classes $n_1\cdot n_2 \gg \lambda^2$

extra term for ≥ two collinear directions, <u>only</u> factorization violating term (more later)