Soft-Collinear Effective Theory

I. Stewart

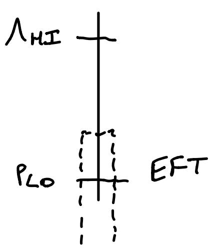
- EFT treatment of Soft & Collinear IR physics for hard collisions in QCD (or decays with large E released)

$\Rightarrow$ jets, energetic hadrons, soft partons / hadrons

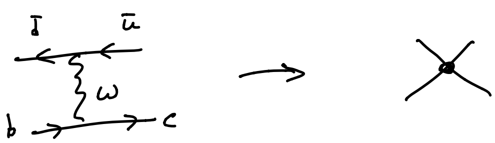
eg. $e^+e^- \to$ 2-jets, $e^-p \to e^- X$ (DIS), $pp \to H$ + 1-jet, $B \to \pi\pi$, jet substructure, ... [many many more]

Concepts: Factorization, Wilson Lines, Sudakov Double Logs, ...

Decoupling Effects from heavy or offshell particles are suppressed / decouple $\quad p_{LO} \ll \Lambda_{HI}$

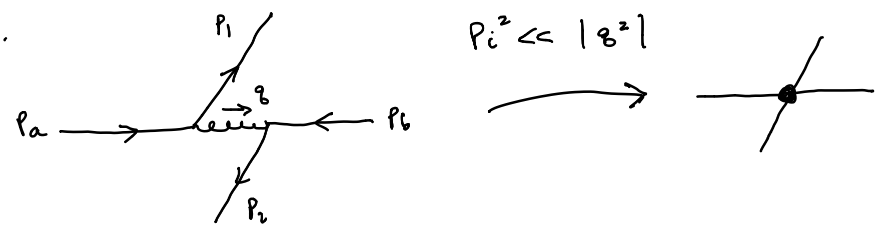


eg



$$\sim \frac{(\bar\psi\psi)(\bar\psi\psi)}{M_W^2} \qquad M_W^2 \gg p_i^2$$

eg.



$$\sim \frac{1}{q^2}(\bar\psi..\psi)(\bar\psi..\psi)$$

say $p_i^2 = 0$ on-shell, $\quad q = p_a - p_1 = n_a E_a - n_1 E_1$

$n_a = (1, \hat{z})$, $n_a^2 = 0$ $\quad q^2 = -2E_aE_1\, n_a\cdot n_1$

$n_1 = (1, \hat{n})$, $n_1^2 = 0$ $\quad = -2E_aE_1(1 - \hat{z}\cdot\hat{n})$

$q^2 \sim Q^2$ "hard" $\quad$ large if energies big & deflection angles large

EFT
- degrees of freedom $\to$ what fields $\quad$ low energy / nearly onshell modes
- symmetries $\to$ constrain operators $\quad$ [Lorentz, Gauge, Global ...]
- expansions $\to$ power counting $\quad$ (importance of operators, leading order description)

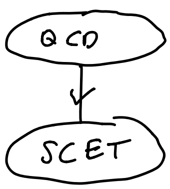
$\mathcal{L}_{EFT} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots$

often in mass dimension of operators, but not so in SCET

(∞ # operators, but only specific subset needed at given order)

- Power counting handles powers $\frac{p_{LO}}{\Lambda_{HI}} \ll 1$
- Renormalization group handles logs $\ln\left(\frac{p_{LO}}{\Lambda_{HI}}\right)$ which may be large as $\ln(\cdot\cdot) \sim 1$

Matching SCET is a "top-down EFT" [Heweak, HQET, NRQCD, SCET, ...]



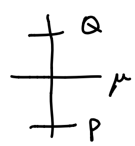
$\Lambda_{QCD}^2, p_{IR}^2 \ll Q^2$ ← hard collision

$$\mathcal{L}^{(k)}_{SCET} = \sum_i C_i(\mu)\, O_i^{(k)}(\mu)$$

Calculate C, construct O

$C_i(\mu)$ ↑ short. dist. (offshell)

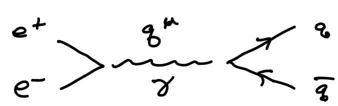
$O_i^{(k)}(\mu)$ ↑ long dist. (~ on-shell)



renormalization scale is cutoff that splits scales

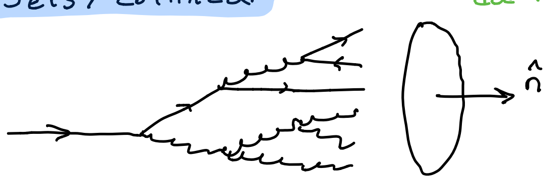
- $\mathcal{L}_{QCD}$ & $\mathcal{L}_{SCET}$ have same IR, differ in UV
- $C_i(\mu)$ does not depend on IR scales (masses in EFT, Λ_{QCD}, IR regulators, ...)

degrees of freedom consider $e^+e^- \to 2$ jets



$Q^2 = q^2$

Jets / collinear due to collinear (& soft) enhancements in QCD



- collimated radiation in direction $\hat{n}$
- $E_{Jet} \sim Q$

Let $n^\mu = (1, \hat{n})$
$\bar{n}^\mu = (1, -\hat{n})$
$n^2 = \bar{n}^2 = 0\,, \quad n\cdot\bar{n} = 2$

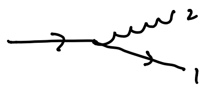
$$p^\mu = \underbrace{\bar{n}\cdot p}_{p^-} \frac{n^\mu}{2} + \underbrace{n\cdot p}_{p^+} \frac{\bar{n}^\mu}{2} + p_\perp^\mu$$

$$p^2 = n\cdot p\, \bar{n}\cdot p + \underbrace{p_\perp^2}_{-\vec{p}_\perp^{\,2}}$$

Collinear ?

1 massless particle : $p^\mu = \bar{n}\cdot p \dfrac{n^\mu}{2}$

2 massless :



$$p_i^\mu = \underbrace{\bar{n}\cdot p_i}_{\sim Q} \frac{n^\mu}{2} + \underbrace{p_{i\perp}^\mu}_{\sim \lambda Q} + \underbrace{n\cdot p_i}_{\sim \lambda^2 Q} \frac{\bar{n}^\mu}{2}$$

$\sim Q$ large; $\sim \lambda Q$, $\lambda \ll 1$, collimated; $\sim \lambda^2 Q$ nearly on-shell

on-shell
$n\cdot p_i = -\dfrac{p_{i\perp}^2}{\bar{n}\cdot p_i}$

$\lambda \ll 1$ — dimensionless power counting parameter

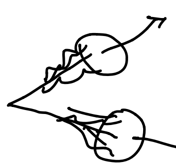
n-Collinear Fields : quark ξ_n, gluon A_n^μ

$$p^\mu \sim Q(\lambda^2, 1, \lambda) \qquad (n\cdot p,\ \bar{n}\cdot p,\ \perp)$$

energetic hadron : $p_\perp \sim \Lambda_{QCD} \Rightarrow \lambda \sim \dfrac{\Lambda_{QCD}}{Q}$ — energetic quarks & gluons confine into single hadron

jet of hadrons: $1 \gg \lambda \gg \dfrac{\Lambda_{QCD}}{Q}$

2-jets

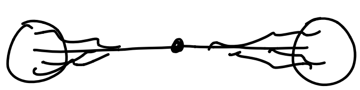


$n_1, \bar{n}_1$ — $\xi_{n_1}, A_{n_1}^\mu$

$n_2, \bar{n}_2$ — $\xi_{n_2}, A_{n_2}^\mu$

back-to-back jets :

$n_2 = \bar{n} = (1, -\hat{n})$
$\bar{n}_2 = n$



$n_1 = n = (1, \hat{n})$
$\bar{n}_1 = \hat{n}$

Soft $\quad p_s^\mu \sim Q\lambda^\alpha$ $\quad$ all components small & homogeneous

Value of α depends on what we measure

eg1 Mass in (large enough) region a, $M_a^2 = \left(\sum_{i\in a} p_i^\mu\right)^2$

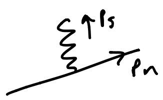
[mass of R=1 jet, hemisphere mass, ...]

n-collinear + n-collinear $\quad (p_n + p_n')^2 = 2\, p_n\cdot p_n' \sim Q^2\lambda^2$

$$\begin{matrix} + & - \\ - & + \\ \perp & \perp \end{matrix}$$

$\therefore$ demand $M_a^2 \sim Q^2\lambda^2 \ll Q^2 \sim E_{jet}^2$ $\quad$ [collimated jet]

collinear + soft



$(p_n+p_s)^2 = 2\,p_n\cdot p_s = \bar n\cdot p_n\; n\cdot p_s + \cdots$ $\quad$ (suppressed) $\sim Q^2\lambda^\alpha$

$\lambda^0 * \lambda^\alpha$

$\therefore\; \alpha = 2$ to contribute "ultrasoft"

eg2 Transverse Momenta, broadening $B_\perp = \sum_{i\in a} |\vec p_i^{\,\perp}| \ll Q$ $\quad \sim\lambda$

Σ collinear ✓

soft $\Rightarrow$ $\alpha = 1$ "soft"

Go to Slides # 1 to # 14 $\quad$ Discussion after # 14 $\quad$ Slides # 15 - 17

trades $\bar n\cdot A_n \to W_n$

$W_n^\dagger W_n = \mathbb{1} = W_n W_n^\dagger$

$[\, i\bar n\cdot D_n\, W_n\,] = 0$ $\quad \therefore \quad i\bar n\cdot D_n\, W_n\, \Phi = W_n\, i\bar n\cdot\partial_n\, \Phi$ $\quad$ (collinear gauge singlet)

$W_n^\dagger\, i\bar n\cdot D_n\, W_n = i\bar n\cdot\partial_n$ $\quad$ as operator

$i\bar n\cdot D_n = W_n\, i\bar n\cdot\partial_n\, W_n^\dagger$