Introduction to the
Soft - Collinear Effective Theory

An effective field theory for energetic hadrons & jets

Lecture 1

Methods of Effective Field Theory & Lattice Field Theory
FGZ-PH Summer School, Munich, Germany
July 2017

Outline (Lecture I)

- EFT concepts
  Intro to SCET
  SCET degrees of freedom

  Done on the Board
  (See the lecture notes below.)

- SCETɪ , momentum scales and regions
- Field power counting in SCET
- Wilson lines, W, from off shell propagators
- Gauge Symmetry
- Hard-Collinear Factorization
- eg. Deep Inelastic Scattering

Soft - Collinear Effective Theory

I. Stewart

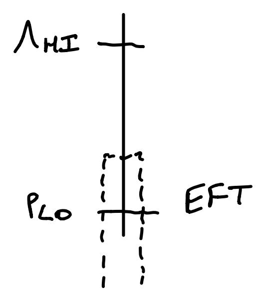
- EFT treatment of Soft & Collinear IR physics for hard collisions in QCD (or decays with large E released)

$\Rightarrow$ jets, energetic hadrons, soft partons / hadrons

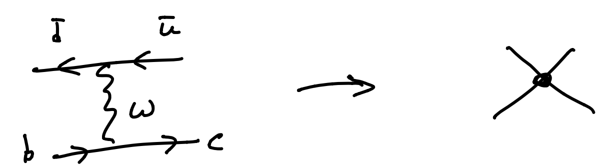
eg. $e^+e^- \to$ 2-jets, $e^- p \to e^- X$ (DIS), $pp \to H + 1$-jet, $B \to \pi\pi$, jet substructure, ... [many many more]

Concepts: Factorization, Wilson Lines, Sudakov Double Logs, ...

<u>Decoupling</u> Effects from heavy or <u>offshell</u> particles are suppressed / decouple $\qquad p_{LO} \ll \Lambda_{HI}$

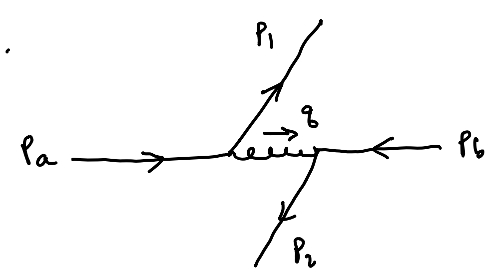


eg

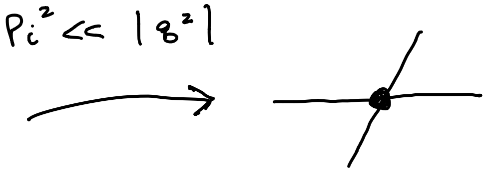


$\sim \dfrac{(\bar\psi\psi)(\bar\psi\psi)}{M_W^2} \qquad M_W^2 \gg p_i^2$

eg.



$p_i^2 \ll |q^2|$



$\sim \dfrac{1}{q^2} (\bar\psi \cdot\cdot \psi)(\bar\psi \cdot\cdot \psi)$

say $p_i^2 = 0$ on-shell, $\qquad q = p_a - p_1 = n_a E_a - n_1 E_1$

$n_a = (1, \hat{z})$, $n_a^2 = 0$ $\qquad q^2 = -2E_aE_1\, n_a \cdot n_1$

$n_1 = (1, \hat{n})$, $n_1^2 = 0$ $\qquad = -2E_aE_1\,(1 - \hat{z}\cdot\hat{n})$

$q^2 \sim Q^2$ "hard" $\qquad$ large if energies big & deflection angles large

<u>EFT</u>
- degrees of freedom $\to$ what fields $\quad$ low energy / nearly onshell modes
- symmetries $\to$ constrain operators [Lorentz, Gauge, Global ...]
- expansions $\to$ power counting (importance of operators, leading order description)

$\mathcal{L}_{EFT} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots$

often in mass dimension of operators, but not so in SCET

(∞ # operators, but only specific subset needed at given order)

- Power counting handles powers $\dfrac{p_{LO}}{\Lambda_{HI}} \ll 1$
- Renormalization group handles logs $\ln\left(\dfrac{p_{LO}}{\Lambda_{HI}}\right)$ which may be large as $\ln(\cdot\cdot) \sim 1$

Matching SCET is a "top-down EFT" [Heweak, HQET NRQCD, SCET, ...]

QCD

$\downarrow$ $\quad \Lambda_{QCD}^2, p_{IR}^2 \ll Q^2$ ← hard collision

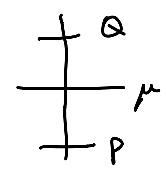
SCET

$$\mathcal{L}^{(k)}_{SCET} = \sum_i C_i(\mu)\, O_i^{(k)}(\mu)$$

Calculate C, construct O

$C_i(\mu)$ ↑ short. dist. (offshell)

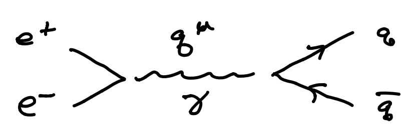
$O_i^{(k)}(\mu)$ ↑ long dist. ($\sim$ on-shell)



renormalization scale is cutoff that splits scales

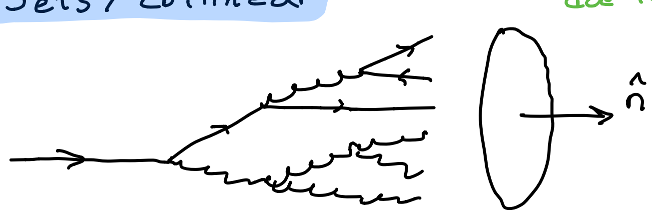
- $\mathcal{L}_{QCD}$ & $\mathcal{L}_{SCET}$ have same IR, differ in UV
- $C_i(\mu)$ does not depend on IR scales (masses in EFT, Λ_{QCD}, IR regulators, ...)

degrees of freedom consider $e^+e^- \to 2$ jets



$Q^2 = q^2$

Jets / collinear due to collinear (& soft) enhancements in QCD



- collimated radiation in direction $\hat{n}$
- $E_{Jet} \sim Q$

Let $n^\mu = (1, \hat{n})$

$\bar{n}^\mu = (1, -\hat{n})$

$n^2 = \bar{n}^2 = 0\ ,\quad n\cdot\bar{n} = 2$

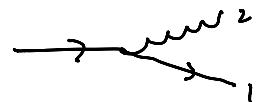
$$p^\mu = \underbrace{\bar{n}\cdot p}_{p^-} \frac{n^\mu}{2} + \underbrace{n\cdot p}_{p^+} \frac{\bar{n}^\mu}{2} + p_\perp^\mu$$

$$p^2 = n\cdot p\, \bar{n}\cdot p + \underbrace{p_\perp^2}_{-\vec{p}_\perp^{\,2}}$$

Collinear ?

1 massless particle : $p^\mu = \bar{n}\cdot p \frac{n^\mu}{2}$

2 massless :



$$p_i^\mu = \underbrace{\bar{n}\cdot p_i}_{\sim Q} \frac{n^\mu}{2} + \underbrace{p_{i\perp}^\mu}_{\sim \lambda Q} + \underbrace{n\cdot p_i}_{\sim \lambda^2 Q} \frac{\bar{n}^\mu}{2}$$

large

$\lambda \ll 1$

collimated

nearly on-shell

on-shell

$$n\cdot p_i = -\frac{p_{i\perp}^2}{\bar{n}\cdot p_i}$$

$\boxed{\lambda \ll 1}$ $\boxed{\text{dimensionless power counting parameter}}$

$\boxed{\text{n-Collinear Fields}}$:

quark ξ_n

gluon A_n^μ

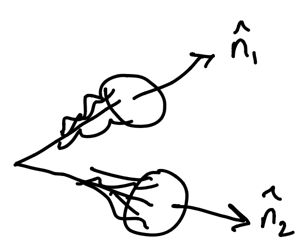
$$p^\mu \sim Q(\overset{n\cdot p}{\lambda^2}, \overset{\bar{n}\cdot p}{1}, \overset{\perp}{\lambda})$$

energetic hadron : $p_\perp \sim \Lambda_{QCD} \Rightarrow \lambda \sim \frac{\Lambda_{QCD}}{Q}$

energetic quarks & gluons confine into single hadron

jet of hadrons: $1 \gg \lambda \gg \frac{\Lambda_{QCD}}{Q}$

2-jets



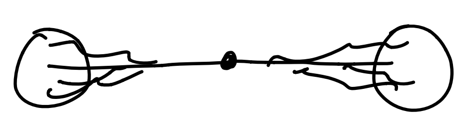
$n_1, \bar{n}_1$ $\quad \xi_{n_1}, A_{n_1}^\mu$

$n_2, \bar{n}_2$ $\quad \xi_{n_2}, A_{n_2}^\mu$

back-to-back jets :

$n_2 = \bar{n} = (1, -\hat{n})$

$\bar{n}_2 = n$



$n_1 = n = (1, \hat{n})$

$\bar{n}_1 = \hat{n}$

Soft $p_s^\mu \sim Q\lambda^\alpha$ all components small & homogeneous

Value of α depends on what we measure

eg1 Mass in (large enough) region a, $M_a^2 = \left(\sum_{i\in a} p_i^\mu\right)^2$

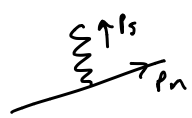
[mass of $R=1$ jet, hemisphere mass, ...]

n-collinear + n-collinear $\quad (p_n + p_n')^2 = 2\, p_n\cdot p_n' \sim Q^2\lambda^2$

$$\begin{array}{cc} + & - \\ - & + \\ \perp & \perp \end{array}$$

$\therefore$ demand $M_a^2 \sim Q^2\lambda^2 \ll Q^2 \sim E_{jet}^2$ [collimated jet]

collinear + soft



$$(p_n+p_s)^2 = 2\,p_n\cdot p_s = \underbrace{\bar n\cdot p_n\; n\cdot p_s}_{\lambda^0 \,*\, \lambda^\alpha} + \underbrace{\cdots}_{\text{suppressed}} \sim Q^2\lambda^\alpha$$

$\therefore \alpha = 2$ to contribute "ultrasoft"

eg2 Transverse Momenta, broadening $B_\perp = \sum_{i\in a} |\vec p_i^{\,\perp}| \ll Q$
$\sim \lambda$

Σ collinear ✓

soft $\Rightarrow \alpha = 1$ "soft"

Go to Slides #1 to #14 | Discussion after #14 | Slides #15 - 17

trades $\bar n\cdot A_n \to W_n$

$W_n^\dagger W_n = \mathbb{1} = W_n W_n^\dagger$

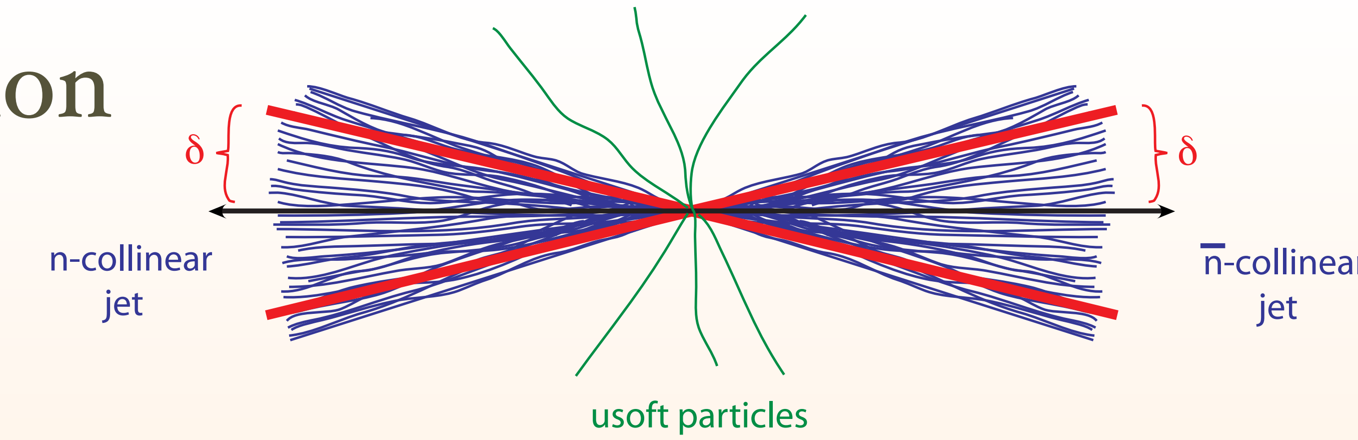
$[i\bar n\cdot D_n W_n] = 0$ $\quad\therefore\quad$ $i\bar n\cdot D_n\, W_n \Phi = W_n\, i\bar n\cdot\partial_n \Phi$

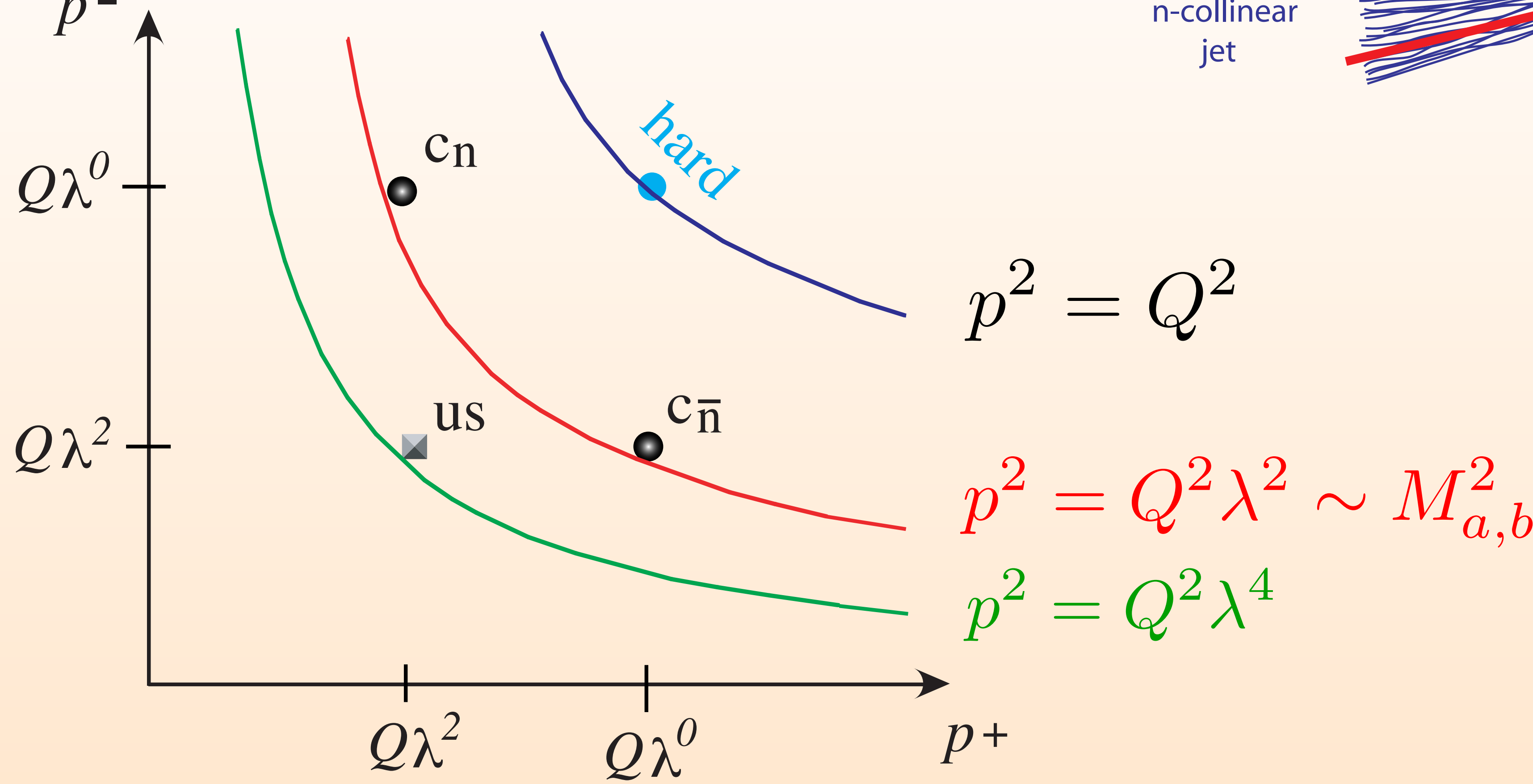
collinear gauge singlet

$W_n^\dagger\, i\bar n\cdot D_n\, W_n = i\bar n\cdot\partial_n$ as operator

$i\bar n\cdot D_n = W_n\, i\bar n\cdot\partial_n\, W_n^\dagger$

Two jets and ultrasoft radiation





$p^2 = p^+ p^-$
for picture

Comments:

1) multiple modes for IR

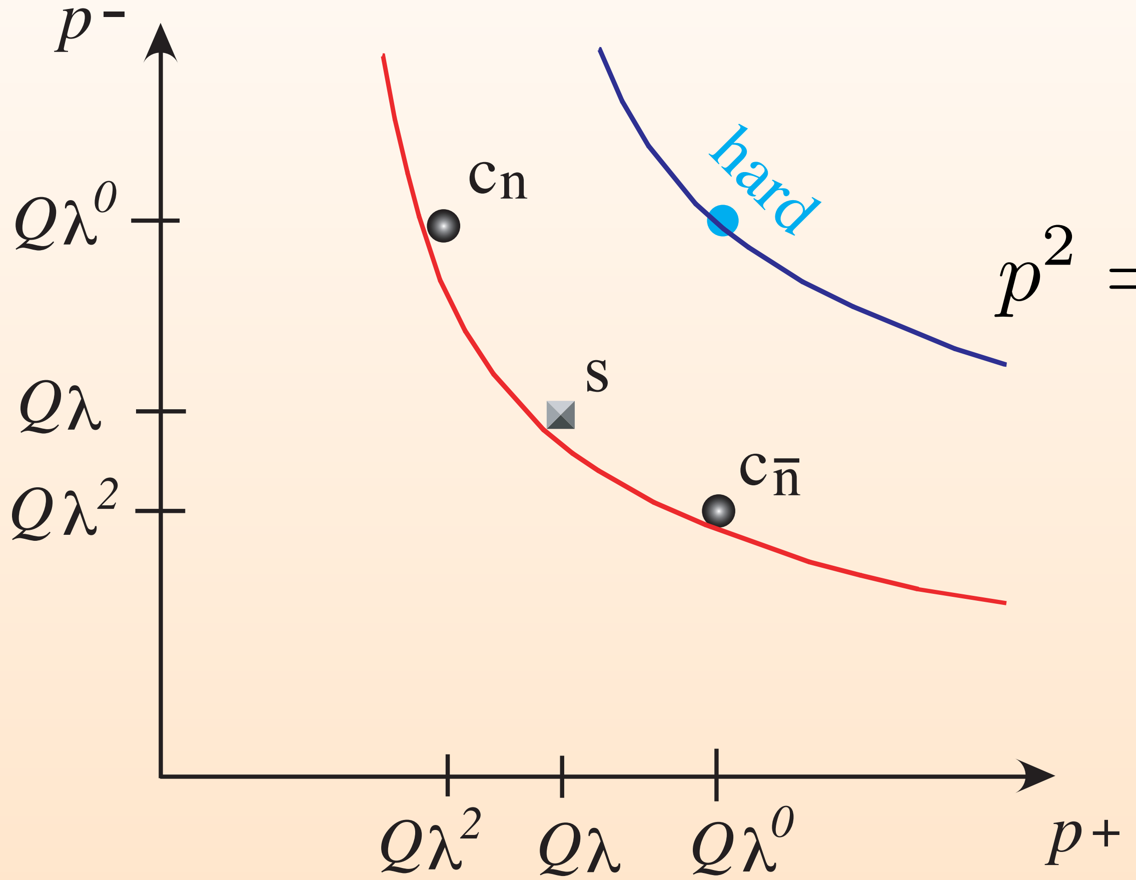
2) integrate out modes above a hyperbola

$\mathrm{SCET_I}$

modes	$p^\mu = (+, -, \perp)$	p^2	fields
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A^\mu_n$
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}},\ A^\mu_{\bar{n}}$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us},\ A^\mu_{us}$

Another SCET: $\mathrm{SCET_{II}}$ (not covered here)

Two jets and soft radiation with $p_\perp$-type measurement



$p^2 = Q^2$

$p^2 = p^+ p^-$
for picture

soft $p_s^\mu \sim Q\lambda$

instead of ultrasoft $p_{us}^\mu \sim Q\lambda^2$

$\mathrm{SCET_{II}}$

modes	$p^\mu = (+, -, \perp)$	p^2	fields
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}},\ A_{\bar{n}}^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$

n-Collinear Propagators

$$p^2 + i\epsilon = \bar{n}\cdot p \;\; n\cdot p - \vec{p}_\perp^{\,2} + i\epsilon$$
$$\sim \;\; \lambda^0 * \lambda^2 \;\; - (\lambda)^2$$

same size

Collinear Fermions

$$\frac{i\,\not{p}}{p^2+i\epsilon} = \frac{i\,\not{n}}{2}\,\frac{\bar{n}\cdot p}{p^2+i\epsilon} + \ldots$$
$$= \frac{i\,\not{n}}{2}\,\frac{1}{n\cdot p - \frac{\vec{p}_\perp^{\,2}}{\bar{n}\cdot p} + i\epsilon\,\mathrm{sign}(\bar{n}\cdot p)} + \ldots$$

thus we expect

$$\underbrace{\int d^4x}_{\lambda^{-4}}\, e^{ip\cdot x}\,\langle 0|T\underbrace{\xi_n(x)\bar{\xi}_n(0)}_{\text{must be }\lambda^2}|0\rangle = \frac{i\,\not{n}}{2}\,\underbrace{\frac{\bar{n}\cdot p}{p^2+i\epsilon}}_{\lambda^{-2}}$$

so $\boxed{\xi_n \sim \lambda}$

power counting for the field

$$d^4x \sim (dp^+ dp^- d^2p_\perp)^{-1}$$
$$\lambda^2 \quad \lambda^0 \quad (\lambda)^2$$

This also implies: $\not{n}\,\xi_n = 0$ since $\not{n}^2 = n^2 = 0$

Projection: Take $\xi_n = \frac{\not{n}\bar{\not{n}}}{4}\psi$ for spin

$$\boxed{\frac{\not{n}\bar{\not{n}}}{4}\xi_n = \xi_n \quad , \quad \not{n}\xi_n = 0}$$

For spinors: QCD SCET $p^+ \ll p^-$

$$u_n = \frac{\not{n}\bar{\not{n}}}{4}\, u^{\mathrm{QCD}}$$

$$u_+(p) = |p+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{p^-} \\ \sqrt{p^+}e^{i\,\phi_p} \\ \sqrt{p^-} \\ \sqrt{p^+}e^{i\,\phi_p}\end{pmatrix} \Longrightarrow u_n^+ = \sqrt{\frac{p^-}{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}$$

$$e^{\pm i\phi_p} = \frac{p_\perp^1 \pm i p_\perp^2}{\sqrt{p^+p^-}}$$

$$u_-(p) = |p-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{p^+}e^{i\,\phi_p} \\ -\sqrt{p^-} \\ -\sqrt{p^+}e^{i\,\phi_p} \\ \sqrt{p^-}\end{pmatrix} \Longrightarrow u_n^- = \sqrt{\frac{p^-}{2}}\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}$$

Check:

$$\sum_s u_n^s \bar{u}_n^s = \frac{\not{n}\bar{\not{n}}}{4}\sum_s u^s\bar{u}^s\,\frac{\bar{\not{n}}\not{n}}{4} = \frac{\not{n}\bar{\not{n}}}{4}\,\not{p}\,\frac{\bar{\not{n}}\not{n}}{4} = \frac{\not{n}}{2}\,\bar{n}\cdot p$$

agrees with numerator of propagator $\quad i\frac{\not{n}}{2}\,\frac{\bar{n}\cdot p}{p^2+i\epsilon}$

Gauge Fields for SCET$_{\mathrm{I}}$

Collinear Gluons - same propagator as QCD

covariant gauges

$$\int d^4x\, e^{ip\cdot x}\, \langle 0|T A_n^\mu(x) A_n^\nu(0)|0\rangle = \frac{-i}{p^2}\Big(g^{\mu\nu} - \tau \frac{p^\mu p^\nu}{p^2}\Big)$$

solution

$$\boxed{(A_n^+, A_n^-, A_n^\perp) \sim (\lambda^2, 1, \lambda) \sim p^\mu}$$

components scale differently

Usoft Gluon $\quad A_{us}^\mu \sim (\lambda^2, \lambda^2, \lambda^2) \sim p_{us}^\mu$

Usoft Quark $\quad q_{us} \sim \lambda^3$

Power Counting Summary

Type	$(p^+, p^-, p^\perp)$	Fields	Field Scaling
collinear	$(\lambda^2, 1, \lambda)$	$\xi_{n,p}$	λ
		$(A^+_{n,p}, A^-_{n,p}, A^\perp_{n,p})$	$(\lambda^2, 1, \lambda)$
soft	$(\lambda, \lambda, \lambda)$	$q_{s,p}$	$\lambda^{3/2}$
		$A^\mu_{s,p}$	λ
usoft	$(\lambda^2, \lambda^2, \lambda^2)$	q_{us}	λ^3
		A^μ_{us}	λ^2

Power counting of fields and derivatives gives a power counting for operators

Power counting of operators yields a power counting for any Feynman graph

The power counting can be associated entirely to vertices and
is then gauge invariant

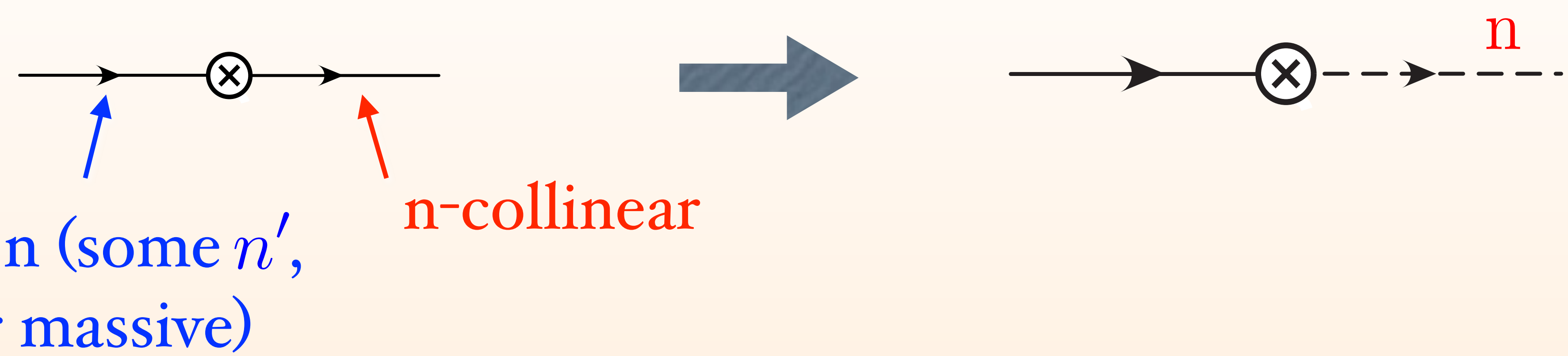
$$(A_n^+, A_n^-, A_n^\perp) \sim (\lambda^2, 1, \lambda) \sim p^\mu$$

Currents

QCD — SCET

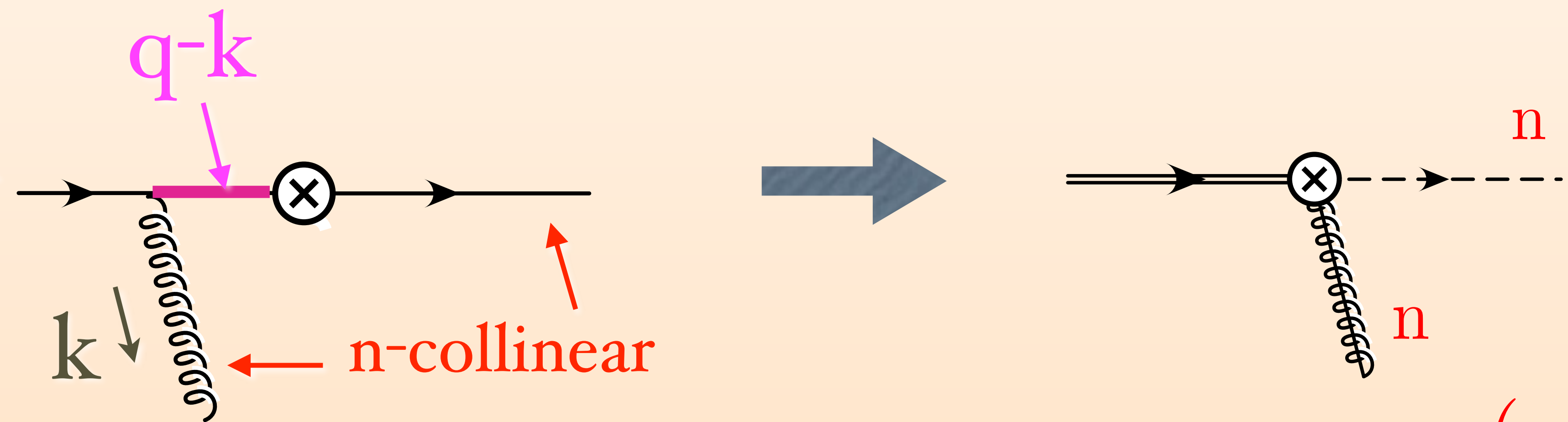
$\bar{\psi}\,\Gamma\,\psi$

no gluons



$\bar{\xi}_n\;\Gamma\;\psi$

one gluon



$$\bar{\xi}_n \frac{(-g\bar{n}\cdot A_n)}{-\bar{n}\cdot k}\Gamma\psi$$

offshell $\quad (q-k)^2 = q^2 + k^2 - 2q\cdot k = -\bar{n}\cdot k\, n\cdot q + \ldots$

gluon field $\quad \not{A}^a = \bar{n}\cdot A^a\,\frac{\not{n}}{2} + \ldots$

graph = $\bar{u}_n\Gamma\frac{i(\not{q}-\not{k}+m)}{(q-k)^2-m^2}ig\not{A}^aT^a\gamma^\mu u(q)$ = ... (Homework) ...

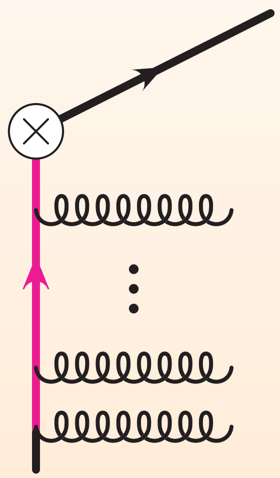
$$= \frac{(-g\bar{n}\cdot A_n^a)}{-\bar{n}\cdot k}\,\bar{u}_n\Gamma T^a u(q)$$

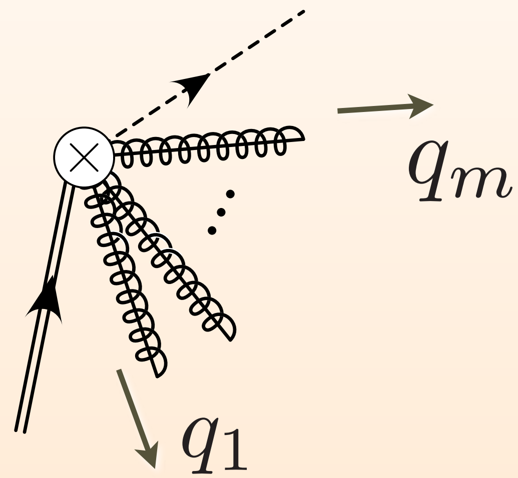
Currents

add any number of gluons

$\Rightarrow$ $\bar{\xi}_n W_n \,\Gamma\, \psi$ get a Wilson line W

offshell





$$= g^m \sum_{\text{perms}} \frac{(\bar{n}^{\mu_m} T^{A_m})\cdots(\bar{n}^{\mu_1} T^{A_1})}{[\bar{n}\cdot q_1][\bar{n}\cdot(q_1+q_2)]\cdots[\bar{n}\cdot\sum_{i=1}^{m} q_i]}$$

$\sim \lambda^0$ no cost to add these gluons

momentum space Wilson line $$W_n = \sum_k \sum_{\text{perms}} \frac{(-g)^k}{k!}\left(\frac{\bar{n}\cdot A_{\bar{n},q_1}\cdots\bar{n}\cdot A_{\bar{n},q_k}}{[\bar{n}\cdot q_1][\bar{n}\cdot(q_1+q_2)]\cdots[\bar{n}\cdot\sum_{i=1}^{k} q_i]}\right)$$

position space Wilson line $$W_n(y,-\infty) = P\,\exp\left(ig\int_{-\infty}^{0} ds\,\bar{n}\cdot A_n(y+s\bar{n})\right)$$

$W_n \sim \lambda^0$

(more on Homework)

SCET Operator: $$\underbrace{\bar{\xi}_n W_n}_{\bar{\chi}_n} \,\Gamma\, \psi$$

this is generic, gives an operator "building block"

like a "parton" or "jet" field

Gauge symmetry

$$U(x) = \exp\left[i\alpha^A(x) T^A\right]$$

need to consider U's which leave us in the EFT

collinear $\quad i\partial^\mu \mathcal{U}_c(x) \sim p_c^\mu \mathcal{U}_c(x) \leftrightarrow A^\mu_{n,q}$

usoft $\quad i\partial^\mu U_{us}(x) \sim p_{us}^\mu U_{us}(x) \leftrightarrow A^\mu_{us}$

Object	Collinear $\mathcal{U}_c$	Usoft U_{us}
ξ_n	$\mathcal{U}_c\, \xi_n$	$U_{us}\, \xi_n$
$g A_n^\mu$	$\mathcal{U}_c\, g A_n^\mu\, \mathcal{U}_c^\dagger + \mathcal{U}_c\left[i\mathcal{D}^\mu, \mathcal{U}_c^\dagger\right]$	$U_{us}\, g A_n^\mu\, U_{us}^\dagger$
W	$\mathcal{U}_c\, W$	$U_{us}\, W\, U_{us}^\dagger$
q_{us}	q_{us}	$U_{us}\, q_{us}$
$g A_{us}^\mu$	$g A_{us}^\mu$	$U_{us} g A_{us}^\mu U_{us}^\dagger + U_{us}[i\partial^\mu, U_{us}^\dagger]$
Y	Y	$U_{us}\, Y$

our current is invariant: $\quad (\bar\xi_n W)\Gamma\psi$

$$\to (\bar\xi_n \mathcal{U}_c^\dagger \mathcal{U}_c W)\Gamma\psi = (\bar\xi_n W)\Gamma\psi$$

$$\to (\bar\xi_n U_{us}^\dagger U_{us} W) U_{us}^\dagger \Gamma U_{us}\psi = (\bar\xi_n W)\Gamma\psi$$

Building Blocks: collinear gauge invariant

quark $\chi_n = W_n^\dagger \xi_n$

gluon $\mathcal{B}_{n\perp}^\mu = \frac{1}{g}\big[W_n^\dagger i D_{n\perp}^\mu W_n\big] = \frac{1}{g}\Big[\frac{1}{i\bar{n}\cdot\partial_n} W_n^\dagger [i\bar{n}\cdot D_n, iD_{n\perp}^\mu] W_n\Big]$

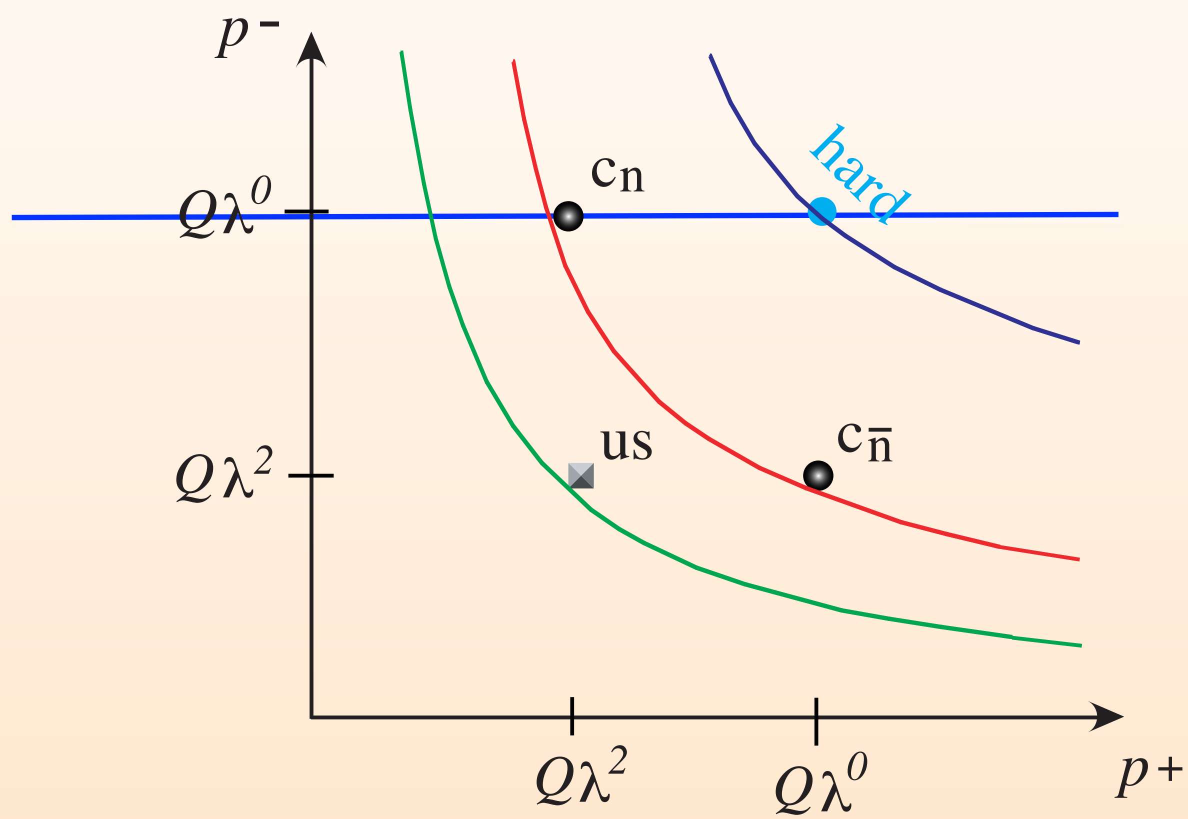
field strength
+ adjoint Wilson line

$$= A_{n\perp}^\mu - \frac{k_\perp^\mu}{\bar{n}\cdot k}\,\bar{n}\cdot A_n(k) + \ldots$$

Wilson Coefficients and Hard-Collinear Factorization

$$\begin{array}{lll} & & \underline{(+,-,\perp)} \\ \text{hard:} & p^\mu \sim & (1\,,\,1\,,\,1) \\ \text{collinear:} & p^\mu \sim & (\lambda^2, 1, \lambda) \end{array}$$

can exchange momenta



$i\bar{n}\cdot\partial_n \sim \lambda^0$

Constrained by gauge invariance:

$C(i\bar{n}\cdot\partial_n)$ coefficients depend on large collinear momenta

eg. $$C(i\bar{n}\cdot\partial_n)\ \chi_n = \int d\omega\, C(\omega)\ \delta(\omega - i\bar{n}\cdot\partial_n)\chi_n$$

only the product is gauge invariant $\qquad \chi_n = W_n^\dagger\, \xi_n$

implies convolutions between coefficients and operators $\qquad \int d\omega\ C(\omega,\mu)\ O(\omega,\mu)$

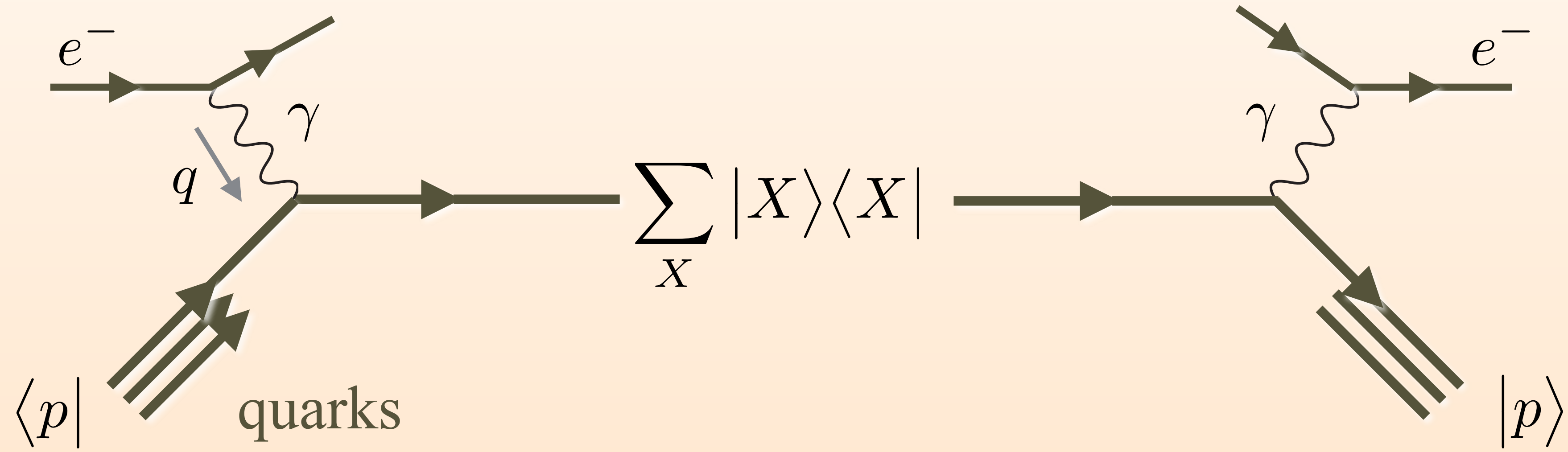
Deep Inelastic Scattering $e^- p \to e^- X$

inclusive factorization

[full analysis requires bit more knowledge, eg. SCET Lagrangian, here we cover the key conceptual part, skipping softs, prefactors, tensor indices, etc.]

$$\sigma = (\text{prefactor})\; L_{\mu\nu}^{\text{leptons}} \sum_X (2\pi)^4 \delta^{(4)}(p_p + q - p_X)\, \langle p | J^\mu(0) | X \rangle \langle X | J^\nu(0) | p \rangle$$

$$J^\mu = \bar{\psi}\gamma^\mu\psi$$



$q = (0,0,0,Q) = \frac{Q}{2}(\bar{n} - n)$ picked a frame (Breit frame), Bjorken $x = \frac{Q^2}{2p_p \cdot q}$

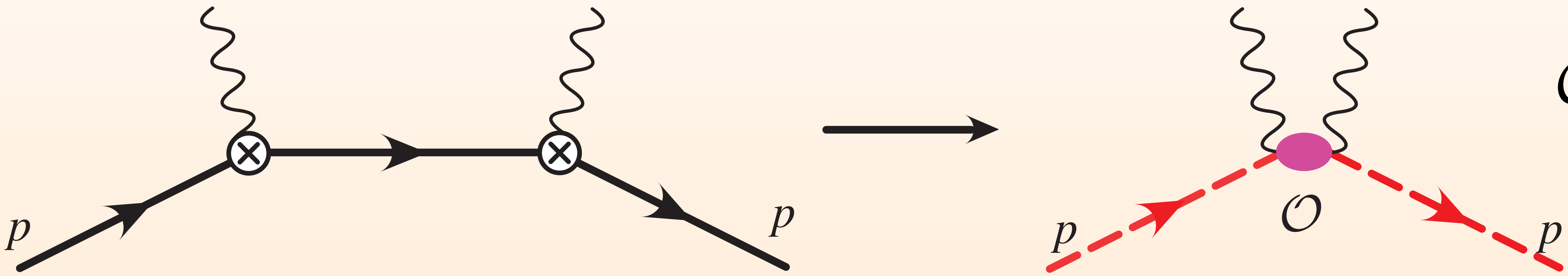
$q^2 = -Q^2$ spacelike $\qquad Q^2 \gg \Lambda_{\rm QCD}^2 \qquad \lambda = \frac{\Lambda_{\rm QCD}}{Q} \ll 1$

Proton $\quad p_p^\mu = \underbrace{\frac{n^\mu}{2}\bar{n}\cdot p_p}_{\text{big}} + \underbrace{\frac{\bar{n}^\mu}{2}\frac{m_p^2}{\bar{n}\cdot p_p}}_{\text{small}}$ $\quad$ n-collinear $\qquad \bar{n}\cdot p_p = \frac{Q}{x}$

X $\quad p_X^\mu = p_p^\mu - q^\mu = \frac{n^\mu}{2}\frac{Q(1-x)}{x} + \frac{\bar{n}^\mu}{2}Q$ $\quad$ hard, offshell

Deep Inelastic Scattering $\quad e^- p \to e^- X$

QCD $\qquad\qquad\qquad\qquad$ SCET



$$\mathcal{O} = \bar\chi_n \frac{\bar{n}\!\!\!/}{2} \chi_n$$

$$\mathcal{O} \sim \lambda^2$$

(twist 2)

Add arbitrary perturbative corrections (any order in α_s)

$$\mathcal{L}_{\rm int}^{\rm hard} = \int d\omega\, d\cancel{\omega'}\; C(\omega,\cancel{\omega'})\; \bar\chi_n \frac{\bar{n}\!\!\!/}{2} \delta(\cancel{w'} + i\bar{n}\cdot \overleftarrow{\partial}_n)\delta(w - i\bar{n}\cdot\partial_n)\chi_n$$

forward matrix element fixes $\omega' = \omega$

$$\sigma = (\text{prefactor}) \int d\omega\; {\rm Im}C(\omega,\mu)\; \langle p|\bar\chi_n \frac{\bar{n}\!\!\!/}{2}\delta(\omega - i\bar{n}\cdot\partial_n)\; \chi_n|p\rangle(\mu)$$

$$= (\text{prefactor}) \int \frac{d\xi}{\xi}\; H\Big(\frac{\xi}{x}, Q, \mu\Big)\; f(\xi,\mu)$$

$$\omega = \xi\, \bar{n}\cdot p_p$$

hard perturbative corrections $\qquad\qquad$ parton distribution function

A more detailed set of SCET lecture notes can be found under "textbooks" in the 8.EFTx course.

All suggested homework problems can be accessed through the chapter for this school in 8.EFTx. The homework requires long answer (equation) solutions and is computer graded, so you will get immediate feedback.

To access the materials in 8.EFTx:

first sign up for an edX account here,

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