Operator Formalism for Glauber Exchange ①

based on 1601.04695 [1]
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ESI workshop
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Outline :

- Lagrangian for Glauber Exchange $\mathcal{L}_G^{(0)}$, properties
- Eikonalization or Not
- Resummation $\ln x$

$\boxed{\text{Sideboard}}$

$$\mathcal{L}_G^{(0)} = \sum_n \sum_{i,j=q,g} O_n^{iB} \frac{1}{P_\perp^2} O_s^{j_n B} + \sum_{n,n'} \sum_{i,j=q,g} O_n^{iB} \frac{1}{P_\perp^2} O_s^{BC} \frac{1}{P_\perp^2} O_{n'}^{jC}$$

(2 rapidities) (3 rapidities)

will discuss highlighted things

$$O_n^{qB} = \bar{\chi}_n T^B \frac{\bar{n}\!\!\!/}{2} \chi_n \,, \quad O_n^{gB} = \frac{i}{2} f^{BCD} \mathcal{B}_{n\perp\mu}^C \frac{\bar{n}}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{n\perp}^{D\mu}$$

similar for $O_{\bar{n}}$'s

$$O_s^{q_n B} = 8\pi\alpha_s \bar{\psi}_s^n T^B \frac{n\!\!\!/}{2} \psi_s^n \,, \quad O_s^{g_n B} = 8\pi\alpha_s \frac{i}{2} f^{BCD} \mathcal{B}_{s\perp\mu}^{nC} \frac{n}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger) \mathcal{B}_{s\perp}^{nD\mu}$$

$$O_s^{BC} = 8\pi\alpha_s \Big\{ P_\perp^\mu \mathcal{S}_n^T \mathcal{S}_{\bar{n}} \mathcal{P}_{\perp\mu} - \mathcal{P}_{\perp\mu} g \widetilde{\mathcal{B}}_{s\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} - \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \widetilde{\mathcal{B}}_{s\perp}^{\bar{n}\mu} \mathcal{P}_{\perp\mu}$$
$$- g \widetilde{\mathcal{B}}_{s\perp}^{n\mu} \mathcal{S}_n^T \mathcal{S}_{\bar{n}} g \widetilde{\mathcal{B}}_{s\perp\mu}^{\bar{n}} - \frac{n_\mu \bar{n}_\nu}{2} \mathcal{S}_n^T \, i g \widetilde{G}_s^{\mu\nu} \mathcal{S}_{\bar{n}} \Big\}^{BC}$$

where

$$\chi_n = W_n^\dagger \xi_n \quad W_n = W_n[\bar{n}\cdot A_n] \,, \quad \psi_s^n = S_n^\dagger \psi_s \quad S_n = S_n[n\cdot A_s]$$

$$g\mathcal{B}_{n\perp}^\mu = [W_n^\dagger i D_{n\perp}^\mu W_n] \,, \quad g\mathcal{B}_{s\perp}^{n\mu} = [S_n^\dagger i D_{s\perp}^\mu S_n]$$

$$\widetilde{\mathcal{B}}_{s\perp}^{n\,AB} = -i f^{ABC} \mathcal{B}_{s\perp}^{nC}$$

$$\widetilde{G}_s^{\mu\nu\,AB} = -i f^{ABC} G_s^{\mu\nu C}$$

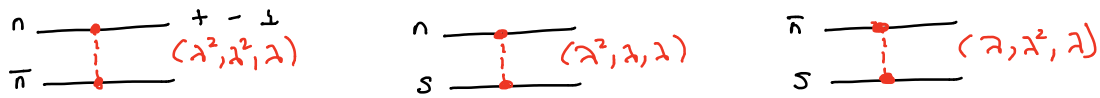
S_n fundamental Wilson line $\qquad \mathcal{S}_n$ adjoint Wilson line

Present but suppressed above: rapidity regulator $|k_z|^{-\eta}$, multipole expansion

Glauber Scaling

$p^\mu \sim Q(\lambda^a, \lambda^b, \lambda) \quad a+b>2$

Mediate Fwd Scattering $\quad s \gg t$



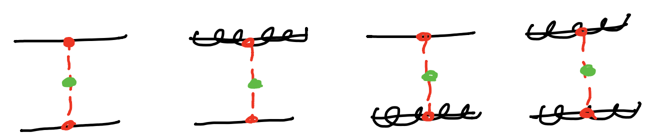
$\frac{1}{k_\perp^2}$ [Coulomb like but 2-d, instantaneous in z & t]

Ops. Constructed by int. out hard & Glauber QCD ⇒ SCET

Universality

$i = q, g$

$j = q, g$

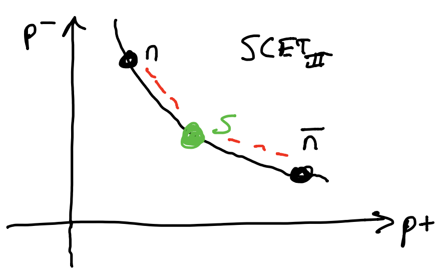


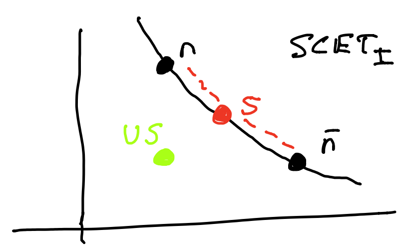
& O_s^{BC} pure glue

Interactions

$$\mathcal{L}_{II}^{(0)} = \mathcal{L}_s^{(0)} + \sum_n \mathcal{L}_n^{(0)} + \mathcal{L}_G^{(0)}(\{\xi_n, A_n\}, \psi_s, A_s)$$

Same for $SCET_I$





- no double counting
- 0-bin subtractions
- rapidity regulator

$\mathcal{L}_G^{(0)}$ violates Factorization

→ couples modes at λ^0

→ $i\pi$'s

→ determines Wilson Line directions / universality

Power Counting (use sideboard!)

$$\mathcal{L}_G^{(0)} = \sum_n \sum_{i,j=q,g} \underbrace{O_n^{iB}}_{\lambda^2} \underbrace{\frac{1}{P_\perp^2}}_{\lambda^{-2}} \underbrace{O_s^{j_n B}}_{\lambda^3} \,{=}\, \lambda^3 \quad + \sum_{n,n'} \sum_{i,j=q,g} \underbrace{O_n^{iB}}_{\lambda^2} \underbrace{\frac{1}{P_\perp^2}}_{\lambda^{-2}} \underbrace{O_s^{BC}}_{\lambda^2} \underbrace{\frac{1}{P_\perp^2}}_{\lambda^{-2}} \underbrace{O_{n'}^{jC}}_{\lambda^2} = \lambda^2$$

$$O_n^{qB} = \overset{\lambda}{\bar\chi_n} T^B \frac{\bar{n}\!\!\!/}{2} \overset{\lambda}{\chi_n}\,, \qquad O_n^{gB} = \frac{i}{2} f^{BCD} \overset{\lambda}{\mathcal{B}^C_{n\perp\mu}} \frac{\bar n}{2}\cdot \overset{\lambda^0}{(\mathcal{P}+\mathcal{P}^\dagger)} \overset{\lambda}{\mathcal{B}^{D\mu}_{n\perp}}$$

similar for $O_{\bar n}$'s

$$O_s^{q_n B} = 8\pi\alpha_s \underbrace{\bar\psi_s^n}_{\lambda^{3/2}} T^B \frac{n\!\!\!/}{2} \underbrace{\psi_s^n}_{\lambda^{3/2}} = \lambda^3\,, \qquad O_s^{g_n B} = 8\pi\alpha_s \frac{i}{2} f^{BCD} \underbrace{\mathcal{B}^{nC}_{s\perp\mu}}_{\lambda} \frac{n}{2}\cdot \underbrace{(\mathcal{P}+\mathcal{P}^\dagger)}_{\lambda} \underbrace{\mathcal{B}^{nD\mu}_{s\perp}}_{\lambda} = \lambda^3$$

$$O_s^{BC} = 8\pi\alpha_s \Big\{ \underbrace{\mathcal{P}_\perp^\mu}_{\lambda} S_n^T S_{\bar n} \underbrace{\mathcal{P}_{\perp\mu}}_{\lambda} = \lambda^2 \ + \ \dots \Big\}$$

Power counting for any graph (without state scaling)

any α_s^k, any power

gauge invariant

$\sim \lambda^\delta$

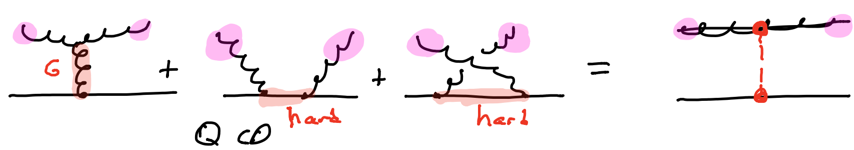
$$\delta = \delta^{topo} + \sum_k \underbrace{(k-2)V_k^{n\bar n}}_{\text{3 rapidity}} + \underbrace{(k-3)\left(V_k^{ns} + V_k^{\bar n s}\right)}_{\text{2-rapidity}} + (k-4)\left(V_k^n + V_k^{\bar n} + V_k^s\right) + (k-8)V_k^{us}$$

[$\delta^{topo} = 6 - N^n - N^{\bar n} - N^{ns} - N^{\bar n s} + 2u$ is topological factor having to do with connectedness of various sectors]

examples on page 184 of [1]

Aside (time permitting)

$\mathcal{B}^\mu_{n\perp}$ matching



$$B^\mu_{n\perp} = A^\mu_{n\perp} - \frac{k_\perp^\mu}{\bar n\cdot k}\, \bar n\cdot A_n(k) + \dots$$

eqtns. of motion

$p\cdot A(p) = 0$ remove $n\cdot A_n$

$p^2 = 0$

important to get $\bar n\cdot A_n$ terms correctly

SCET Glauber $\neq$ CSS Glauber

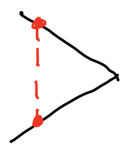
- goals differ

  classic CSS aims to prove absence of Glauber region
  not define it as a contribution that can be independently calculated. ↳ as we do with $\mathcal{L}_G^{(0)}$ in SCET

- SCET : expand first

  CSS : deform contour, see where we are trapped, then expand

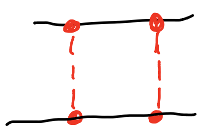
eg.



$\neq 0$ in SCET , no G region in CSS

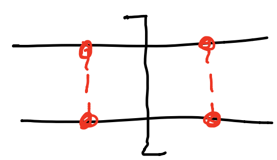
↳ $\displaystyle\int \frac{d^dk \;\; |k_z|^{-\eta}}{(k^+ + \dots)(k^- + \dots)(k_\perp^2)}$ ← $k^\pm$ contours don't converge at ∞

eg.



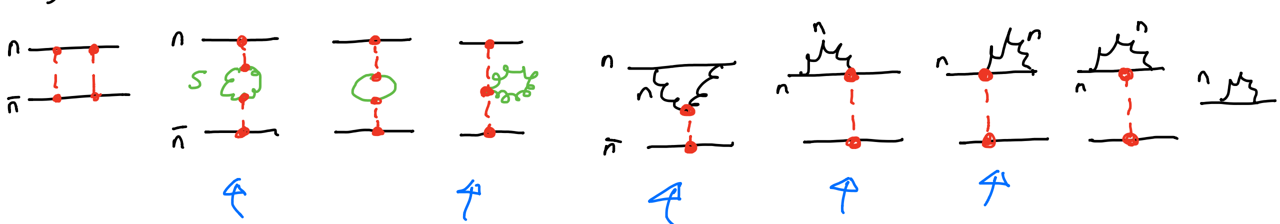
$\neq 0$ in SCET , $\sim (-i\pi) \displaystyle\int \frac{d^2k_\perp}{k_\perp^2 (k_\perp - q_\perp)^2}$

only encodes cut needed for unitarity



Graphs = Σ interact. $\mathcal{L}^{SCET}$ $\underset{\text{one-loop}}{=}$ $A_G + A_S + A_n + A_{\bar{n}}$ (+ sometimes hard)

eg.

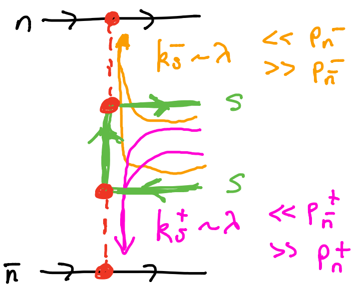


rapidity divergent

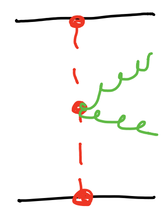
+ $\bar{n}$ analogs

- exactly reproduces leading power QCD amplitude
- finite part of 1-loop soft graphs = 2-loop Γ_{cusp}

Note:



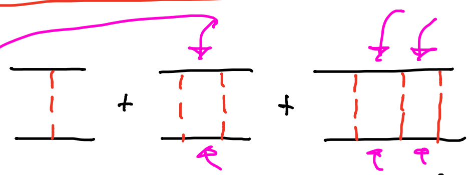
soft's from T-product



direct soft's

(don't write momenta)

$\boxed{\text{Eikonal or not}}$

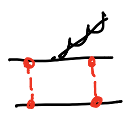
•  $+ \cdots \;\propto\; \mathrm{FT}\left[\,\tilde G(b_\perp) - 1\,\right]$

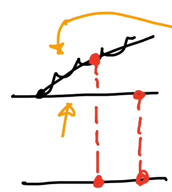
$$\tilde G(b_\perp) = e^{i\phi(b_\perp)}, \qquad \phi(b_\perp) = -g^2\, T_1^A T_2^A \int \frac{d^{d-2}q_\perp}{q_\perp^2}\, e^{i q_\perp \cdot b_\perp}$$

eikonal scattering

$$\frac{1}{\left(k^+ + \left\{p^+ - \frac{(k_\perp + p_\perp)^2}{p^-}\right\} + i0\right)} \Rightarrow \frac{1}{(k^+ + i0)}$$

- effectively eikonal for log-div. cases (needed rap. reg.)
- collapses to instantaneous in t & z as $\eta \to 0$

•  $= 0$ (interrupted collapse)

• 

not eikonal, k^+ poles on both sides

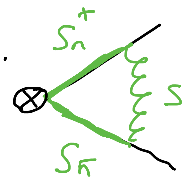
eikonal

$\Rightarrow$ these properties leads to shockwave picture, Balitsky's Wilson Line EFT, B-JIMWLK

Cheshire Glauber

• like the Cheshire Cat, we don't see Glauber in Hard Matching Calcs.

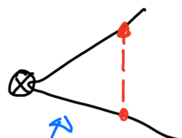
eg.



naive $\tilde{S} = \int \frac{d^d k \quad |k_z|^{-\eta}}{(k^2 - m_{IR}^2)(n\cdot k + i0)(\bar n \cdot k - i0)}$

$= (\cdots) + i\pi\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{m^2}\right)$

true $S = \tilde{S} - S^{(G)} = (\cdots)$ only



collinear, but effectively eikonal

$G = S^{(G)}$

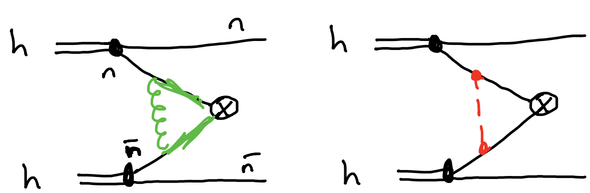
$S + G = (\tilde{S} - S^{(G)}) + G = \tilde{S}$

(A) only G carries info about soft Wilson line directions, $S = \tilde{S} - S^{(G)}$ does not care

(B) can absorb this Glauber into soft Wilson Lines if they are taken with proper physical directions, & just use $\tilde{S}$

(done for any SCET matching calculation)

Same in Active - Active Graphs

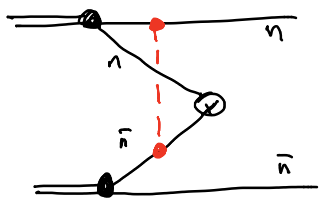


$\Rightarrow$ define "Active" lines as those that effectively eikonalize. Rest are "Spectator"

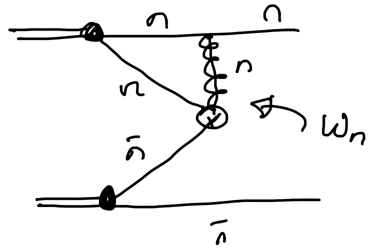
Active-Spectator

$G = C_n^{(G)}$ here

$C_n + G = \tilde{C}_n$



G

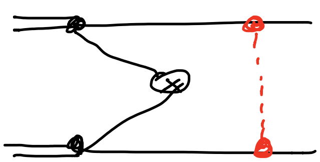


$C_n = \tilde{C}_n - C_n^{(G)}$

(A) no Wilson line direction dependence in C_n, all in G

(B) can absorb this Glauber into collinear Wilson line, so direction is determined by G graph

Spectator-Spectator



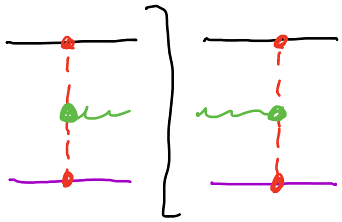
no soft or collinear analogs

Cancel when we take $|A|^2$ & integrate over $\Delta p_\perp$ of spectators

$\boxed{\text{Resummation}}$

- sum $\ln \frac{s}{t}$, $\ln x$ in DIS, Fwd. Scattering, DY, ...

- eg.



$+$ virtual graphs $+$ higher orders in $s, n, \bar{n}$

$$\sum_X |\langle p| O_n |X\rangle|^2 \qquad\qquad \sum_{X_s} |\langle 0| O_{s(1,1)} |X_s\rangle|^2 \qquad\qquad \cdots$$

$$= \int d^2q_\perp\, d^2q_\perp'\ \underbrace{C_n(q_\perp, p^-, \nu)}_{n\text{-collinear}}\ \underbrace{S(q_\perp, q_\perp', \nu)}_{\text{soft}}\ \underbrace{C_{\bar{n}}(q_\perp', p^+, \nu)}_{\bar{n}\text{-collinear}}$$

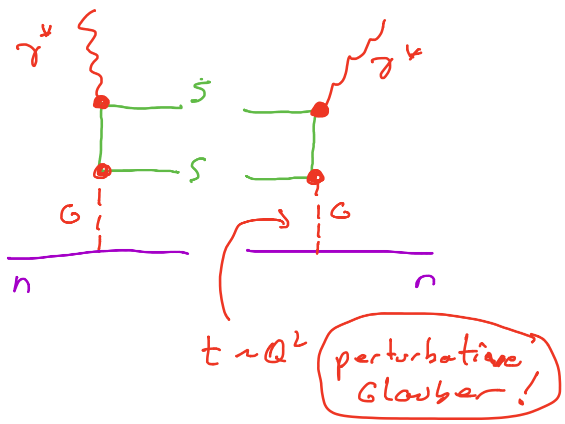
$$\nu \frac{\partial}{\partial \nu} S(q_\perp, q_\perp', \nu) = \int d^2k_\perp\ \underline{\gamma^{BFKL}(q_\perp, k_\perp)}\ S(k_\perp, q_\perp', \nu)$$

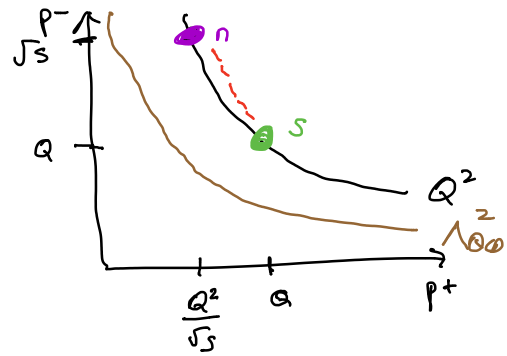
Operator based in SCET $\longrightarrow$ recycle, exploit universality

$\boxed{\text{DIS}}$ with D. Neill, A. Pathak, S. Marzani, I. Rothstein

$s \gg Q^2$

$x \ll 1$





$$\frac{d\sigma}{dQ^2 dx} \propto \int \frac{dz}{z} \int d^2 q_\perp' \, S(q_\perp', q_\perp, x P^-, \nu) \, C_n(q_\perp', z P^-, \nu)$$

match onto PDF: $C_n = H_n(\mu) \otimes f_n(\mu)$

RGE in ν sums $\ln x \to$ BFKL

RGE in μ sums $\ln \frac{Q}{\Lambda_{QCD}} \to$ DGLAP

In SCET:

- gauge invariant defn of soft function (can extend to higher orders)
- usual EFT story for scheme dependence
- could potentially go beyond NLL