Introduction to the
Soft - Collinear Effective Theory

An effective field theory for energetic hadrons & jets

Lecture 1

Methods of Effective Field Theory & Lattice Field Theory
FGZ-PH Summer School, Munich, Germany
July 2017

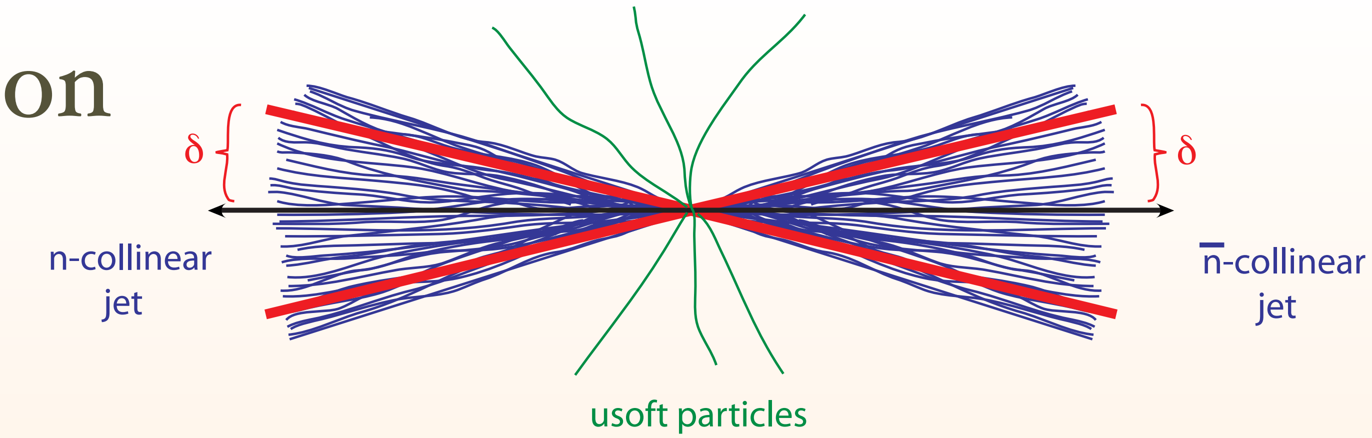
Outline (Lecture I)

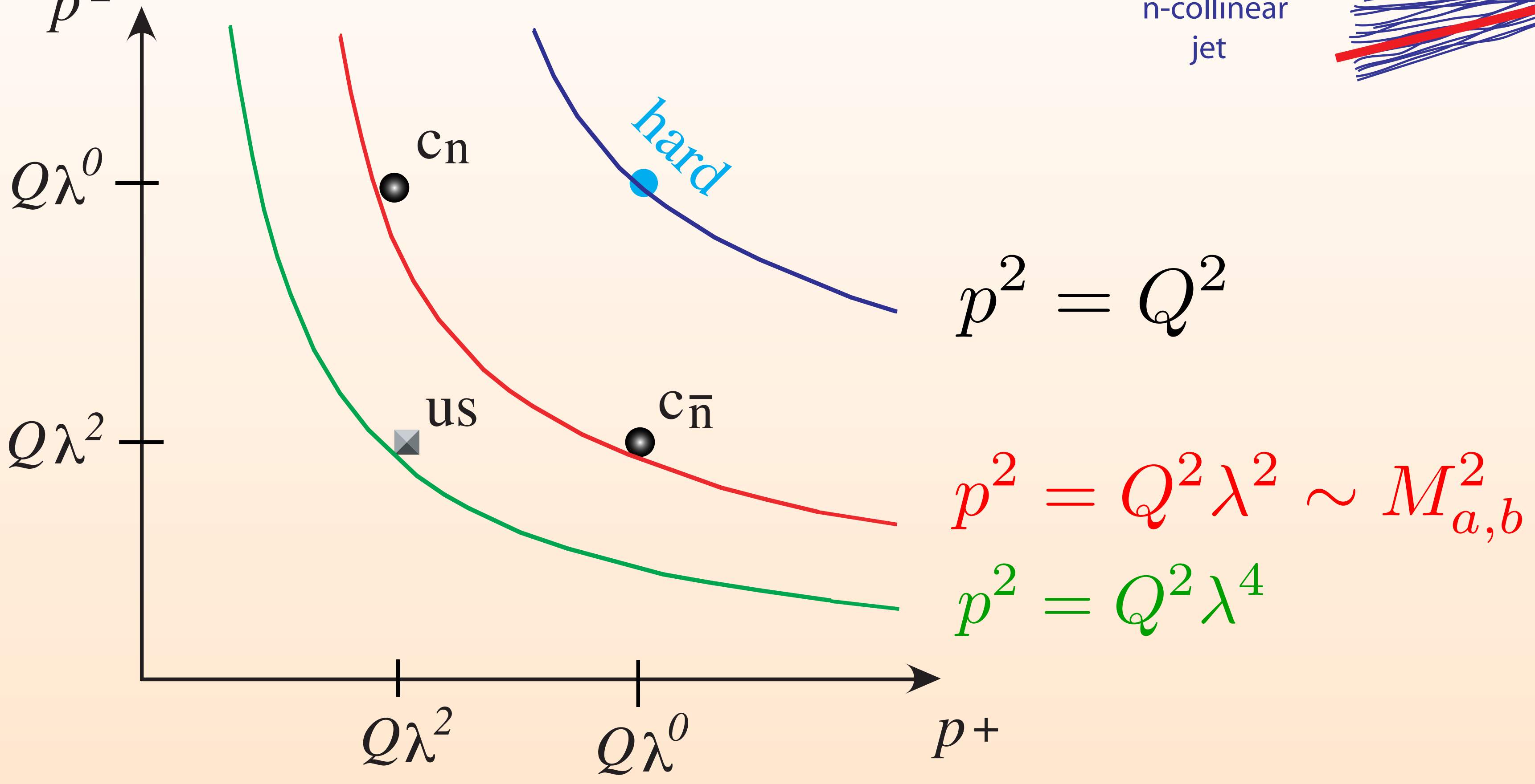
- EFT concepts
  Intro to SCET
  SCET degrees of freedom

  $\Big\}$ Done on the Board
  (See separate lecture notes.)

- SCETı , momentum scales and regions
- Field power counting in SCET
- Wilson lines, W, from off shell propagators
- Gauge Symmetry
- Hard-Collinear Factorization
- eg. Deep Inelastic Scattering

Two jets and ultrasoft radiation





$$p^2 = p^+ p^-$$

for picture

Comments:

1) multiple modes for IR

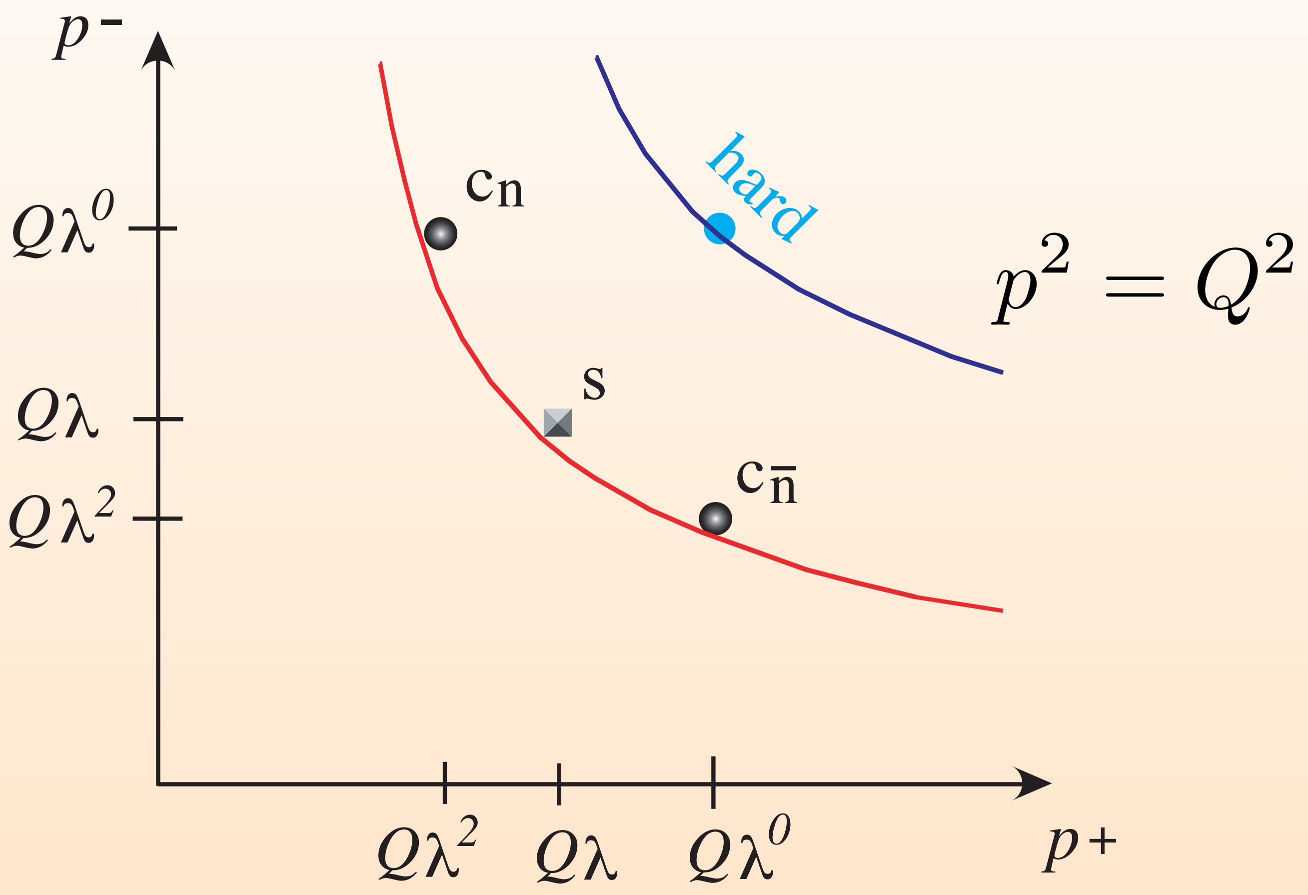
2) integrate out modes above a hyperbola

SCET$_\text{I}$

modes	$p^\mu = (+,-,\perp)$	p^2	fields
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us},\ A_{us}^\mu$

Another SCET: $\mathrm{SCET}_{\mathrm{II}}$ (not covered here)

Two jets and soft radiation with $p_\perp$-type measurement



$p^2 = p^+ p^-$
for picture

soft $p_s^\mu \sim Q\lambda$

instead of ultrasoft $p_{us}^\mu \sim Q\lambda^2$

$\mathrm{SCET}_{\mathrm{II}}$

modes	$p^\mu = (+, -, \perp)$	p^2	fields
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$

n-Collinear Propagators

$$p^2 + i\epsilon = \bar{n}\cdot p\ \ n\cdot p - \vec{p}_\perp^{\,2} + i\epsilon$$

$$\sim\ \ \lambda^0 * \lambda^2\ \ - (\lambda)^2$$

same size

Collinear Fermions

$$\frac{i\not{p}}{p^2+i\epsilon} = \frac{i\not{n}}{2}\,\frac{\bar{n}\cdot p}{p^2+i\epsilon} + \ldots$$

$$= \frac{i\not{n}}{2}\,\frac{1}{n\cdot p - \frac{\vec{p}_\perp^{\,2}}{\bar{n}\cdot p} + i\epsilon\,\mathrm{sign}(\bar{n}\cdot p)} + \ldots$$

thus we expect

$$\int \underbrace{d^4x}_{\lambda^{-4}}\, e^{ip\cdot x}\, \langle 0|T\underbrace{\xi_n(x)\bar{\xi}_n(0)}_{\text{must be }\lambda^2}|0\rangle = \frac{i\not{n}}{2}\,\underbrace{\frac{\bar{n}\cdot p}{p^2+i\epsilon}}_{\lambda^{-2}}$$

so $\boxed{\xi_n \sim \lambda}$

power counting for the field

$$d^4x \sim (dp^+ dp^- d^2p_\perp)^{-1}$$

$$\lambda^2 \quad \lambda^0 \quad (\lambda)^2$$

This also implies: $\not{n}\,\xi_n = 0$ since $\not{n}^2 = n^2 = 0$

Projection:

Take $\xi_n = \frac{\slashed{n}\slashed{\bar{n}}}{4}\psi$ for spin

$$\boxed{\frac{\slashed{n}\slashed{\bar{n}}}{4}\xi_n = \xi_n \quad , \qquad \slashed{n}\xi_n = 0}$$

For spinors:

QCD

SCET $\quad p^+ \ll p^-$

$$u_n = \frac{\slashed{n}\slashed{\bar{n}}}{4}\; u^{\mathrm{QCD}}$$

$$u_+(p) = |p+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{p^-}\\ \sqrt{p^+}e^{i\,\phi_p}\\ \sqrt{p^-}\\ \sqrt{p^+}e^{i\,\phi_p}\end{pmatrix} \Longrightarrow u_n^+ = \sqrt{\frac{p^-}{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$$

$$e^{\pm i\phi_p} = \frac{p_\perp^1 \pm i p_\perp^2}{\sqrt{p^+p^-}}$$

$$u_-(p) = |p-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{p^+}e^{i\,\phi_p}\\ -\sqrt{p^-}\\ -\sqrt{p^+}e^{i\,\phi_p}\\ \sqrt{p^-}\end{pmatrix} \Longrightarrow u_n^- = \sqrt{\frac{p^-}{2}}\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}$$

Check:

$$\sum_s u_n^s \bar{u}_n^s = \frac{\slashed{n}\slashed{\bar{n}}}{4}\sum_s u^s\bar{u}^s\,\frac{\slashed{\bar{n}}\slashed{n}}{4} = \frac{\slashed{n}\slashed{\bar{n}}}{4}\,\slashed{p}\,\frac{\slashed{\bar{n}}\slashed{n}}{4} = \frac{\slashed{n}}{2}\,\bar{n}\cdot p$$

agrees with numerator of propagator $\quad i\frac{\slashed{n}}{2}\,\frac{\bar{n}\cdot p}{p^2+i\epsilon}$

Gauge Fields for $\mathrm{SCET_I}$

Collinear Gluons - same propagator as QCD

covariant gauges

$$\int d^4x\, e^{ip\cdot x}\, \langle 0|T A_n^\mu(x) A_n^\nu(0)|0\rangle = \frac{-i}{p^2}\Big(g^{\mu\nu} - \tau \frac{p^\mu p^\nu}{p^2}\Big)$$

solution

$$\boxed{(A_n^+, A_n^-, A_n^\perp) \sim (\lambda^2, 1, \lambda) \sim p^\mu}$$

components
scale
differently

Usoft Gluon $A_{us}^\mu \sim (\lambda^2, \lambda^2, \lambda^2) \sim p_{us}^\mu$

Usoft Quark $q_{us} \sim \lambda^3$

Power Counting Summary

Type	$(p^+, p^-, p^\perp)$	Fields	Field Scaling
collinear	$(\lambda^2, 1, \lambda)$	$\xi_{n,p}$	λ
		$(A_{n,p}^+, A_{n,p}^-, A_{n,p}^\perp)$	$(\lambda^2, 1, \lambda)$
soft	$(\lambda, \lambda, \lambda)$	$q_{s,p}$	$\lambda^{3/2}$
		$A_{s,p}^\mu$	λ
usoft	$(\lambda^2, \lambda^2, \lambda^2)$	q_{us}	λ^3
		A_{us}^μ	λ^2

Power counting of fields and derivatives gives a power counting for operators

Power counting of operators yields a power counting for any Feynman graph

The power counting can be associated entirely to vertices and is then gauge invariant

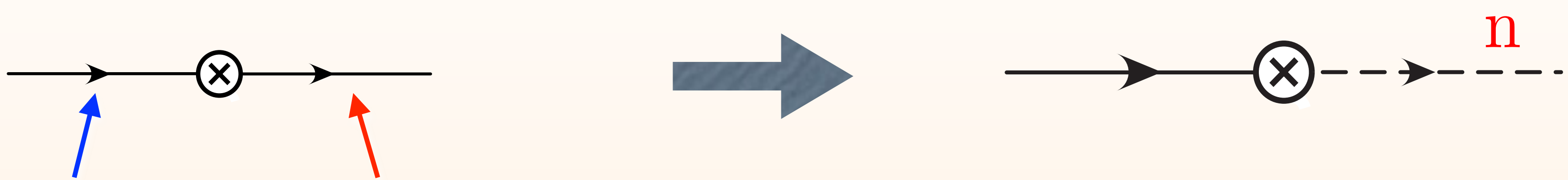
$$(A_n^+, \boxed{A_n^-}, A_n^\perp) \sim (\lambda^2, \boxed{1}, \lambda) \sim p^\mu$$

Currents

QCD | SCET

$\bar\psi\,\Gamma\,\psi$

no gluons

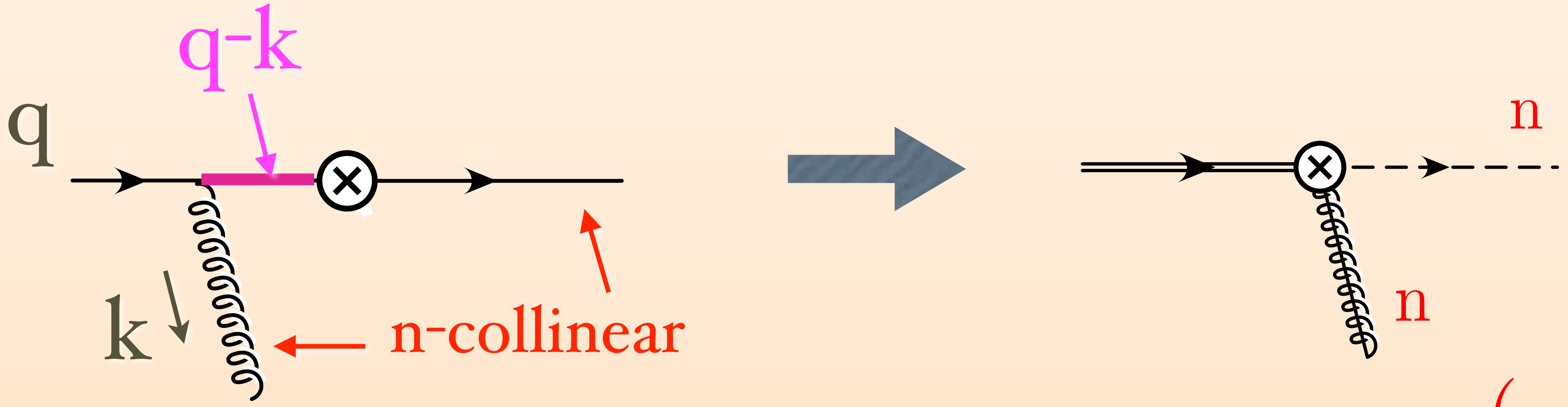


not n (some n', or massive)

n-collinear

$\bar\xi_n\;\Gamma\;\psi$

one gluon



n-collinear

$$\bar\xi_n\,\frac{(-g\bar n\!\cdot\! A_n)}{-\bar n\!\cdot\! k}\Gamma\psi$$

offshell $(q-k)^2 = q^2 + k^2 - 2q\cdot k = -\bar n\cdot k\, n\cdot q + \ldots$

gluon field $\not{A}^a = \bar n\cdot A^a\,\frac{\not{n}}{2} + \ldots$

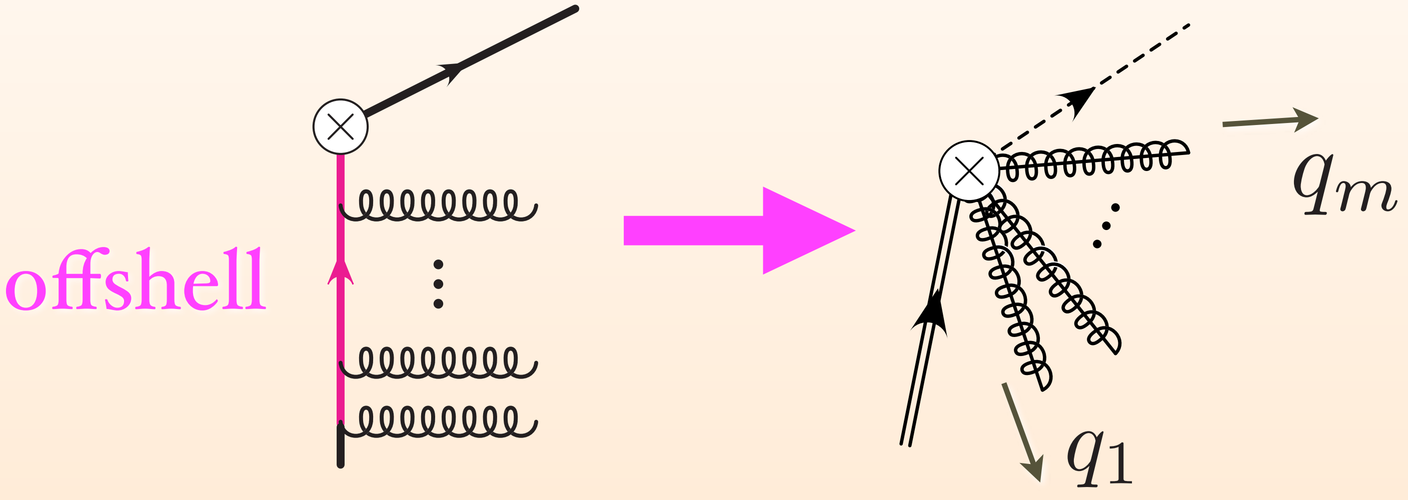
graph = $\bar u_n \Gamma \frac{i(\not{q}-\not{k}+m)}{(q-k)^2-m^2} ig\not{A}^a T^a \gamma^\mu u(q)$ = ... (Homework) ...

$$= \frac{(-g\bar n\cdot A_n^a)}{-\bar n\!\cdot\! k}\,\bar u_n \Gamma T^a u(q)$$

Currents

add any number of gluons

→ $\bar\xi_n W_n \, \Gamma \, \psi$ get a Wilson line W



$$= g^m \sum_{\text{perms}} \frac{(\bar n^{\mu_m} T^{A_m})\ldots(\bar n^{\mu_1} T^{A_1})}{[\bar n\cdot q_1][\bar n\cdot(q_1+q_2)]\cdots[\bar n\cdot\sum_{i=1}^m q_i]}$$

$\sim \lambda^0$ no cost to add these gluons

momentum space Wilson line $W_n = \sum_k \sum_{\text{perms}} \frac{(-g)^k}{k!}\left(\frac{\bar n\cdot A_{\bar n,q_1}\cdots\bar n\cdot A_{\bar n,q_k}}{[\bar n\cdot q_1][\bar n\cdot(q_1+q_2)]\cdots[\bar n\cdot\sum_{i=1}^k q_i]}\right)$

position space Wilson line $W_n(y,-\infty) = P\,\exp\left(ig\int_{-\infty}^0 ds\,\bar n\cdot A_n(y+s\bar n)\right)$

$W_n \sim \lambda^0$

(more on Homework)

SCET Operator: $\underbrace{\bar\xi_n W_n}_{\bar\chi_n} \, \Gamma \, \psi$

this is generic, gives an operator "building block"

like a "parton" or "jet" field

Gauge symmetry

$$U(x) = \exp\left[i\alpha^A(x)T^A\right]$$

need to consider U's
which leave us in the EFT

collinear $\quad i\partial^\mu \mathcal{U}_c(x) \sim p_c^\mu \mathcal{U}_c(x) \leftrightarrow A_{n,q}^\mu$

usoft $\quad i\partial^\mu U_{us}(x) \sim p_{us}^\mu U_{us}(x) \leftrightarrow A_{us}^\mu$

Object	Collinear $\mathcal{U}_c$	Usoft U_{us}
ξ_n	$\mathcal{U}_c\, \xi_n$	$U_{us}\, \xi_n$
gA_n^μ	$\mathcal{U}_c\, gA_n^\mu\, \mathcal{U}_c^\dagger + \mathcal{U}_c\left[i\mathcal{D}^\mu, \mathcal{U}_c^\dagger\right]$	$U_{us}\, gA_n^\mu\, U_{us}^\dagger$
W	$\mathcal{U}_c\, W$	$U_{us}\, W\, U_{us}^\dagger$
q_{us}	q_{us}	$U_{us}\, q_{us}$
gA_{us}^μ	gA_{us}^μ	$U_{us} gA_{us}^\mu U_{us}^\dagger + U_{us}[i\partial^\mu, U_{us}^\dagger]$
Y	Y	$U_{us}\, Y$

our current
is invariant:

$$(\bar{\xi}_n W)\Gamma\psi \quad \to (\bar{\xi}_n \mathcal{U}_c^\dagger \mathcal{U}_c W)\Gamma\psi = (\bar{\xi}_n W)\Gamma\psi$$

$$\to (\bar{\xi}_n U_{us}^\dagger U_{us} W) U_{us}^\dagger \Gamma U_{us} \psi = (\bar{\xi}_n W)\Gamma\psi$$

Building Blocks:

collinear gauge invariant

quark $$\chi_n = W_n^\dagger \xi_n$$

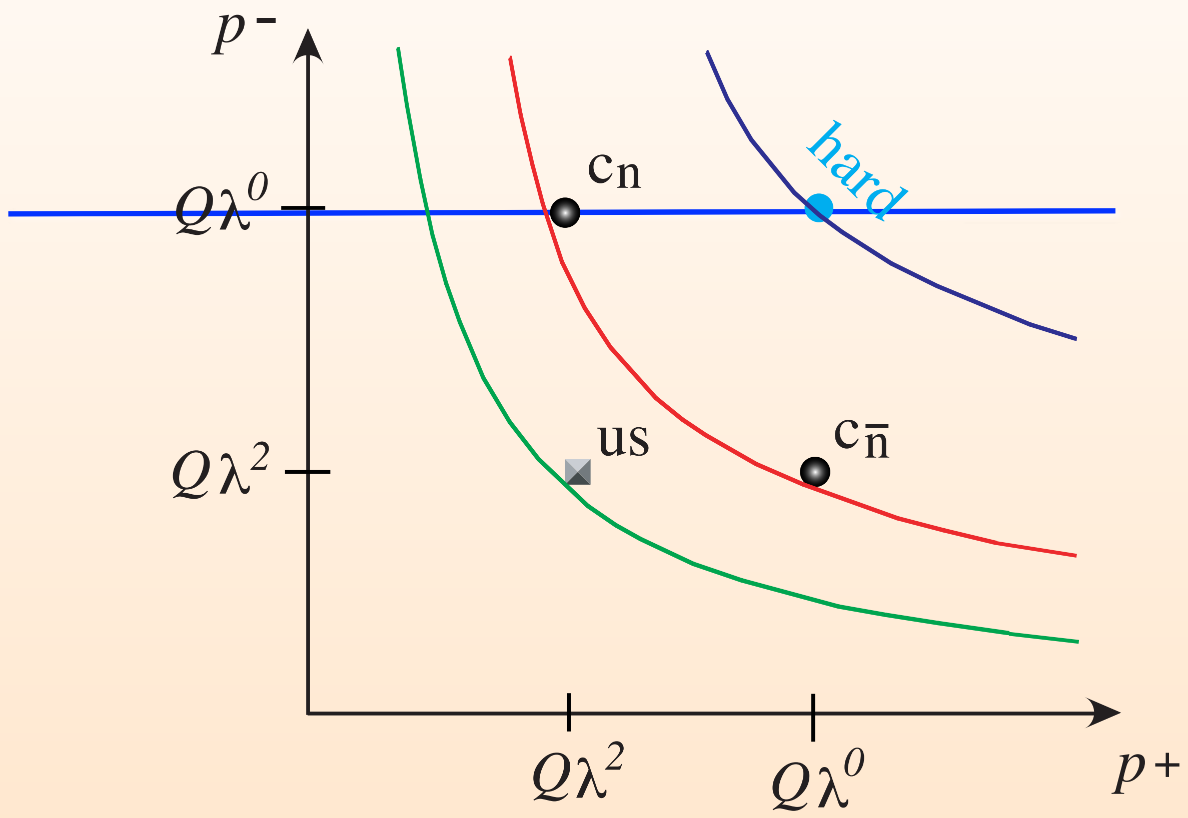
gluon $$\mathcal{B}_{n\perp}^{\mu} = \frac{1}{g}\big[W_n^\dagger iD_{n\perp}^{\mu} W_n\big] = \frac{1}{g}\Big[\frac{1}{i\bar{n}\cdot\partial_n} W_n^\dagger [i\bar{n}\cdot D_n, iD_{n\perp}^{\mu}] W_n\Big]$$

field strength
+ adjoint Wilson line

$$= A_{n\perp}^{\mu} - \frac{k_\perp^\mu}{\bar{n}\cdot k}\,\bar{n}\cdot A_n(k) + \ldots$$

Wilson Coefficients and Hard-Collinear Factorization

$$\begin{array}{ll} & \quad\;\; (+,\,-,\,\perp) \\ \hline \text{hard:} & p^\mu \sim (1\,,\,1\,,\,1) \\ \text{collinear:} & p^\mu \sim (\lambda^2,\,1,\,\lambda) \end{array}$$



can exchange momenta

$i\bar n\cdot\partial_n \sim \lambda^0$

Constrained by gauge invariance:

$C(i\bar n\cdot\partial_n)$ coefficients depend on large collinear momenta

eg. $$C(i\bar n\cdot\partial_n)\;\chi_n = \int\! d\omega\, C(\omega)\,\delta(\omega - i\bar n\cdot\partial_n)\chi_n$$

only the product is gauge invariant $\chi_n = W_n^\dagger\,\xi_n$

implies convolutions between coefficients and operators $$\int\! d\omega\;C(\omega,\mu)\;O(\omega,\mu)$$

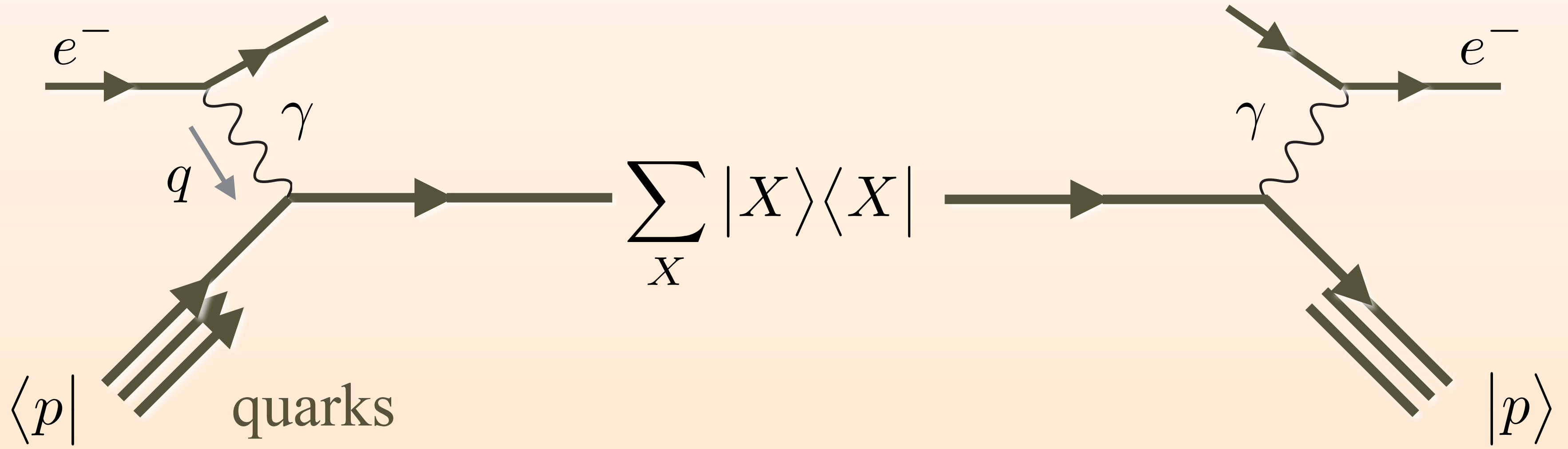
Deep Inelastic Scattering $e^- p \to e^- X$

inclusive
factorization

[full analysis requires bit more knowledge, eg. SCET Lagrangian, here we cover the key conceptual part, skipping softs, prefactors, tensor indices, etc.]

$$\sigma = (\text{prefactor})\ L_{\mu\nu}^{\text{leptons}} \sum_X (2\pi)^4 \delta^{(4)}(p_p + q - p_X)\, \langle p|J^\mu(0)|X\rangle\langle X|J^\nu(0)|p\rangle$$

$$J^\mu = \bar\psi\gamma^\mu\psi$$



$$q = (0,0,0,Q) = \frac{Q}{2}(\bar n - n)$$ picked a frame (Breit frame), Bjorken $x = \dfrac{Q^2}{2p_p \cdot q}$

$$q^2 = -Q^2 \text{ spacelike} \qquad Q^2 \gg \Lambda_{\text{QCD}}^2 \qquad \lambda = \frac{\Lambda_{\text{QCD}}}{Q} \ll 1$$

Proton $$p_p^\mu = \frac{n^\mu}{2}\bar n\cdot p_p + \frac{\bar n^\mu}{2}\frac{m_p^2}{\bar n \cdot p_p}$$ n-collinear $$\bar n\cdot p_p = \frac{Q}{x}$$

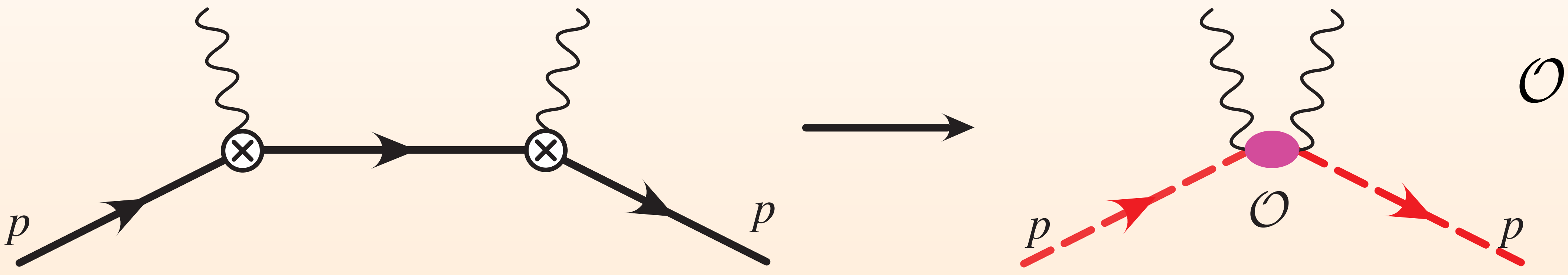
big small

X $$p_X^\mu = p_p^\mu - q^\mu = \frac{n^\mu}{2}\frac{Q(1-x)}{x} + \frac{\bar n^\mu}{2}Q$$ hard, offshell

Deep Inelastic Scattering $e^- p \to e^- X$

QCD

SCET



$$\mathcal{O} = \bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \chi_n$$

$$\mathcal{O} \sim \lambda^2$$

(twist 2)

Add arbitrary perturbative corrections (any order in α_s)

$$\mathcal{L}_{\rm int}^{\rm hard} = \int d\omega\, d\cancel{\omega'}\; C(\omega, \cancel{\omega'})\; \bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \delta(\cancel{w'} + i\bar{n} \cdot \overleftarrow{\partial}_n) \delta(w - i\bar{n} \cdot \partial_n) \chi_n$$

forward matrix element fixes $\omega' = \omega$

$$\sigma = (\text{prefactor}) \int d\omega\; \text{Im} C(\omega, \mu)\; \langle p | \bar{\chi}_n \frac{\bar{n}\!\!\!/}{2} \delta(\omega - i\bar{n} \cdot \partial_n)\; \chi_n | p \rangle(\mu)$$

$$= (\text{prefactor}) \int \frac{d\xi}{\xi}\; H\Big(\frac{\xi}{x}, Q, \mu\Big)\; f(\xi, \mu)$$

$$\omega = \xi\, \bar{n} \cdot p_p$$

hard perturbative corrections

parton distribution function

A more detailed set of SCET lecture notes can be found under "textbooks" in the 8.EFTx course.

All suggested homework problems can be accessed through the chapter for this school in 8.EFTx. The homework requires long answer (equation) solutions and is computer graded, so you will get immediate feedback.

To access the materials in 8.EFTx:
first sign up for an edX account here,
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