Lecture #2 — ① pg 5–7 Handwritten ② Slides, Lecture 2 1–20 ③ pg 8–9 Handwritten

More Hard Operators

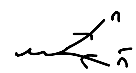
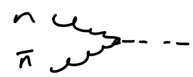
power counting, symmetry & matching imply $\mathcal{O}$ built from

χ_n, $\mathcal{B}_{n\perp}^{\mu}$

$P_\perp^\mu$ } often suppressed

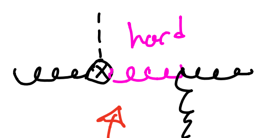
& Soft Fields

[Note: true at any order
other collinear ops
elliminated by operator identities
& eqtns. of motion.]

Example	Operators		
$e^+e^- \to 2$ jets	$\bar{\chi}_n \, \gamma_\perp^\mu \, \chi_{\bar n}$		Amplitude
$gg \to H$	$\mathcal{B}_{n\perp}^{\mu} \, \mathcal{B}_{\bar n \perp \mu} \, H$		Ampl.
gluon PDF	$\mathrm{tr}\left[\mathcal{B}_{n\perp}^{\mu} \, \delta(\omega - i\bar n\cdot\partial) \, \mathcal{B}_{n\perp\mu}\right]$		Ampl^2

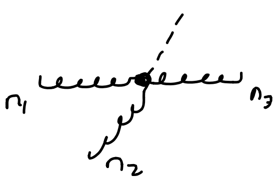
$pp \to H + 1$-jet (remove top) & $Q \sim M_H$ modes

$gg \to Hg$



$qg \to Hq$, $q\bar q \to Hg$

Amplitude



$$\mathcal{B}_{n_1\perp}^{a_1\mu_1} \, \mathcal{B}_{n_2\perp}^{a_2\mu_2} \, \mathcal{B}_{n_3\perp}^{a_3\mu_3} \, H \, T_{\mu_1\mu_2\mu_3} \, (i f^{a_1 a_2 a_3})$$

how many operators?

↑ no $d^{a_1 a_2 a_3}$ by charge conjugation

Helicity basis: natural in SCET since we have direction to use $\hat n$

$$\mathcal{B}_{n\pm}^{a} \equiv - \epsilon_{\mp}^{\mu}(n,\bar n) \, \mathcal{B}_{n\mu}^{\perp}, \quad \epsilon_{\mp} = \frac{1}{\sqrt{2}}(0,1,\pm i,0)$$

Allowed

$\mathcal{B}$	$\mathcal{B}$	$\mathcal{B}$	
+	+	+	
+	+	−	
−	−	+	} Wilson Coeff fixed by Parity
−	−	−	

2 non-trivial $C(\mu)$'s

[note: no evanescent operators in leading power SCET due to helicity conservation]

Easy to exploit modern spinor-helicity results.

[see 1508.02397 for more on helicity operators in SCET.]

SCET $\mathcal{L}$

SCET$_I$ $(\alpha = 2)$

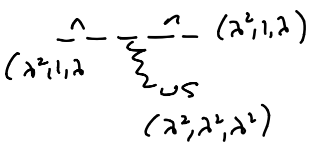
- For isolated interactions which are purely n-collinear or purely ultrasoft $\Longrightarrow$ just full QCD $\mathcal{L}$ for each sector.

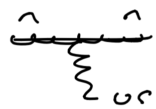
usoft: nothing to expand

n-collinear: boost everything $\overset{+\ -\ \perp}{(\lambda^2, 1, \lambda)} \to (\lambda, \lambda, \lambda)$ some

- SCET describes interactions between sectors

For $\mathcal{L}^{(0)}$

1) 



usoft leave collinear $\sim$ on-shell

2) hard interactions produce collinear quarks with $\not{n}\,\xi_n = 0$

[hard int. breaks boost arguement]

$$\mathcal{L}^{(0)}_{SCET} = \mathcal{L}^{(0)}_{hard} + \mathcal{L}^{(0)}$$ ← dynamics, find this

$$\psi = \left(\frac{\not{n}\bar{\not{n}}}{4} + \frac{\bar{\not{n}}\not{n}}{4}\right)\psi = \xi_n + \varphi_n$$

$$\mathcal{L}_{QCD} = \bar{\psi} i \not{D} \psi = \bar{\xi}_n \frac{\bar{\not{n}}}{2} in\cdot D\, \xi_n + \bar{\varphi}_n \frac{\not{n}}{2} i\bar{n}\cdot D\, \varphi_n + \bar{\xi}_n i\not{D}_\perp \varphi_n + \bar{\varphi}_n i\not{D}_\perp \xi_n$$

e.o.m. $\delta / \delta\bar{\varphi}_n \Rightarrow$

$$\varphi_n = \frac{1}{i\bar{n}\cdot D}\, i\not{D}_\perp \frac{\bar{\not{n}}}{2}\, \xi_n$$

$\lambda \quad * \quad \lambda$ — smaller than ξ_n for hard production

$$\mathcal{L}_{QCD} = \bar{\xi}_n \left(in\cdot D + i\not{D}_\perp \frac{1}{i\bar{n}\cdot D} i\not{D}_\perp \right) \frac{\bar{\not{n}}}{2} \xi_n$$ still QCD

Expand

- couple only to ξ_n in path integral $\int \xi_n$
- $$i n\cdot D = \underset{\lambda^2}{i n\cdot\partial} + \underset{\lambda^2}{g\, n\cdot A_n} + \underset{\lambda^2}{g\, n\cdot A_{s}}$$

  $$i D_\perp = \underset{\lambda}{i\partial_{n\perp}} + \underset{\lambda}{g A_{n\perp}} + \dots$$

  $$i\bar n\cdot D = i\bar n\cdot\partial_n + g\,\bar n\cdot A_n + \dots$$

multipole expansion
(SCET notes for details)

$A^\perp_{us} \ll A_{n\perp}$

$i\partial^\perp_{us} \ll i\partial^\perp_n$

$\bar n\cdot A_{us} \ll \bar n\cdot A_n$

$i\bar n\cdot\partial_{us} \ll i\bar n\cdot\partial_n$

$$\mathcal{L}^{(0)}_{n\xi} = \bar\xi_n \left(i n\cdot D + i\not{D}_{n\perp} \frac{1}{i\bar n\cdot D_n} i\not{D}_{n\perp} \right) \frac{\not{\bar n}}{2}\xi_n$$

gluons $\mathcal{L}^{(0)}_{ng} = \mathcal{L}^{(0)}_{ng}\left[n\cdot D,\ D_{n\perp},\ \bar n\cdot D_n \right]$ too

(+ gauge fixing & ghosts)

If we drop $n\cdot A_{us}$ these are QCD Lagrangians

[<u>Higher Orders</u> eg. $$\mathcal{L}^{(1)} = \underset{\lambda}{(\bar\xi_n W_n)}\, \underset{\lambda^2}{i\not{D}^{us}_\perp} \left(W_n^\dagger \frac{1}{i\bar n\cdot D_n} \underset{\lambda}{i\not{D}_{n\perp}} \frac{\not{\bar n}}{2} \underset{\lambda}{\xi_n} \right) = \lambda^5$$

eg. $$\mathcal{L}^{(1)} = \underset{\lambda}{(\bar\xi_n W_n)}\, g\, \underset{\lambda}{\mathcal{B}_{n\perp}}\, \underset{\lambda^3}{q_{us}} + \text{h.c.} \qquad = \lambda^5$$]

Parton Distributions & Quasi Distributions

momentum space: $f_{i/p}\left(\xi = \frac{\omega}{\bar{n}\cdot p}\right) = \langle p | \left(\bar{\xi}_n^i W_n\right) \frac{\bar{\not{n}}}{2} \delta(\omega - i\bar{n}\cdot\partial_n) \left(W_n^\dagger \xi_n^i\right) | p \rangle$

collinear fields = QCD fields here (purely collinear m.elt.), $\xi_n^i = \psi^i$

Same definition in position space:

$$f_{i/p}(\xi) = \int \frac{dy^+}{2\pi} e^{-iy^+(\bar{n}p\,\xi)} \langle p | \bar{\psi}^i(y^+\bar{n})\, W_n(y^+\bar{n}, 0) \frac{\bar{\not{n}}}{2} \psi^i(0) | p \rangle$$

↑ non-perturbative function!

finite spacetime separation on light-cone $[0, y^+]$

- Problem for Lattice QCD: time-dependent, intrinsically Minkowski

Moments - can be calculated on Lattice $\qquad W_n (i\bar{n}\cdot\partial)^k W_n^\dagger = i\bar{n}\cdot D$

$$\int d\xi\, \xi^{N-1} f_{i/p}(\xi) = \langle p | (\bar{\xi}_n^i W_n) \frac{\bar{\not{n}}}{2} \frac{1}{\bar{n}\cdot p} \left(\frac{i\bar{n}\cdot\partial}{\bar{n}\cdot p}\right)^{N-1} (W_n^\dagger \xi_n^i) | p \rangle$$

$$= \frac{\bar{n}^{\mu_1} \cdots \bar{n}^{\mu_N}}{2(\bar{n}\cdot p)^N} \langle p | \bar{\psi}^i_{(0)} \gamma_{\mu_1} (iD_{\mu_2}) \cdots (iD_{\mu_N}) \psi^i(0) | p \rangle$$

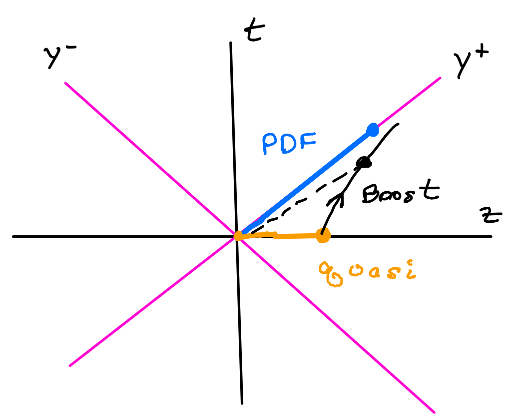
local operator with symmetric-traceless projection (twist-2)

Issue: complication of UV mixing restricts $N \leq 4$

Quasi-Distribution - one idea for direct calculations

(Ji 2013)

- calculate distribution with a spacelike separation
- As $P^z \gg \Lambda_{QCD}$ it is boosted towards lightcone
- match to light-cone PDF



quasi-PDF: $\qquad$ space like separation $[0, z]$

$$\tilde{q}_{i/p}(\xi, P_z) = \int \frac{dz}{2\pi} e^{-i(\xi P_z) z} \langle p | \bar{\psi}^i(z) \frac{\gamma^z}{2} W_z(z, 0) \psi^i(0) | p \rangle$$

$$\equiv \tilde{q}(z, P_z)$$

renormalization (non-singlet u-d avoids mixing with gluon operators)

bare

$$q(\xi,\epsilon) = \int \frac{dx}{x}\, Z\left(\frac{\xi}{x},\epsilon,\mu\right) q(x,\mu)$$ ← typically $\overline{MS}$

$$\tilde{q}(z,P_z,\epsilon) = \tilde{Z}(z,P_z,\epsilon,\tilde{\mu})\, \tilde{q}(z,P_z,\tilde{\mu})$$ ← lattice $\overline{MS}$, RI MOM, ...

matching $P_z \gg \Lambda_{QCD}$

$$\tilde{q}(\xi,P_z,\tilde{\mu}) = \int_{-1}^{1} \frac{dx}{|x|}\, C\left(\frac{\xi}{x},\frac{\tilde{\mu}}{P_z},\frac{\mu}{P_z}\right) q(x,\mu) + \mathcal{O}\left(\frac{M^2}{P_z^2},\frac{\Lambda_{QCD}^2}{P_z^2}\right)$$

Calculate conversion perturbatively

- Early results available, improvements are work in progress (chiral extrap., statistical uncertainties grow for large P_z)

Other ideas: Gradient Flow, Pseudo-PDF with γ^0, Current-Current Correlator