

# Theory Uncertainties for Higgs Searches using Jet Bins

Iain Stewart  
MIT

LBL Higgs Jamboree, Oct.2011

Based On:

arXiv:1107.2117      I.S. & F. Tackmann

(input to LHC Higgs Xsec working group for summer 2011 recommendations, “BNL accord”)

arXiv:1012.4480      + C.Berger, C.Marcantonini, W.Waalewijn

# Outline

- Introduction: Jet-bins in Higgs Searches
- Theory Uncertainties & Correlations
- Using Fixed Order Calculations for Jet Bins
- Exploiting Log Resummation for Jet Bins

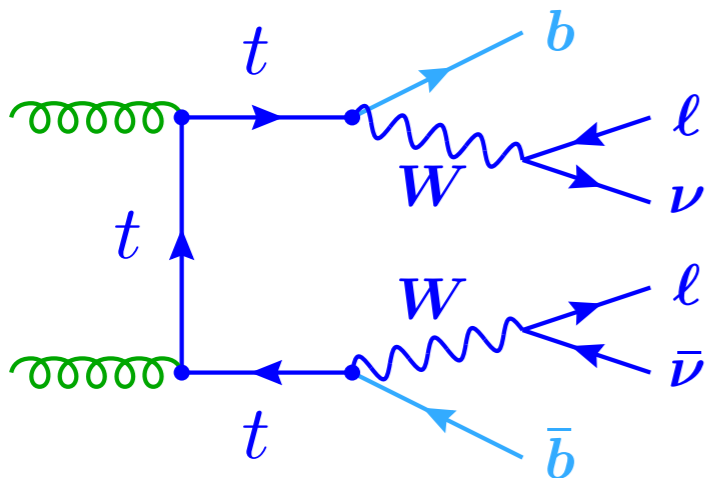
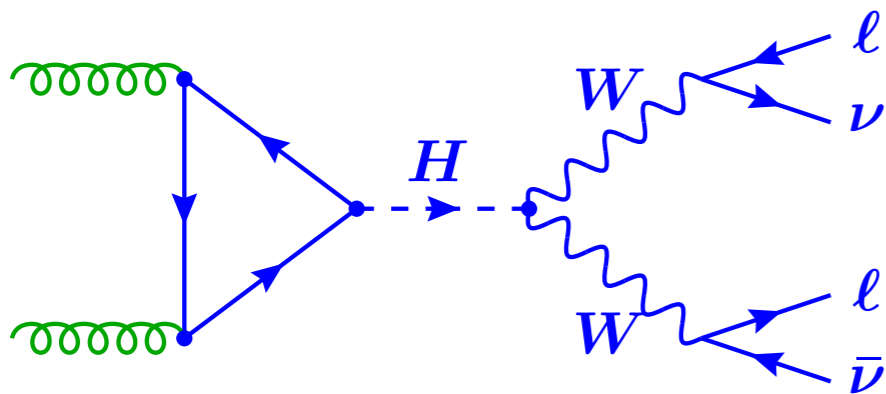
# Use jet bins to maximize sensitivity

- backgrounds vary with # of jets

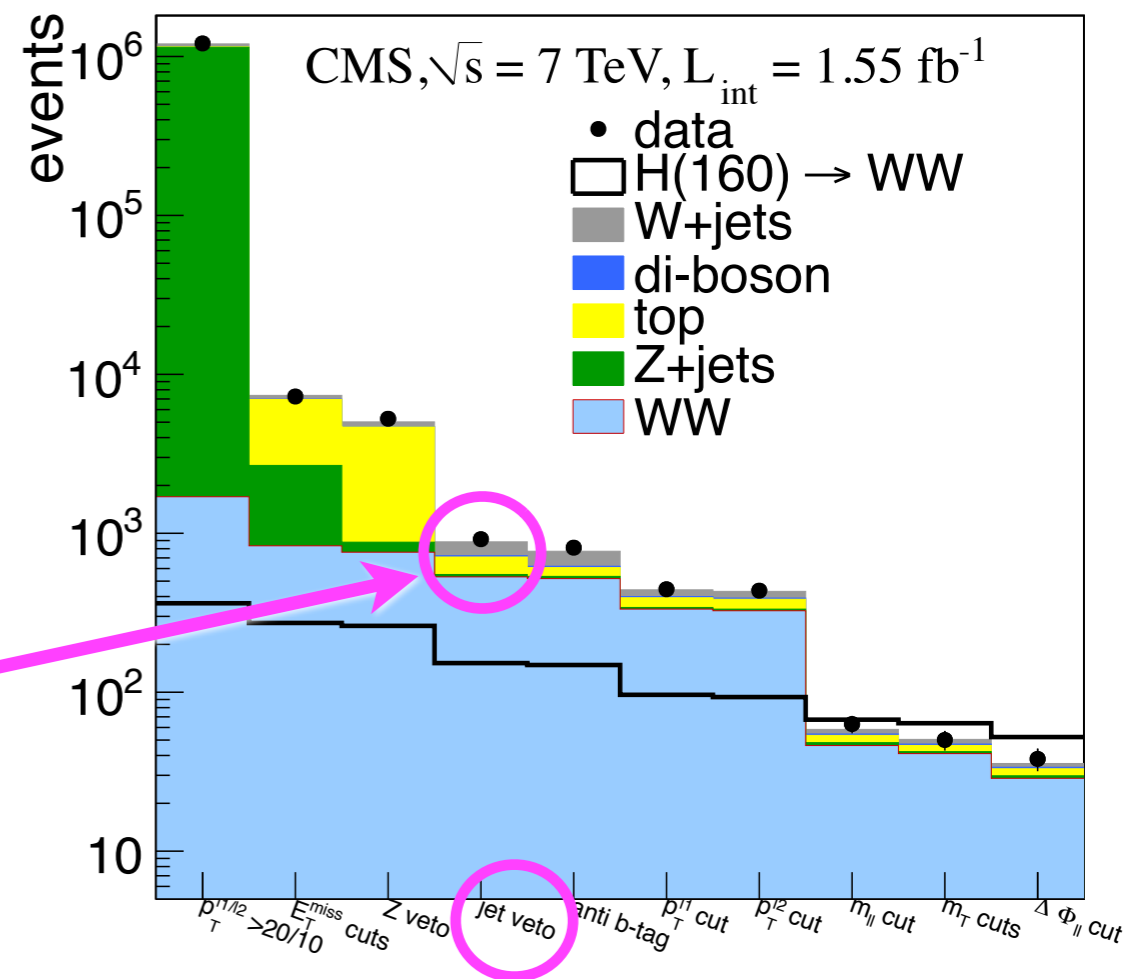
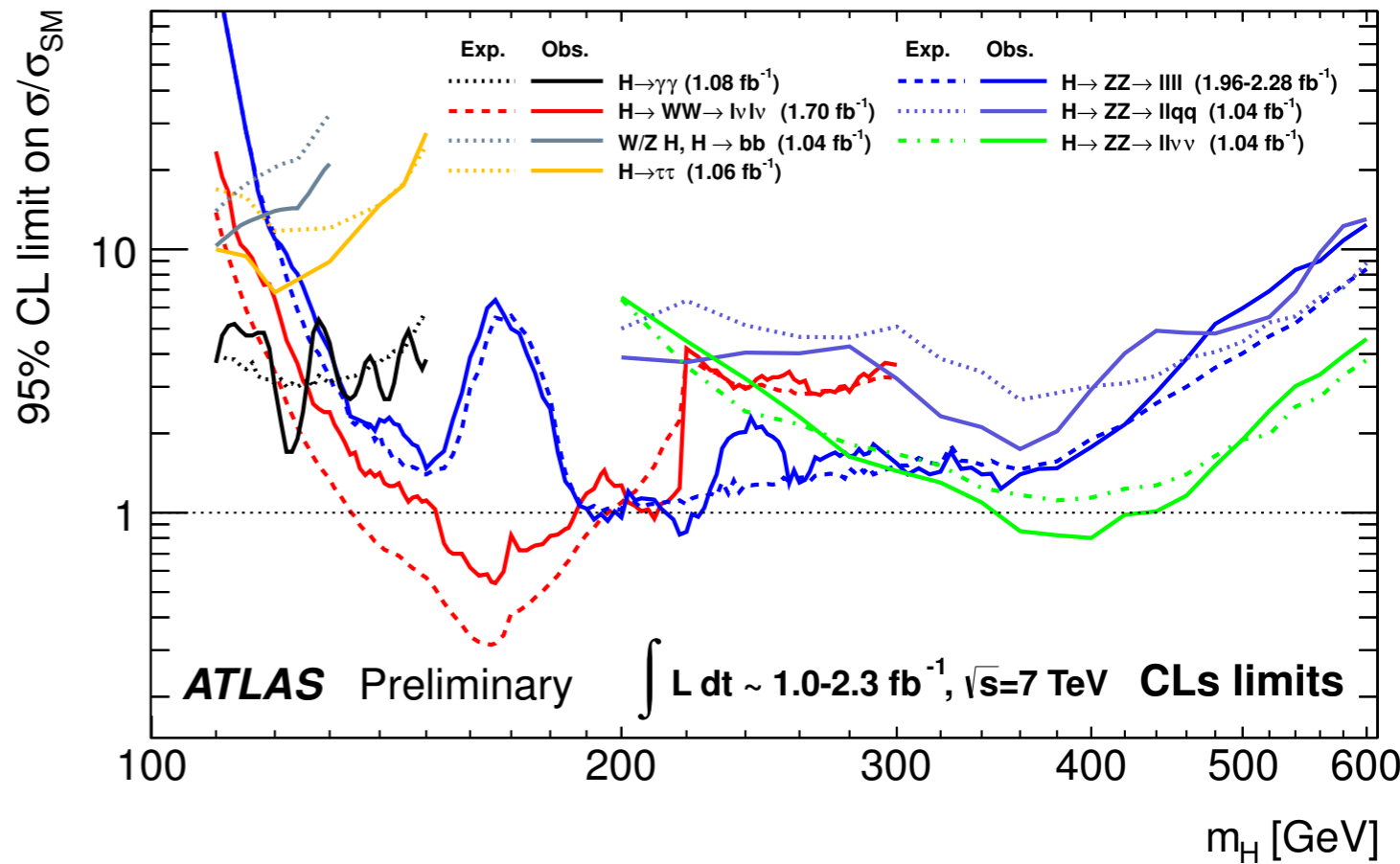
$$\begin{array}{ll}
 H \rightarrow WW \rightarrow \ell\nu\ell\bar{\nu} & H \rightarrow \tau\tau \\
 H \rightarrow WW \rightarrow \ell\nu jj & H \rightarrow \gamma\gamma
 \end{array}$$

eg. large top background in

$$H \rightarrow WW$$



o-jet bin  
vetoes  
b-jets



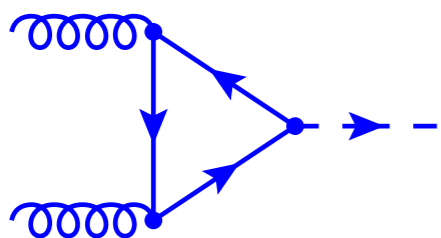
# Gluon fusion: $gg \rightarrow H$

# Vector-boson fusion: $qq \rightarrow qqH$

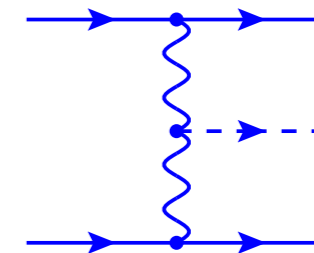
$\sigma_{\text{total}}$  to NNLO

(see Sally's talk)

$\sigma_{\text{total}}$  to NNLO\*



PDF +  $\alpha_s$  uncertainty  
(see Joey's talk)



perturbative uncertainty:  
 $\sim 2\%$

$\Delta_{\text{total}} \sim 8\%$

perturbative uncertainties

## exclusive jet cross sections

$$pp \rightarrow H + 0 \text{ jets} \rightarrow \sigma_0 \pm \Delta_0$$

$$pp \rightarrow H + 1 \text{ jet} \rightarrow \sigma_1 \pm \Delta_1$$

$$pp \rightarrow H + 2 \text{ jets} \rightarrow \sigma_2 \pm \Delta_2$$

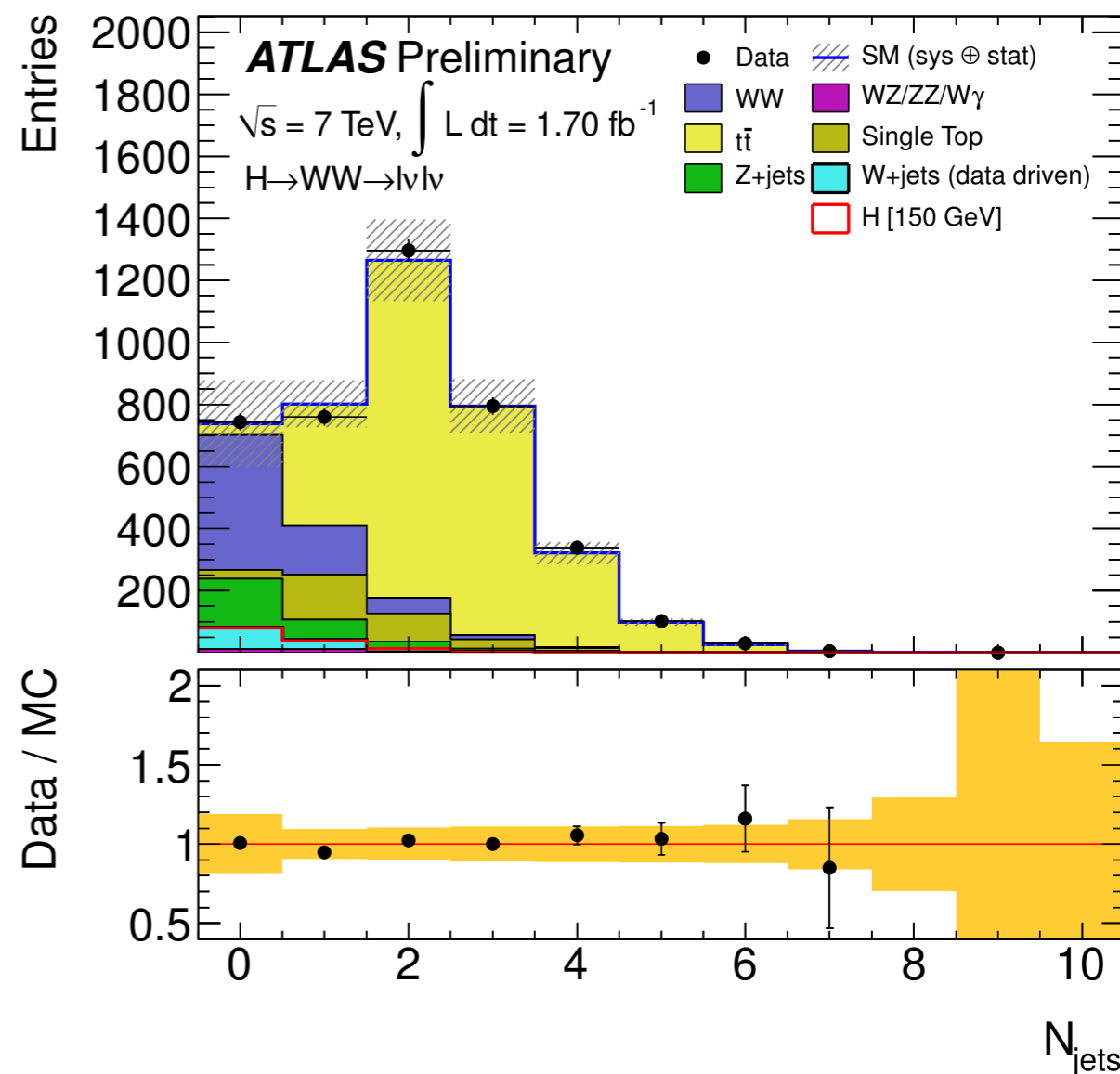
⋮

sum is  $\rightarrow \sigma_{\text{total}} \pm \Delta_{\text{total}}$

My talk:

$\Delta_i = ?$

correlations = ?

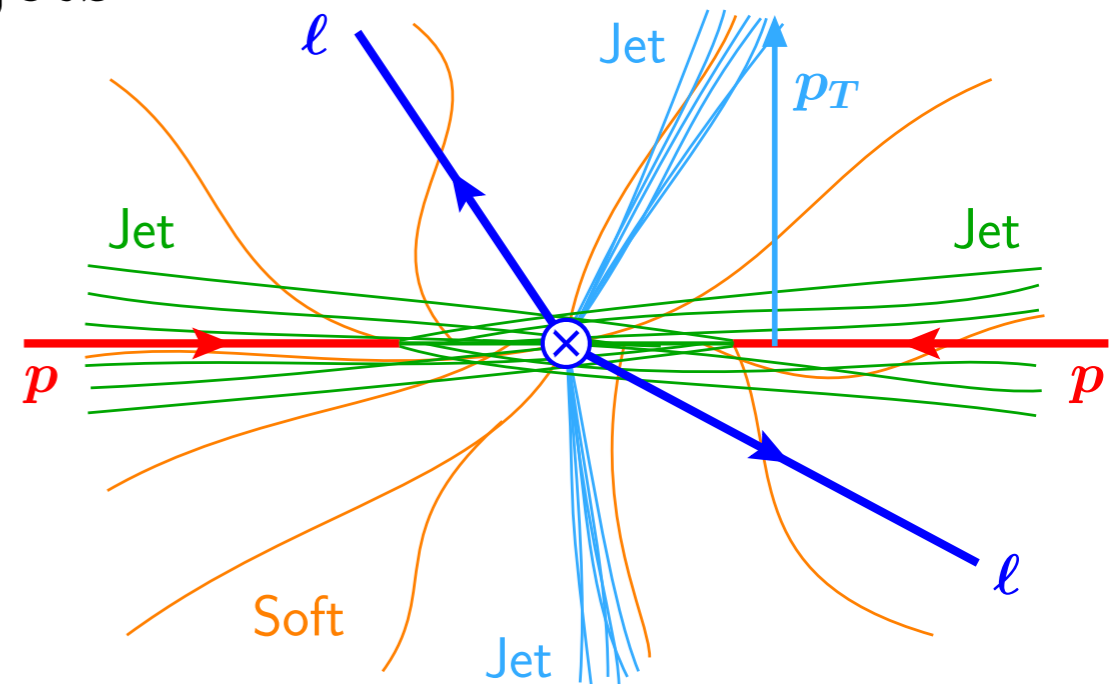


# Vetoing Jets : eg. $H \rightarrow WW + 0$ jets

Search for jets and require  $p_T^{\text{jet}} < p_T^{\text{cut}}$

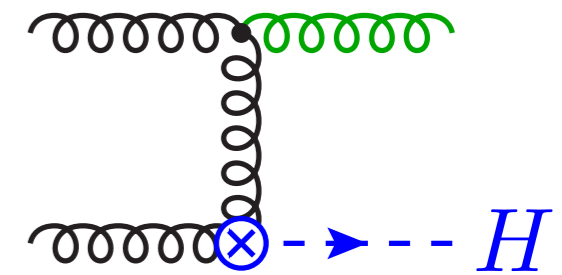
Tevatron:  $p_T^{\text{cut}} \simeq 20 \text{ GeV}$

LHC:  $p_T^{\text{cut}} \simeq 25 \text{ GeV}$



Jet Veto changes form of perturbation theory

Even if hard signal process  $gg \rightarrow H$  contains no jets, jet veto affects cross section by restricting ISR



$t$ -channel singularities produce Sudakov double logarithms

$$\sigma_0 = \sigma(p_T^{\text{cut}}) = \sigma_B \left( 1 - \frac{3\alpha_s}{\pi} 2 \ln^2 \frac{p_T^{\text{cut}}}{m_H} + \dots \right)$$

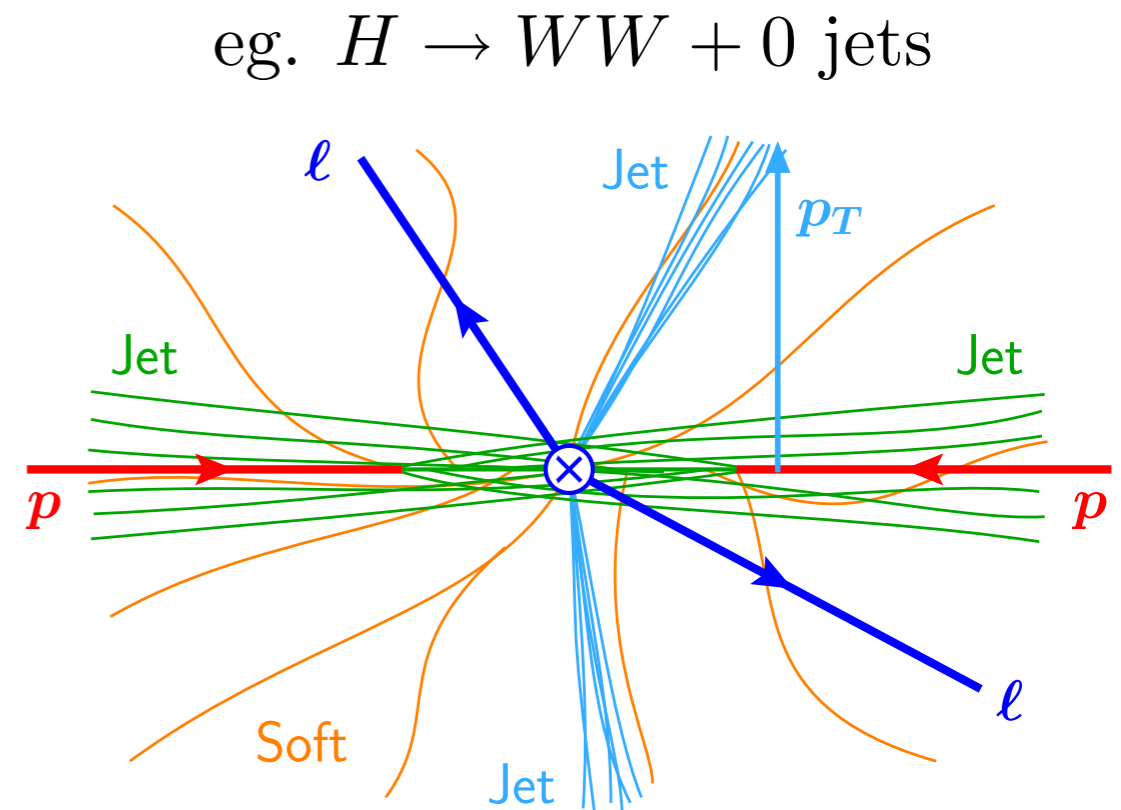
$\Rightarrow$  Perturbative corrections get large at small  $p_T^{\text{cut}} \ll m_H$

# Vetoing Jets :

Search for jets and require  $p_T^{\text{jet}} < p_T^{\text{cut}}$

Tevatron:  $p_T^{\text{cut}} \simeq 20 \text{ GeV}$

LHC:  $p_T^{\text{cut}} \simeq 25 \text{ GeV}$



Jet Veto changes form of perturbation theory

$$\sigma_0 \sim 1 + \alpha_s L^2 + \alpha_s^2 L^4 + \dots$$

$$+ \alpha_s L + \alpha_s^2 L^3 + \dots$$

$$+ \alpha_s + \alpha_s^2 L^2 + \dots$$

$$+ \alpha_s^2 L + \dots$$

$$+ \alpha_s^2 + \dots$$

$$L = \ln \frac{p_T^{\text{cut}}}{m_H}$$

$$\sigma_{\text{total}} = \underbrace{\int_0^{p_T^{\text{cut}}} dp_T \frac{d\sigma}{dp_T}}_{\sigma_0(p_T^{\text{cut}})} + \underbrace{\int_{p_T^{\text{cut}}} dp_T \frac{d\sigma}{dp_T}}_{\sigma_{\geq 1}(p_T^{\text{cut}})}$$

← inclusive jet cross section  
 $pp \rightarrow H + \geq 1 \text{ jet}$

- Added uncertainty  $\Delta_{\text{cut}}$  from our ability to predict  $p_T^{\text{cut}}$  dependence  
 (“large logs” or “particle migration between bins”)

- Cancels when adding  $\sigma_0$  and  $\sigma_{\geq 1}$   
**anti-correlated**  $\begin{pmatrix} \Delta_{\text{cut}}^2 & -\Delta_{\text{cut}}^2 \\ -\Delta_{\text{cut}}^2 & \Delta_{\text{cut}}^2 \end{pmatrix}$

- Extension to multiple exclusive jet bins:

$$\sigma_0(p_T^{\text{cut}}), \sigma_1(p_T^{\text{cut}}, p_{T2}^{\text{cut}}), \sigma_2(p_{T2}^{\text{cut}}, p_{T3}^{\text{cut}}), \dots$$

How do we compute  $\Delta_{\text{cut}}$  ?

(A) “Direct Exclusive Scale Variation?” vary  $\mu_F, \mu_R$  in  $\sigma_i$ 's  $\longrightarrow \Delta_i$

consider  $\sigma_0(\mu)$ , vary  $\mu \in [m_H/2, 2m_H]$  to get  $\Delta_0$  etc.

- uncertainties here are 100% correlated so that

$$\sigma_{\text{total}} = \sigma_0 + \sigma_1 + \dots \quad \text{gets back its uncertainty } \Delta_{\text{total}}$$

- **does not** account for  $\Delta_{\text{cut}}$
- due to numerical cancellations can underestimate uncertainties

$$\sigma_{\text{total}} \simeq \sigma_B [1 + \alpha_s + \alpha_s^2 + \mathcal{O}(\alpha_s^3)] \quad \text{large K-factor}$$

$$\sigma_{\geq 1}(p_T^{\text{cut}}) \simeq \sigma_B [\alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3 L^6)] \quad \text{large logs}$$

$$L = \ln(p_T^{\text{cut}}/m_H)$$

large cancellation for  $\sigma_0(p_T^{\text{cut}}) = \sigma_{\text{total}} - \sigma_{\geq 1}(p_T^{\text{cut}})$   
for some range of  $p_T^{\text{cut}}$

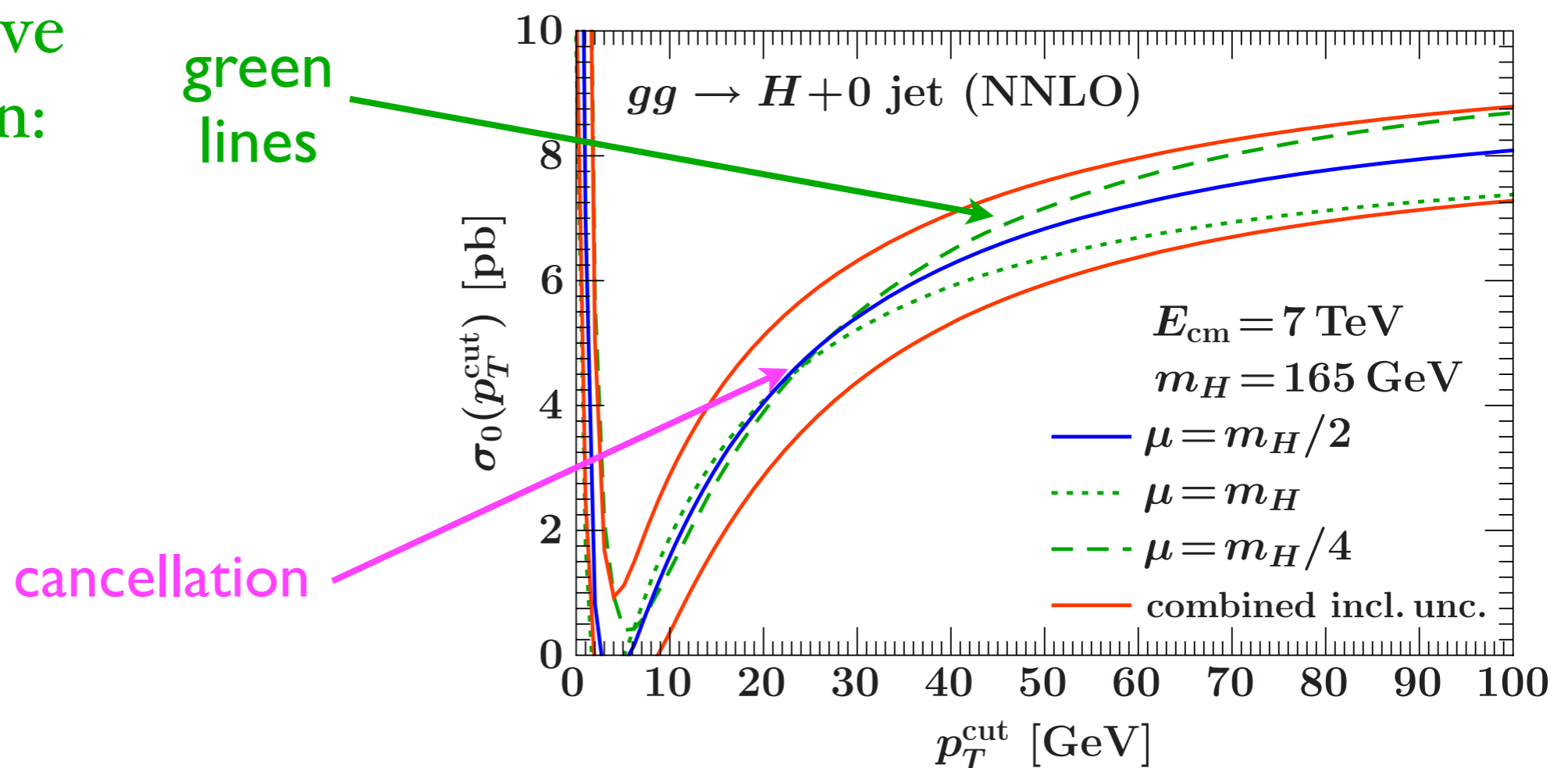


For example, at LHC for  $m_H = 165 \text{ GeV}$  and  $E_{\text{cm}} = 7 \text{ TeV}$

$$\sigma_{\text{total}} = (3.32 \text{ pb}) [1 + 9.5 \alpha_s + 35 \alpha_s^2 + \mathcal{O}(\alpha_s^3)]$$

$$\sigma_{\geq 1}(p_T^{\text{jet}} \geq 30 \text{ GeV}) = (3.32 \text{ pb}) [5.1 \alpha_s + 28 \alpha_s^2 + \mathcal{O}(\alpha_s^3)] .$$

Direct Exclusive  
Scale Variation:



All plots: MCFM for spectra, FeHiP for NNLO cross section,  
MSTW pdfs, anti-kT jets with  $R=0.5$

## (B) “Combined Inclusive Scale Variation”

IS, Tackmann, arXiv:1107.2117

- Treat inclusive cross-section uncertainties as independent

$$\Delta_{\text{total}}, \Delta_{\geq 1}, \Delta_{\geq 2}, \dots$$

- For  $p_T^{\text{cut}}$  uncertainty use:  $\Delta_{\text{cut}} = \Delta_{\geq 1}$

Propagate errors to get uncertainty for  $\sigma_0(p_T^{\text{cut}}) = \sigma_{\text{total}} - \sigma_{\geq 1}(p_T^{\text{cut}})$

eg.  $\{\sigma_0, \sigma_{\geq 1}\}$   $\begin{pmatrix} \Delta_{\geq 1}^2 + \Delta_{\text{total}}^2 & -\Delta_{\geq 1}^2 \\ -\Delta_{\geq 1}^2 & \Delta_{\geq 1}^2 \end{pmatrix}$  has anti-correlation

$$\sigma_{\text{total}} \simeq \sigma_B [1 + \alpha_s + \alpha_s^2 + \mathcal{O}(\alpha_s^3)] \quad \text{large K-factor}$$

$$\sigma_{\geq 1}(p_T^{\text{cut}}) \simeq \sigma_B [\alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3 L^6)] \quad \text{large logs}$$

treated as independent series

$$L = \ln(p_T^{\text{cut}}/m_H)$$

estimate for logs obtained from  $\sigma_{\geq 1}(p_T^{\text{cut}})$

For example, at LHC for  $m_H = 165 \text{ GeV}$  and  $E_{\text{cm}} = 7 \text{ TeV}$

$$\sigma_{\text{total}} = (3.32 \text{ pb}) [1 + 9.5 \alpha_s + 35 \alpha_s^2 + \mathcal{O}(\alpha_s^3)]$$

$$\sigma_{\geq 1}(p_T^{\text{jet}} \geq 30 \text{ GeV}) = (3.32 \text{ pb}) [5.1 \alpha_s + 28 \alpha_s^2 + \mathcal{O}(\alpha_s^3)] .$$

Direct Exclusive  
Scale Variation

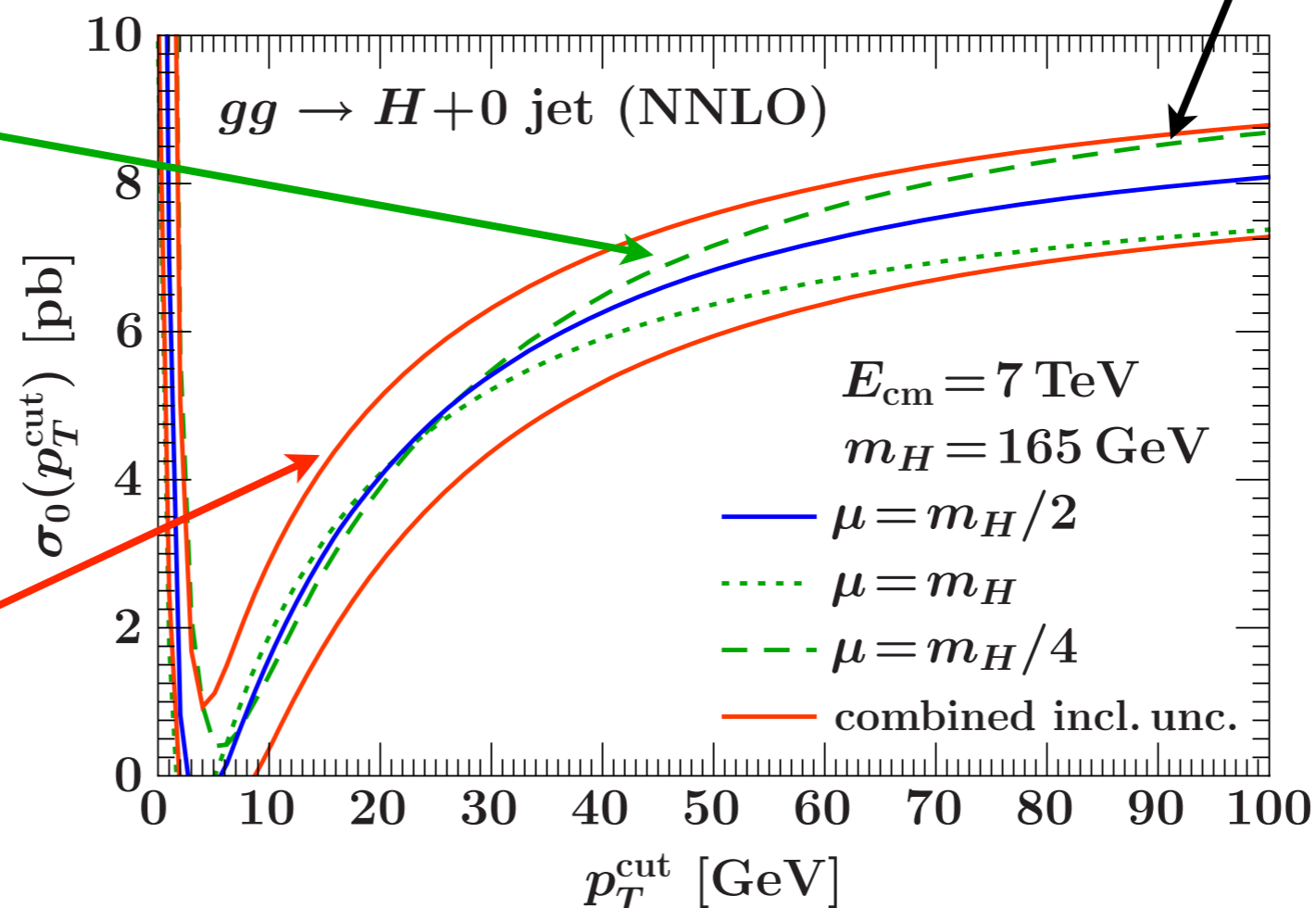
agree when  
cut is turned  
off

green lines

Combined Inclusive  
Scale Variation

$$\Delta_0^2 = \Delta_{\text{total}}^2 + \Delta_{\geq 1}^2$$

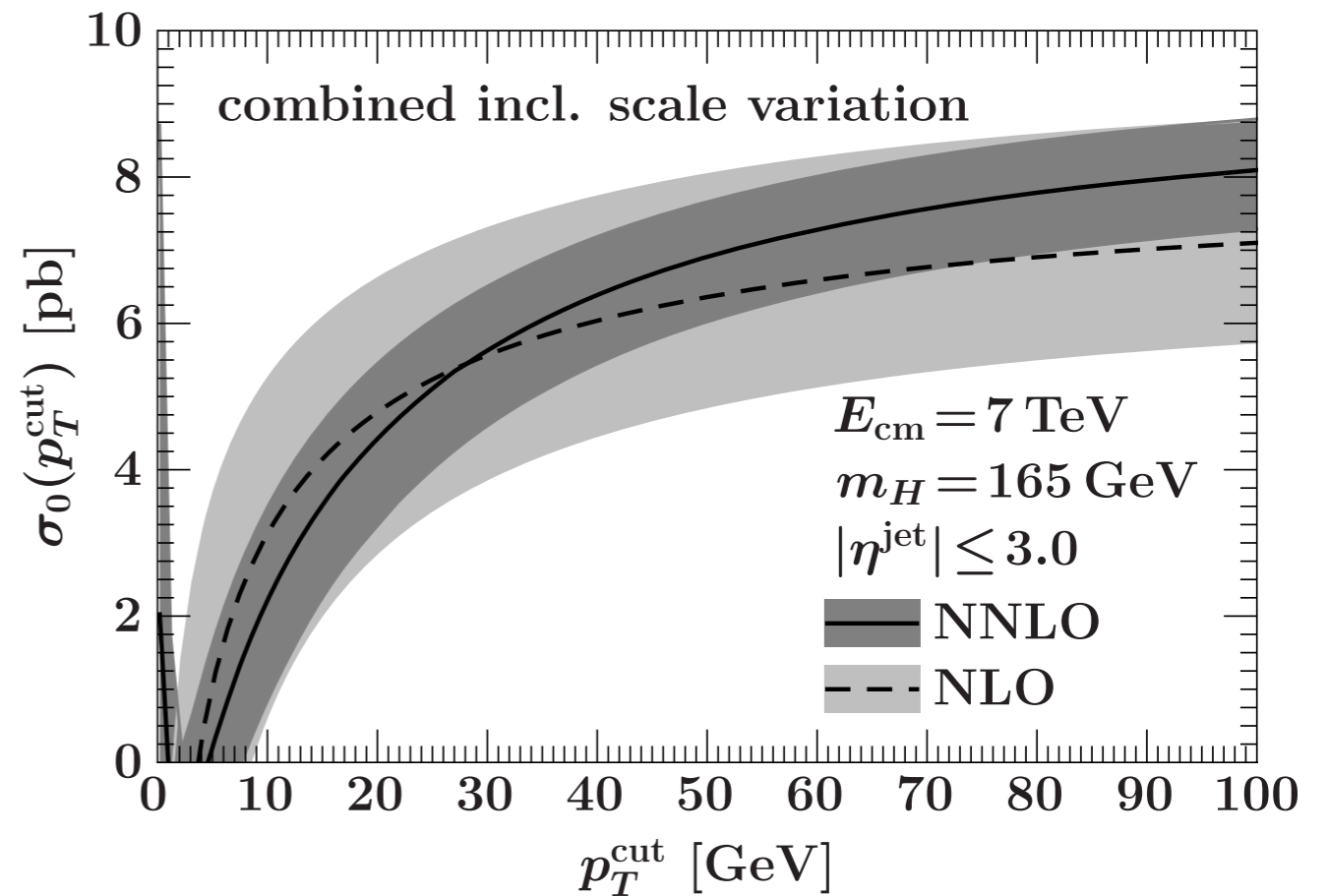
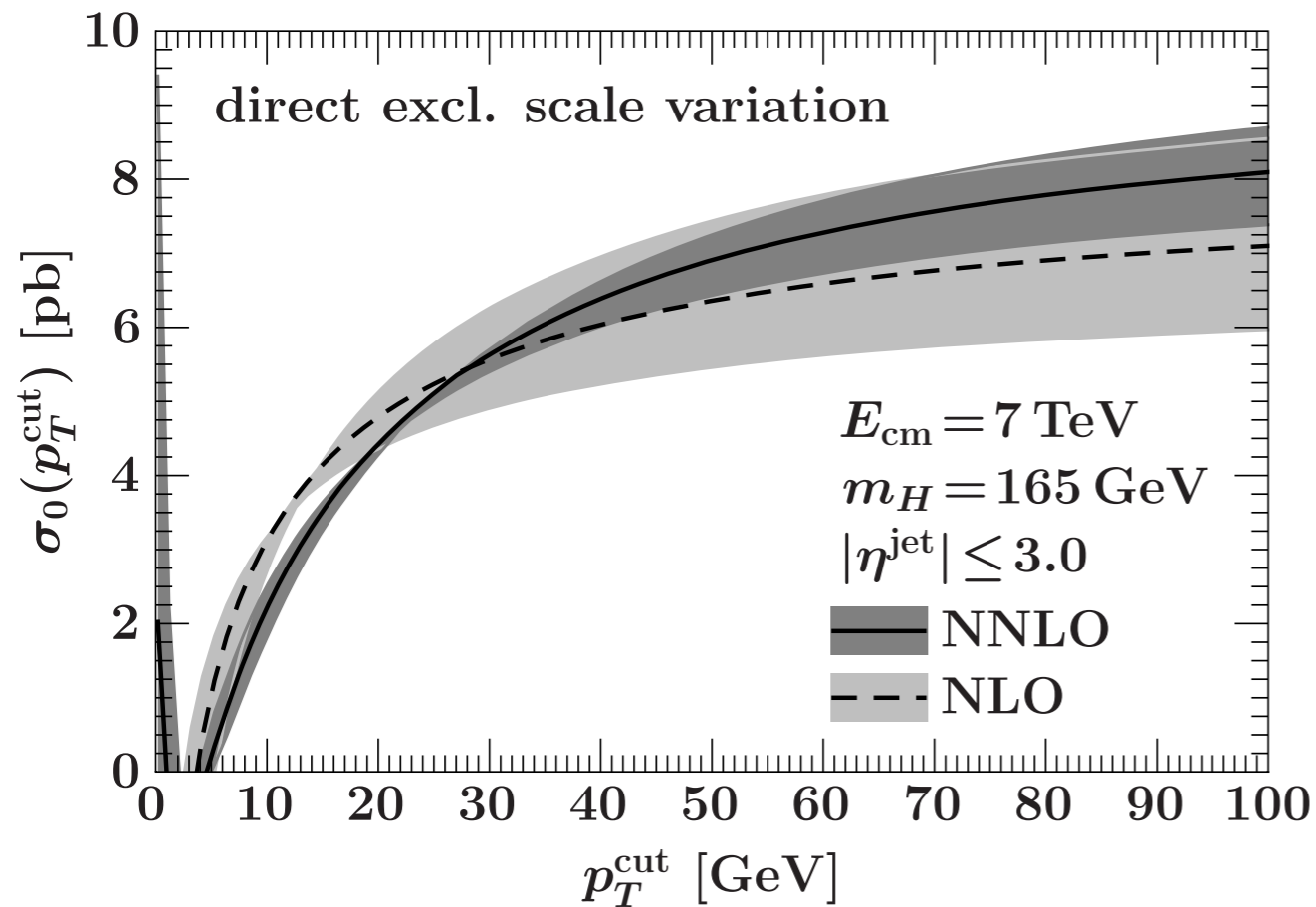
red lines



these plots only vary  $\mu_R = \mu_F$  (varying  $\mu_F$  alone is quite small for Higgs)

Quite generic: same pattern at Tevatron  
similar plots if we vary rapidity cuts  
similar plots for other processes

# Convergence (NLO to NNLO)



at  $p_T^{\text{cut}} = 30 \text{ GeV}$  and NNLO

$$\delta(\sigma_{\text{total}}) = 8.6\%$$

$$\delta(\sigma_{\geq 1}) = 19\%$$

$$\delta(\sigma_0) = 2.4\%$$

$$\rho(\sigma_0, \sigma_{\geq 1}) = +100\%$$

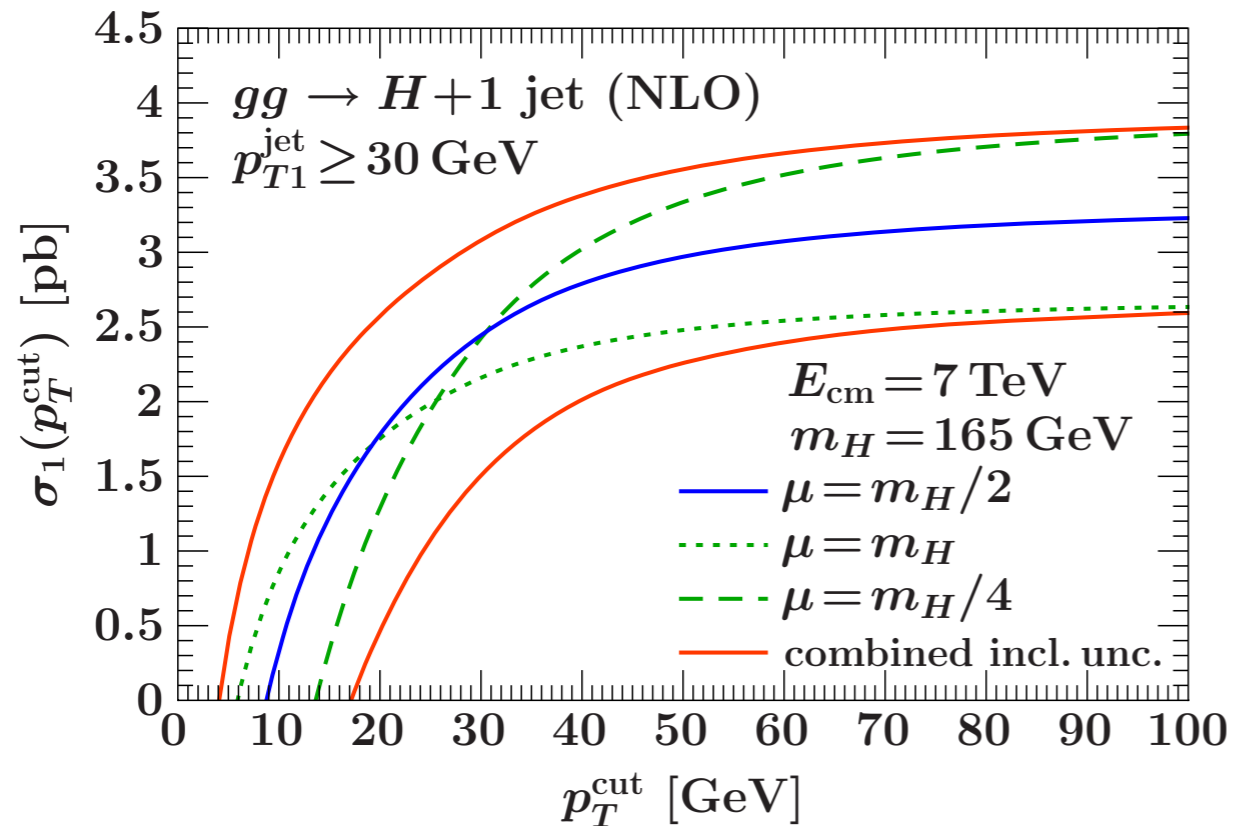
$$\delta(\sigma_{\text{total}}) = 8.6\%$$

$$\delta(\sigma_{\geq 1}) = 19\%$$

$$\delta(\sigma_0) = 18\%$$

$$\rho(\sigma_0, \sigma_{\geq 1}) = -64\%$$

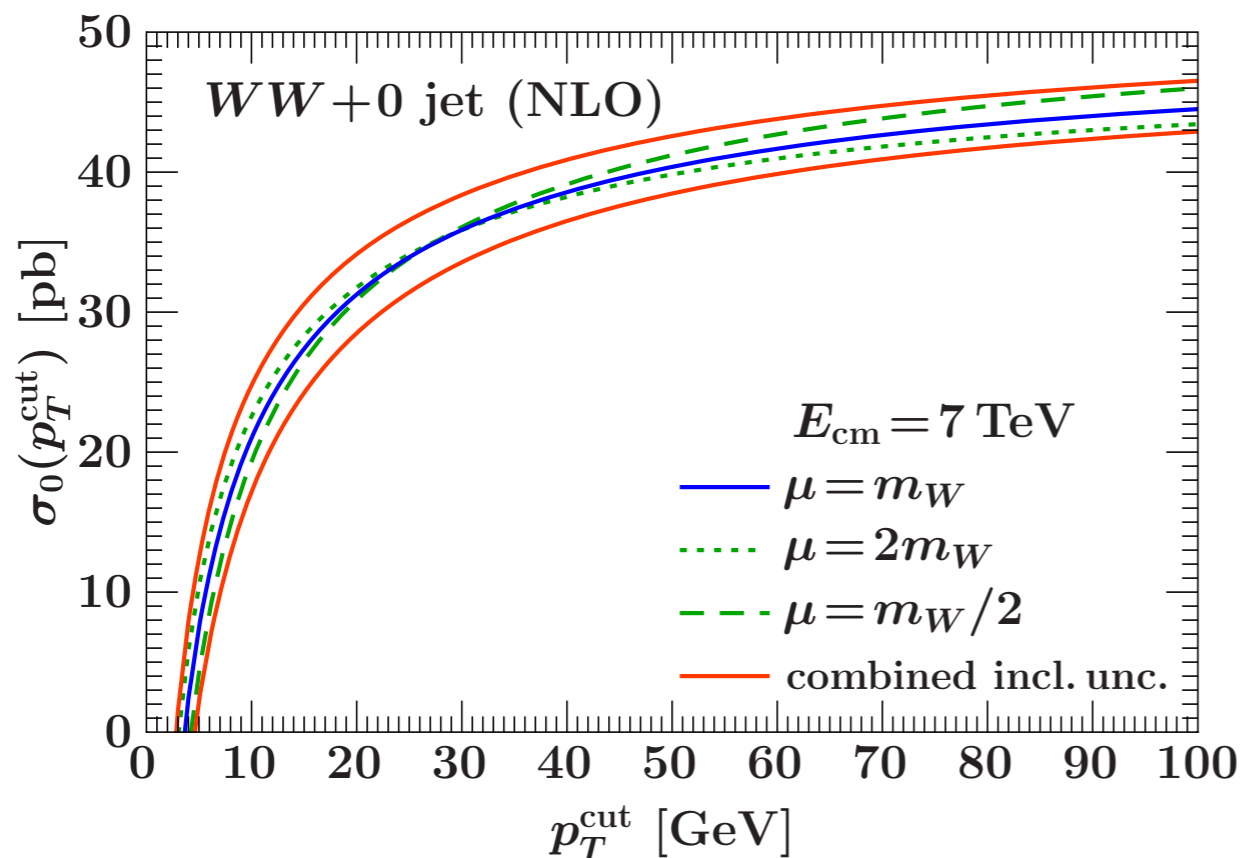
# other examples



$$\sigma_1(\mu)$$

$\Delta_{\geq 1}, \Delta_{\geq 2}$  independent

$$\sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}$$



uncertainties for background  
 processes in jet-bins  
 should be estimated the same way

eg. Numbers for  $\{\sigma_{\text{total}}, \sigma_0, \sigma_1\}$

$$\begin{array}{l}
 p_T^{\text{jet}} \geq 30 \text{ GeV} \\
 p_{T2}^{\text{jet}} \geq 30 \text{ GeV}
 \end{array}
 \begin{pmatrix}
 \Delta_{\text{total}}^2 & \Delta_{\text{total}}^2 & 0 \\
 \Delta_{\text{total}}^2 & \Delta_{\text{total}}^2 + \Delta_{\geq 1}^2 & -\Delta_{\geq 1}^2 \\
 0 & -\Delta_{\geq 1}^2 & \Delta_{\geq 1}^2 + \Delta_{\geq 2}^2
 \end{pmatrix}$$

start with:

$$\sigma_{\text{total}} = (8.70 \pm 0.75) \text{ pb}, \quad 8.6\%$$

$$\sigma_{\geq 1} = (3.29 \pm 0.62) \text{ pb} \quad 18.8\%$$

$$\sigma_{\geq 2} = (0.85 \pm 0.49) \text{ pb}, \quad 57\%$$

$$\sigma_0 = \sigma_{\text{tot}} - \sigma_{\geq 1}$$

$$\sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}$$

propagate to get:

$$\delta(\sigma_0) = 18\%$$

$$\rho(\sigma_0, \sigma_{\text{total}}) = 0.77$$

$$\rho(\sigma_0, \sigma_1) = -0.50$$

$$\delta(\sigma_1) = 32\%$$

$$\rho(\sigma_1, \sigma_{\geq 2}) = -0.62$$

or consider jet fractions:

$$f_0 = \frac{\sigma_0}{\sigma_{\text{total}}}$$

$$\delta(f_0) = 13\%$$

$$\rho(f_0, \sigma_{\text{total}}) = 0.42$$

$$\rho(f_0, f_1) = -0.80$$

$$\delta(f_1) = 33\%$$

$$\rho(f_1, \sigma_{\text{total}}) = -0.26$$

$$f_1 = \frac{\sigma_1}{\sigma_{\text{total}}}$$

# (C) Use Resummed Predictions to get Uncertainties

this will allow us to include both types of uncertainties (correlated & uncorrelated) from methods (A) and (B)



# Jet Vetoes

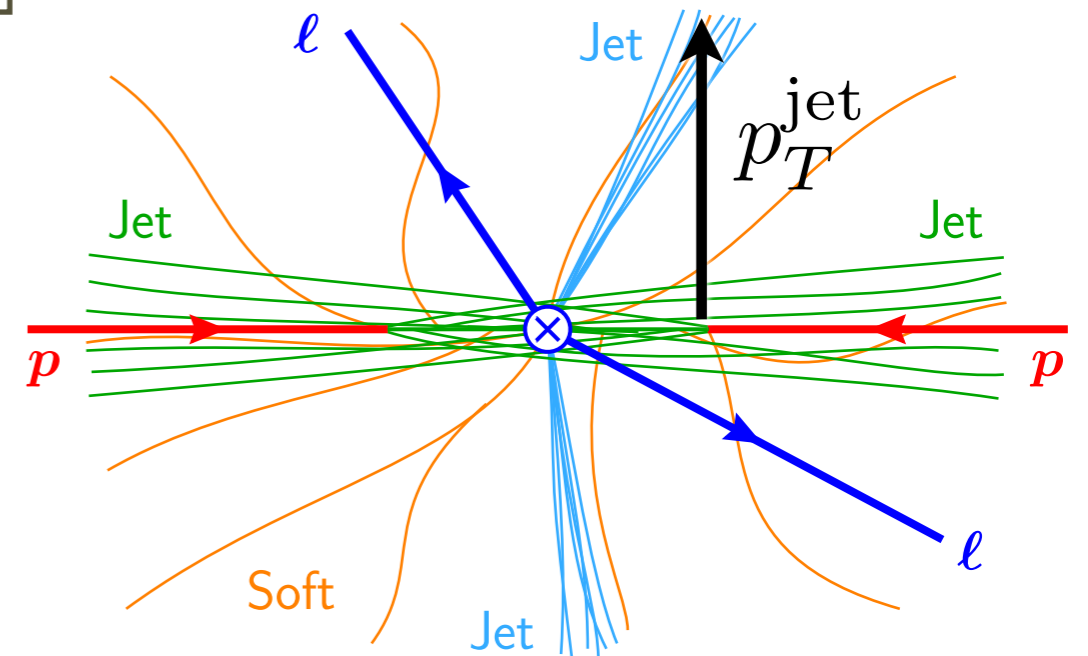
## Conventional: Jet Algorithm

- Search for jets and require  $p_T^{\text{jet}} < p_T^{\text{cut}}$

Tevatron:  $p_T^{\text{cut}} \simeq 20 \text{ GeV}$

LHC:  $p_T^{\text{cut}} \simeq 25 \text{ GeV}$

- Complicated phase-space restrictions



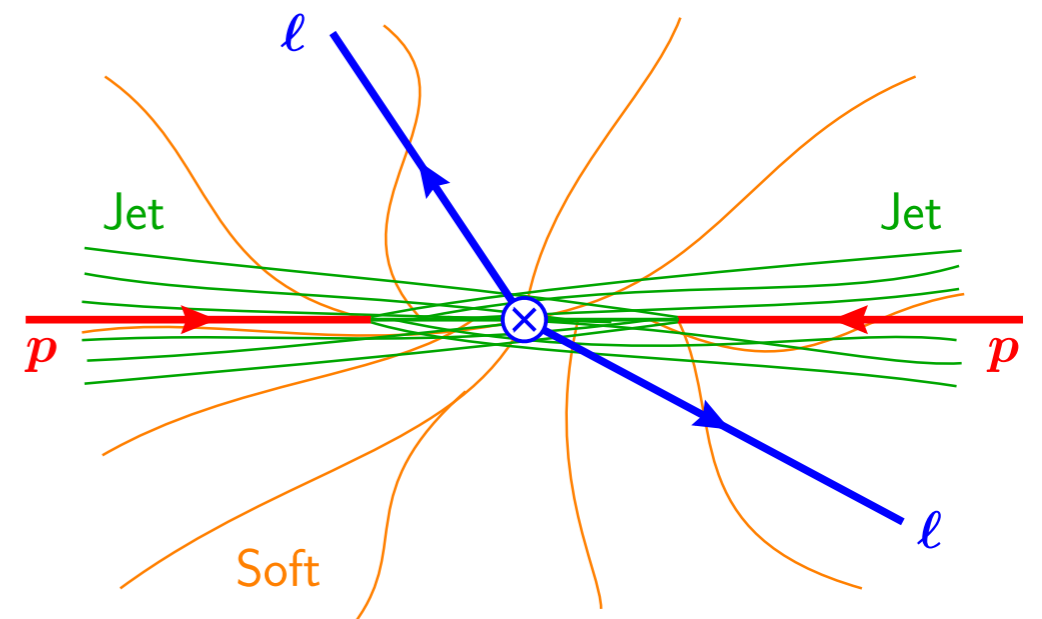
## Alternative: Event Shape

- Measure beam thrust for each event

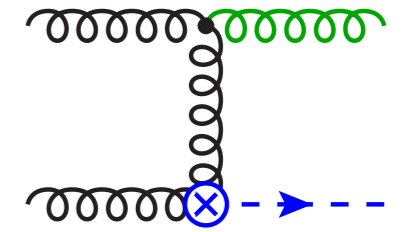
$$\mathcal{T}_{\text{cm}} = \sum_k |\vec{p}_{kT}| e^{-|\eta_k|} = \sum_k (E_k - |p_k^z|)$$

and require  $\mathcal{T}_{\text{cm}} < \mathcal{T}_{\text{cm}}^{\text{cut}}$

- Nice for higher order calculations



# Jet veto restricts ISR, gives double logs



$$L = \ln \frac{p_T^{\text{cut}}}{m_H} \quad \text{or} \quad L = \ln \frac{\mathcal{T}_{\text{cm}}^{\text{cut}}}{m_H}$$

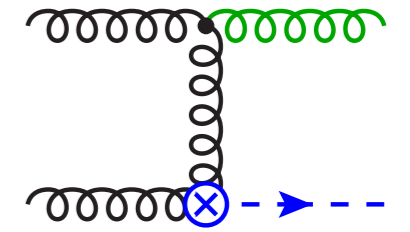
	LO	NLO				
$\sigma_{0\text{-jet}} =$	1	$+\alpha_s L^2$	$+\alpha_s^2 L^4$	$+\alpha_s^3 L^6$	$+\dots$	LL
		$+\alpha_s L$	$+\alpha_s^2 L^3$	$+\alpha_s^3 L^5$	$+\dots$	
		$+\alpha_s n_1(p_T^{\text{cut}})$	$+\alpha_s^2 L^2$	$+\alpha_s^3 L^4$	$+\dots$	
			$+\alpha_s^2 L$	$+\alpha_s^3 L^3$	$+\dots$	
			$+\alpha_s^2 n_2(p_T^{\text{cut}})$	$+\alpha_s^3 L^2$	$+\dots$	
				$+\alpha_s^3 L$	$+\dots$	
				$+\alpha_s^3$	$+\dots$	

## Parton Shower

eg. Pythia is LL (+ tuning)

eg. MC@NLO is NLO+LL

# Jet veto restricts ISR, gives double logs



$$L = \ln \frac{p_T^{\text{cut}}}{m_H} \quad \text{or} \quad L = \ln \frac{\mathcal{T}_{\text{cm}}^{\text{cut}}}{m_H}$$

	LO	NLO	NNLO			
$\sigma_{0\text{-jet}} =$	1	$+\alpha_s L^2$	$+\alpha_s^2 L^4$	$+\alpha_s^3 L^6$	$+\dots$	LL
		$+\alpha_s L$	$+\alpha_s^2 L^3$	$+\alpha_s^3 L^5$	$+\dots$	NLL
		$+\alpha_s n_1(p_T^{\text{cut}})$	$+\alpha_s^2 L^2$	$+\alpha_s^3 L^4$	$+\dots$	NNLL
			$+\alpha_s^2 L$	$+\alpha_s^3 L^3$	$+\dots$	NNLL
			$+\alpha_s^2 n_2(p_T^{\text{cut}})$	$+\alpha_s^3 L^2$	$+\dots$	
				$+\alpha_s^3 L$	$+\dots$	
				$+\alpha_s^3$	$+\dots$	

Calculation:

NNLL + NNLO

Berger et.al.

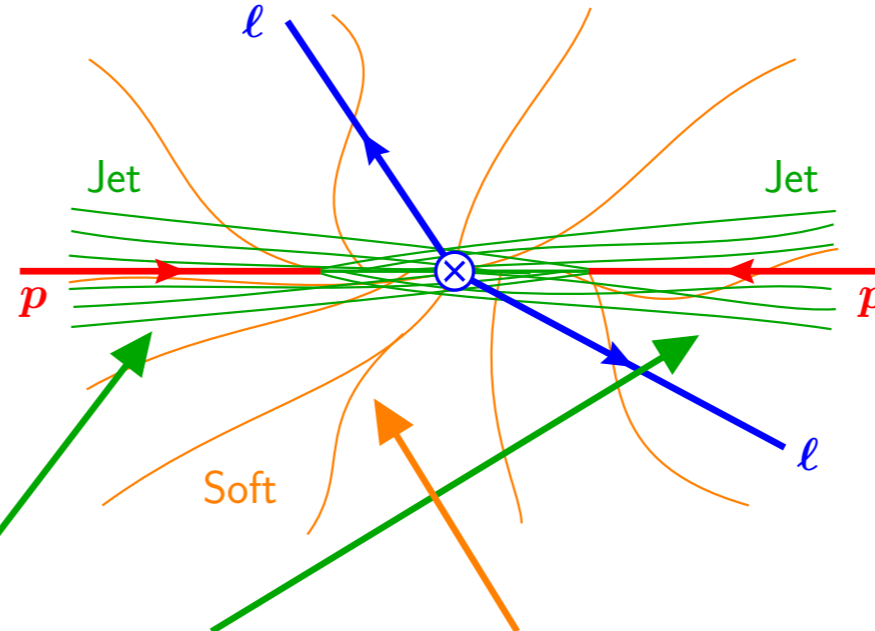
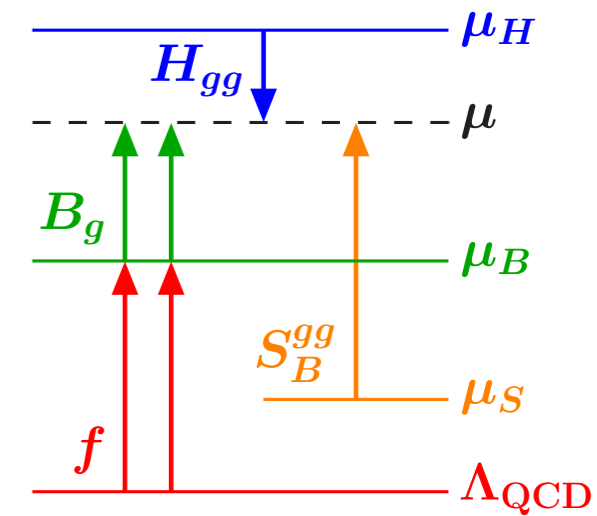
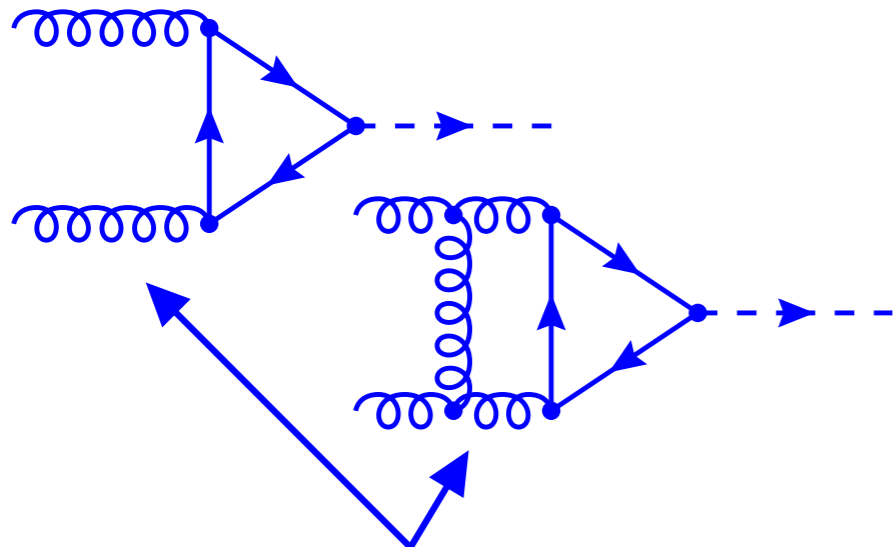
two orders of summation  
beyond LL shower programs

# (C) Use Resummed Predictions to get Uncertainties

this will allow us to include both types of uncertainties (correlated & uncorrelated) from methods (A) and (B)

- Idea:
- reweigh MC@NLO or POWHEG to NNLO (what you do now)  
for central values for  $p_T^{\text{cut}}$
  - resummed calculation has two sources of uncertainty, one is correlated with  $\Delta_{\text{total}}$ , one gives  $\Delta_{\text{cut}}$
  - given these as % errors for spectra in  $\mathcal{T}_{\text{cm}}$ , reweigh a MC sample to apply these errors for  $p_T^{\text{cut}}$

# NNLL + NNLO calculation



$$\frac{d\sigma^s}{d\mathcal{T}_{\text{cm}}} = H_{gg}(\mu) \int dt_a dt_b B_g(t_a, \mu) B_g(t_b, \mu) S_B^{gg} \left( \mathcal{T}_{\text{cm}} - \frac{t_a + t_b}{m_H}, \mu \right)$$

$$B_i(t, x) = \int \frac{d\xi}{\xi} \mathcal{I}_{ij}(t, x/\xi) f_j(\xi)$$

Function	describes	at the scale
Hard $H_{gg}$	hard virtual radiation	$ \mu_H  \simeq m_H$
Beam $B_g$	virtual & real energetic ISR	$\mu_B \simeq \sqrt{\mathcal{T}_{\text{cm}} m_H}$
Soft $S_B^{gg}$	virtual & real soft radiation	$\mu_S \simeq \mathcal{T}_{\text{cm}}$

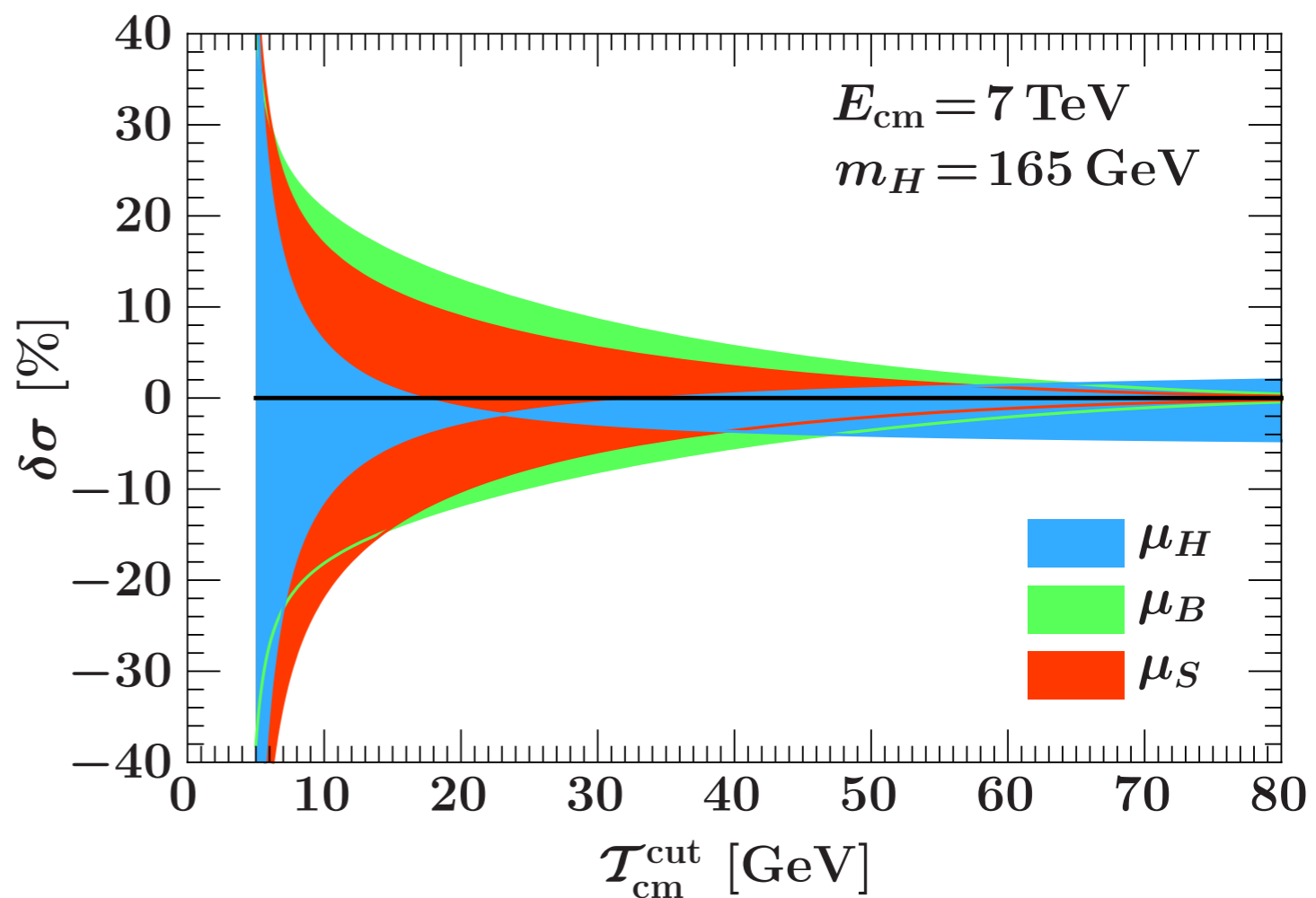
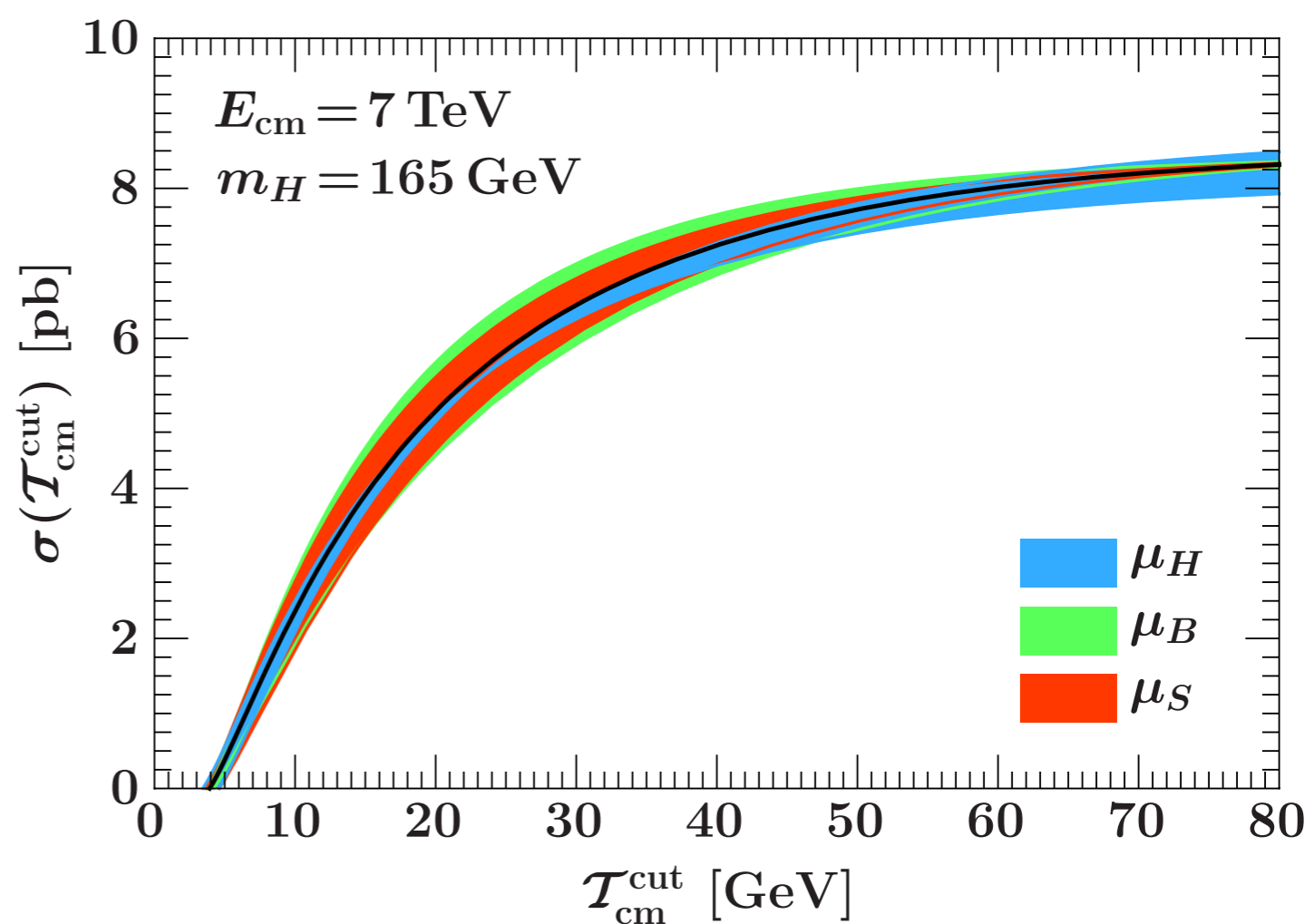
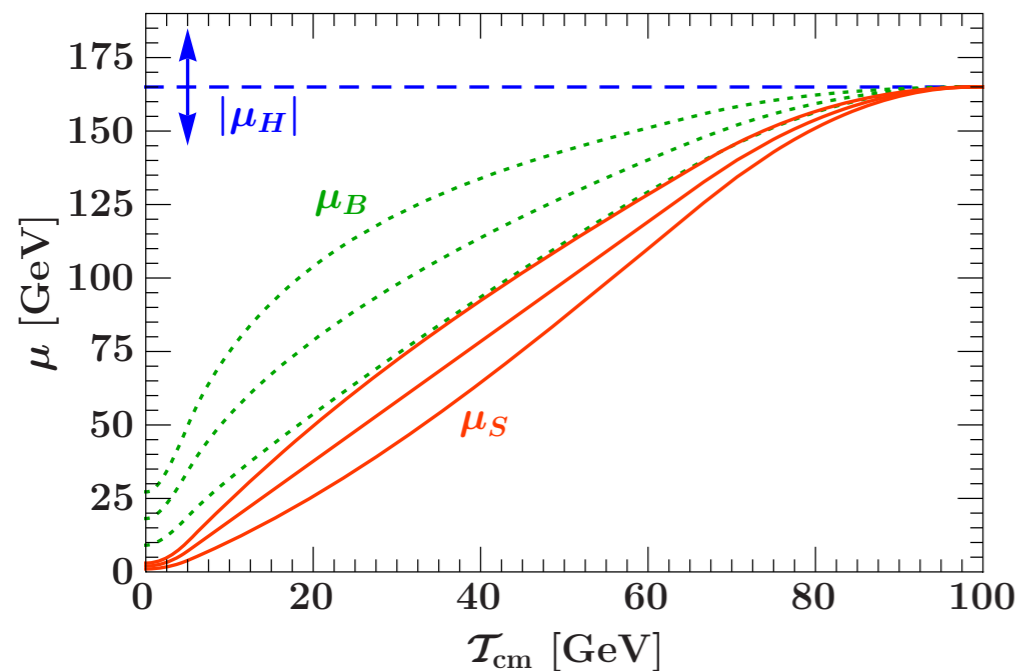
} logs give sensitivity to smaller scales

Perturbation theory at each scale contributes to uncertainties

Small  $\mathcal{T}_{\text{cm}}^{\text{cut}}$

individual scale variations

- three separate scale variations
- $\mu_H = \mu_{H0}$  100% correlated with  $\sigma_{\text{total}}$
- $\mu_B$  and  $\mu_S$  give  $\Delta_{\text{cut}} = \Delta_{SB}$  (dominate for small  $\mathcal{T}_{\text{cm}}^{\text{cut}}$ )



$$C = C_{SB} + C_H$$

$$C_H = \begin{pmatrix} \Delta_{H\text{tot}}^2 & \Delta_{H\text{tot}}\Delta_{H0} & \Delta_{H\text{tot}}\Delta_{H\geq 1} \\ \Delta_{H\text{tot}}\Delta_{H0} & \Delta_{H0}^2 & \Delta_{H0}\Delta_{H\geq 1} \\ \Delta_{H\text{tot}}\Delta_{H\geq 1} & \Delta_{H0}\Delta_{H\geq 1} & \Delta_{H\geq 1}^2 \end{pmatrix}$$

$$\Delta_{H\text{tot}} = \Delta_{H0} + \Delta_{H\geq 1}$$

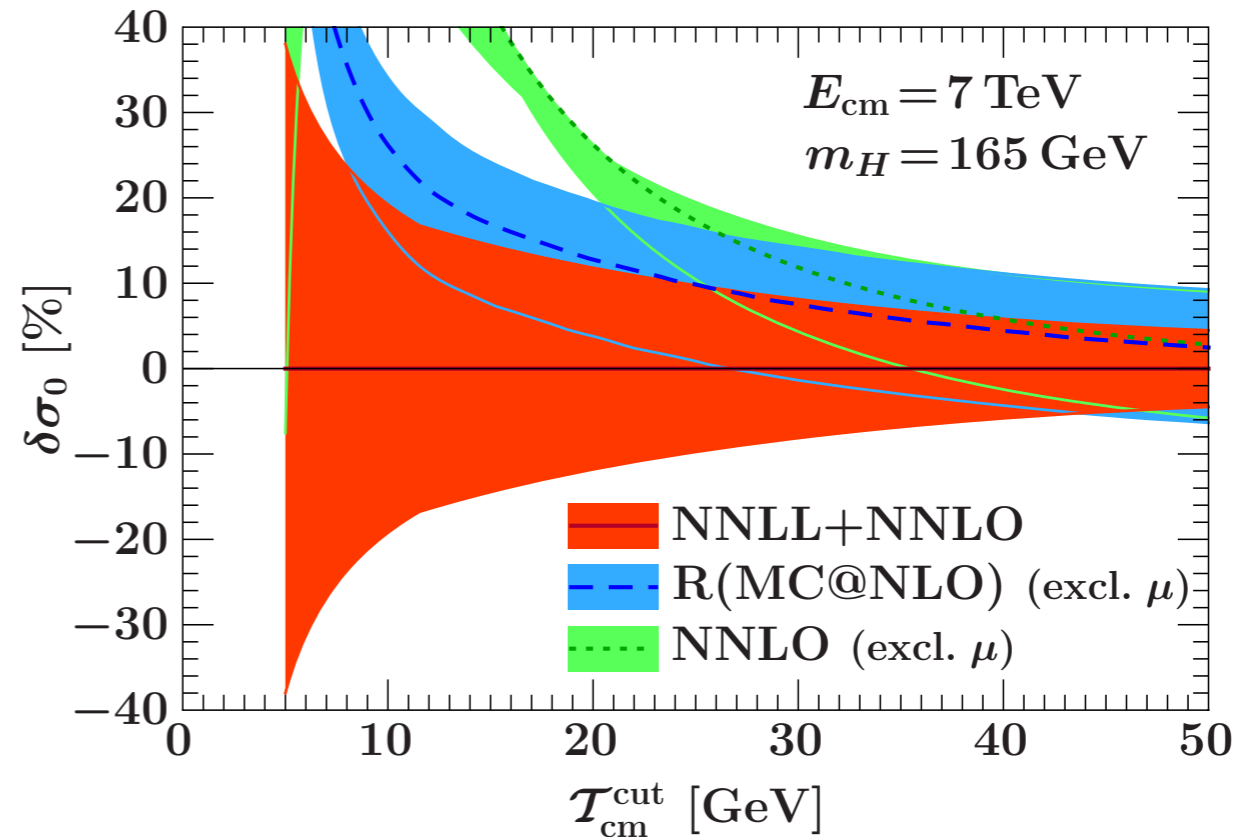
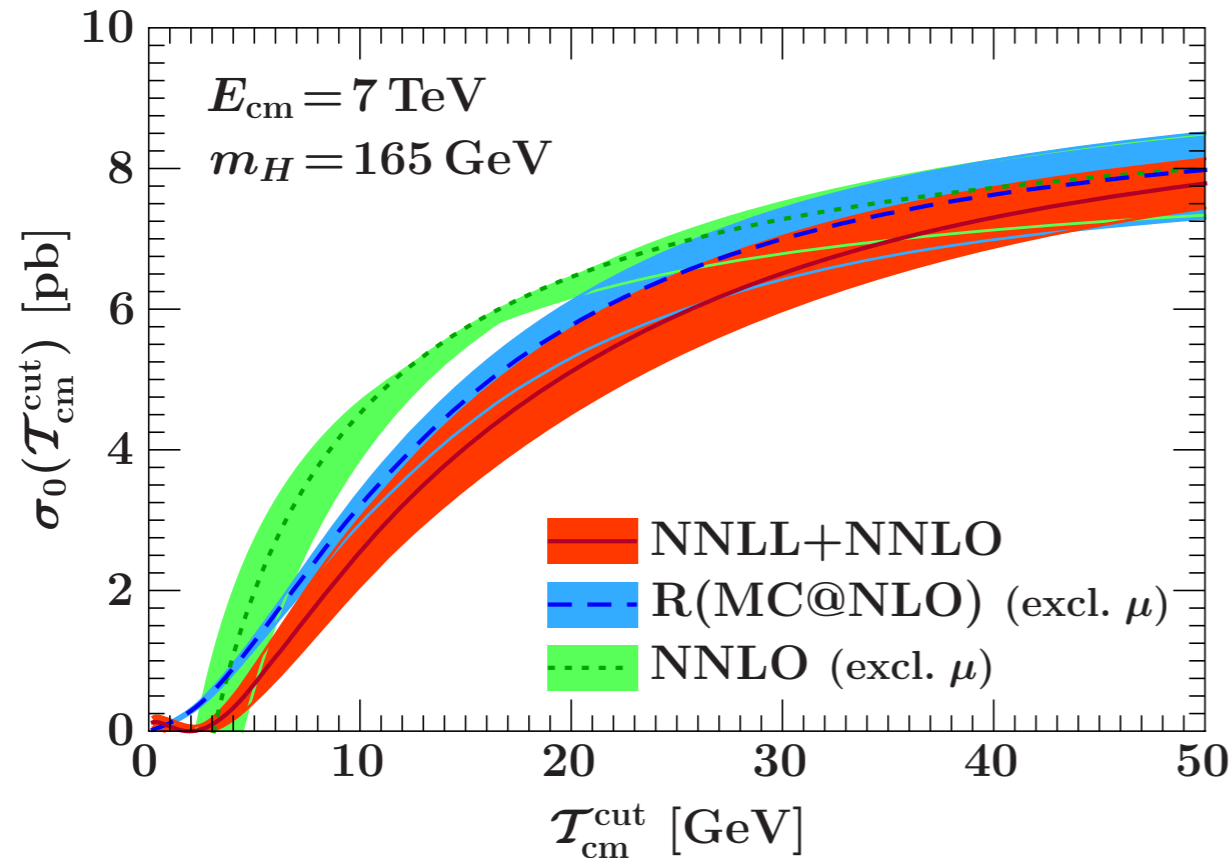
$$C_{SB} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta_{SB}^2 & -\Delta_{SB}^2 \\ 0 & -\Delta_{SB}^2 & \Delta_{SB}^2 \end{pmatrix}$$

Small  $\mathcal{T}_{\text{cm}}^{\text{cut}}$

like small  $p_T^{\text{cut}}$

direct exclusive scale variation shown for NNLO & MC@NLO

combined NNLL scale variations shown



- logs are large, **NNLL** central value lower than **NNLO**
- reweigh **MC@NLO** to match **NNLO** value/uncertainty at 200GeV  
Central value is nearer NNLL. **Uncertainty** is only for norm.
- direct exclusive uncertainties here are too small (we discussed that...)

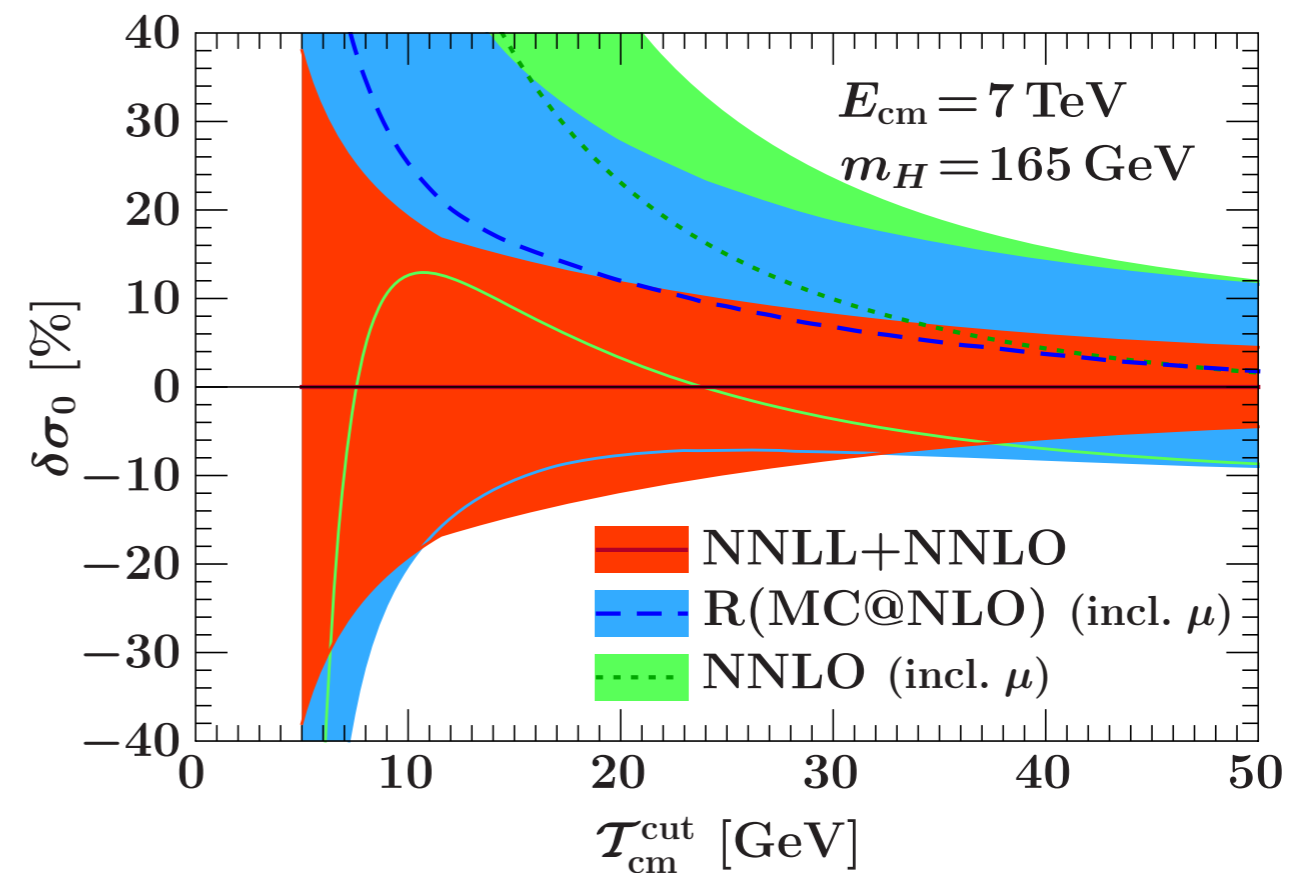
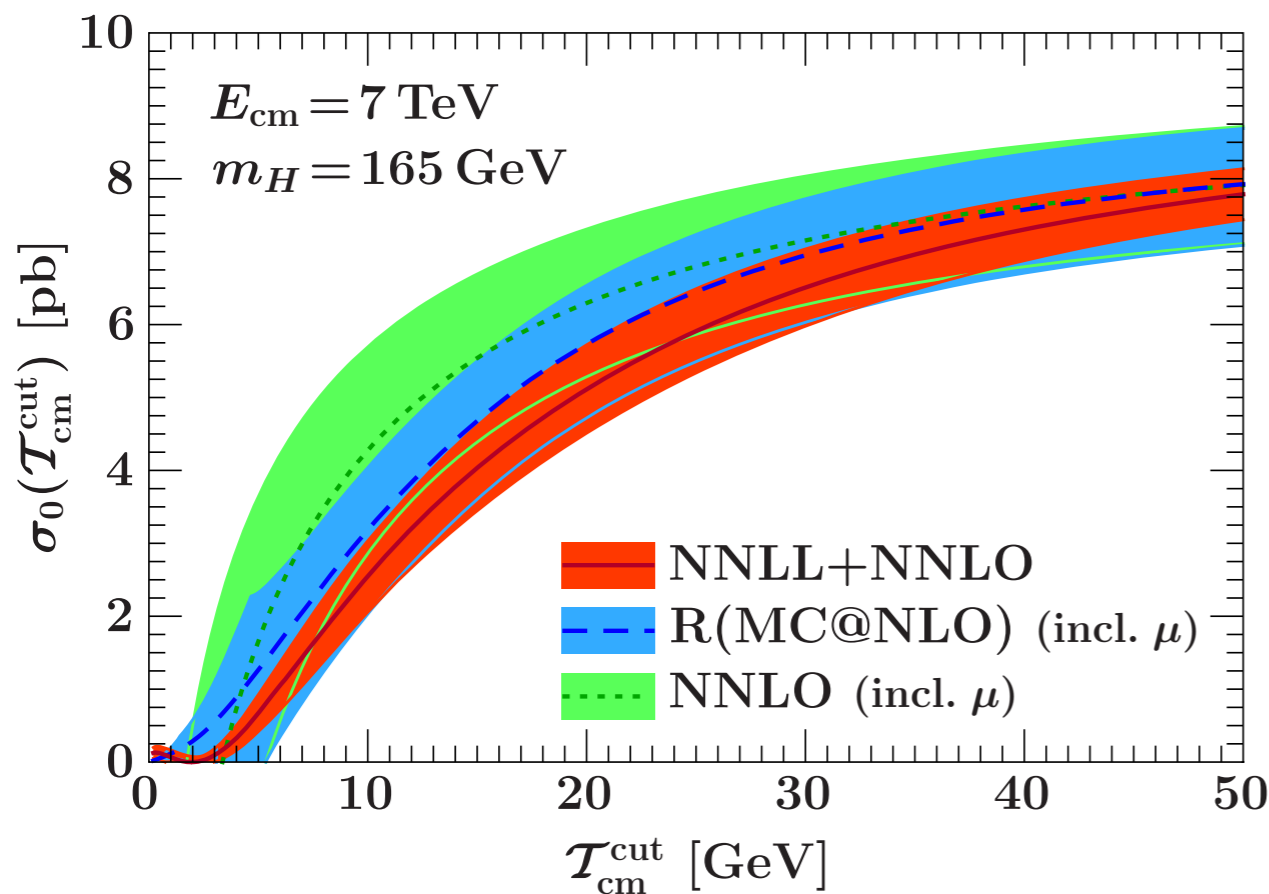


Small  $\mathcal{T}_{\text{cm}}^{\text{cut}}$

like small  $p_T^{\text{cut}}$

combined inclusive scale variation shown for NNLO & MC@NLO

combined NNLL scale variations shown



- NNLO band largely overlaps NNLL result
- reweigh MC@NLO to match NNLO incl. uncertainties (full spectrum). Overlaps nicely.
- This factor of two improvement in uncertainty is what one would expect if a similar reweighing exercise is done for  $p_T^{\text{jet}}$

# Discussion

- Reweighting will reduce theory errors. Logical next step. I have tables for LHC@ 7 TeV,  $m_H = 165$  GeV. (And Tevatron for many  $m_H$ 's because they already started the reweighting during the summer rush.) Other  $m_H$  table's for the LHC are straightforward to produce.
- I) Will you quote experimental results/limits for exclusive jet cross section? (bkgnd+signal, signal) at Hadron level (corrected to perfect detector). ie. Quote experimental results without any jet-bin bias from theory. (Useful to theorists for benchmarking.)  
  
II) Can you make a plot to show the effect of theory uncertainties? eg. quote what the result would look like with zero theory errors. This would be a strong motivation for theorists to do better, once they see clearly the places they are loosing.

## From CMS:

2011/08/22

Table 3: Summary of all systematic uncertainties (relative). This is just an indicative table, since the precise values depend on the final state and jet-bin.

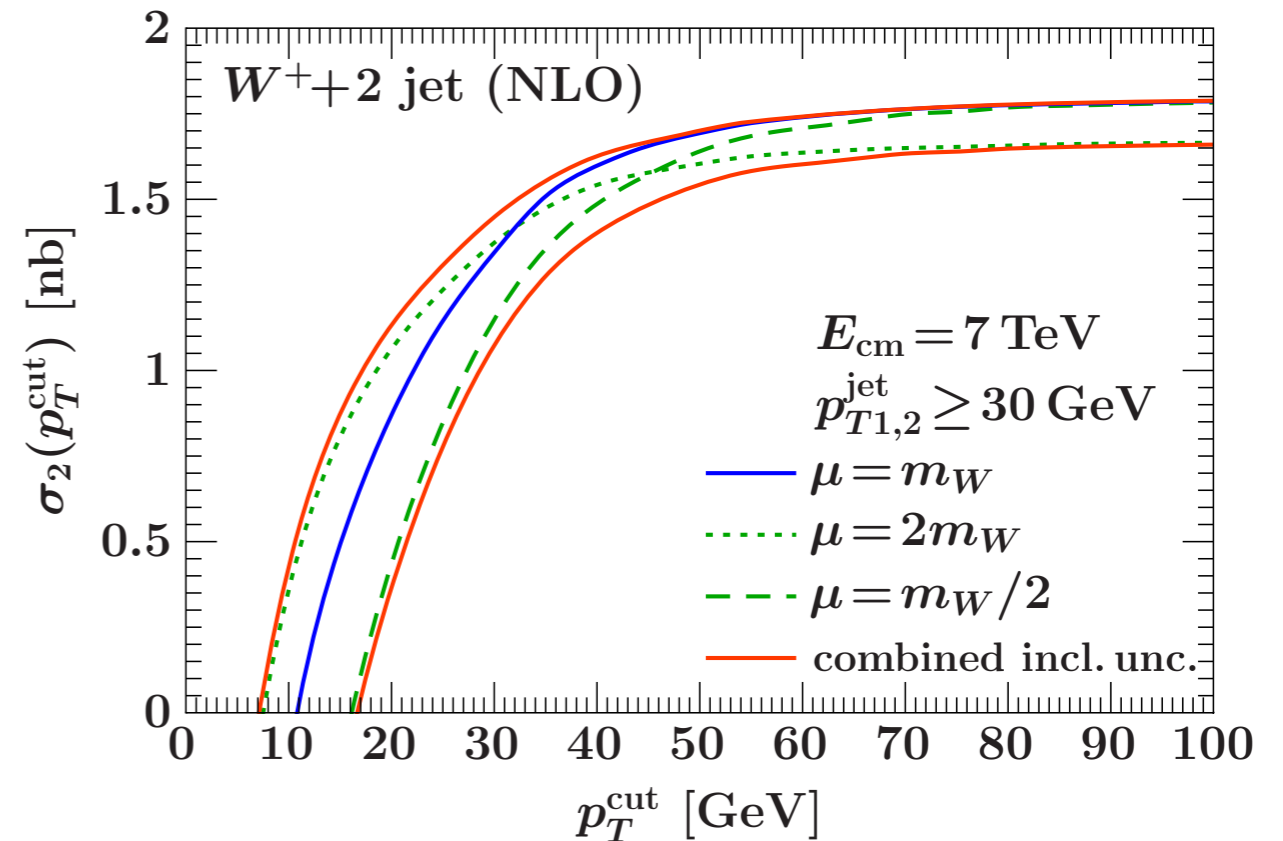
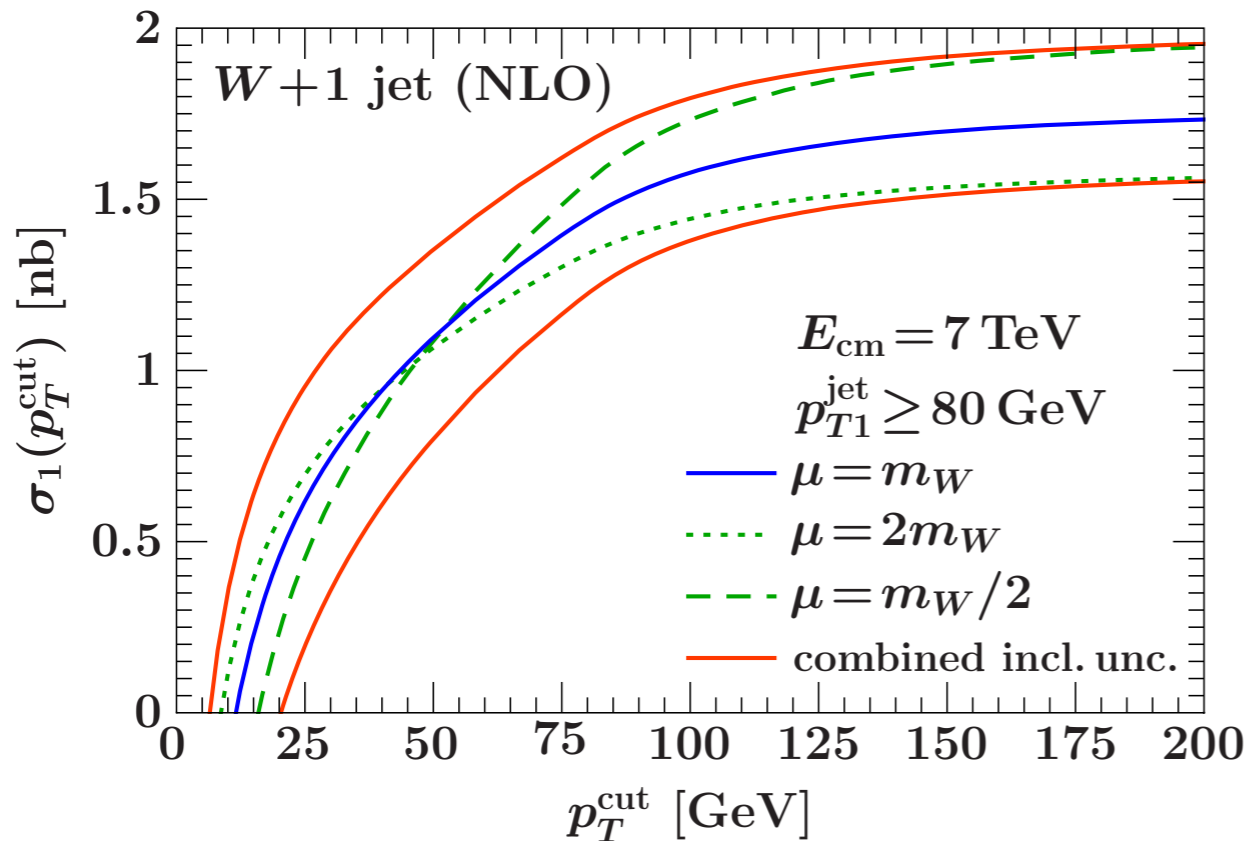
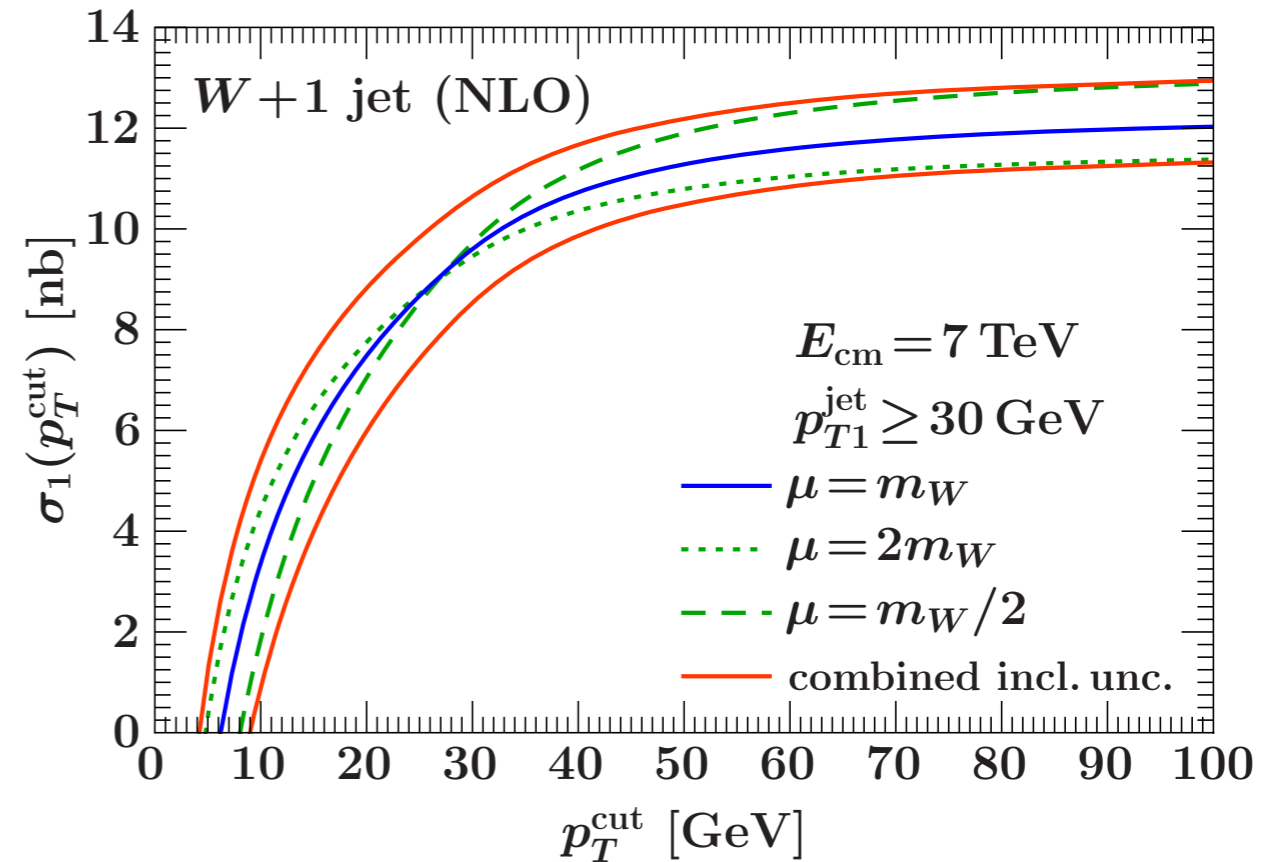
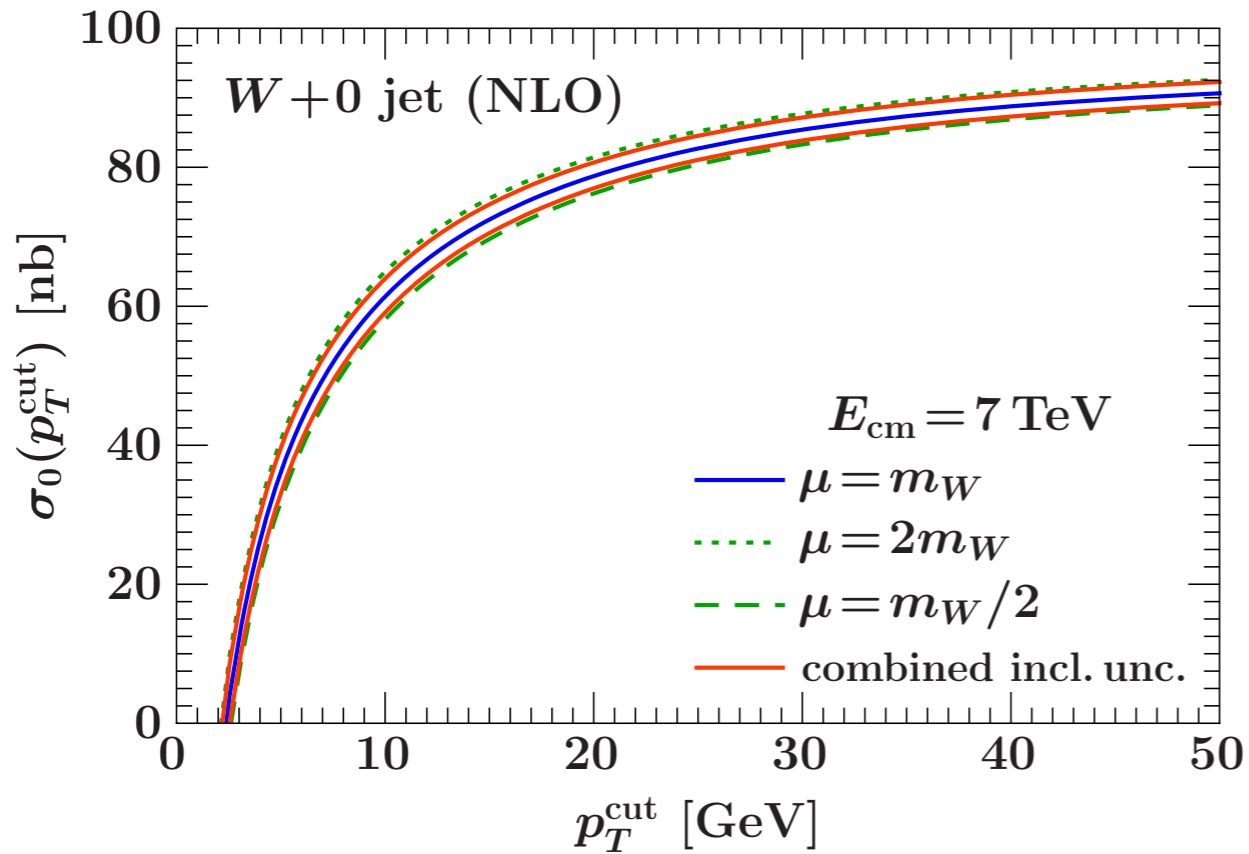
Source	H $\rightarrow$ W <sup>+</sup> W <sup>-</sup>	qq $\rightarrow$ W <sup>+</sup> W <sup>-</sup>	gg $\rightarrow$ W <sup>+</sup> W <sup>-</sup>	non-Z resonant WZ/ZZ	top	DY	W + jets	V(W/Z) + $\gamma$
Luminosity	4.5	—	—	4.5	—	—	—	4.5
Trigger efficiencies	1.5	1.5	1.5	1.5	—	—	—	1.5
Muon efficiency	1.5	1.5	1.5	1.5	—	—	—	1.5
Electron id efficiency	2.5	2.5	2.5	2.5	—	—	—	2.5
Momentum scale	1.5	1.5	1.5	1.5	—	—	—	1.5
$E_T^{\text{miss}}$ resolution	2.0	2.0	2.0	2.0	2.0	3.0	—	1.0
Jet counting	7-20	—	5.5	5.5	—	—	—	5.5
Higgs cross section	5-15	—	—	—	—	—	—	—
WZ/ZZ cross section	—	—	—	3.0	—	—	—	—
qq $\rightarrow$ WW norm.	—	15	—	—	—	—	—	—
gg $\rightarrow$ WW norm.	—	—	50	—	—	—	—	—
W + jets norm.	—	—	—	—	—	—	36	—
top norm.	—	—	—	—	25	—	—	—
Z/ $\gamma^*$ $\rightarrow$ $l^+l^-$ norm.	—	—	—	—	—	60	—	—
Monte Carlo statistics	1.0	1.0	1.0	4.0	6.0	20.0	20.0	10.0

The uncertainty on the signal efficiency is estimated to be  $\sim 20\%$  and is dominated by the theoretical uncertainty in the jet veto efficiency determination. The uncertainty on the background estimations in the H  $\rightarrow$  W<sup>+</sup>W<sup>-</sup> signal region is  $\sim 15\%$ , which is dominated by the statistical uncertainties of the background control regions in data.

## Theory Plans:

- A calculation of the Higgs + 0-jet cross section at one higher order (N3LL) is feasible. “Only” a missing 2 loop calculation. This will help reduce the perturbative uncertainty.
- Similar resummed calculations for Higgs + 1 jet are already in progress.

# Backup



$$\sigma_0 = \sigma_{\text{total}} - \sigma_{\geq 1}, \quad f_0 = \frac{\sigma_0}{\sigma_{\text{total}}},$$

$$\sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}, \quad f_1 = \frac{\sigma_1}{\sigma_{\text{total}}}.$$

$$\begin{pmatrix} \Delta_{\text{total}}^2 & \Delta_{\text{total}}^2 & 0 \\ \Delta_{\text{total}}^2 & \Delta_{\text{total}}^2 + \Delta_{\geq 1}^2 & -\Delta_{\geq 1}^2 \\ 0 & -\Delta_{\geq 1}^2 & \Delta_{\geq 1}^2 + \Delta_{\geq 2}^2 \end{pmatrix}$$

## relative uncertainties

$$\delta(\sigma_0)^2 = \frac{1}{f_0^2} \delta_{\text{total}}^2 + \left(\frac{1}{f_0} - 1\right)^2 \delta_{\geq 1}^2$$

$$\delta(\sigma_1)^2 = \left(\frac{1-f_0}{f_1}\right)^2 \delta_{\geq 1}^2 + \left(\frac{1-f_0}{f_1} - 1\right)^2 \delta_{\geq 2}^2$$

$$\delta(f_0)^2 = \left(\frac{1}{f_0} - 1\right)^2 (\delta_{\text{total}}^2 + \delta_{\geq 1}^2),$$

$$\delta(f_1)^2 = \delta_{\text{total}}^2 + \left(\frac{1-f_0}{f_1}\right)^2 \delta_{\geq 1}^2 + \left(\frac{1-f_0}{f_1} - 1\right)^2 \delta_{\geq 2}^2,$$

## correlation coefficients

$$\rho(\sigma_0, \sigma_{\text{total}}) = \left[1 + \frac{\delta_{\geq 1}^2}{\delta_{\text{total}}^2} (1-f_0)^2\right]^{-1/2},$$

$$\rho(\sigma_0, \sigma_1) = -\left[1 + \frac{\delta_{\text{total}}^2}{\delta_{\geq 1}^2} \frac{1}{(1-f_0)^2}\right]^{-1/2} \\ \times \left[1 + \frac{\delta_{\geq 2}^2}{\delta_{\geq 1}^2} \left(1 - \frac{f_1}{1-f_0}\right)^2\right]^{-1/2},$$

$$\rho(\sigma_0, \sigma_{\geq 2}) = 0,$$

$$\rho(\sigma_1, \sigma_{\text{total}}) = 0,$$

$$\rho(\sigma_1, \sigma_{\geq 2}) = -\left[1 + \frac{\delta_{\geq 1}^2}{\delta_{\geq 2}^2} \left(1 - \frac{f_1}{1-f_0}\right)^{-2}\right]^{-1/2}.$$

$$\rho(f_0, \sigma_{\text{total}}) = \left[1 + \frac{\delta_{\geq 1}^2}{\delta_{\text{total}}^2}\right]^{-1/2},$$

$$\rho(f_0, f_1) = -\left(1 + \frac{1-f_0}{f_1} \frac{\delta_{\geq 1}^2}{\delta_{\text{total}}^2}\right) \left(\frac{1}{f_0} - 1\right) \frac{\delta_{\text{total}}^2}{\delta(f_0)\delta(f_1)},$$

$$\rho(f_1, \sigma_{\text{total}}) = -\frac{\delta_{\text{total}}}{\delta(f_1)}.$$

# Validation? Other options?

- Drell-Yan pairs from  $\gamma^*$ ,  $Z^*$  with a jet veto should be used for validation.
- Directly measure beam thrust (important on its own). And UE is no harder than it is for HT.

