Introduction to the Soft - Collinear Effective Theory

Lecture II

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Methods of Effective Field Theory & Lattice Field Theory
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Outline (Lecture I)

- EFT concepts
  Intro to SCET
  SCET degrees of freedom

  } Done on the Board
  (See separate lecture notes.)

- SCETI , momentum scales and regions
- Field power counting in SCET
- Wilson lines, W, from off shell propagators
- Gauge Symmetry
- Hard-Collinear Factorization
- eg. Deep Inelastic Scattering

Outline (Lecture II)

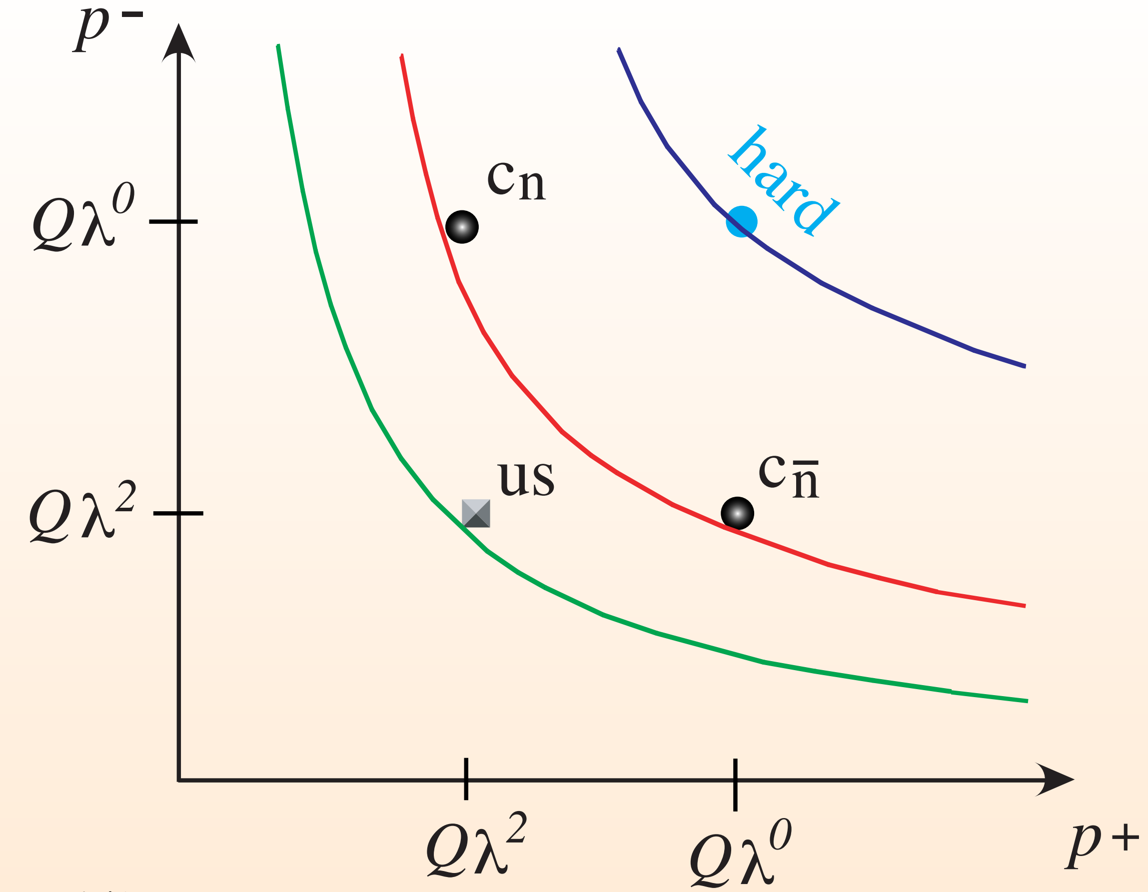
- Review from Lecture I
- Hard Operator Examples
- SCET Lagrangian

} Done on the Board
(See separate lecture notes.)

- Sudakov Resummation from RGE
  (& One-loop Matching Example)
- Soft-Collinear Factorization
- $e^+e^- \rightarrow$ dijets & Factorization
- Quasi Parton Distribution Function

} On board (separate lecture notes.)

$\mathrm{SCET_I}$ summary



usoft & collinear modes

$q_{us} \sim \lambda^3$ $\qquad$ $\xi_n \sim \lambda$

$A^\mu_{us} \sim \lambda^2$ $\qquad$ $(A_n^+, A_n^-, A_n^\perp) \sim (\lambda^2, 1, \lambda) \sim p_c^\mu$

covariant derivatives:

$$iD_\perp^{n\mu} = i\partial^\mu_{n\perp} + gA_n^{\perp\mu} \qquad iD^\mu_{us} = i\partial^\mu + gA^\mu_{us}$$

$$i\bar n\cdot D_n = i\bar n\cdot\partial_n + g\bar n\cdot A_n$$

LO $\mathrm{SCET_I}$ Lagrangian:

$$\mathcal{L}^{(0)}_{n\xi} = \bar\xi_n \Big\{ n\cdot iD_{us} + gn\cdot A_n + i\not{D}^n_\perp \frac{1}{i\bar n\cdot D_n} i\not{D}^n_\perp \Big\} \frac{\bar{\not{n}}}{2} \xi_n$$

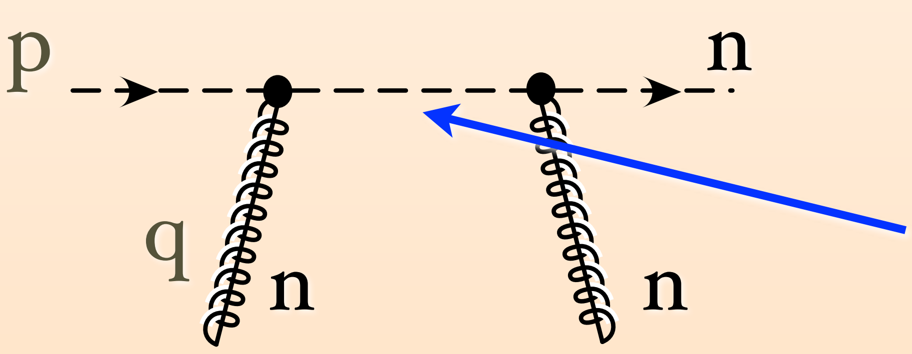
$$\mathcal{L}^{(0)}_{ng} = \mathcal{L}^{(0)}_{ng}(D^\mu_{n\perp}, \bar n\cdot D_n, in\cdot D_{us} + gn\cdot A_n)\,, \qquad \mathcal{L}^{(0)}_{us} = \mathcal{L}^{\rm QCD}(q_{us}, A^\mu_{us})$$

Properties of $\mathcal{L}^{(0)}_{n\xi} = \bar{\xi}_n \Big\{ n\cdot iD_{us} + g n\cdot A_n + i\not{D}^n_\perp \frac{1}{i\bar{n}\cdot D_n} i\not{D}^n_\perp \Big\} \frac{\not{\bar{n}}}{2}\xi_n$

1) has particles and antiparticles, pair creation & annihilation

$$\frac{i\not{n}}{2}\frac{\theta(\bar{n}\cdot p)}{n\cdot p + \frac{p_\perp^2}{\bar{n}\cdot p} + i\epsilon} + \frac{i\not{n}}{2}\frac{\theta(-\bar{n}\cdot p)}{n\cdot p + \frac{p_\perp^2}{\bar{n}\cdot p} - i\epsilon} = \frac{i\not{n}}{2}\frac{\bar{n}\cdot p}{n\cdot p\,\bar{n}\cdot p + p_\perp^2 + i\epsilon} = \frac{i\not{n}}{2}\frac{\bar{n}\cdot p}{p^2 + i\epsilon}$$

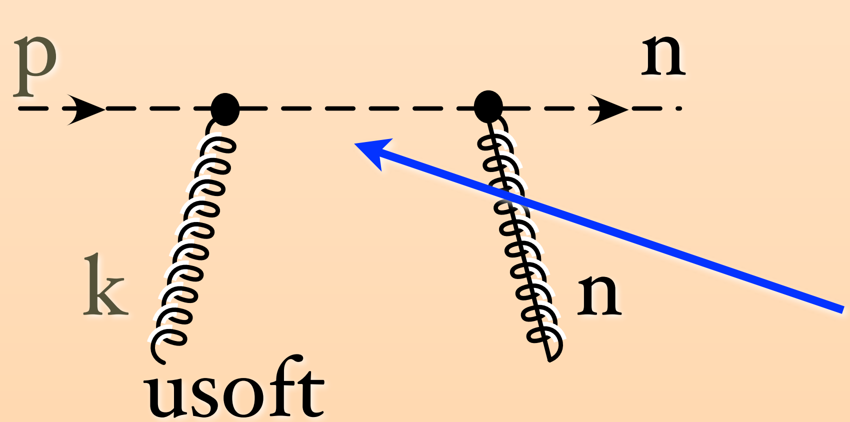
2) all components of A^μ_n couple to ξ_n



$$\frac{i\not{n}}{2}\frac{\bar{n}\cdot(p+q)}{(p+q)^2 + i\epsilon}$$

all components of
p & q appear

3) only $n\cdot A_{us}$ couple at LO, only depends on $n\cdot k_{us}$ momentum



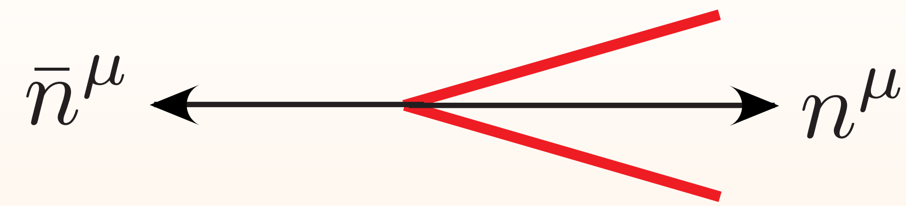
$$\frac{i\not{n}}{2}\frac{\bar{n}\cdot p}{\bar{n}\cdot p\, n\cdot(p+k) + p_\perp^2 + i\epsilon} = \frac{i\not{n}}{2}\frac{\bar{n}\cdot p}{\bar{n}\cdot p\, n\cdot k + p^2 + i\epsilon} = \frac{i\not{n}}{2}\frac{1}{n\cdot k + i\epsilon}$$

onshell $p^2 = 0$

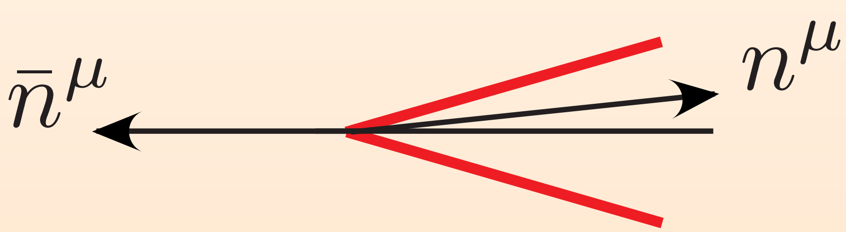
eikonal

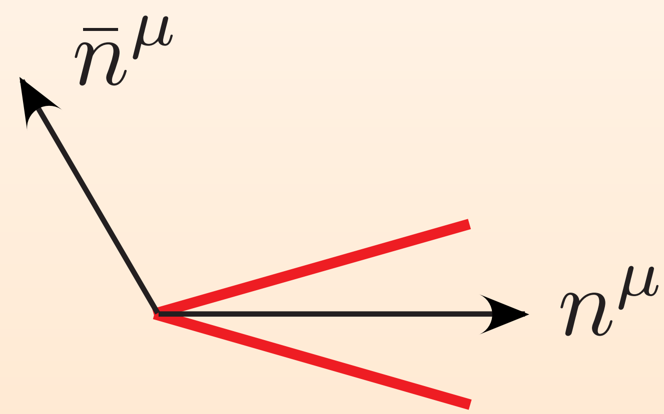
Symmetries: Gauge Invariance ✓

Reparameterization Invariance (RPI)



$n\ ,\ \ \bar{n}$ break Lorentz invariance, restored within collinear cone by RPI, three types





longitudinal boost

$$\text{(I)} \begin{cases} n_\mu \to n_\mu + \Delta_\mu^\perp \\ \bar{n}_\mu \to \bar{n}_\mu \end{cases} \qquad \text{(II)} \begin{cases} n_\mu \to n_\mu \\ \bar{n}_\mu \to \bar{n}_\mu + \varepsilon_\mu^\perp \end{cases} \qquad \text{(III)} \begin{cases} n_\mu \to (1+\alpha)\, n_\mu \\ \bar{n}_\mu \to (1-\alpha)\, \bar{n}_\mu \end{cases}$$

$\Delta_\mu^\perp \sim \lambda$ $\qquad$ $\varepsilon_\mu^\perp \sim \lambda^0$ $\qquad$ $\alpha \sim \lambda^0$

unique

$$\Rightarrow \quad \mathcal{L}_{n\xi}^{(0)} = \bar{\xi}_n \Big\{ n \cdot iD_{us} + g n \cdot A_n + i\not{D}_\perp^n \frac{1}{i\bar{n} \cdot D_n} i\not{D}_\perp^n \Big\} \frac{\not{\bar{n}}}{2} \xi_n$$

Together:

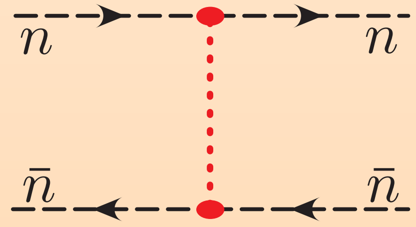
$$\mathcal{L}^{(0)}_{\mathrm{SCET_I}} = \mathcal{L}^{(0)}_{us} + \sum_n \left(\mathcal{L}^{(0)}_{n\xi} + \mathcal{L}^{(0)}_{ng} \right) + \mathcal{L}^{(0)}_{\mathrm{Glauber}}$$

Full QCD
for q_{us}, A^{μ}_{us}

sum over distinct RPI
equivalence classes
$n_1 \cdot n_2 \gg \lambda^2$

extra term encoding
Glauber gluon exchange,
only factorization
violating term

$$\mathcal{L}^{(0)}_{\mathrm{Glauber}}(\xi_{n_1}, A_{n_1}, \xi_{n_2}, A_{n_2}, \ldots, q_s, A_s)$$



$\frac{1}{k_\perp^2}$

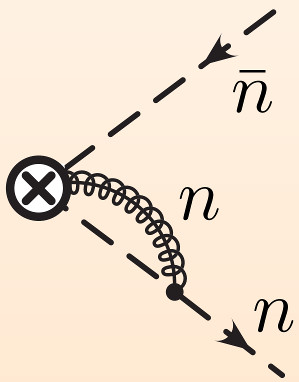
see arXiv:1601.04695

Sudakov Logs & RGE (Renormalization Group Equations)

UV renormalization in SCET

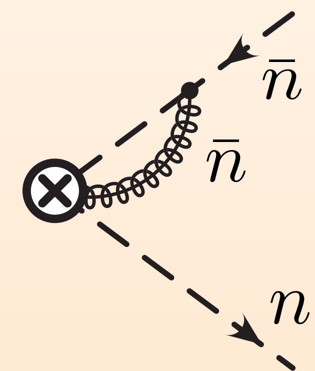
eg. $e^+e^- \to$ dijets $\qquad \bar\chi_n \gamma_\perp^\mu \chi_{\bar n} = (\bar\xi_n W_n)\gamma_\perp^\mu (W_{\bar n}^\dagger \xi_{\bar n})$

(Feynman gauge, UV: $d = 4 - 2\epsilon$, IR: $p^2 \neq 0,\ \bar p^2 \neq 0$)

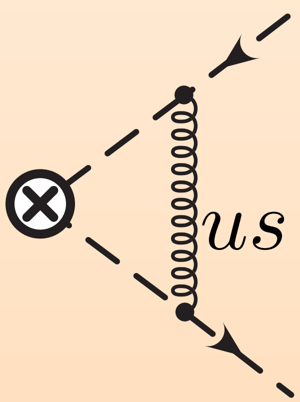


$$= \frac{\alpha_s C_F}{4\pi}\Big[\frac{2}{\epsilon^2} + \frac{2}{\epsilon} - \frac{2}{\epsilon}\ln\Big(\frac{-p^2}{\mu^2}\Big) + \dots\Big]$$

$$\int \frac{d^dk}{(2\pi)^d}\frac{\bar n\cdot(k+p)}{\bar n\cdot k\,(p+k)^2 k^2} \text{ - (0-bin)}$$

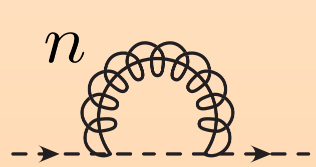


= same with $p^2 \leftrightarrow \bar p^2$

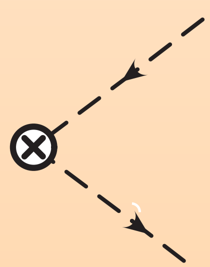


$$= \frac{\alpha_s C_F}{4\pi}\Big[-\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\Big(\frac{(-p^2)(-\bar p^2)}{(-Q^2)\mu^2}\Big) + \dots\Big]$$

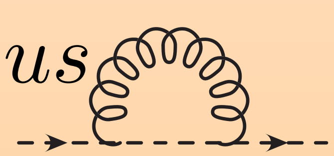
$$\int \frac{d^dk}{(2\pi)^d}\frac{n\cdot\bar n}{(n\cdot k + \frac{p^2}{Q})(\bar n\cdot k + \frac{\bar p^2}{Q})k^2}$$



$\times$



$$= -\frac{\alpha_s C_F}{4\pi}\frac{1}{\epsilon}$$



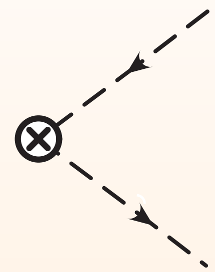
$= 0$

$$\text{sum} = \frac{\alpha_s C_F}{4\pi}\Big[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\Big(\frac{\mu^2}{-Q^2 - i0}\Big) + \frac{3}{\epsilon} + \dots\Big]$$

$$\text{sum} = \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-Q^2 - i0}\right) + \frac{3}{\epsilon} + \ldots\right]$$

$\overline{\text{MS}}$
counterterm

$$(Z_C - 1) \times$$



$$= \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-Q^2 - i0}\right) - \frac{3}{\epsilon} + \ldots\right]$$

$$C^{\text{bare}} = Z_C\, C(\mu)$$

RGE:

$$0 = \mu\frac{d}{d\mu}\,C^{\text{bare}} = \left[\mu\frac{d}{d\mu}\,Z_C\right]C(\mu) + Z_C\left[\mu\frac{d}{d\mu}\,C(\mu)\right] \quad \Rightarrow \quad \mu\frac{d}{d\mu}\,C(\mu) = \gamma_C\, C(\mu)$$

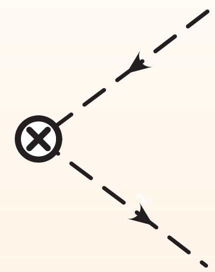
$$\gamma_C = \left(-Z_C^{-1}\right)\mu\frac{d}{d\mu}\,Z_C = (-1)\frac{C_F}{4\pi}\left[(-2\epsilon\,\alpha_s)\left(\cancel{\frac{-2}{\epsilon^2}} - \frac{2}{\epsilon}\ln\frac{\mu^2}{-Q^2} - \frac{3}{\epsilon}\right) + \alpha_s\left(\cancel{\frac{-4}{\epsilon}}\right)\right]$$

$$\mu\frac{d}{d\mu}\alpha_s = -2\epsilon\,\alpha_s + \ldots$$

$$= -\frac{\alpha_s(\mu)}{4\pi}\left[4C_F\ln\frac{\mu^2}{-Q^2} + 6C_F\right] \qquad \text{finite}$$

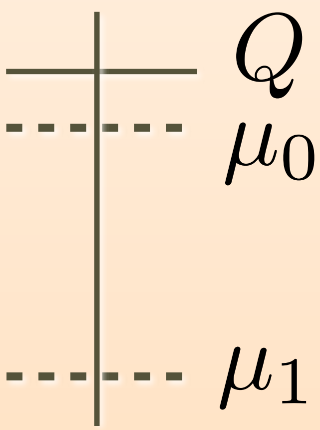
$$\text{sum} = \frac{\alpha_s C_F}{4\pi}\left[\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-Q^2 - i0}\right) + \frac{3}{\epsilon} + \ldots\right]$$

$\overline{\text{MS}}$ counterterm

$(Z_C - 1) \times$  $= \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\ln\left(\frac{\mu^2}{-Q^2 - i0}\right) - \frac{3}{\epsilon} + \ldots\right]$

$$C^{\text{bare}} = Z_C\, C(\mu)$$

RGE: square the amplitude: $H = \left|C(\mu)\right|^2$



$$\mu\frac{d}{d\mu}H(Q,\mu) = \left(\gamma_C + \gamma_C^*\right)H(Q,\mu) = -\frac{\alpha_s(\mu)}{2\pi}\Big[\underbrace{8C_F\ln\frac{\mu}{Q}} + \underbrace{6C_F}\Big]H(Q,\mu)$$

leading log (LL) term — needed at NLL

$$H(Q,\mu_1) = H(Q,\mu_0)\exp\left[-\#\,\alpha_s\ln^2\frac{\mu_1}{Q} + \ldots\right]$$ frozen coupling

$$H(Q,\mu_1) = H(Q,\mu_0)\exp\left[-\#\,\frac{1}{\alpha_s(\mu_0)}f\left(\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_0)}\right) + \ldots\right]$$ running coupling

Complete Solution derived in Homework

Sudakov Form Factor

$\bar{\chi}_n\gamma_\perp^\mu\chi_{\bar{n}}$ restricts radiation, Sudakov = no emission probability

One-Loop Matching Calculation

IR: $p^2 = \bar{p}^2 \neq 0$

both theories

QCD $\qquad \mathcal{L}_{\rm QCD} \quad + \quad J = \bar{\psi}\gamma^\mu\psi$

↓

SCET $\qquad \mathcal{L}^{(0)}_{\rm SCET} \quad + \quad C\ \bar{\chi}_n \gamma_\perp^\mu \chi_{\bar{n}}$

find C at $\mathcal{O}(\alpha_s)$

$$\big(\text{1-loop ren. QCD}\big) - \big(\text{1-loop ren. SCET}\big) = C^{\rm 1loop} \langle \bar{\chi}_n \gamma_\perp^\mu \chi_{\bar{n}} \rangle_{\rm tree}$$

One-Loop Matching Calculation

IR: $p^2 = \bar{p}^2 \neq 0$

both theories

QCD → SCET

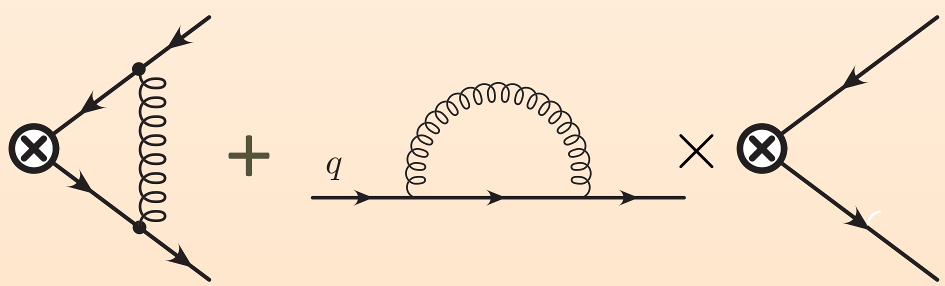
QCD: $\mathcal{L}_{\rm QCD} \quad + \quad J = \bar{\psi}\gamma^\mu\psi$

SCET: $\mathcal{L}^{(0)}_{\rm SCET} \quad + \quad C\, \bar{\chi}_n \gamma^\mu_\perp \chi_{\bar{n}}$

find C at $\mathcal{O}(\alpha_s)$

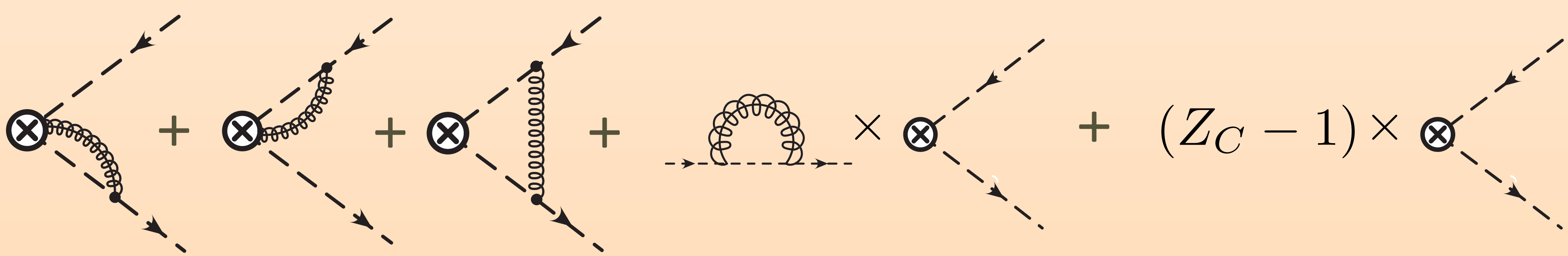
$$(\text{1-loop ren. QCD}) - (\text{1-loop ren. SCET}) = C^{\rm 1loop}\langle \bar{\chi}_n \gamma^\mu_\perp \chi_{\bar{n}}\rangle_{\rm tree}$$

QCD



$$= \frac{\alpha_s C_F}{4\pi}\left[-2\ln^2\frac{p^2}{Q^2} - 3\ln\frac{p^2}{Q^2} - 1 - \frac{2\pi^2}{3}\right]$$

SCET



$$= \frac{\alpha_s C_F}{4\pi}\left[2\ln^2\frac{\mu^2}{-p^2} + 3\ln\frac{\mu^2}{-p^2} - \ln^2\left(\frac{\mu^2 Q^2}{-p^4}\right) + 7 - \frac{5\pi^2}{6}\right]$$

collinear
soft

$$= \frac{\alpha_s C_F}{4\pi}\left[\ln^2\frac{\mu^2}{-Q^2} - 2\ln^2\frac{p^2}{Q^2} - 3\ln\frac{p^2}{Q^2} + 3\ln\frac{\mu^2}{-Q^2} + 7 - \frac{5\pi^2}{6}\right]$$

IR divergences match

One-Loop Matching Calculation

$$\mathrm{QCD} - \mathrm{SCET} = \frac{\alpha_s C_F}{4\pi}\left[-\ln^2\frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$$

$$\Rightarrow \quad C(Q,\mu) = 1 + \frac{\alpha_s(\mu) C_F}{4\pi}\left[-\ln^2\frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$$

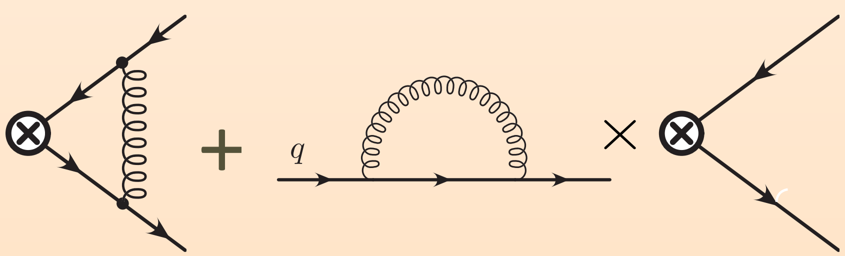
One-Loop Matching Calculation

$$\mathrm{QCD} - \mathrm{SCET} = \frac{\alpha_s C_F}{4\pi}\left[-\ln^2 \frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$$

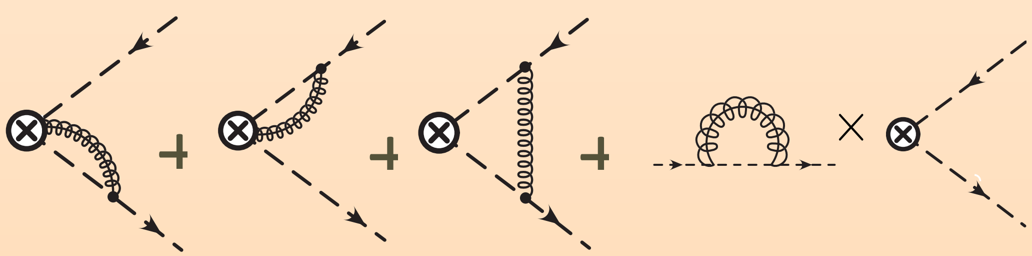
$$\Rightarrow \quad C(Q,\mu) = 1 + \frac{\alpha_s(\mu) C_F}{4\pi}\left[-\ln^2 \frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$$

Once we know how this works, there is a much easier way to get this answer.

Result for C is independent of our choice of IR regulator. Use dim.reg. for IR too.

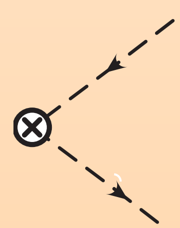


$$= \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon_{\mathrm{IR}}^2} - \frac{2}{\epsilon_{\mathrm{IR}}}\ln\frac{\mu^2}{-Q^2} - \frac{3}{\epsilon_{\mathrm{IR}}} - \ln^2\frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$$



$$\propto \frac{1}{\epsilon_{\mathrm{UV}}^2} - \frac{1}{\epsilon_{\mathrm{IR}}^2}\,,\quad \frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}$$

vanish for $\epsilon_{\mathrm{UV}} = \epsilon_{\mathrm{IR}}$

$(Z_C - 1)\times$ 

$$= \frac{\alpha_s C_F}{4\pi}\left[-\frac{2}{\epsilon_{\mathrm{UV}}^2} - \frac{2}{\epsilon_{\mathrm{UV}}}\ln\left(\frac{\mu^2}{-Q^2 - i0}\right) - \frac{3}{\epsilon_{\mathrm{UV}}} + \ldots\right]$$

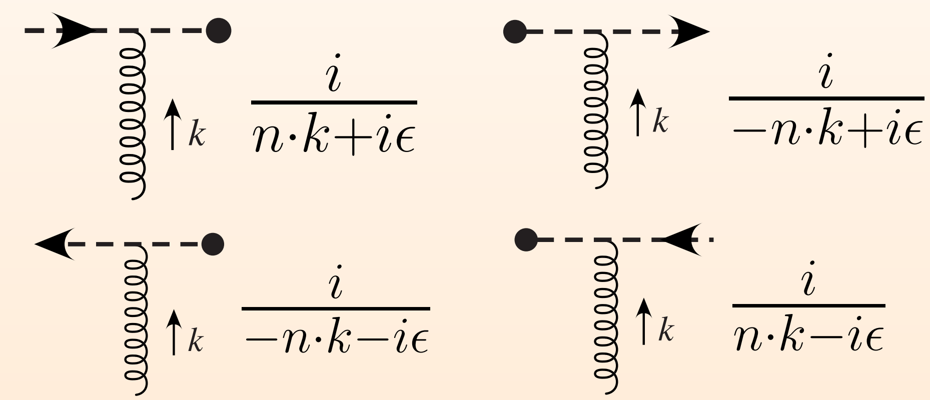
$\mathrm{QCD} - \mathrm{SCET} = \frac{\alpha_s C_F}{4\pi}\left[-\ln^2 \frac{\mu^2}{-Q^2} - 3\ln\frac{\mu^2}{-Q^2} - 8 + \frac{\pi^2}{6}\right]$ = same as above = IR finite part of QCD calculation with this regulator

Ultrasoft - Collinear Factorization

Multipole Expansion:

$$\mathcal{L}_c^{(0)} = \bar{\xi}_n \Big\{ n\cdot iD_{us} + g n\cdot A_n + i\not{D}_\perp^c \frac{1}{i\bar{n}\cdot D_c} i\not{D}_\perp^c \Big\} \frac{\not{\bar{n}}}{2} \xi_n$$

usoft gluons have eikonal Feynman rules and induce eikonal propagators



Field Redefinition:

$$\xi_n \longrightarrow Y_n \xi_n \;, \quad A_n \longrightarrow Y_n A_n Y_n^\dagger$$

$$Y_n(x) = P \exp\Big(ig \int_{-\infty}^{0} ds\, n\cdot A_{us}(x+ns) \Big)$$

$$n\cdot D_{us} Y_n = 0, \quad Y_n^\dagger Y_n = 1$$

gives $\mathcal{L}_{n\xi}^{(0)} = \bar{\xi}_n \Big\{ n\cdot iD_{us} + \ldots \Big\} \frac{\not{\bar{n}}}{2} \xi_n \implies \bar{\xi}_n \Big\{ n\cdot iD_n + i\not{D}_{n\perp} \frac{1}{i\bar{n}\cdot D_n} i\not{D}_{n\perp} \Big\} \frac{\not{\bar{n}}}{2} \xi_n$

similar for $\mathcal{L}_{ng}^{(0)}$

Moves all usoft gluons to operators, simplifies cancellations

Field Theory gives the same results pre- and post- field redefinition, but the organization is different

$$\xi_n \longrightarrow Y_n \xi_n$$

also $W_n \to Y_n W_n Y_n^\dagger$

Ultrasoft - Collinear Factorization:

eg1. $\bar{\chi}_n \gamma_\perp^\mu \chi_{\bar{n}} \implies \bar{\chi}_n \left(Y_n^\dagger Y_{\bar{n}} \right) \gamma_\perp^\mu \chi_{\bar{n}}$

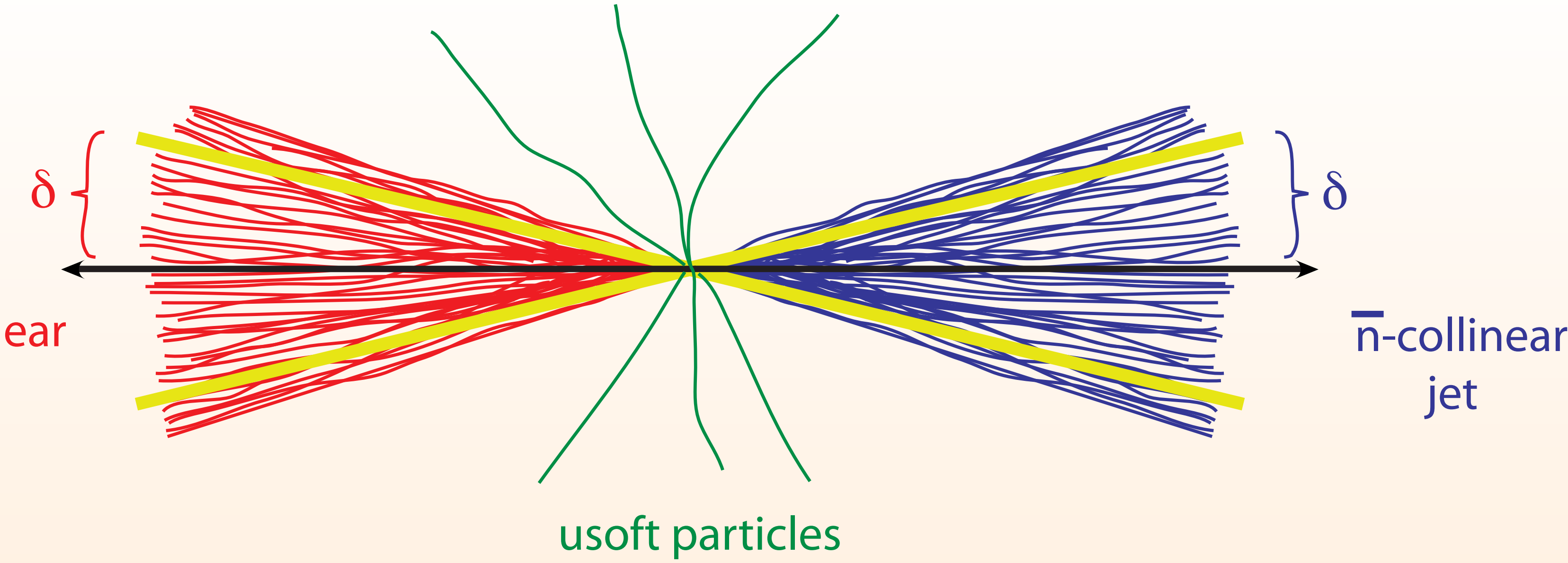
usoft-collinear factorization is simple in SCET

eg2. $\bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \chi_n \implies \bar{\chi}_n \left(Y_n^\dagger Y_n \right) \frac{\bar{n}\!\!\!/}{2} \chi_n = \bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \chi_n$

color transparency

note: not upset by $\delta(\omega - i\bar{n}\cdot\partial_n)$
since ultrasoft gluons carry no $i\bar{n}\cdot\partial_n \sim \lambda^0$ momenta

eg. $e^+e^- \to 2$ jets



event shape in two jet region

$$\frac{d\sigma}{de} = \frac{1}{Q^2} \sum_X Ł_{\mu\nu} \langle 0 | J^{\dagger\nu}(0) | X \rangle \langle X | J^{\mu}(0) | 0 \rangle \delta(e - e(X)) \delta^4(q - p_X)$$

SCET$_\mathrm{I}$

$$J^{(0)} = \int d\omega d\bar{\omega}\, C(\omega, \bar{\omega})\, \bar{\chi}_{n,\omega} \Gamma \chi_{\bar{n},\bar{\omega}}$$

$$= \int d\omega d\bar{\omega}\, C(\omega, \bar{\omega})\, \bar{\chi}_{n,\omega}\, Y_n^\dagger \Gamma Y_{\bar{n}}\, \chi_{\bar{n},\bar{\omega}}$$

$$\chi_{n,\omega} = \delta(\omega - i\bar{n} \cdot \partial_n) \chi_n$$

$$|X\rangle = |X_n X_{\bar{n}} X_{us}\rangle$$

$$\frac{d\sigma}{de} = \frac{1}{Q^2}\sum_{X_{us},X_{\bar n},X_n} \text{Ł}_{\mu\nu}\int [d\omega_i] C(\omega,\bar\omega) C(\omega',\bar\omega') \langle 0|(\tilde Y_{\bar n}^\dagger \Gamma \tilde Y_n)|X_{us}\rangle\langle X_{us}|(Y_n^\dagger \Gamma Y_{\bar n})|0\rangle$$
$$\langle 0|\bar\chi_{\bar n,\bar\omega'}|X_{\bar n}\rangle\langle X_{\bar n}|\chi_{\bar n,\bar\omega}|0\rangle\langle 0|\chi_{n,\omega'}|X_n\rangle\langle X_n|\bar\chi_{n,\omega}|0\rangle\delta(e-e(X))\delta^4(q-p_X)$$

should specify “e” to go further. One example is thrust: $\tau = 1 - T \ll 1$

$$\frac{d\sigma}{d\tau} = \sigma_0 |C(Q,\mu)|^2 \int d\ell^+\, d\ell^- ds\, ds'\, J(s-Q\ell^+,\mu) J(s'-Q\ell^-,\mu) S(\ell^-,\ell^+,\mu)\delta\Big(\tau - \frac{s+s'}{Q^2}\Big)$$

hard perturbative corrections

perturbative jet functions

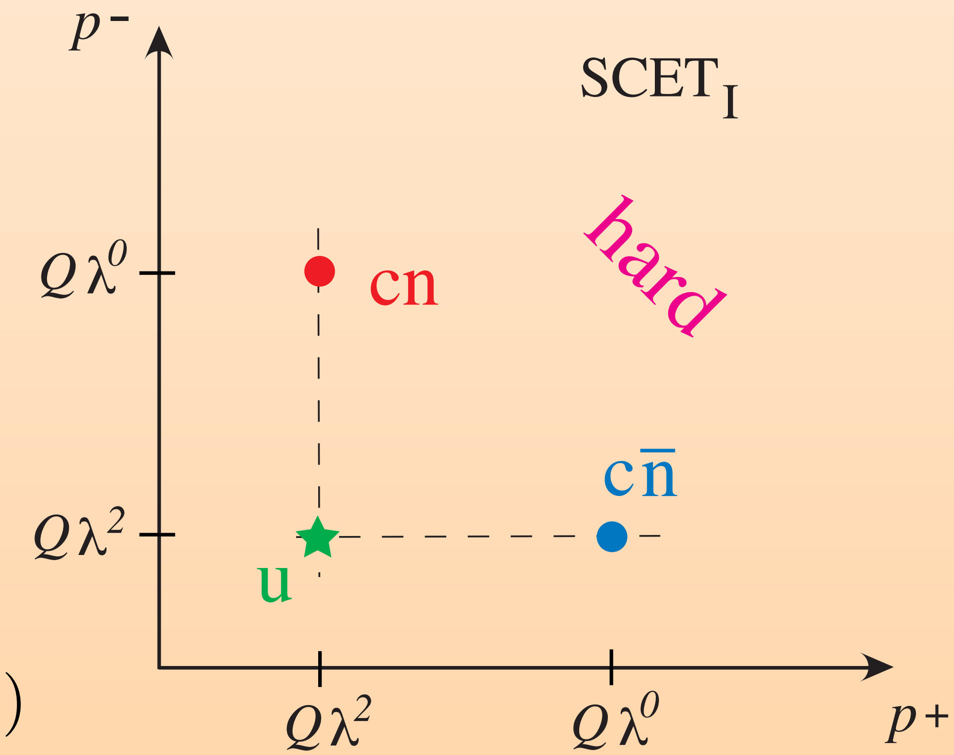
soft function

Homework:
Compute the jet function at one-loop

- sum large $\alpha_s \ln^2\tau$ terms with RGE of C, J, S

- dominant nonperturbative hadronization corrections contained in S:

$$S(\ell^+,\ell^-,\mu) = \int dk\, dk'\, S^{\rm pert}(\ell^+ - k, \ell^- - k', \mu)\, F(k,k')$$



Non-perturbative Factorization:

$$d\sigma = f_a f_b \otimes \hat{\sigma} \otimes F$$

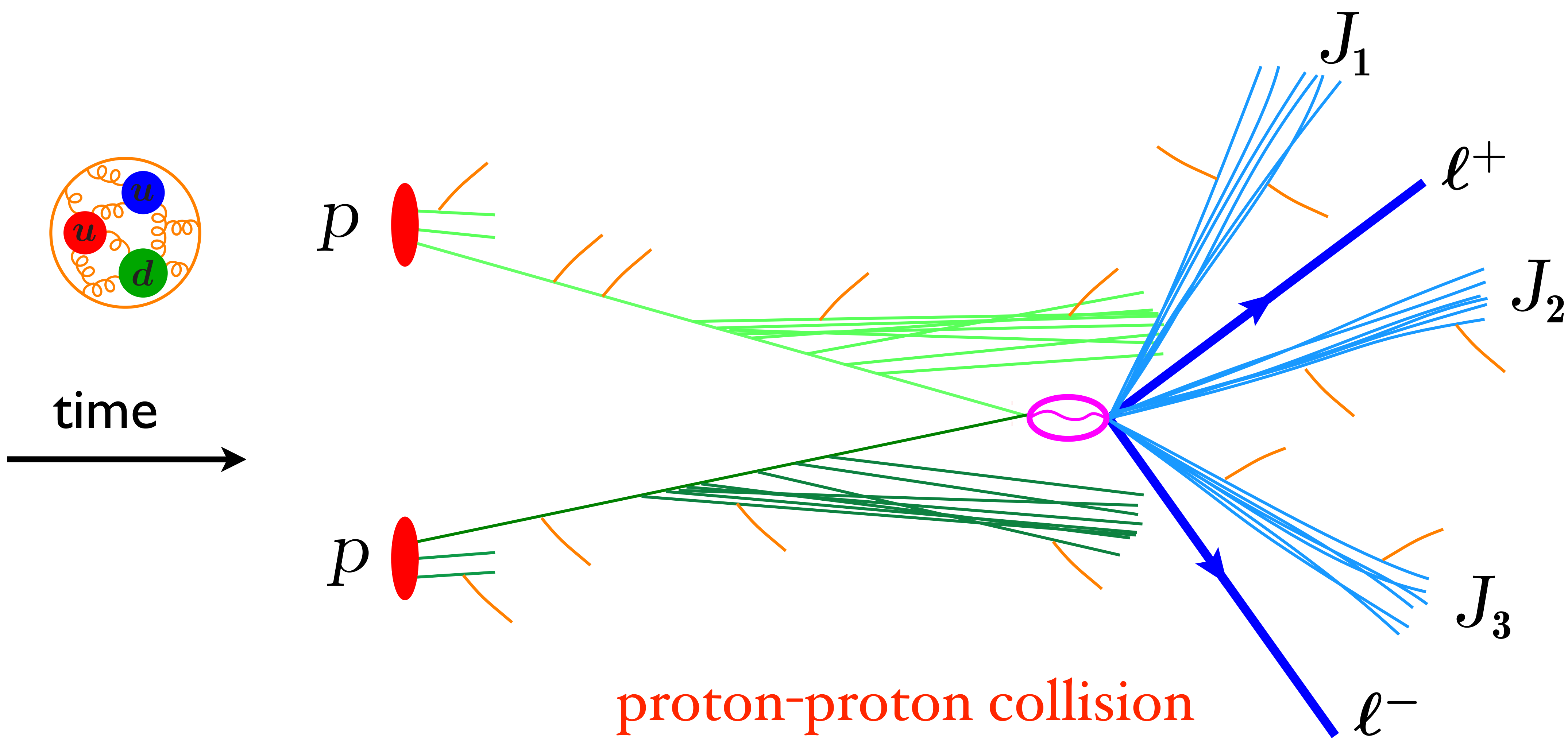
perturbative partonic cross section

parton distributions

hadronization (eg. frag. functions)

eg. Inclusive Higgs production $\qquad pp \to \text{Higgs} + \text{anything}$

$$d\sigma = \int dY \sum_{i,j} \int \frac{d\xi_a}{\xi_a} \frac{d\xi_b}{\xi_b} f_i(\xi_a,\mu) f_j(\xi_b,\mu)\; H_{ij}^{\text{incl}}\Big(\frac{m_H e^{Y}}{E_{\text{cm}}\xi_a}, \frac{m_H e^{-Y}}{E_{\text{cm}}\xi_b}, m_H, \mu\Big)$$



proton-proton collision

Perturbative Factorization: for multi-scale problems

beam		hard		jet		pert. soft

$$\hat{\sigma}_{\rm fact} = \mathcal{I}_a \mathcal{I}_b \otimes H \otimes \prod_i J_i \otimes S$$

μ_B μ_H μ_J μ_S