Theory Predictions & Uncertainties for Higgs Searches using Jet Bins

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MIT

Fermilab Seminar, October 2011

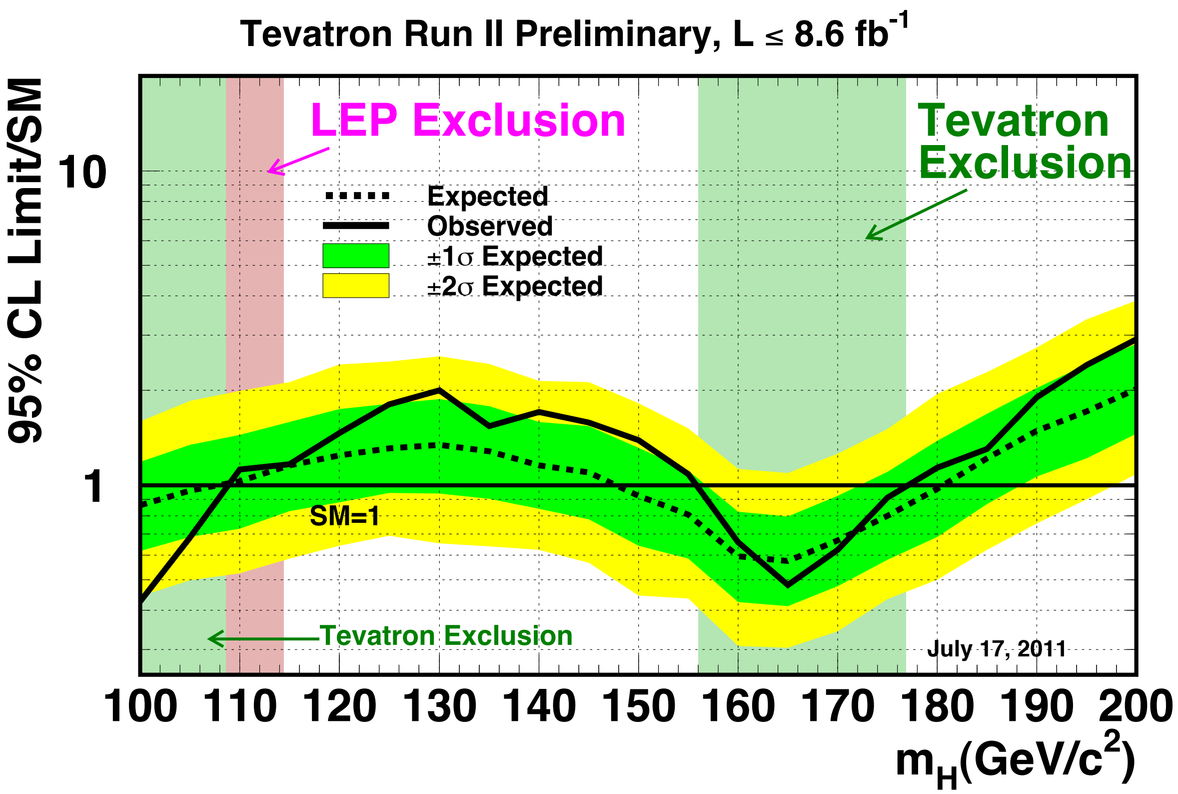
arXiv:1107.2117 I.S. & F. Tackmann

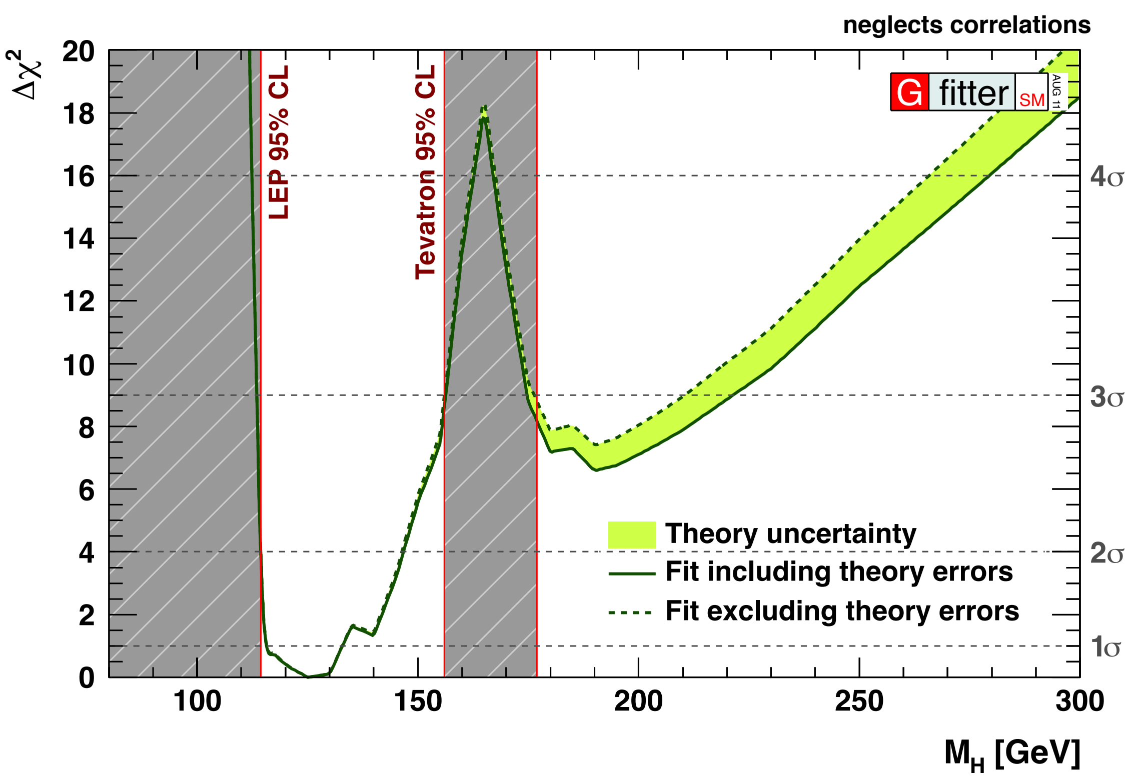
(input to LHC Higgs Xsec working group for summer 2011 recommendations, "BNL accord")

Outline

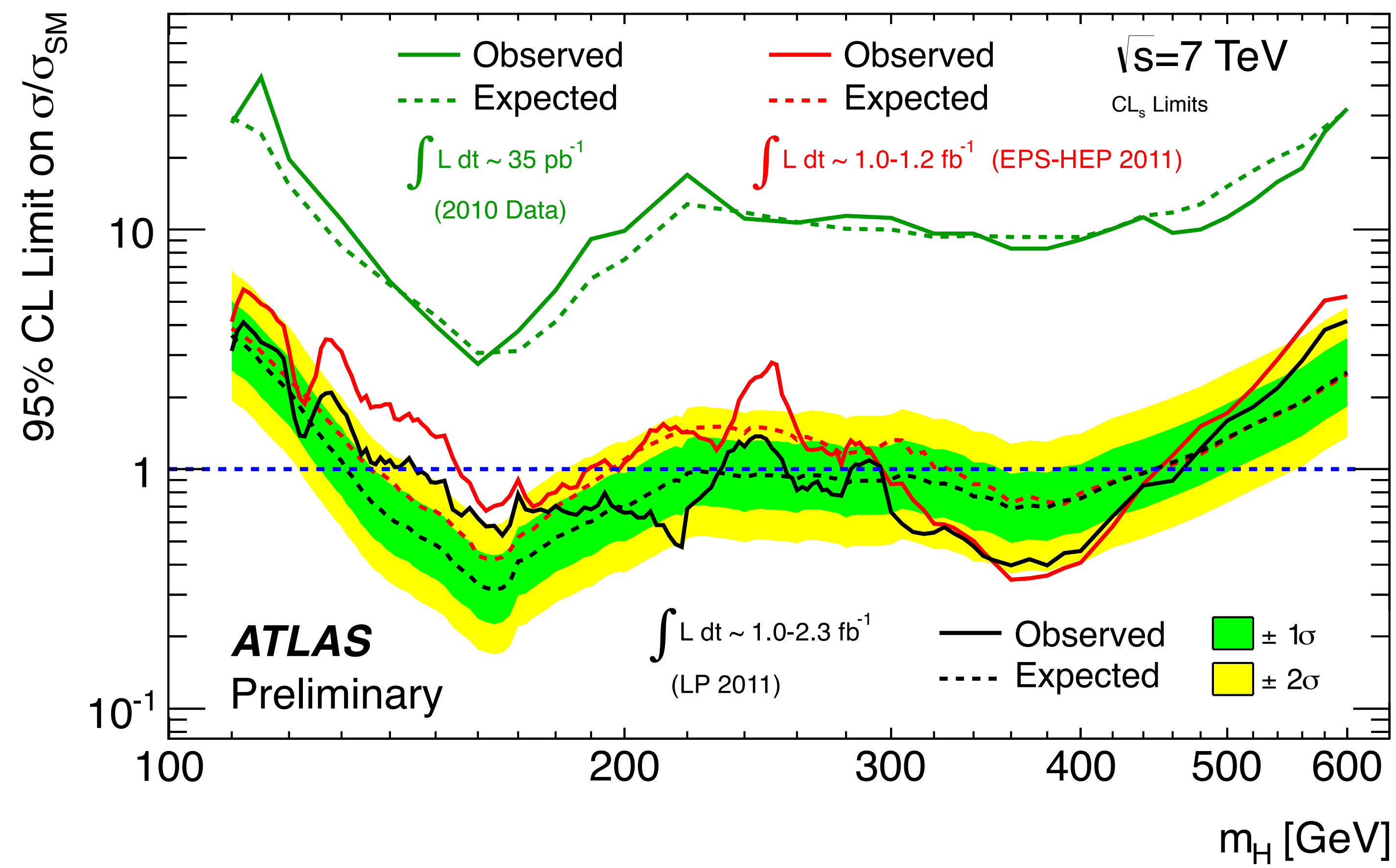
- Introduction: Jet-bins in Higgs Searches
- Theory Predictions, Uncertainties & Correlations
- Using NLO Calculations for Jet Bins
- Exploiting Log Resummation for the 0-Jet Bin
- Extension to NNLL resummation for N-Jet Bin cross sections , with fixed order NLO multi-jet cross sections
- Conclusions

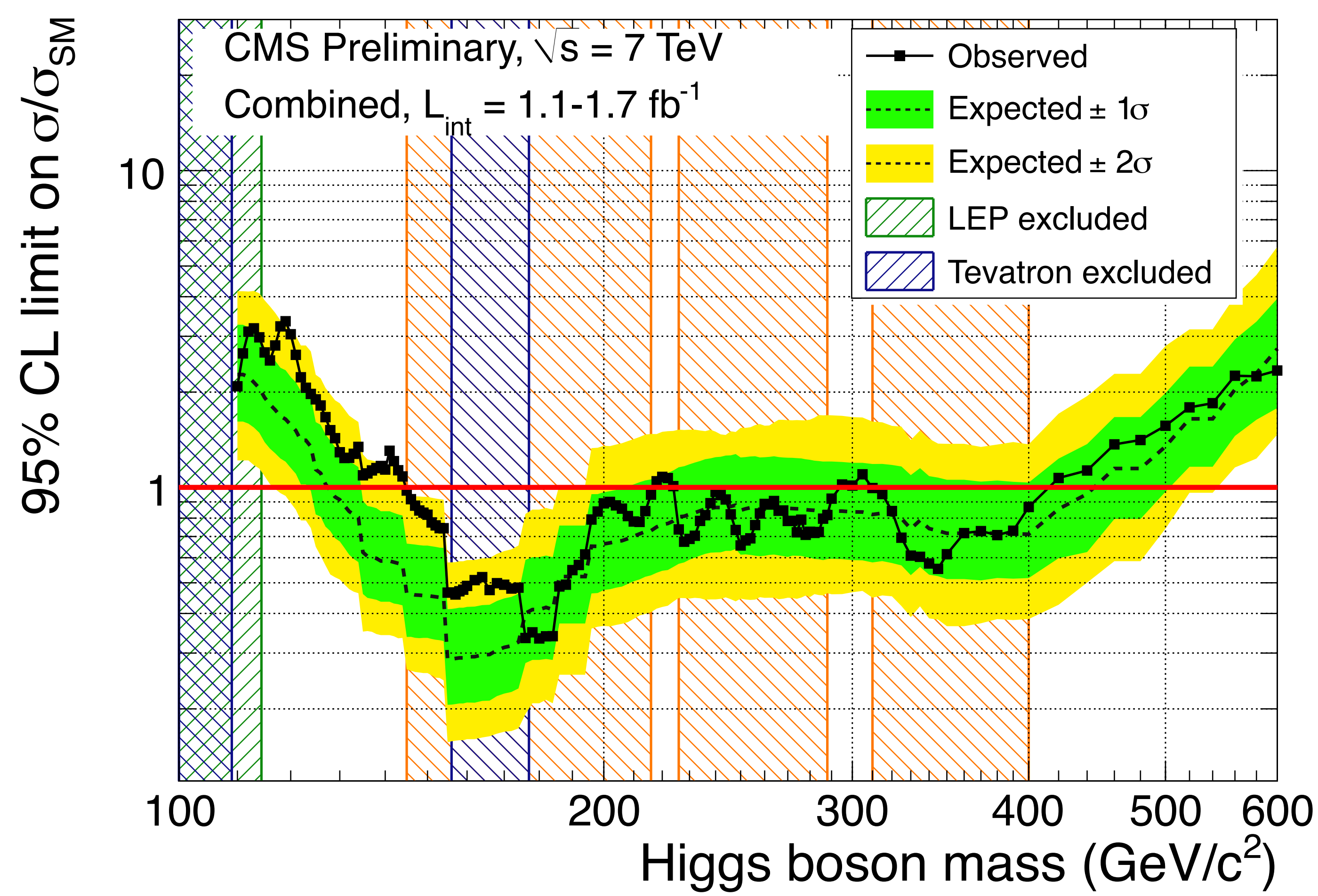
m_H Exclusion in Standard Model



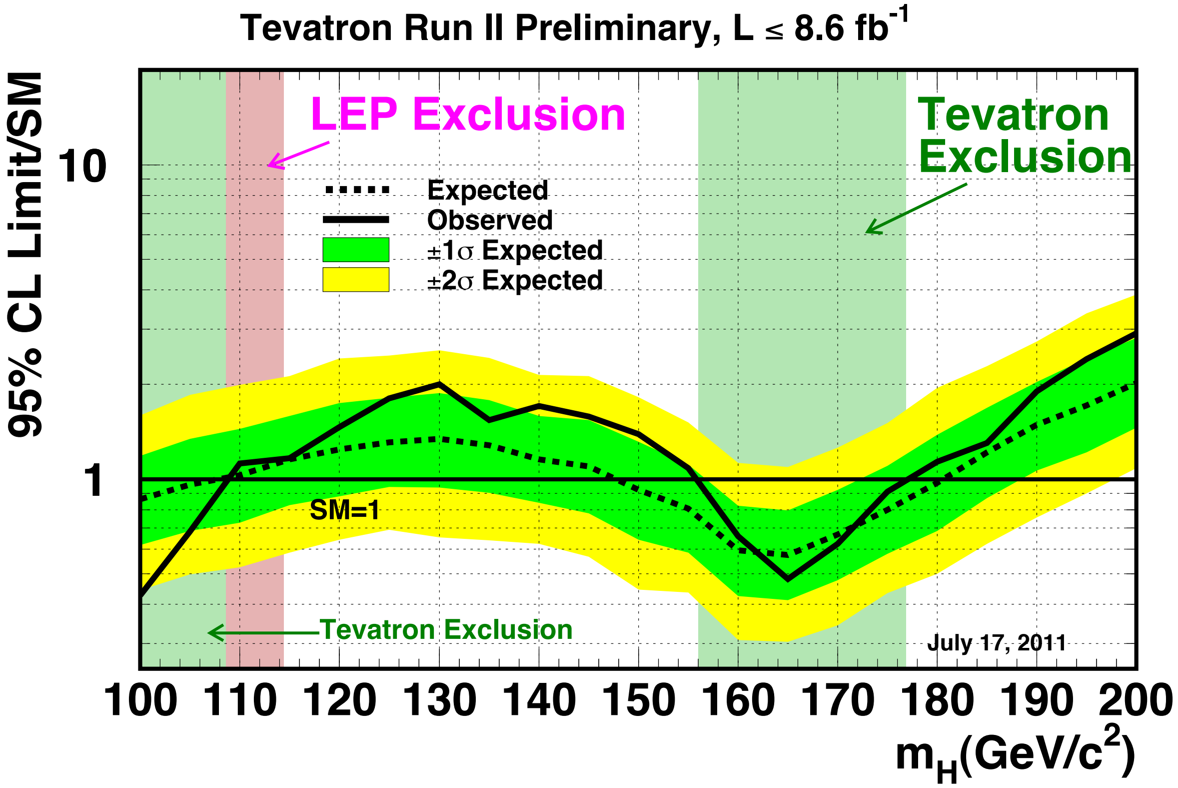


Observed exclusion: 146-230, 256-282, 296-459 GeV





m_H Exclusion in Standard Model



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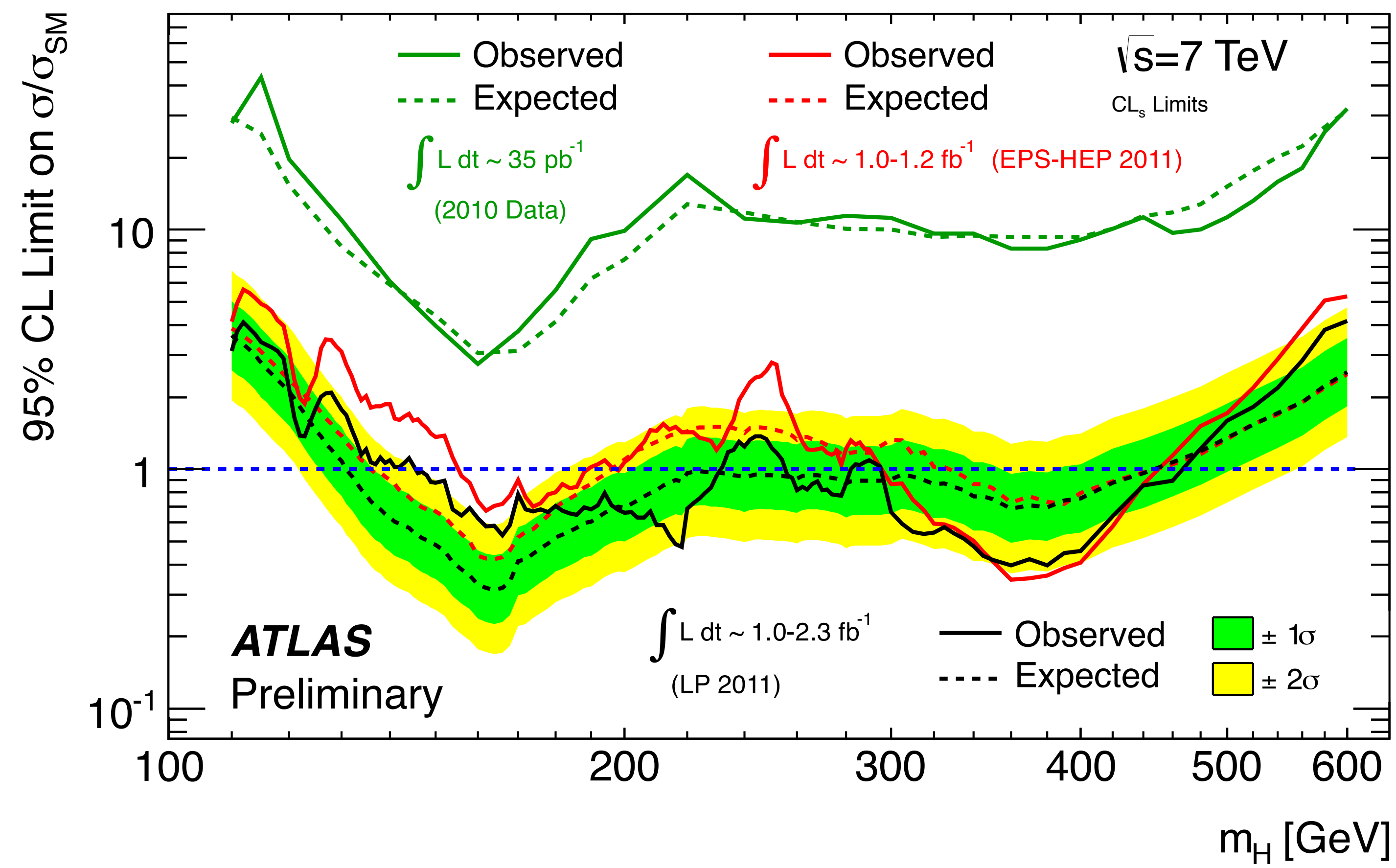
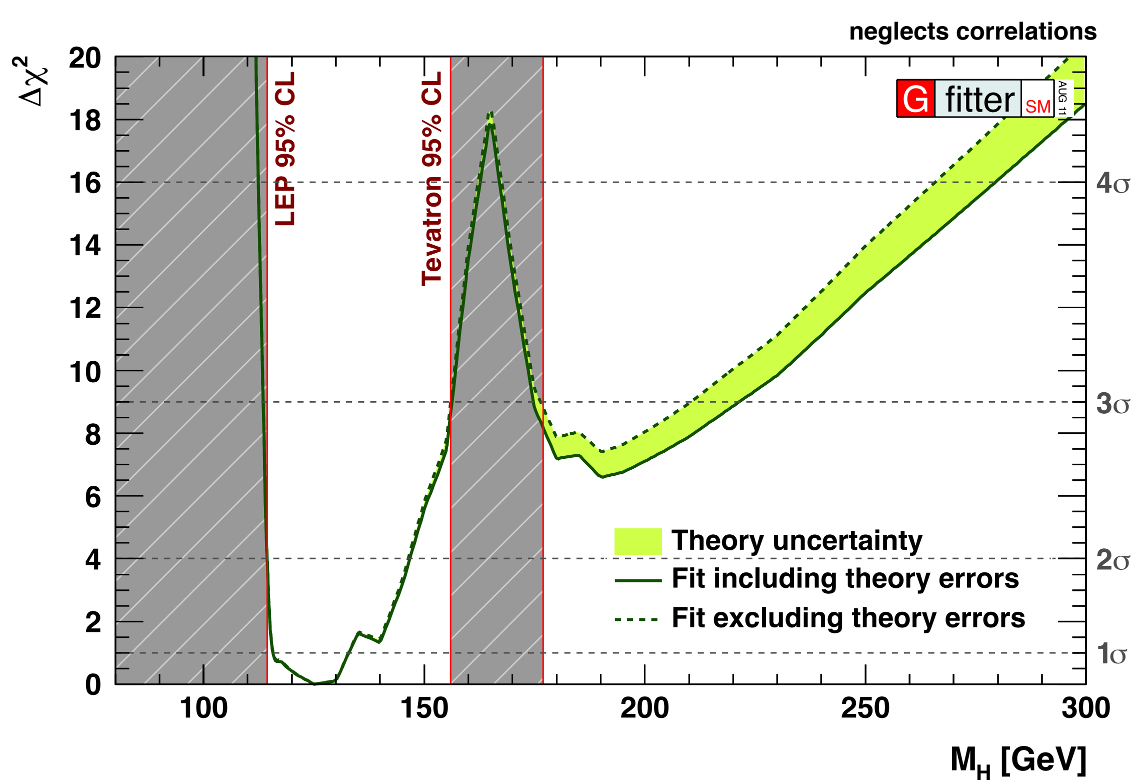
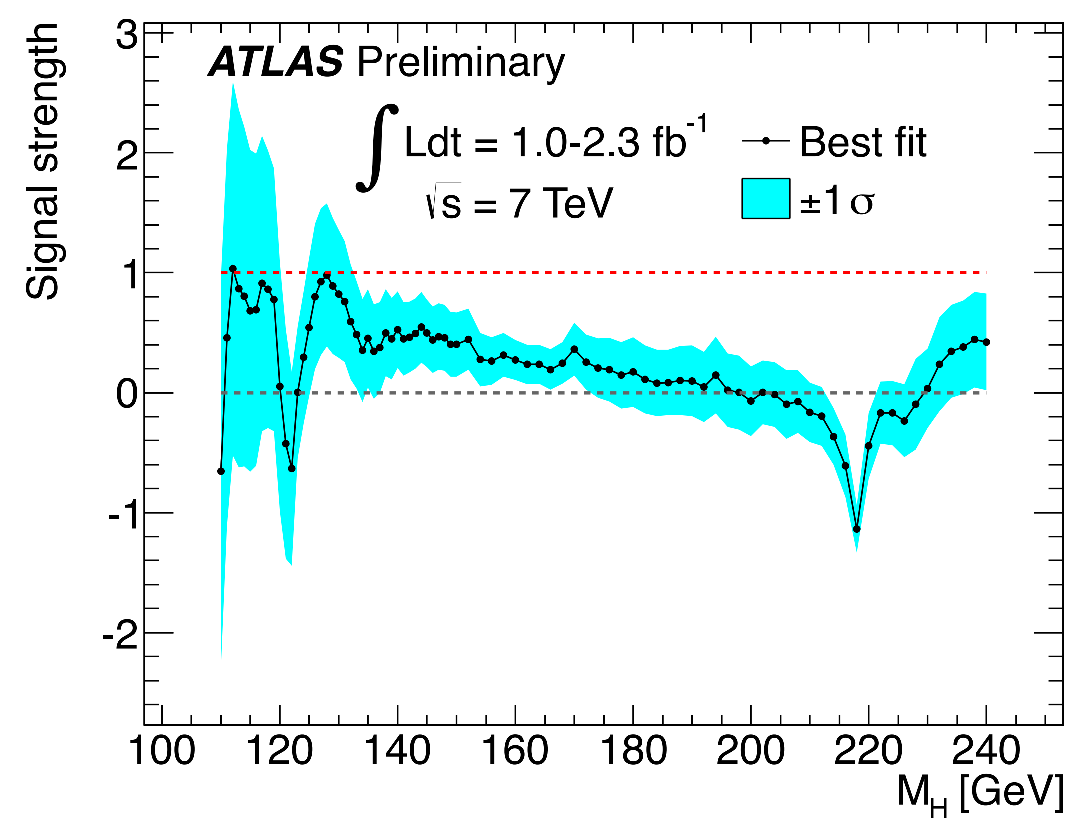
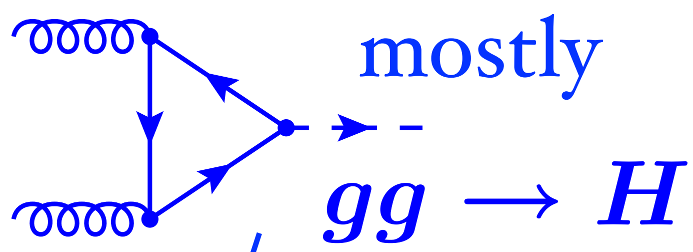
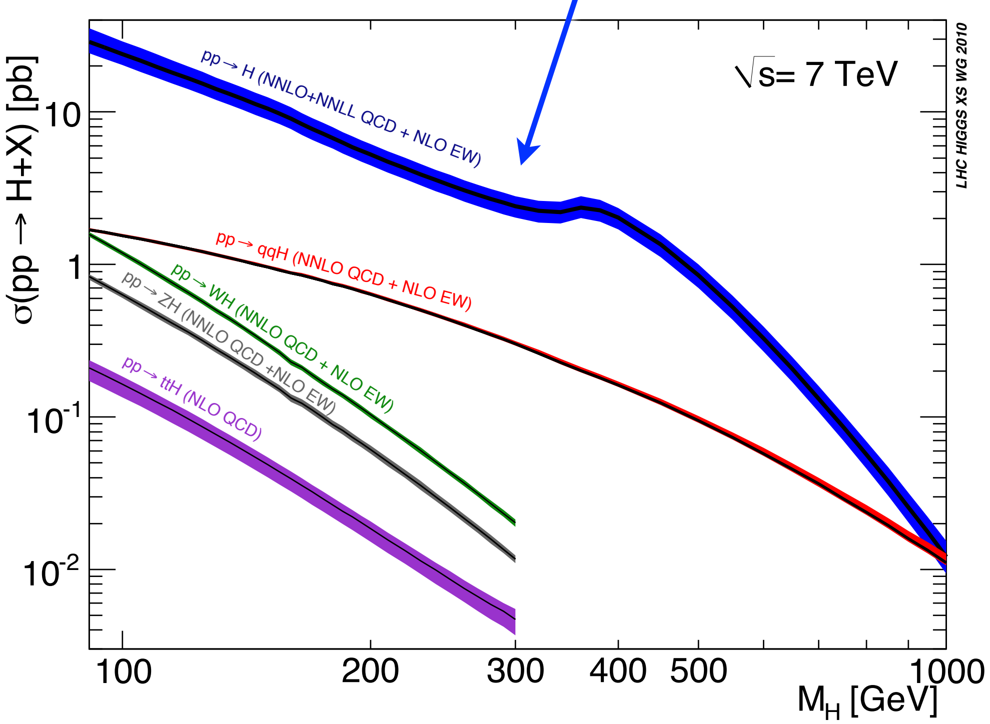


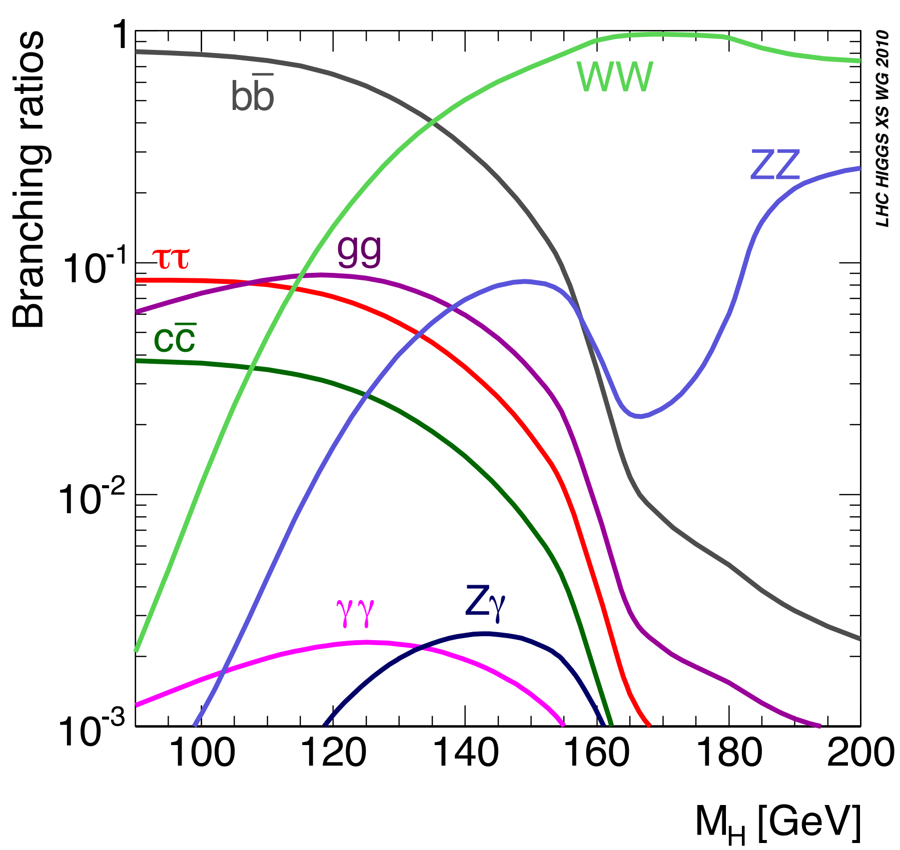


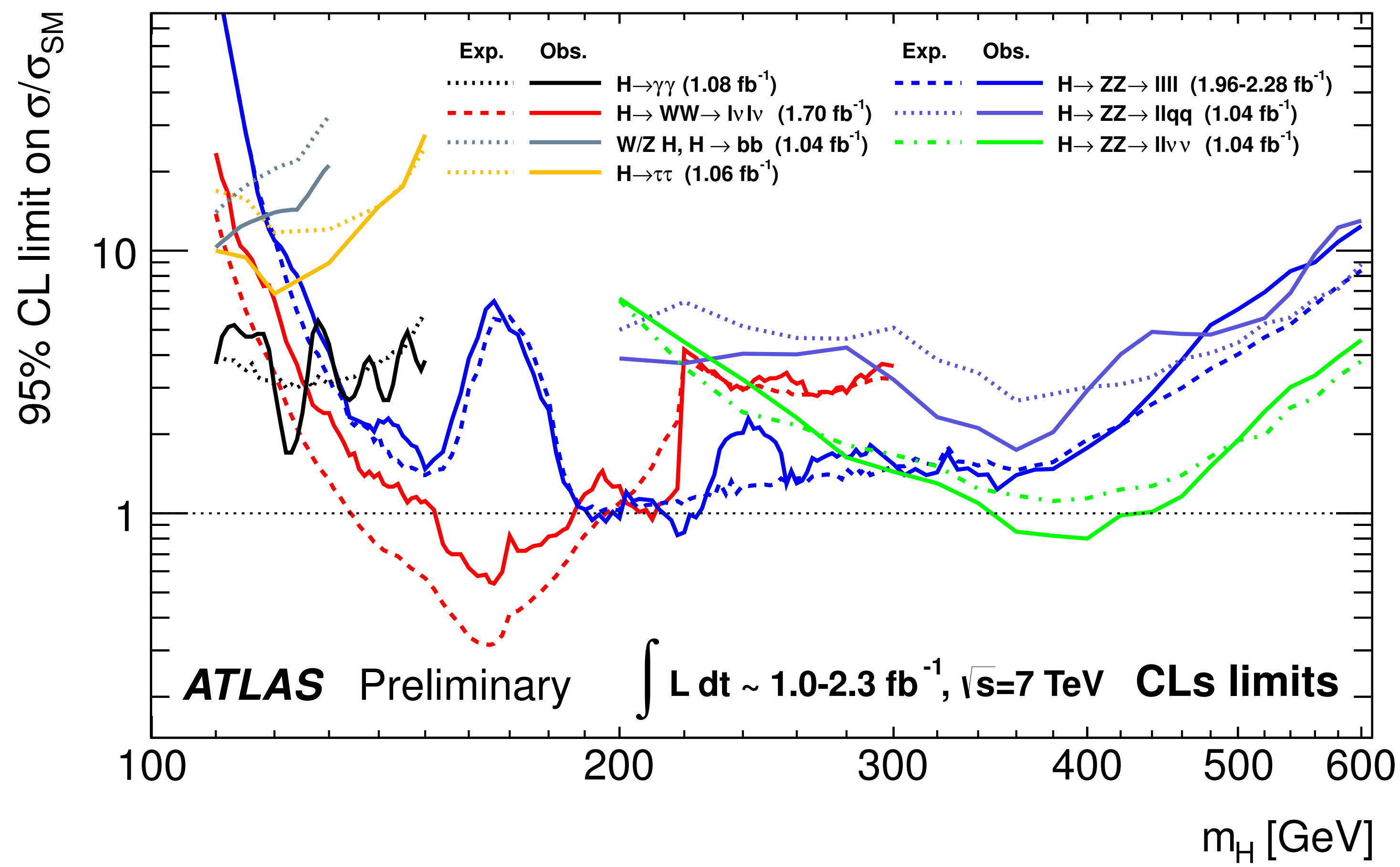
If a signal exists, what is its fit favored cross section?

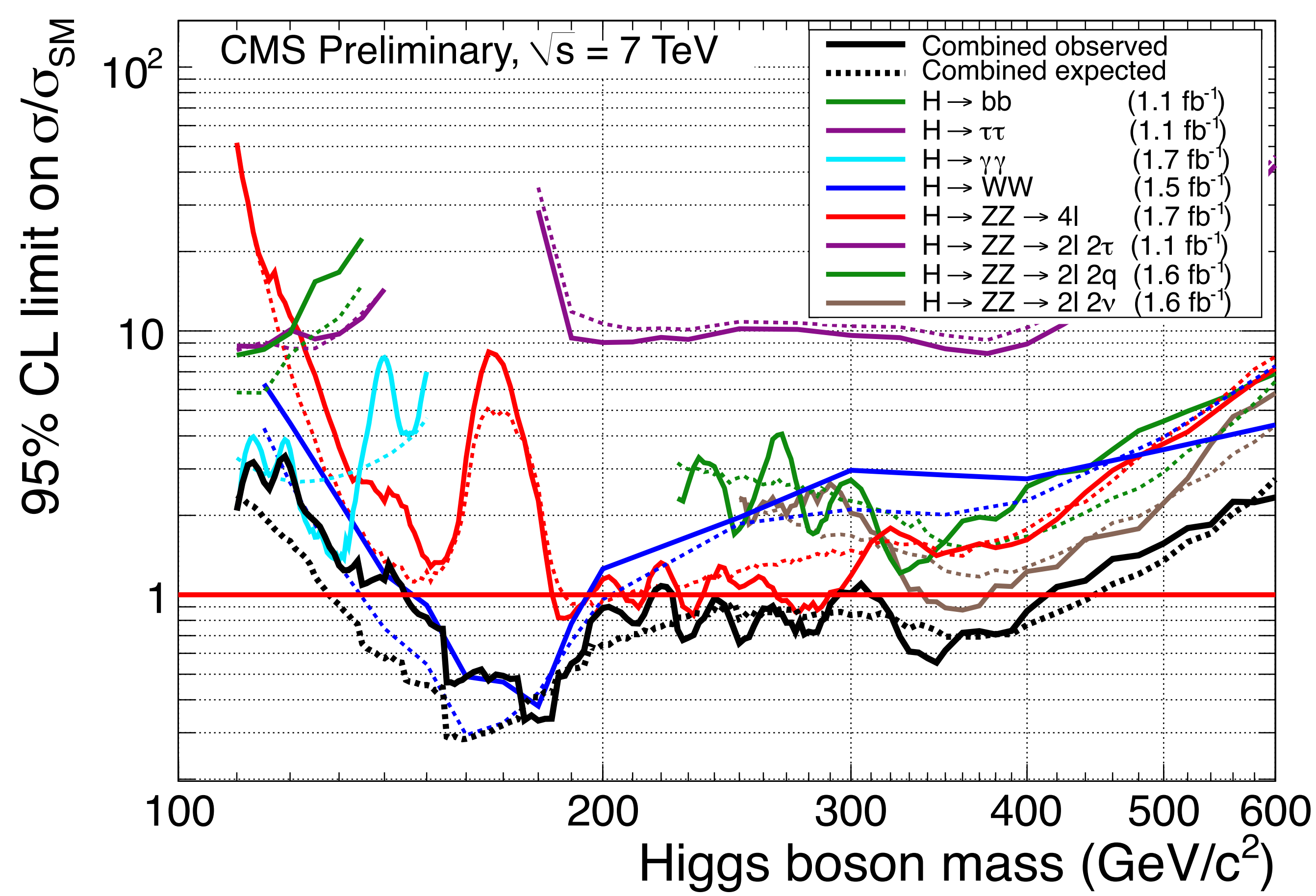


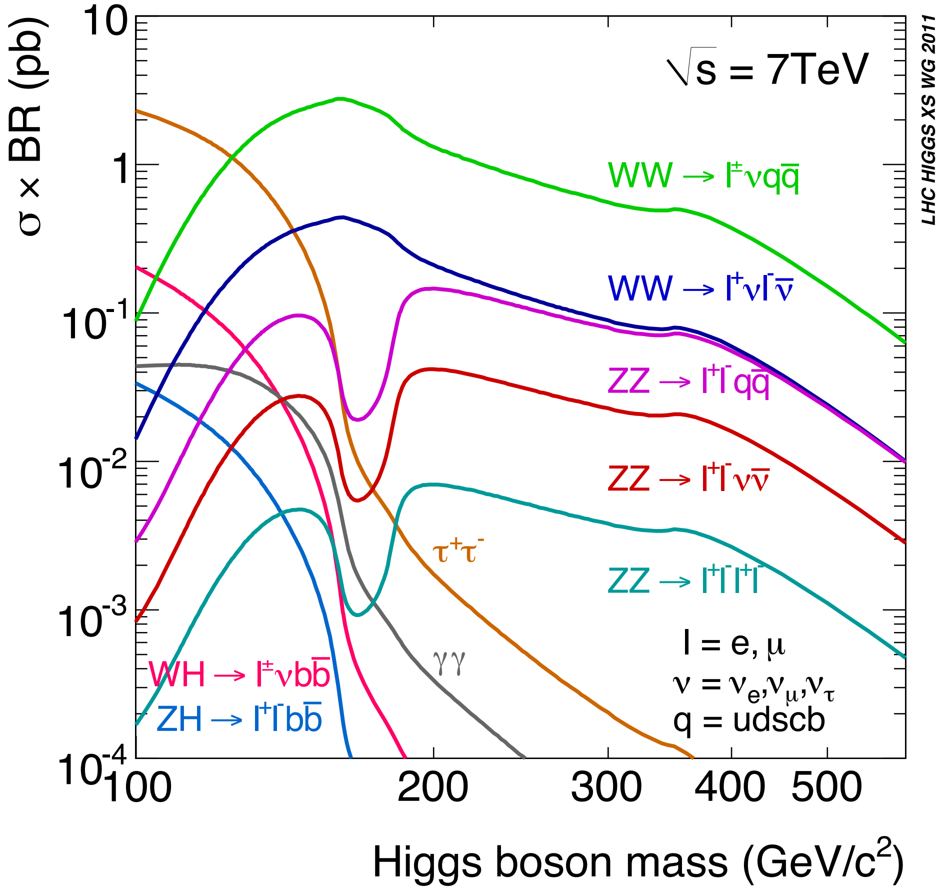

mostly
gg → H


√s= 7 TeV
σ(pp → H+X) [pb]
pp → H (NNLO+NNLL QCD + NLO EW)
pp → qqH (NNLO QCD + NLO EW)
pp → WH (NNLO QCD + NLO EW)
pp → ZH (NNLO QCD +NLO EW)
pp → ttH (NLO QCD)
LHC HIGGS XS WG 2010
M_H [GeV]

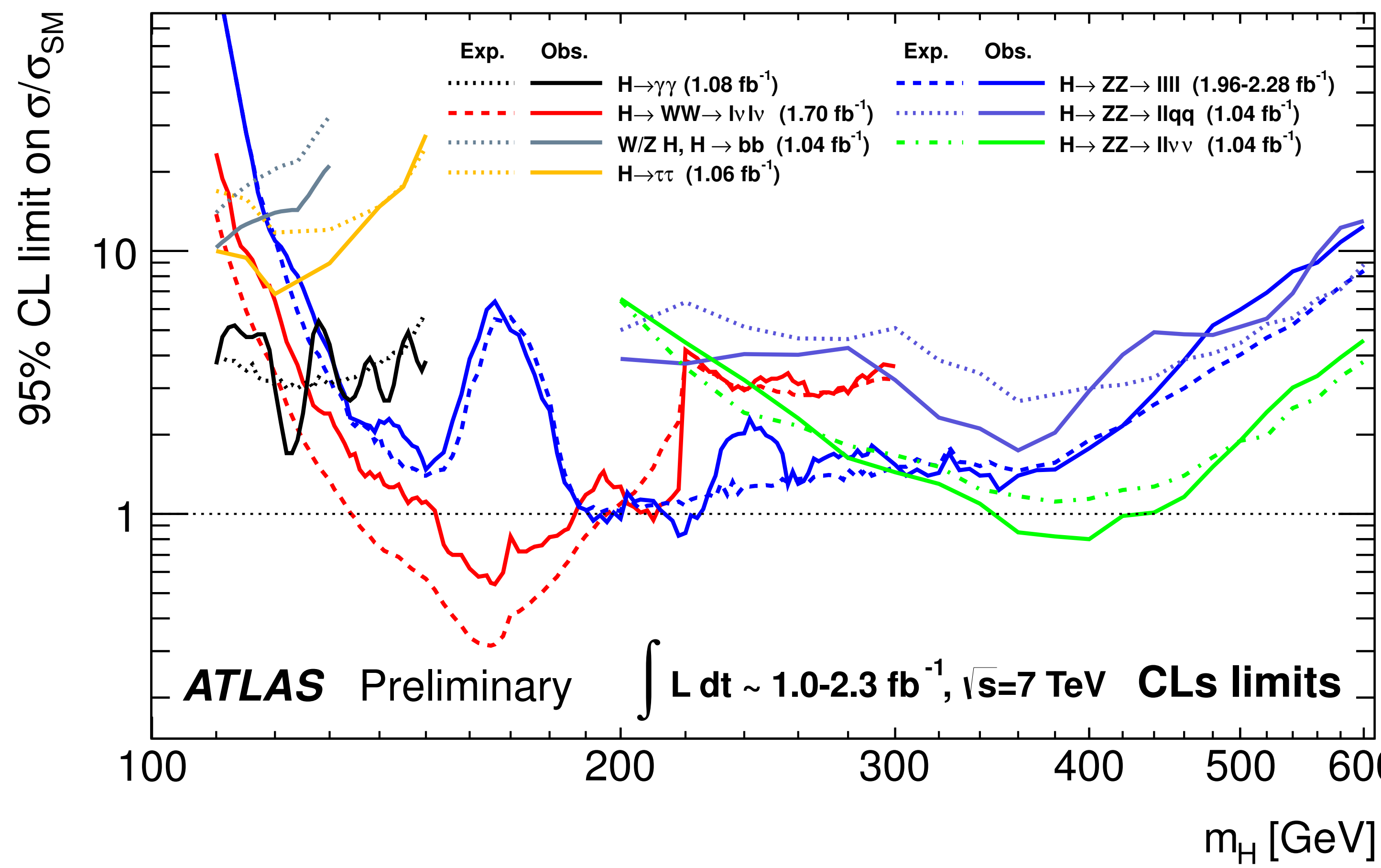

Branching ratios
bb̄
WW
ZZ
ττ
gg
cc̄
γγ
Zγ
LHC HIGGS XS WG 2010
M_H [GeV]


95% CL limit on σ/σ_SM
Exp. Obs.
H→γγ (1.08 fb^-1)
H→ WW→ lνlν (1.70 fb^-1)
W/Z H, H → bb (1.04 fb^-1)
H→ττ (1.06 fb^-1)
H→ ZZ→ llll (1.96-2.28 fb^-1)
H→ ZZ→ llqq (1.04 fb^-1)
H→ ZZ→ llνν (1.04 fb^-1)
ATLAS Preliminary
∫ L dt ~ 1.0-2.3 fb^-1, √s=7 TeV
CLs limits
m_H [GeV]


CMS Preliminary, √s = 7 TeV
95% CL limit on σ/σ_SM
Combined observed
Combined expected
H → bb (1.1 fb^-1)
H → ττ (1.1 fb^-1)
H → γγ (1.7 fb^-1)
H → WW (1.5 fb^-1)
H → ZZ → 4l (1.7 fb^-1)
H → ZZ → 2l 2τ (1.1 fb^-1)
H → ZZ → 2l 2q (1.6 fb^-1)
H → ZZ → 2l 2ν (1.6 fb^-1)
Higgs boson mass (GeV/c^2)



√s = 7TeV
LHC HIGGS XS WG 2011
σ × BR (pb)
WW → l±νqq̄
WW → l+νl-ν̄
ZZ → l+l-qq̄
ZZ → l+l-νν̄
τ+τ-
ZZ → l+l-l+l-
γγ
WH → l±νbb̄
ZH → l+l-bb̄
l = e, μ
ν = νe,νμ,ντ
q = udscb
Higgs boson mass (GeV/c2)



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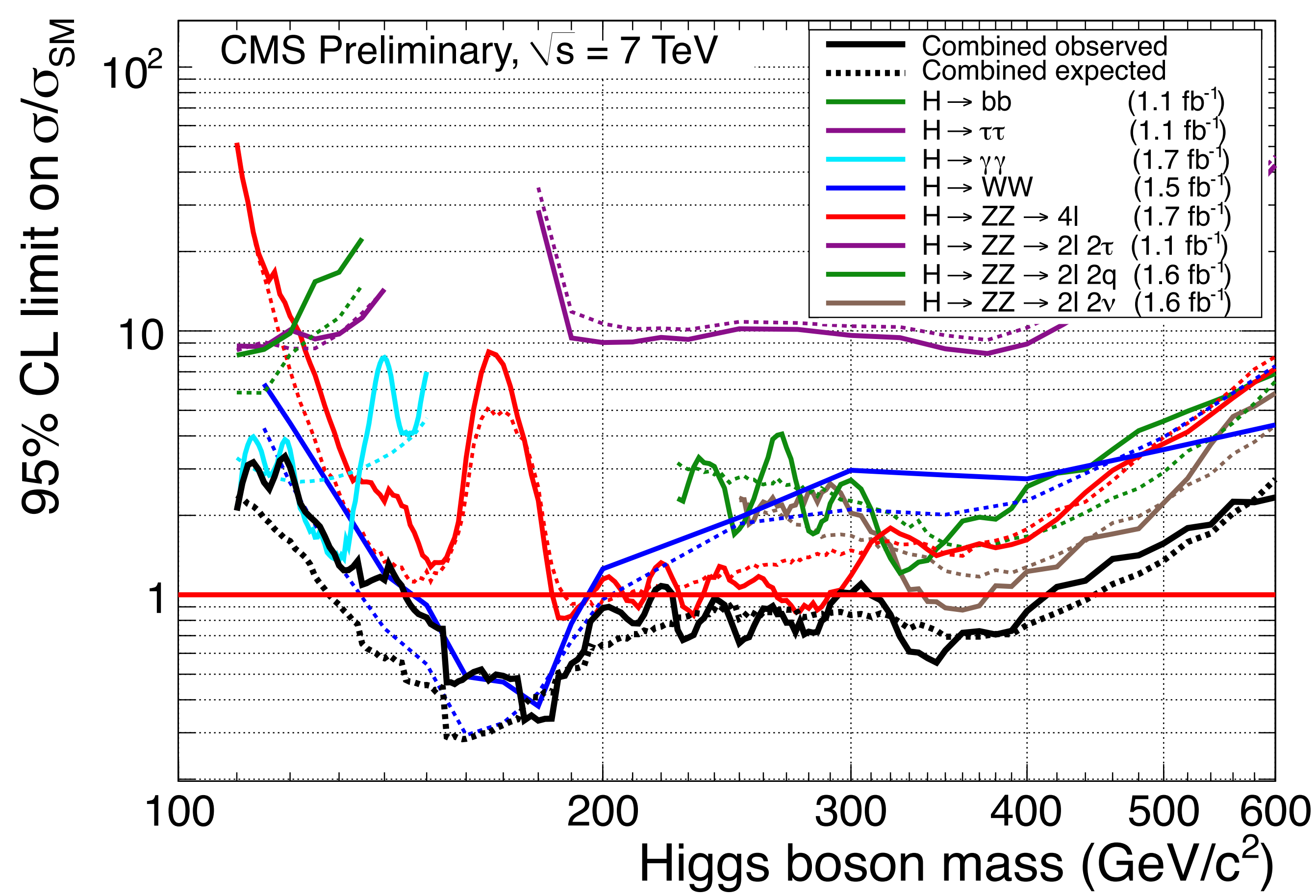
mH [GeV]



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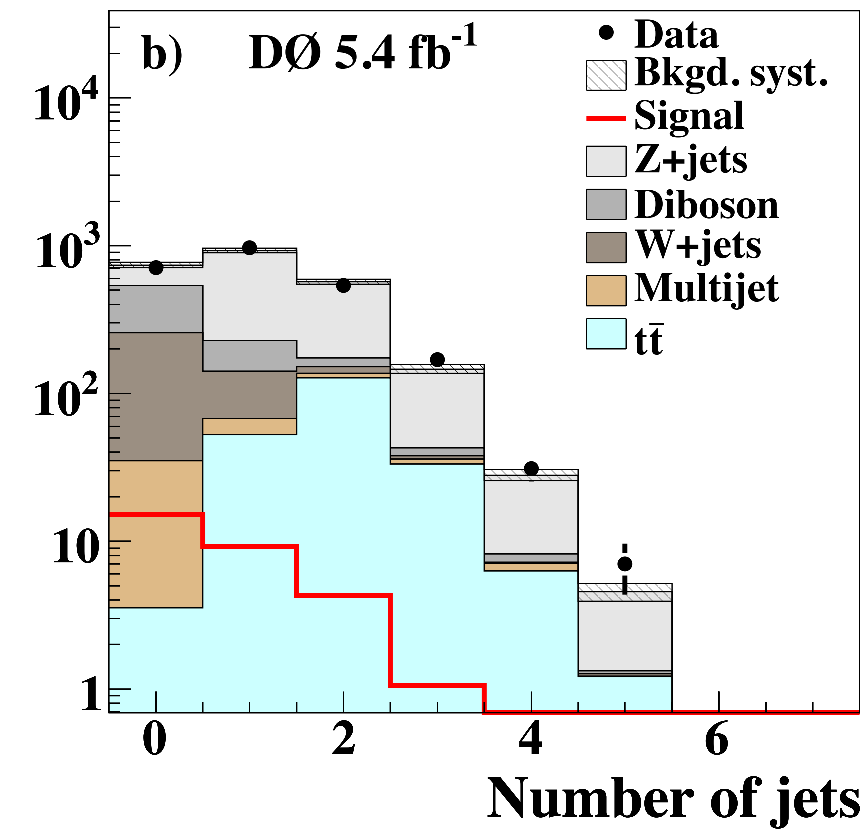
H → ZZ → 2l 2q (1.6 fb-1)
H → ZZ → 2l 2ν (1.6 fb-1)
95% CL limit on σ/σSM
Higgs boson mass (GeV/c2)

Use Jets bins: (exclusive jet σ's)

(σ's with a particular # of jets)

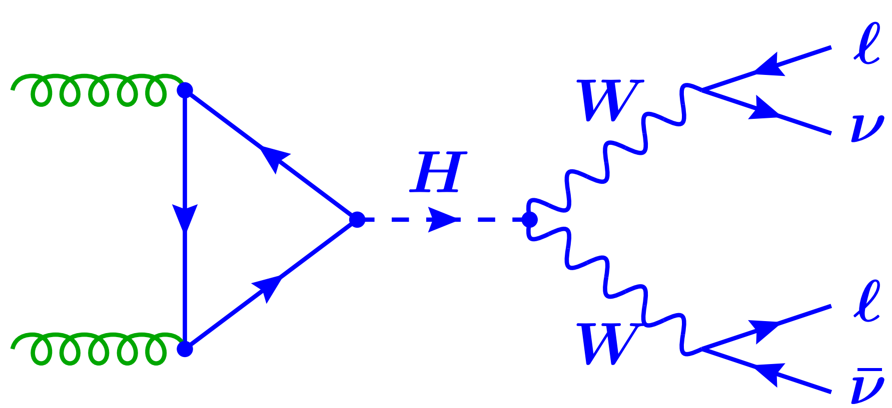
- backgrounds vary with # of jets
- needed to improve sensitivity

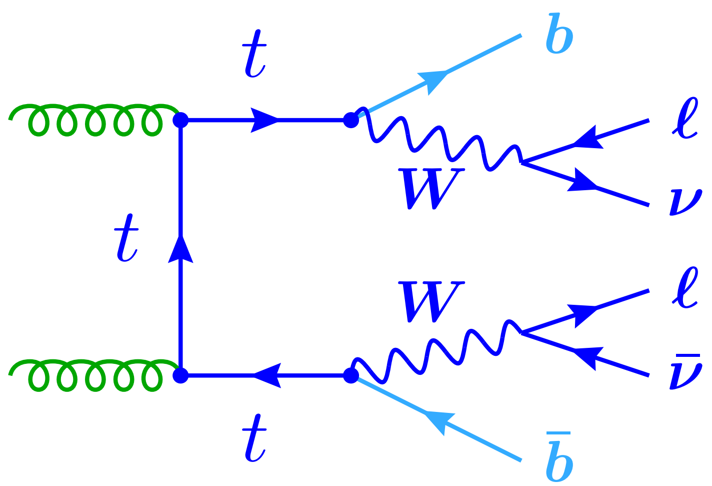
$$H \to WW \to \ell\nu\ell\bar{\nu} \qquad H \to \tau\tau$$
$$H \to WW \to \ell\nu jj \qquad H \to \gamma\gamma$$

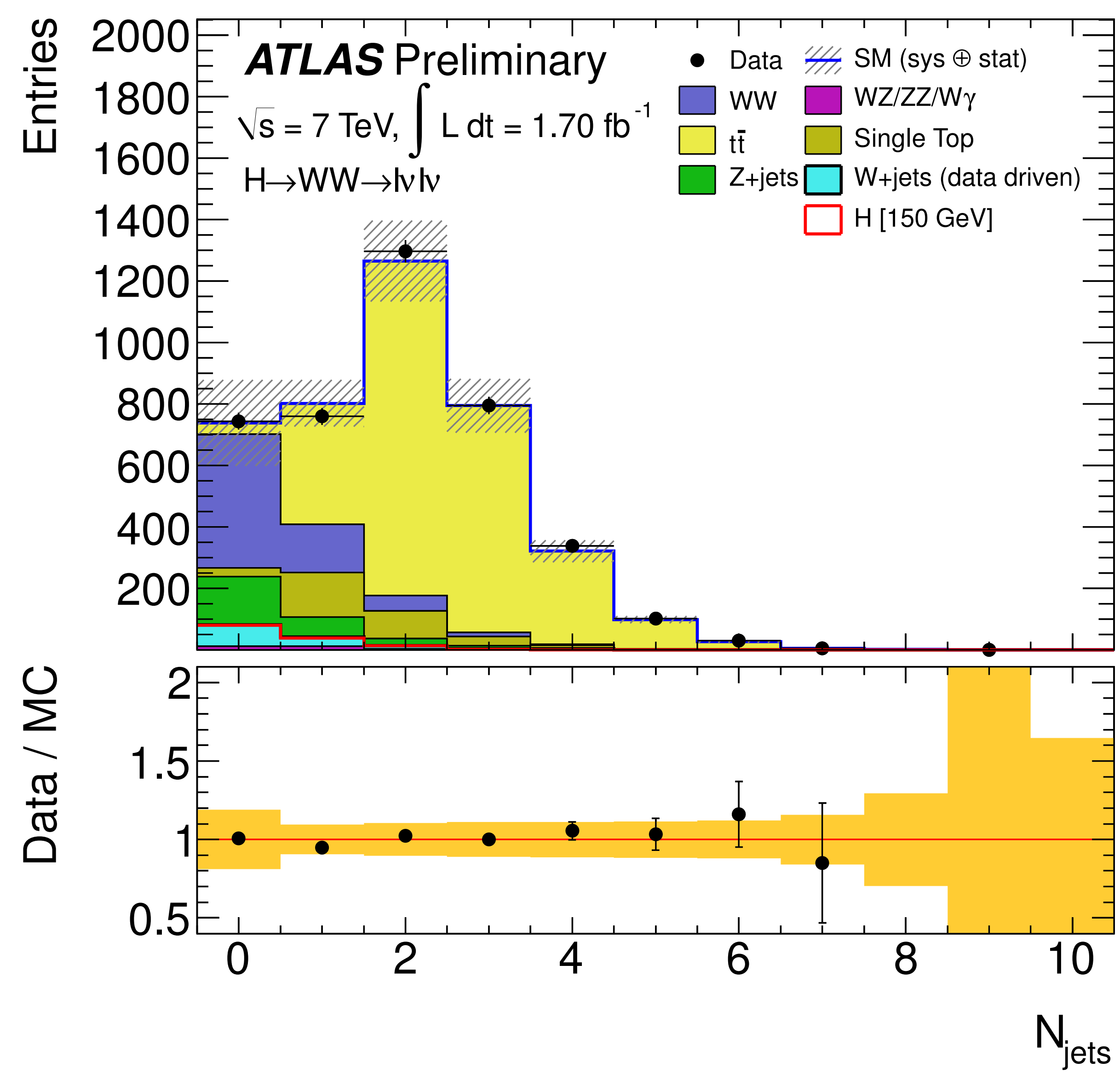


eg. large top background in

$$H \to WW$$







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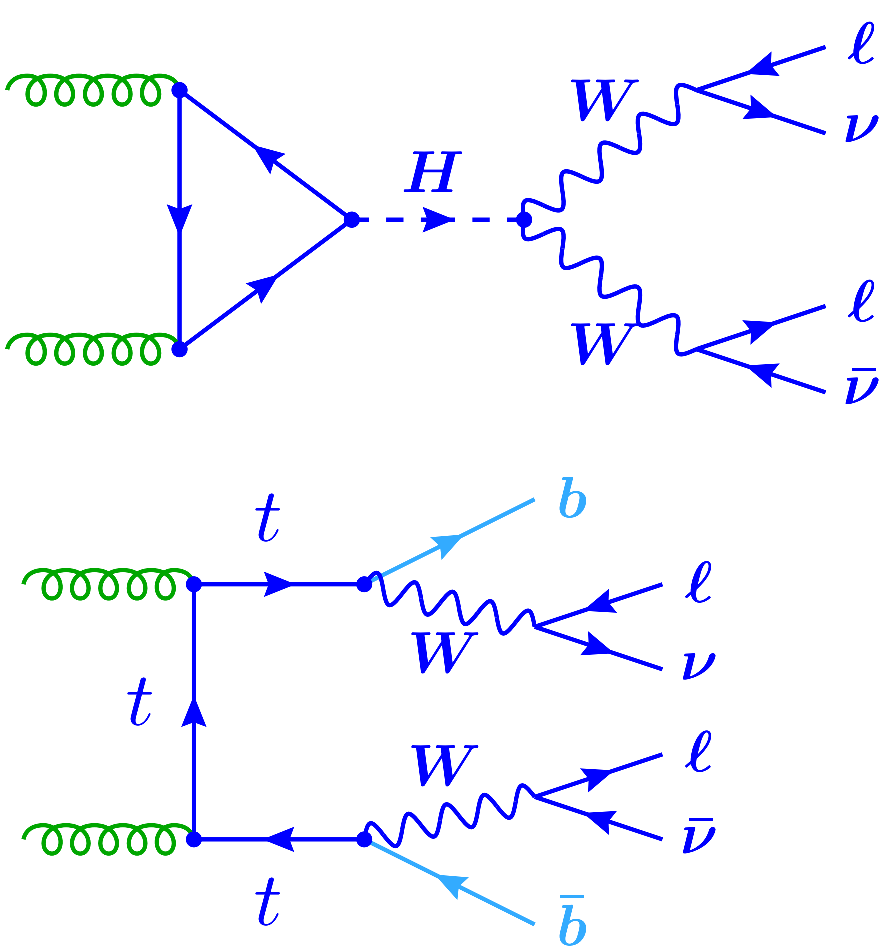

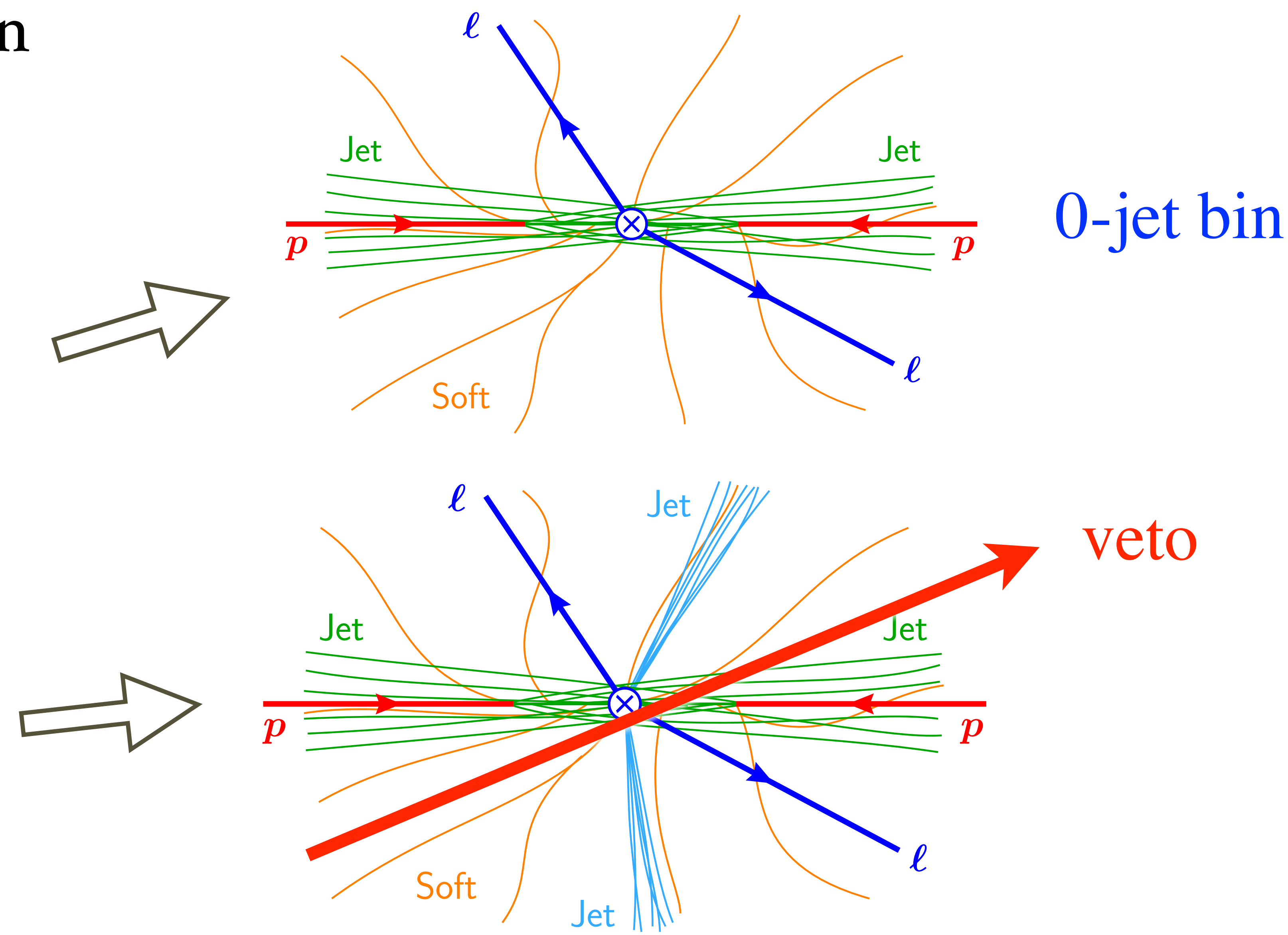

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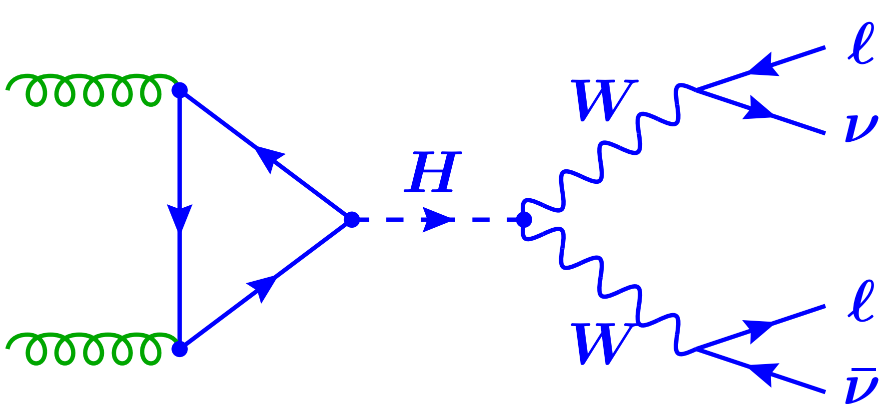
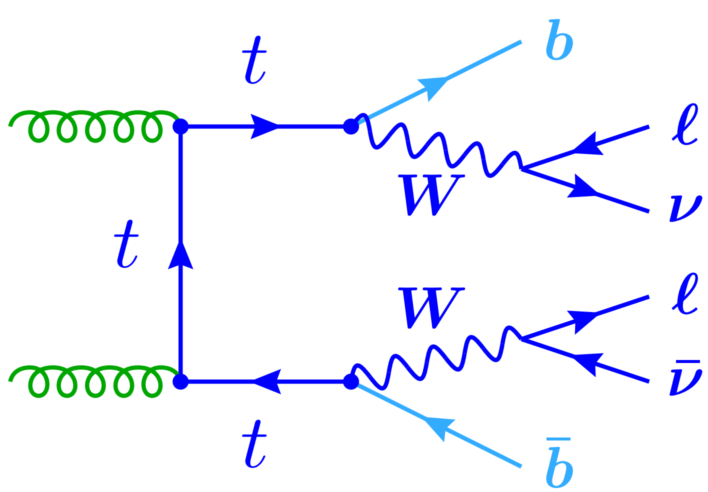
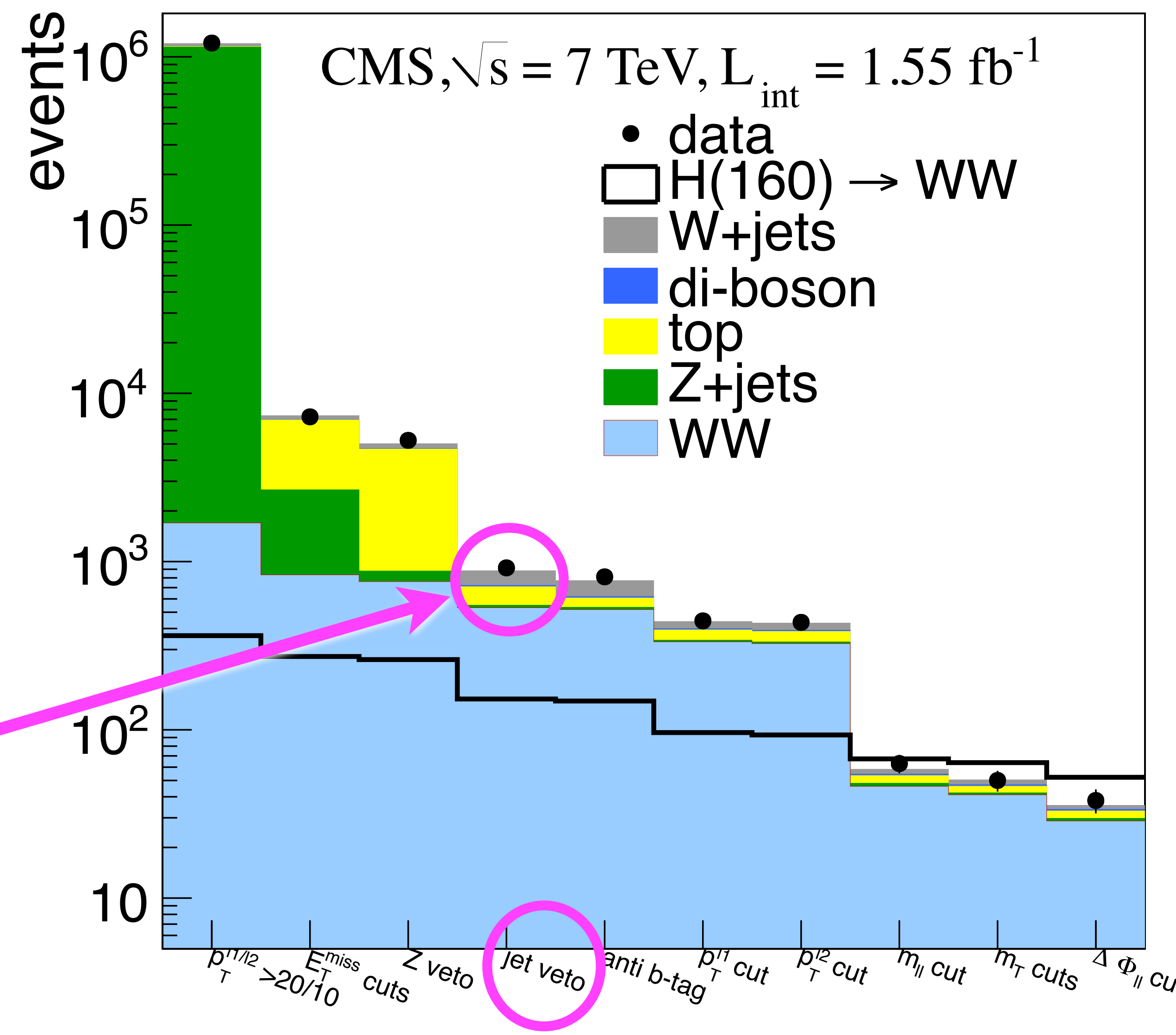




0-jet bin
vetoes
b-jets



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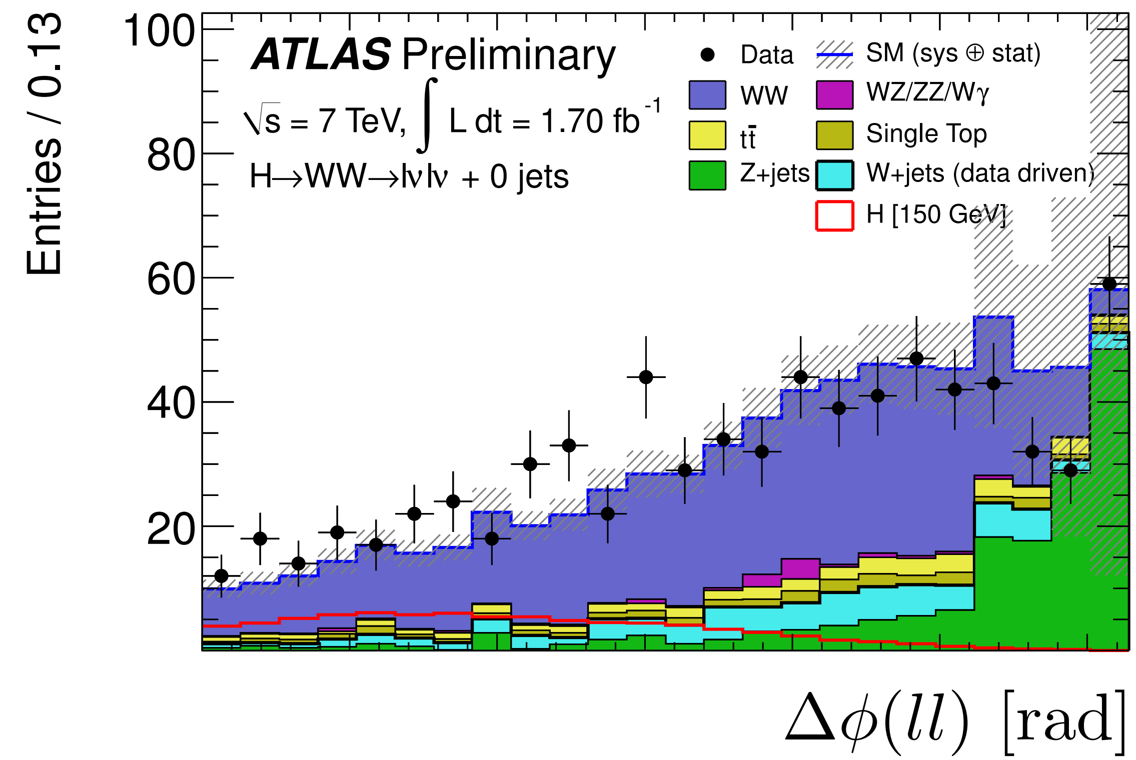

eg. large top background in

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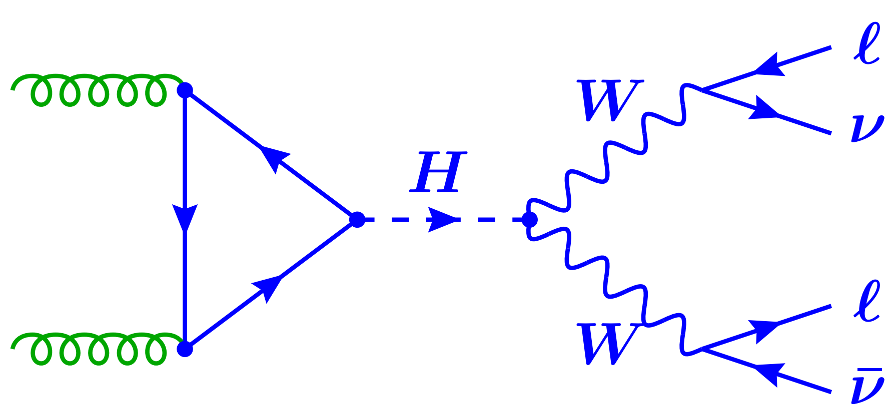

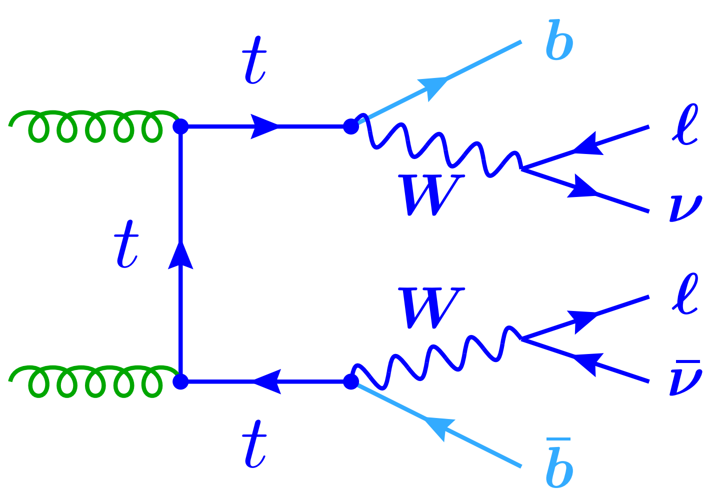

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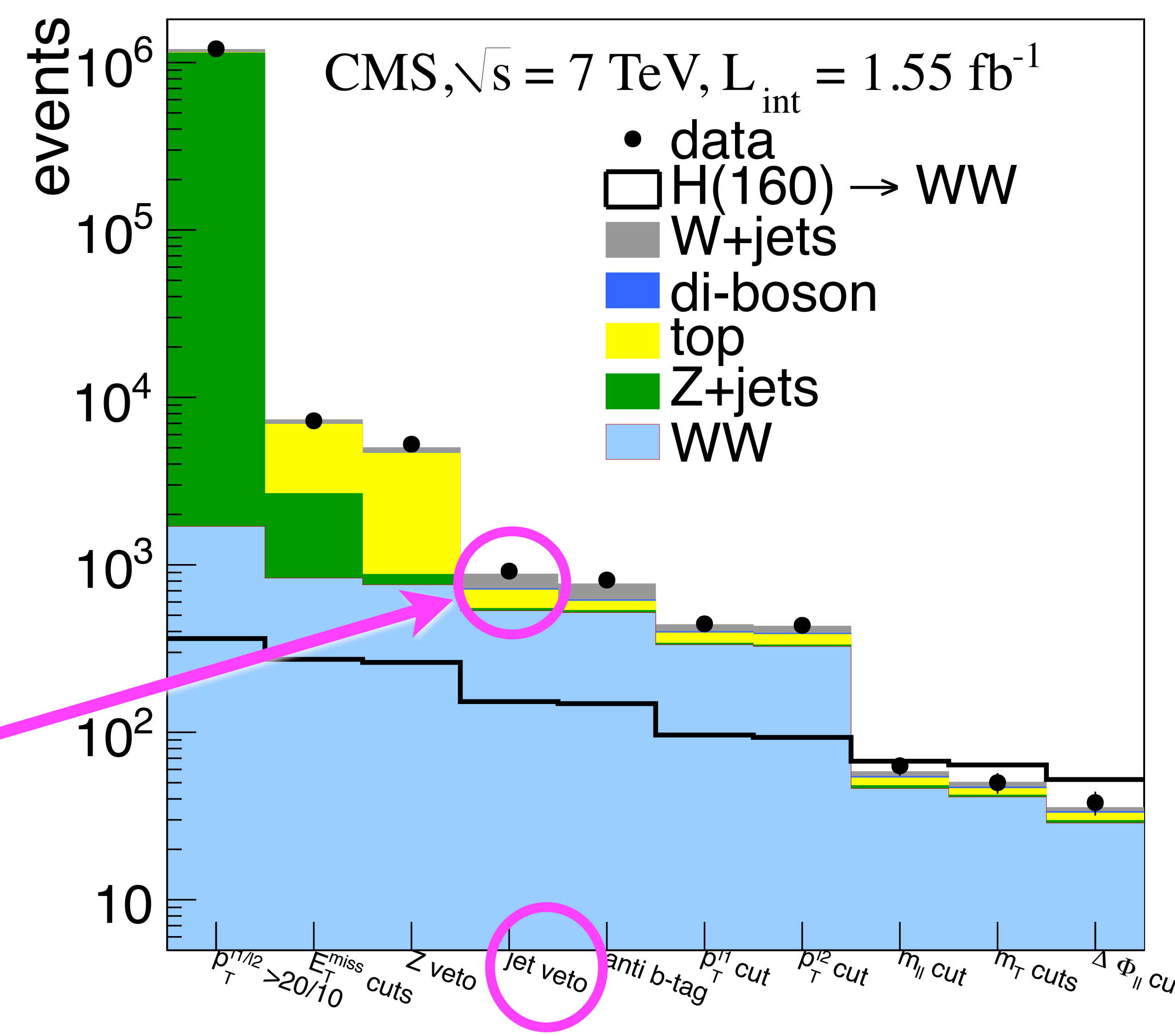


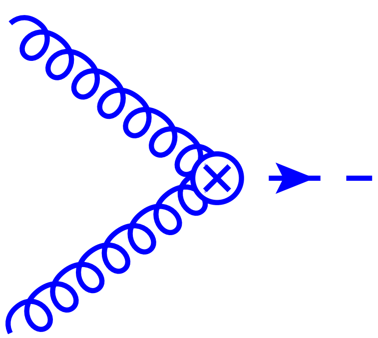
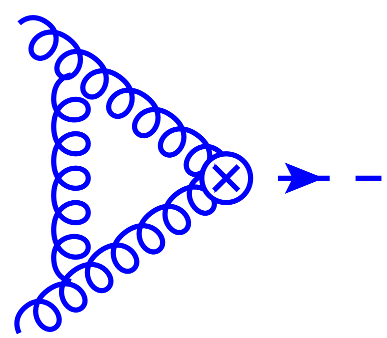
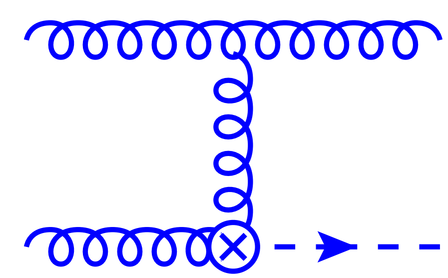
Theory Calculations

Factorization for inclusive Higgs production

[Collins, Soper, Sterman; Bodwin; '80s]

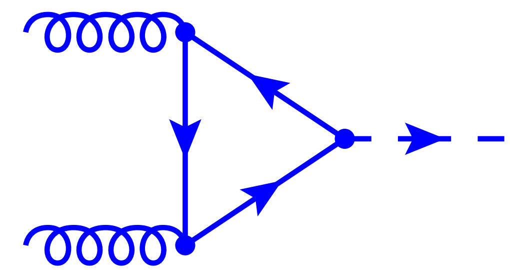
$$\mathrm{d}\sigma^{\mathrm{FO}} = \frac{\sqrt{2}G_F m_H^2}{576\pi E_{\mathrm{cm}}^2} \sum_{i,j} \int \frac{\mathrm{d}\xi_a}{\xi_a} \frac{\mathrm{d}\xi_b}{\xi_b} \, \mathrm{d}\sigma_{ij}^{\mathrm{partonic}}\left(\frac{x_a}{\xi_a}, \frac{x_b}{\xi_b}\right) f_i(\xi_a)\, f_j(\xi_b)$$

Partonic cross section computed in QCD fixed-order perturbation theory

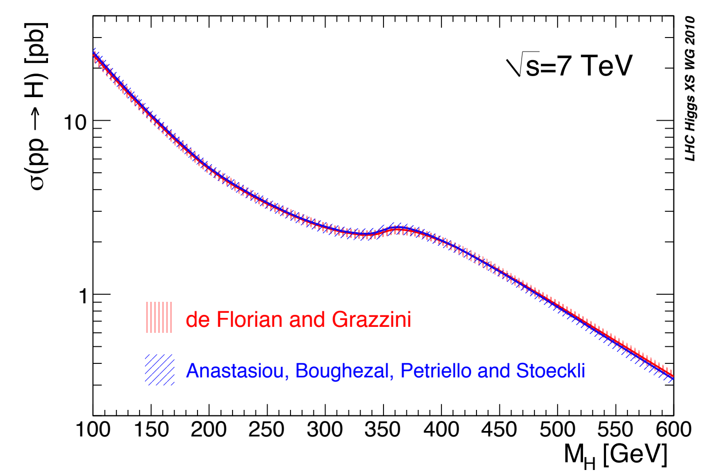
$\mathrm{d}\sigma_{gg}^{\mathrm{partonic}} =$  $+$  $+ \cdots + \int$  $+ \cdots$

Higgs Production to NNLO

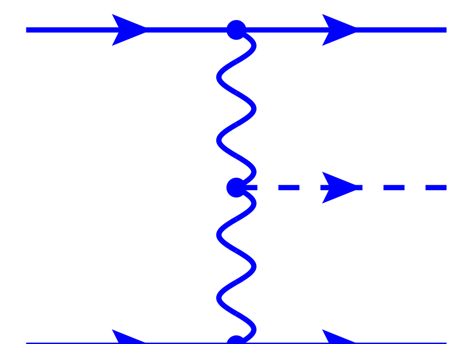
Gluon fusion: $gg \longrightarrow H$



- Total cross section at NNLO including top-mass effects
  [Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven]
  [Pak, Rogal, Steinhauser; Harlander, Mantler, Marzani, Ozeren]
- Electroweak corrections to $\mathcal{O}(\alpha_{\rm em}\alpha_s)$
  [Aglietti, Bonciani, Degrassi, Vicini; Actis, Passarino, Sturm, Uccirati; Anastasiou, Boughezal, Petriello]
- Summation of higher-order threshold and constant terms to N^{3}LL
  [de Florian, Grazzini; Ahrens, Becher, Neubert, Yang]
- FEHiP, HNNLO: Numerical *fully differential* cross section at NNLO
  [Anastasiou, Melnikov, Petriello; Grazzini]



Vector-boson fusion: $qq \longrightarrow qqH$



- Total cross section at NNLO* [Bolzoni, Maltoni, Moch, Zaro]
- HAWK: Numerical *fully differential* cross section at NLO (QCD+EW)
  [Ciccolini, Denner, Dittmaier, Mück]

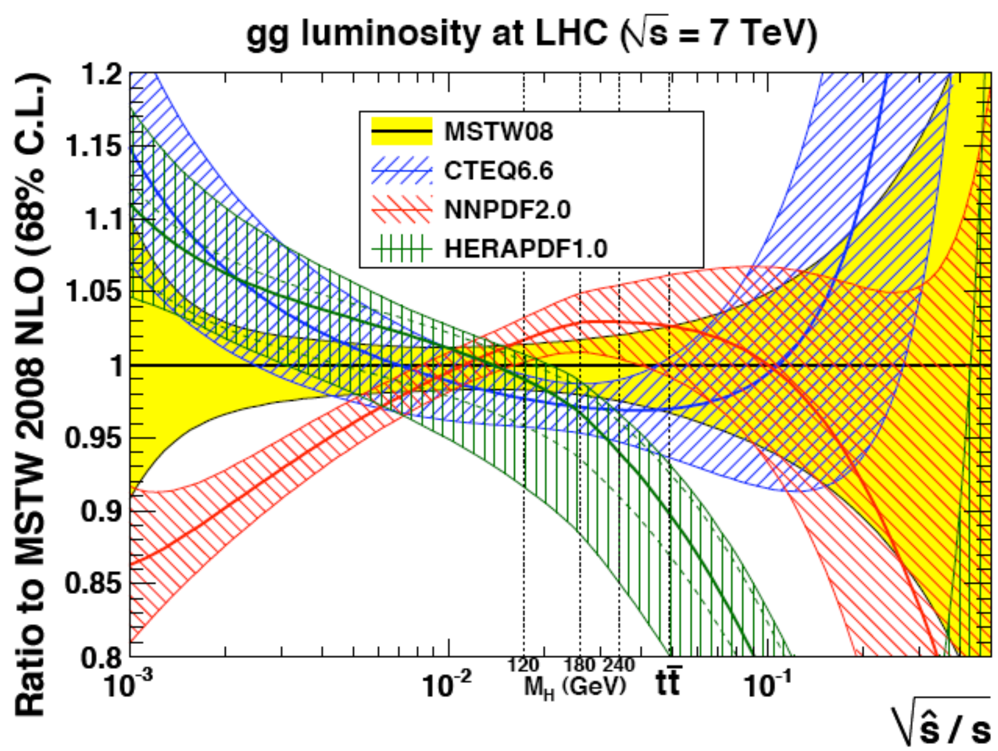
Multiple Particle Final States

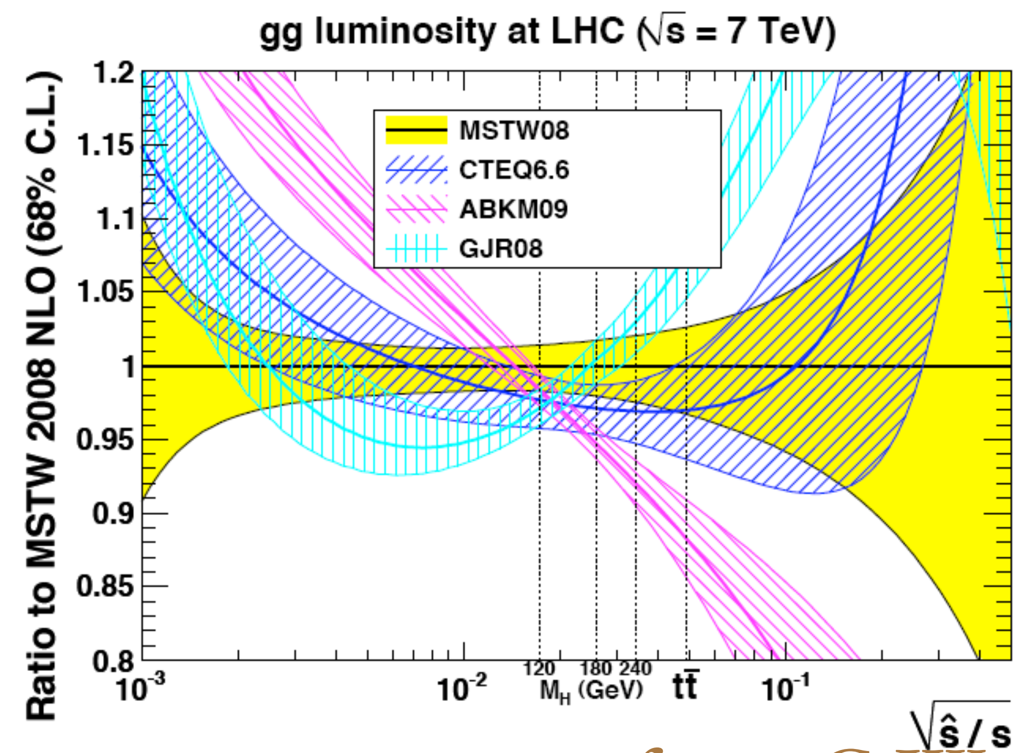
eg. $gg \to H + 2$ jets , . . .

Number of NLO Feynman diagrams explodes with increasing number of particles in the final state

- Unitarity-based methods allow to circumvent Feynman diagrams
- Directly construct NLO helicity amplitudes from “sewing together” lower-point tree amplitudes
- Several NLO programs/libraries
  - MCFM [Campbell, K.Ellis, Williams]
  - Blackhat [Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre]
  - Rocket [K. Ellis, Melnikov, Zanderighi]
  - HelacNLO [Bevilacqua, Czakon, Papadopoulos, Pittau, Worek]
  - Samurai [Mastrolia, Ossola, Reiter, Tramontano]

PDF's





from G. Watt

PDF4LHC recommendation:

Compute MSTW 68% PDF+ α_s errors at NNLO.

Take envelope of CTEQ, MSTW, NNPDF errors at NLO, and divide by MSTW error at NLO. Multiply NNLO errors by this ratio (roughly 2).

Group	Source	Typical Uncertainty
PDFs+α_s	$gg \to H\,,\, t\bar{t}H\,,\, gg \to VV$ (gg)	$8\,\%$
(cross sections)	VBF $H\,,\, VH\,,\, VV$@NLO ($q\bar{q}$)	$4\,\%$
QCD scale	total inclusive $gg \to H$	${}^{+12\,\%}_{-7\,\%}$
	inclusive $gg \to H + \geq 1$ jets	$20\,\%$
	inclusive $gg \to H + \geq 2$ jets	20% (NLO), 70% (LO)
	VBF H	$1\,\%$
	associated VH	$1\,\%$

taken from ATLAS & CMS Higgs combination group

Lets focus on perturbative uncertainties for exclusive σ's

perturbative uncertainties

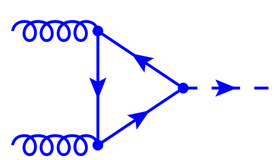
exclusive jet cross sections

$$pp \to H + 0\,\text{jets} \;\longrightarrow\; \sigma_0 \pm \Delta_0$$
$$pp \to H + 1\,\text{jet} \;\longrightarrow\; \sigma_1 \pm \Delta_1$$
$$pp \to H + 2\,\text{jets} \;\longrightarrow\; \sigma_2 \pm \Delta_2$$
$$\vdots$$

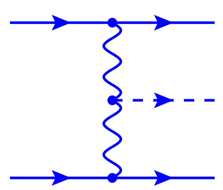
sum is $\longrightarrow \quad \sigma_{\text{total}} \pm \Delta_{\text{total}}$

$\Delta_i = ?$
correlations=?

$\Delta_{\text{total}} \sim 8\%$



$\sim 2\%$



From CMS H→WW analysis:

CMS PAS HIG-11-014

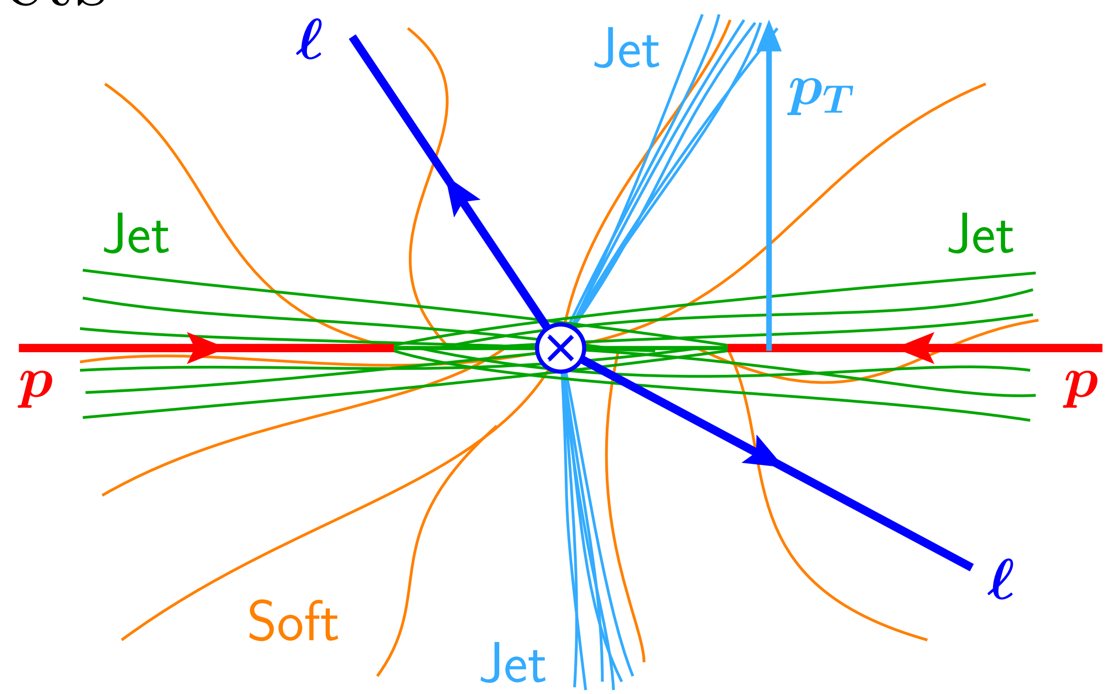
2011/08/22

Table 3: Summary of all systematic uncertainties (relative). This is just an indicative table, since the precise values depend on the final state and jet-bin.

Source	$H \to W^+W^-$	$qq \to W^+W^-$	$gg \to W^+W^-$	non-Z resonant WZ/ZZ	top	DY	W + jets	$V(W/Z) +\gamma$
Luminosity	4.5	—	—	4.5	—	—	—	4.5
Trigger efficiencies	1.5	1.5	1.5	1.5	—	—	—	1.5
Muon efficiency	1.5	1.5	1.5	1.5	—	—	—	1.5
Electron id efficiency	2.5	2.5	2.5	2.5	—	—	—	2.5
Momentum scale	1.5	1.5	1.5	1.5	—	—	—	1.5
E_T^{miss} resolution	2.0	2.0	2.0	2.0	2.0	3.0	—	1.0
Jet counting	7-20	—	5.5	5.5	—	—	—	5.5
Higgs cross section	5-15	—	—	—	—	—	—	—
WZ/ZZ cross section	—	—	—	3.0	—	—	—	—
$qq \to WW$ norm.	—	15	—	—	—	—	—	—
$gg \to WW$ norm.	—	—	50	—	—	—	—	—
W + jets norm.	—	—	—	—	—	—	36	—
top norm.	—	—	—	—	25	—	—	—
$Z/\gamma^* \to \ell^+\ell^-$ norm.	—	—	—	—	—	60	—	—
Monte Carlo statistics	1.0	1.0	1.0	4.0	6.0	20.0	20.0	10.0

The uncertainty on the signal efficiency is estimated to be $\sim 20\%$ and is dominated by the theoretical uncertainty in the jet veto efficiency determination. The uncertainty on the background estimations in the $H \to W^+W^-$ signal region is $\sim 15\%$, which is dominated by the statistical uncertainties of the background control regions in data.

Vetoing Jets : eg. $H \to WW + 0$ jets

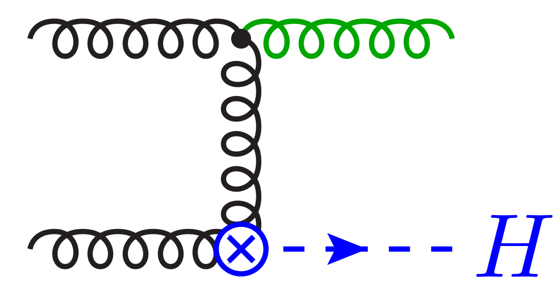


Search for jets and require $p_T^{\mathbf{jet}} < p_T^{\mathbf{cut}}$

Tevatron: $p_T^{\mathbf{cut}} \simeq \mathbf{20\,GeV}$

LHC: $p_T^{\mathbf{cut}} \simeq \mathbf{25\,GeV}$

Jet Veto changes form of perturbation theory



Even if hard signal process $gg \to H$ contains no jets, jet veto affects cross section by restricting ISR

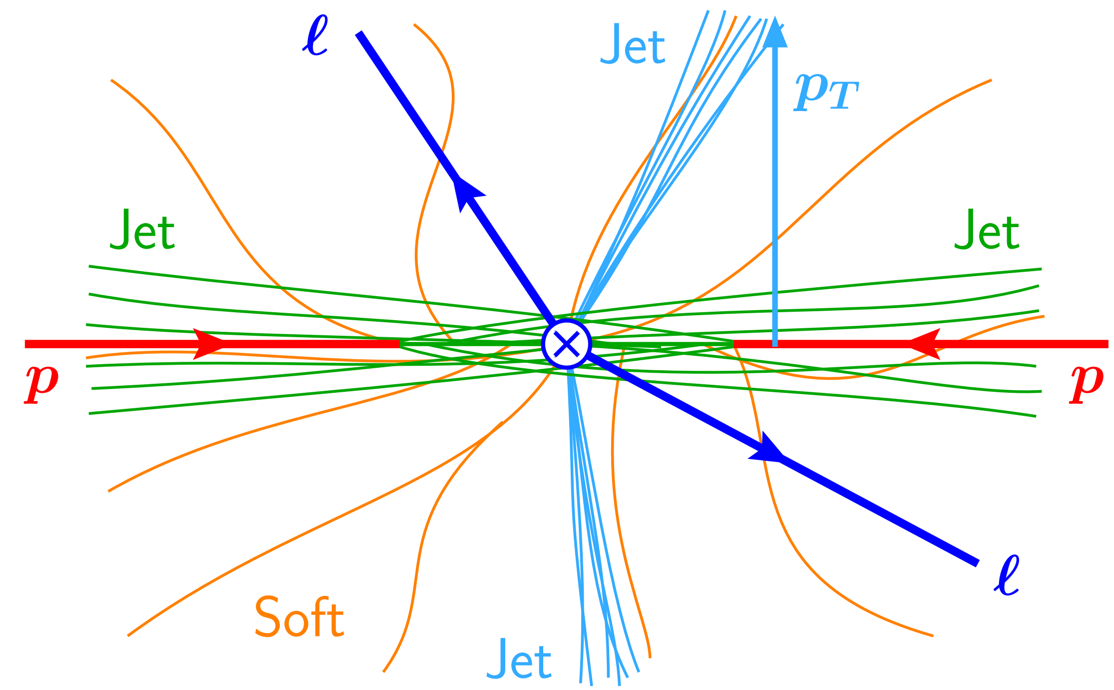
t-channel singularities produce Sudakov double logarithms

$$\sigma_0 = \sigma(p_T^{\mathbf{cut}}) = \sigma_B\left(1 - \frac{3\alpha_s}{\pi}\, 2\ln^2 \frac{p_T^{\mathbf{cut}}}{m_H} + \cdots\right)$$

$\Rightarrow$ Perturbative corrections get large at small $p_T^{\mathbf{cut}} \ll m_H$

Vetoing Jets :

eg. $H \to WW + 0$ jets



Search for jets and require $p_T^{\text{jet}} < p_T^{\text{cut}}$

Tevatron: $p_T^{\text{cut}} \simeq 20\,\text{GeV}$

LHC: $p_T^{\text{cut}} \simeq 25\,\text{GeV}$

Jet Veto changes form of perturbation theory

$$\begin{aligned}
\sigma_0 \sim \; 1 \quad & + \alpha_s L^2 && + \alpha_s^2 L^4 && + \ldots \\
& + \alpha_s L && + \alpha_s^2 L^3 && + \ldots \\
& + \alpha_s && + \alpha_s^2 L^2 && + \ldots \\
& && + \alpha_s^2 L && + \ldots \\
& && + \alpha_s^2 && + \ldots
\end{aligned}$$

$$L = \ln \frac{p_T^{\text{cut}}}{m_H}$$

$$\underbrace{\boldsymbol{\sigma_{\mathrm{total}}} = \int_0^{p_T^{\mathrm{cut}}} \mathbf{d}p_T \frac{\mathbf{d}\boldsymbol{\sigma}}{\mathbf{d}p_T}}_{\boldsymbol{\sigma_0(p_T^{\mathrm{cut}})}} + \underbrace{\int_{p_T^{\mathrm{cut}}} \mathbf{d}p_T \frac{\mathbf{d}\boldsymbol{\sigma}}{\mathbf{d}p_T}}_{\boldsymbol{\sigma_{\geq 1}(p_T^{\mathrm{cut}})}}$$

inclusive jet cross section
$pp \to H + \geq 1\,\mathrm{jet}$

- Added uncertainty Δ_{cut} from our ability to predict p_T^{cut} dependence

  (“large logs” or “particle migration between bins”)

- Cancels when adding σ_0 and $\sigma_{\geq 1}$

  anti-correlated

$$\begin{pmatrix} \Delta^2_{\mathrm{cut}} & -\Delta^2_{\mathrm{cut}} \\ -\Delta^2_{\mathrm{cut}} & \Delta^2_{\mathrm{cut}} \end{pmatrix}$$

- Extension to multiple exclusive jet bins:

$$\sigma_0(p_T^{\mathrm{cut}}),\, \sigma_1(p_T^{\mathrm{cut}}, p_{T2}^{\mathrm{cut}}),\, \sigma_2(p_{T2}^{\mathrm{cut}}, p_{T3}^{\mathrm{cut}}),\, \ldots$$

Δ_{cut} Δ_{cut2}

p_{Tj}^{cut} is cut on j’th largest jet p_T

How do we compute Δ_{cut} ?

We will explore three methods

(A) (B) (C)

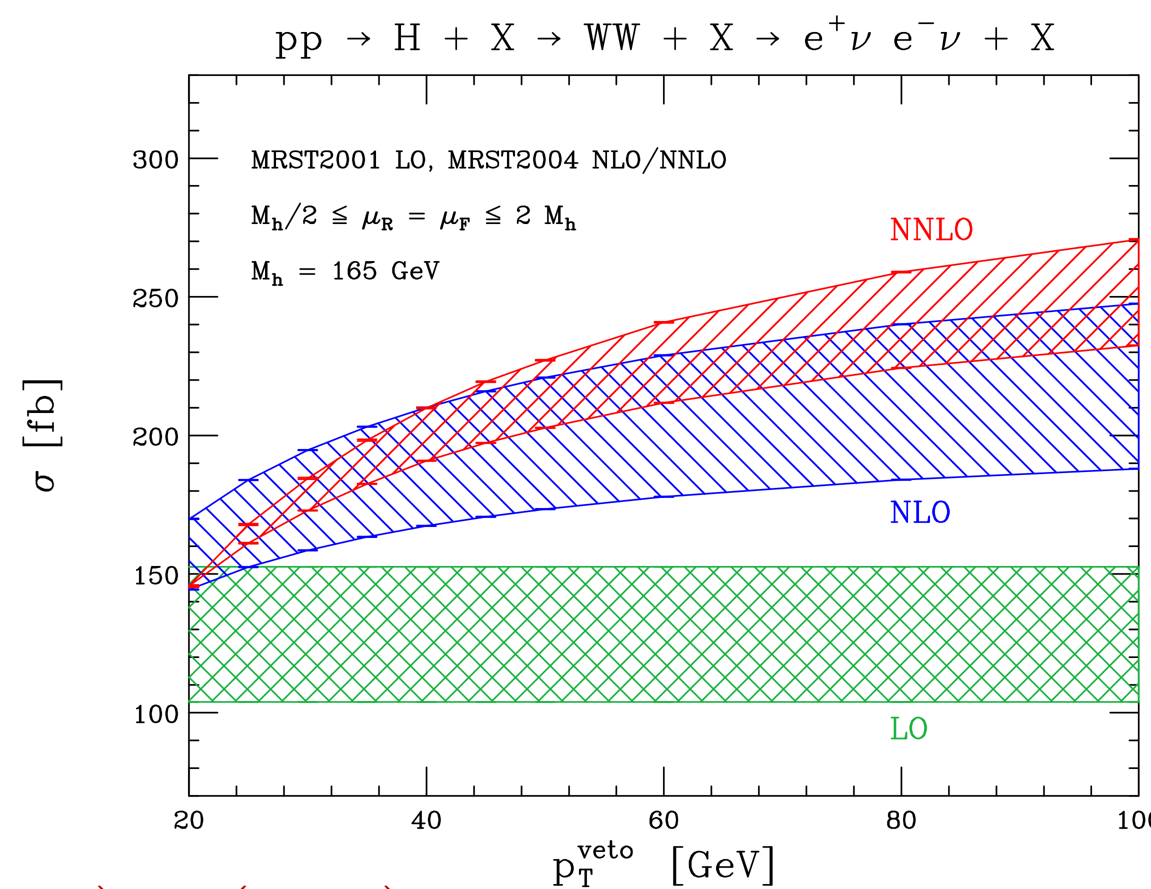
(A) "Direct Exclusive Scale Variation?" vary μ_F, μ_R in σ_i's $\longrightarrow \Delta_i$

consider $\sigma_0(\mu)$, vary $\mu \in [m_H/2, 2m_H]$ to get Δ_0 etc.

- Uncertainties are 100% correlated. Common scale variation for jet bins

$$\sigma_{\text{total}} = \sigma_0 + \sigma_1 + \ldots \quad \text{gets back its uncertainty} \quad \Delta_{\text{total}}$$

- Naively, jet veto appears to improve convergence
- FO expansion gets unstable at small p_T^{cut} and eventually breaks down



eg. Tevatron

$$\frac{\Delta\sigma}{\sigma} = \underbrace{66.5\% \times \begin{pmatrix}+5\% \\ -9\%\end{pmatrix}}_{\text{0 jets}} + \underbrace{28.6\% \times \begin{pmatrix}+24\% \\ -22\%\end{pmatrix}}_{\text{1 jet}} + \underbrace{4.9\% \times \begin{pmatrix}+78\% \\ -41\%\end{pmatrix}}_{\geq \text{2 jets}} = \begin{pmatrix}+14\% \\ -14\%\end{pmatrix}$$

[Anastasiou et al., arXiv:0905.3529]

- *Smaller* uncertainty in 0-jet bin than in inclusive cross section

(A) "Direct Exclusive Scale Variation?" vary μ_F, μ_R in σ_i's $\longrightarrow \Delta_i$

consider $\sigma_0(\mu)$, vary $\mu \in [m_H/2, 2m_H]$ to get Δ_0 etc.

- Uncertainties are 100% correlated. Common scale variation for jet bins

  $\sigma_{\text{total}} = \sigma_0 + \sigma_1 + \ldots$ gets back its uncertainty Δ_{total}

- does not account for Δ_{cut}

- due to numerical cancellations can underestimate uncertainties

$$\boldsymbol{\sigma_{\text{total}} \simeq \sigma_B \big[1 + \alpha_s + \alpha_s^2 + \mathcal{O}(\alpha_s^3)\big]}$$ large K-factor

$$\boldsymbol{\sigma_{\geq 1}(p_T^{\text{cut}}) \simeq \sigma_B \big[\alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3 L^6)\big]}$$ large logs

$\boldsymbol{L = \ln(p_T^{\text{cut}}/m_H)}$

always a large cancellation for $\boldsymbol{\sigma_0(p_T^{\text{cut}}) = \sigma_{\text{total}} - \sigma_{\geq 1}(p_T^{\text{cut}})}$
in some range of p_T^{cut}

For example, at LHC for $m_H = 165\,\mathrm{GeV}$ and $E_{\mathrm{cm}} = 7\,\mathrm{TeV}$ $\qquad \mu_F = \mu_R = \frac{m_H}{2}$

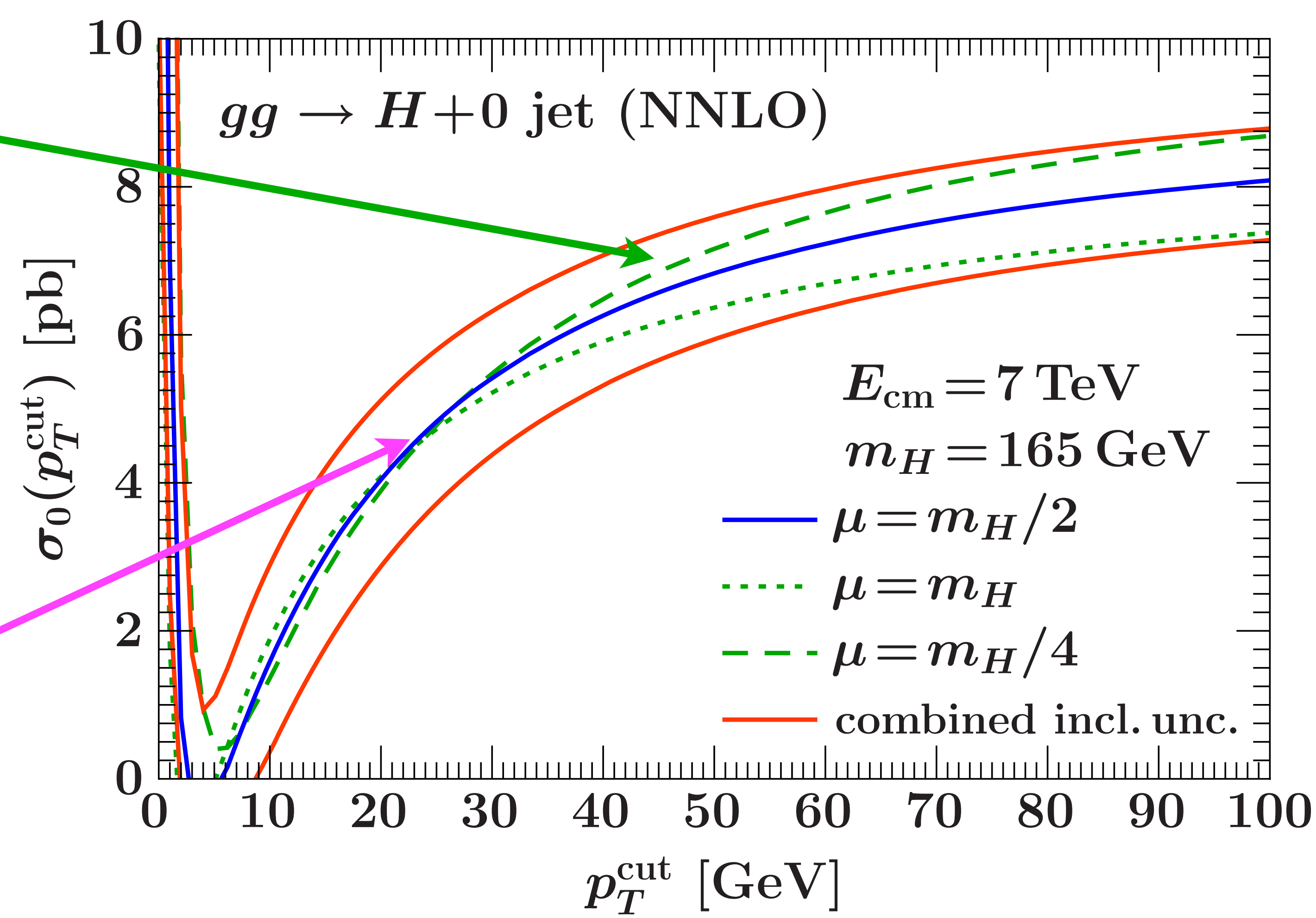
$$\sigma_{\mathrm{total}} = (3.32\,\mathrm{pb})\big[1 + 9.5\,\alpha_s + 35\,\alpha_s^2 + \mathcal{O}(\alpha_s^3)\big]$$

$$\sigma_{\geq 1}(p_T^{\mathrm{jet}} \geq 30\,\mathrm{GeV}) = (3.32\,\mathrm{pb})\big[5.1\,\alpha_s + 28\,\alpha_s^2 + \mathcal{O}(\alpha_s^3)\big]\,.$$

Direct Exclusive Scale Variation:

green lines

cancellation



All my fixed order plots use:
MCFM v6.0 for spectra, FeHiP for NNLO cross section,
MSTW pdfs, anti-kT jets with R=0.5

(B) "Combined Inclusive Scale Variation"

IS, Tackmann, arXiv:1107.2117

- Treat inclusive cross-section uncertainties as independent

$$\Delta_{\text{total}}, \Delta_{\geq 1}, \Delta_{\geq 2}, \ldots$$

$$\boldsymbol{C} = \begin{pmatrix} \boldsymbol{\Delta^2_{\text{total}}} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Delta^2_{\geq 1}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\Delta^2_{\geq 2}} \end{pmatrix}$$

- For p_T^{cut} uncertainty use: $\Delta_{\text{cut}} = \Delta_{\geq 1}$

Propagate errors to get uncertainty for $\sigma_0(p_T^{\text{cut}}) = \sigma_{\text{total}} - \sigma_{\geq 1}(p_T^{\text{cut}})$

eg. $\{\sigma_0, \sigma_{\geq 1}\}$ $\begin{pmatrix} \Delta^2_{\geq 1} + \Delta^2_{\text{total}} & -\Delta^2_{\geq 1} \\ -\Delta^2_{\geq 1} & \Delta^2_{\geq 1} \end{pmatrix}$ has anti-correlation

$\boldsymbol{\sigma_{\text{total}} \simeq \sigma_B \left[1 + \alpha_s + \alpha_s^2 + \mathcal{O}(\alpha_s^3)\right]}$ large K-factor

$\boldsymbol{\sigma_{\geq 1}(p_T^{\text{cut}}) \simeq \sigma_B \left[\alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3 L^6)\right]}$ large logs

$\boldsymbol{L = \ln(p_T^{\text{cut}}/m_H)}$

treated as independent series

estimate for impact of logs obtained from $\sigma_{\geq 1}(p_T^{\text{cut}})$'s μ dependence

For example, at LHC for $m_H = 165\,\mathrm{GeV}$ and $E_{\mathrm{cm}} = 7\,\mathrm{TeV}$

$$\sigma_{\mathrm{total}} = (3.32\,\mathrm{pb})\big[1 + 9.5\,\alpha_s + 35\,\alpha_s^2 + \mathcal{O}(\alpha_s^3)\big]$$

$$\sigma_{\geq 1}(p_T^{\mathrm{jet}} \geq 30\,\mathrm{GeV}) = (3.32\,\mathrm{pb})\big[5.1\,\alpha_s + 28\,\alpha_s^2 + \mathcal{O}(\alpha_s^3)\big]\,.$$

Direct Exclusive Scale Variation

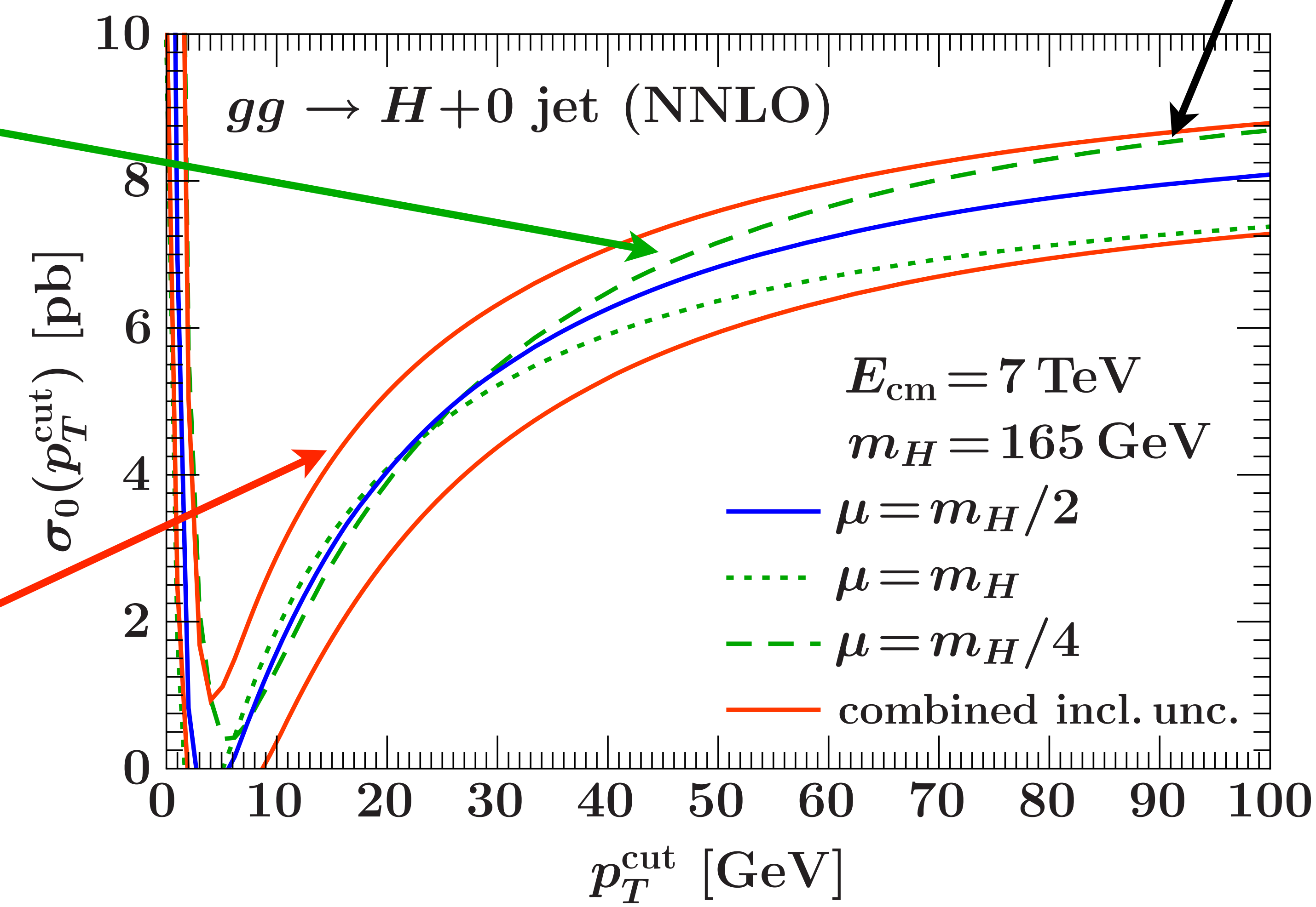
green lines

Combined Inclusive Scale Variation

$$\Delta_0^2 = \Delta_{\mathrm{total}}^2 + \Delta_{\geq 1}^2$$

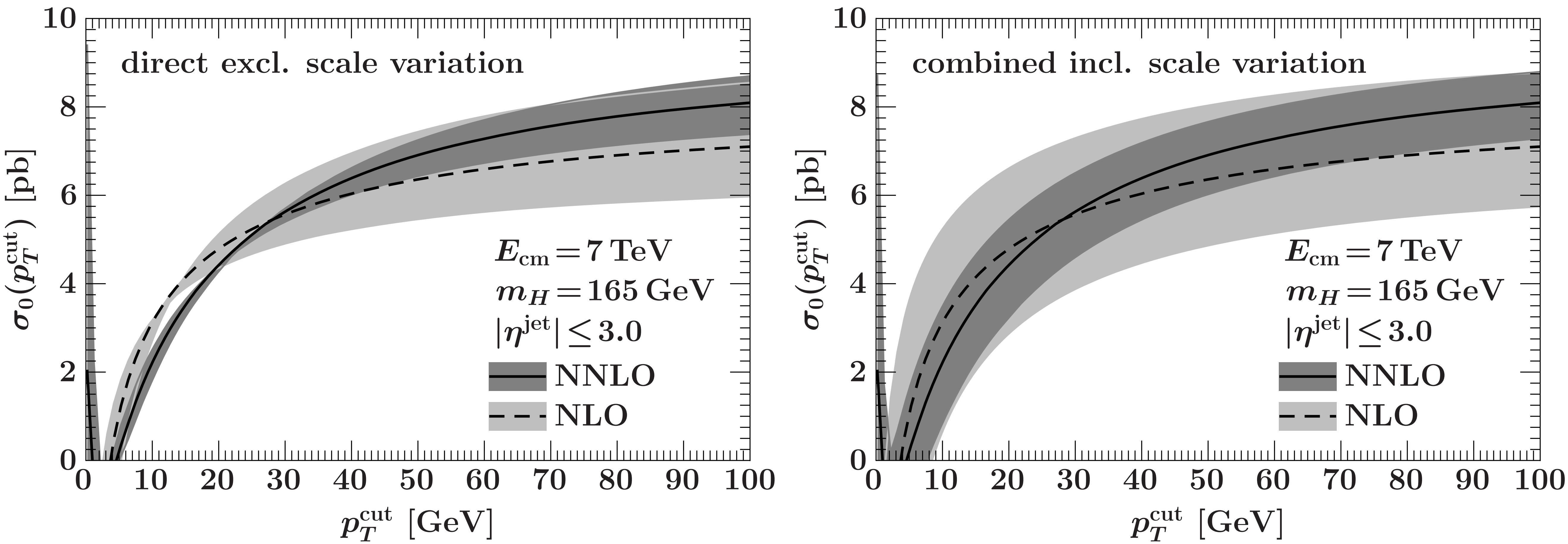
red lines

agree when cut is turned off



these plots only vary $\mu_R = \mu_F$ (varying μ_F alone is quite small for Higgs)

Convergence (NLO to NNLO)



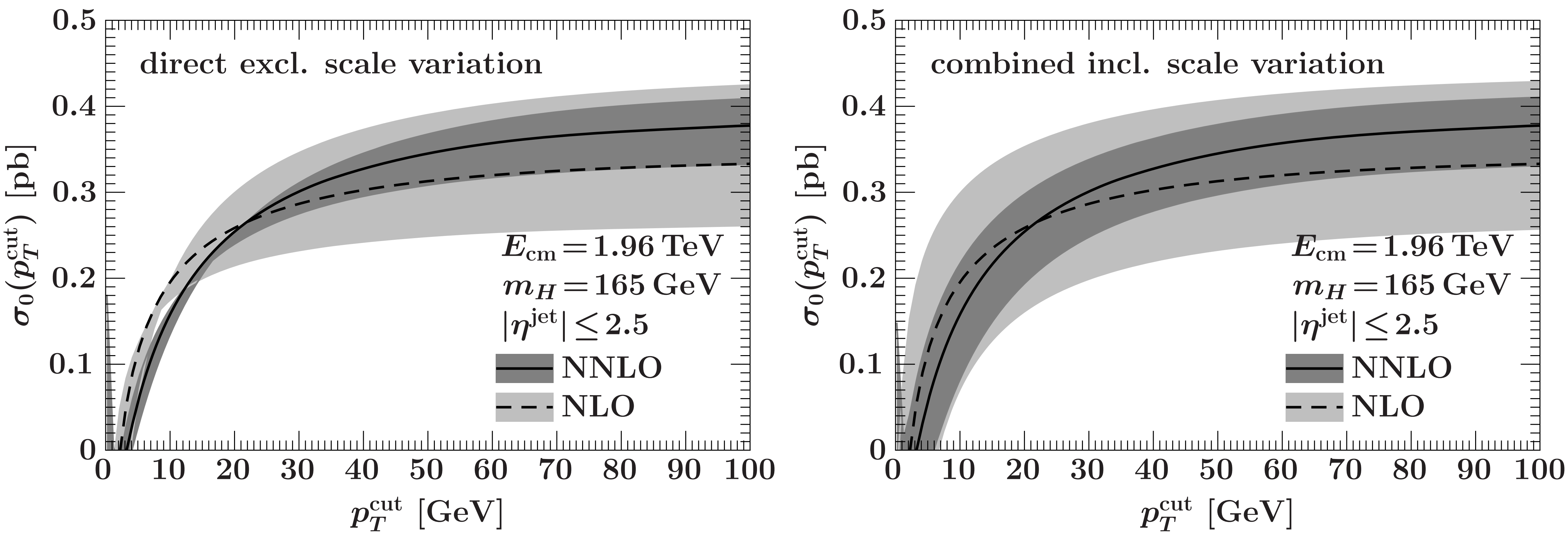
at $p_T^{\rm cut} = 30\,\mathrm{GeV}$ and NNLO

$\delta(\sigma_{\rm total}) = 8.6\%$	$\delta(\sigma_{\rm total}) = 8.6\%$
$\delta(\sigma_{\geq 1}) = 19\%$	$\delta(\sigma_{\geq 1}) = 19\%$
$\delta(\sigma_0) = 2.4\%$	$\delta(\sigma_0) = 18\%$
$\rho(\sigma_0, \sigma_{\geq 1}) = +100\%$	$\rho(\sigma_0, \sigma_{\geq 1}) = -64\%$

Quite generic: same pattern at Tevatron

similar plots if we vary rapidity cuts

similar plots for other processes



eg. $\{\sigma_0, \sigma_1, \sigma_{\geq 2}\}$ **Combined Inclusive Scale Variation**

- Treat inclusive cross-section uncertainties as independent

$$\Delta_{\rm total}, \Delta_{\geq 1}, \Delta_{\geq 2}, \ldots$$

$$C = \begin{pmatrix} \Delta^2_{\rm total} & 0 & 0 \\ 0 & \Delta^2_{\geq 1} & 0 \\ 0 & 0 & \Delta^2_{\geq 2} \end{pmatrix}$$

- For $p_T^{\rm cut}$ uncertainty use: $\Delta_{\rm cut} = \Delta_{\geq 1}, \; \Delta_{\rm cut2} = \Delta_{\geq 2}$

Propagate errors to get uncertainty

$$\boldsymbol{\sigma_0 = \sigma_{\rm total} - \sigma_{\geq 1}\,, \qquad \sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}\,, \qquad \sigma_{\geq 2}}$$

$$\Rightarrow \quad C = \begin{pmatrix} \Delta^2_{\rm total} + \Delta^2_{\geq 1} & -\Delta^2_{\geq 1} & 0 \\ -\Delta^2_{\geq 1} & \Delta^2_{\geq 1} + \Delta^2_{\geq 2} & -\Delta^2_{\geq 2} \\ 0 & -\Delta^2_{\geq 2} & \Delta^2_{\geq 2} \end{pmatrix}$$

$$\sigma_1(p_T^{\rm cut}, p_{T2}^{\rm cut}) \simeq \sigma_B\big[\alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3 L^6)$$
$$-\alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3 L^6)\big]$$

large logs

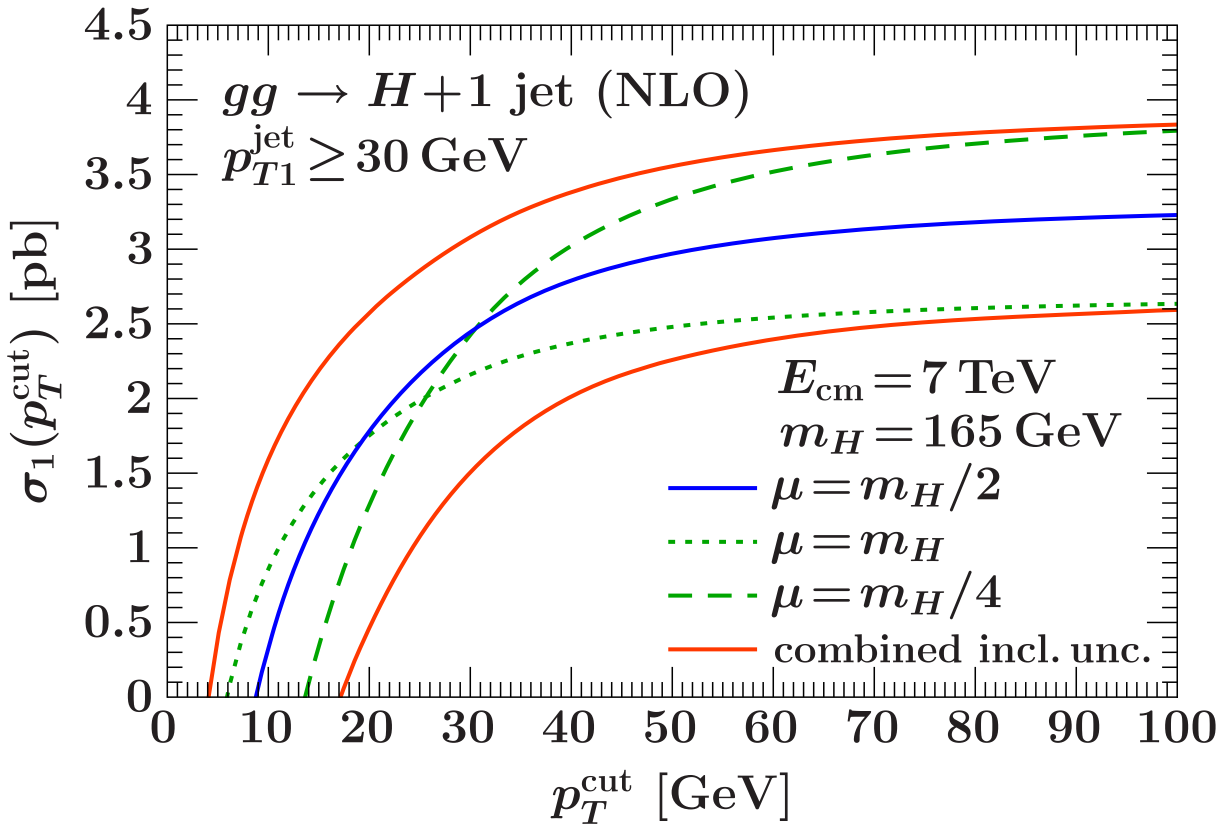
$$\sigma_{\geq 2}(p_{T2}^{\rm cut}) \simeq \sigma_B\big[\alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \mathcal{O}(\alpha_s^3 L^6)\big]$$

$$\boldsymbol{L = \ln(p_T^{\rm cut}/m_H)}$$

$$L = \ln(p_{T2}^{\rm cut}/m_H)$$

estimate $\Delta_{\rm cut2}$ obtained from $\sigma_{\geq 2}(p_{T2}^{\rm cut})$

other examples

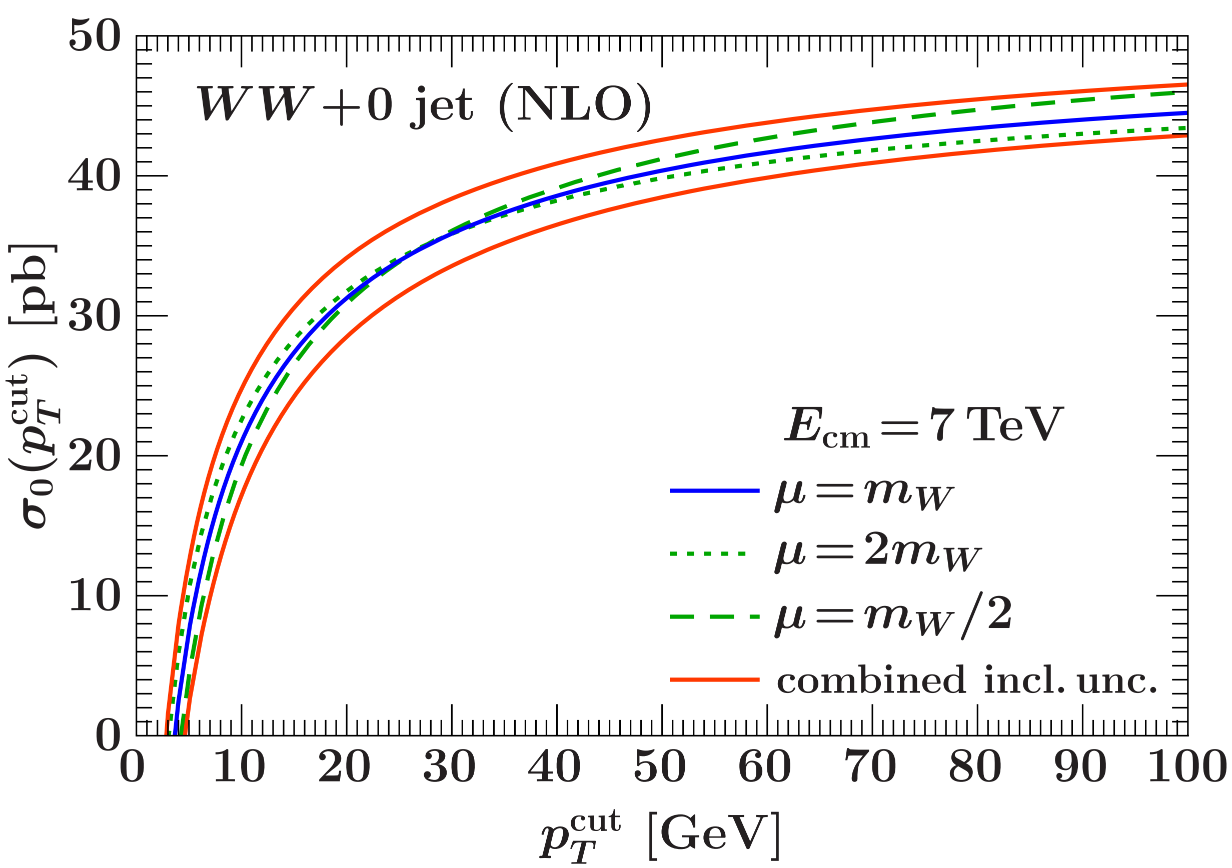


$\sigma_1(\mu)$

$\Delta_{\geq 1}$, $\Delta_{\geq 2}$ independent

$$\sigma_{\geq 1}(p_{T1}^{\rm jet} \geq 30\,\mathrm{GeV}) = (2.00\,\mathrm{pb})\big[1 + 5.4\,\alpha_s + \mathcal{O}(\alpha_s^2)\big]$$

$$\sigma_{\geq 2}(p_{T1}^{\rm jet} \geq 30\,\mathrm{GeV}, p_{T2}^{\rm jet} \geq 30\,\mathrm{GeV}) = (2.00\,\mathrm{pb})\big[3.6\,\alpha_s + \mathcal{O}(\alpha_s^2)\big]\,.$$

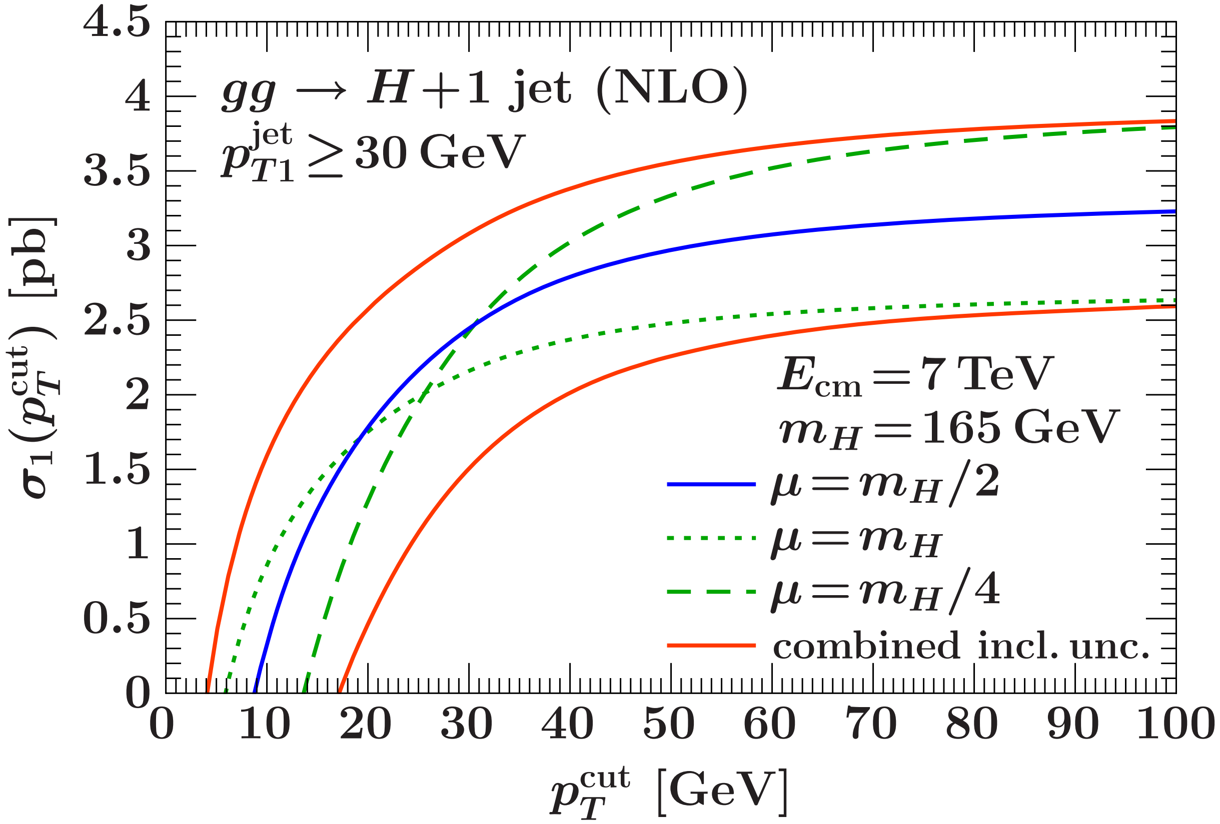


0-jet bin:

K-factor ~ 1.5

(K~ 2 with H search cuts)

other examples

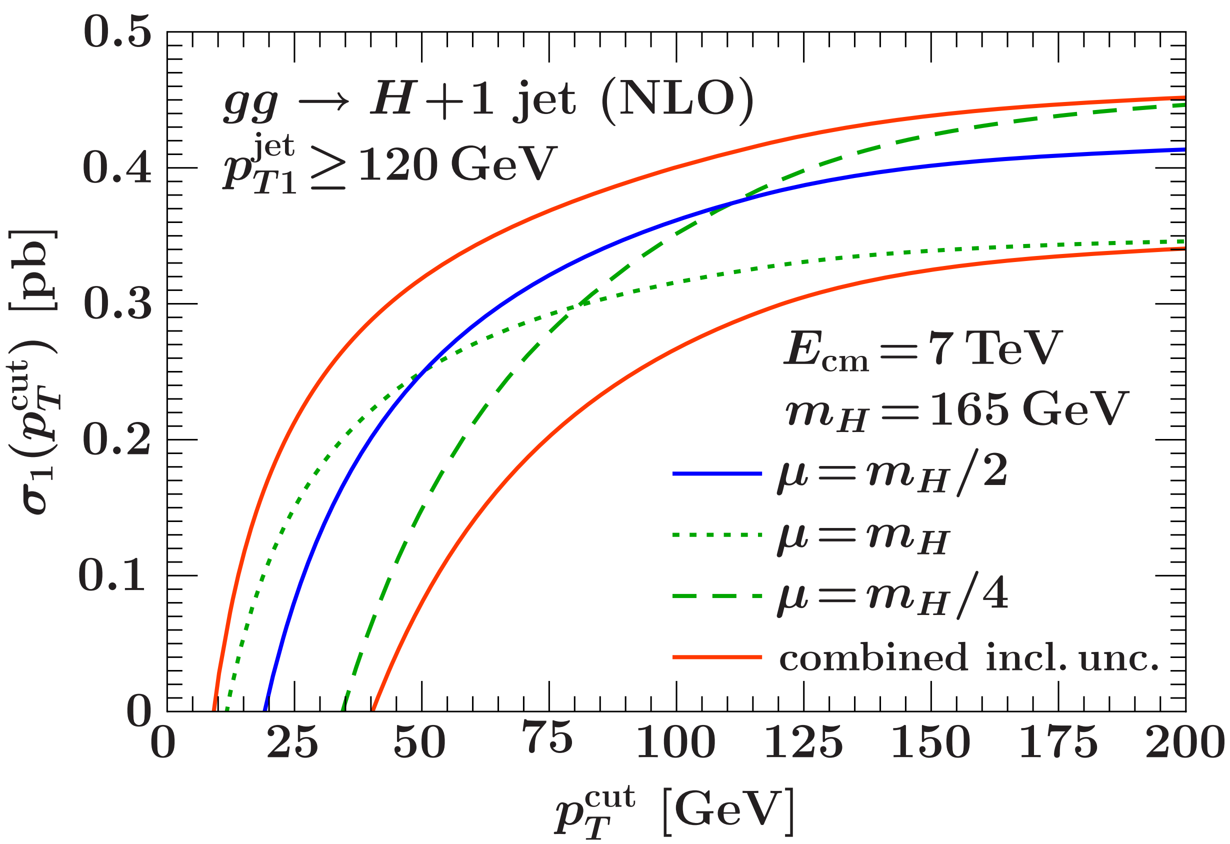


$\sigma_1(\mu)$

$\Delta_{\geq 1},\ \Delta_{\geq 2}$ independent

$$\sigma_{\geq 1}(p_{T1}^{\rm jet} \geq 30\,{\rm GeV}) = (2.00\,{\rm pb})\big[1 + 5.4\,\alpha_s + \mathcal{O}(\alpha_s^2)\big]$$

$$\sigma_{\geq 2}(p_{T1}^{\rm jet} \geq 30\,{\rm GeV}, p_{T2}^{\rm jet} \geq 30\,{\rm GeV}) = (2.00\,{\rm pb})\big[3.6\,\alpha_s + \mathcal{O}(\alpha_s^2)\big]\,.$$



- cancellation occurs for a larger $p_T^{\rm cut}$, as expected
- logs are larger earlier

$$\{\sigma_{\text{total}}, \sigma_0, \sigma_1\} \quad \text{or} \quad \{\sigma_{\text{total}}, f_0, f_1\}$$

$$\sigma_0 = \sigma_{\text{total}} - \sigma_{\geq 1}\,, \qquad f_0 = \frac{\sigma_0}{\sigma_{\text{total}}}\,,$$

$$\sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}\,, \qquad f_1 = \frac{\sigma_1}{\sigma_{\text{total}}}\,.$$

$$\begin{pmatrix} \Delta^2_{\text{total}} & \Delta^2_{\text{total}} & 0 \\ \Delta^2_{\text{total}} & \Delta^2_{\text{total}} + \Delta^2_{\geq 1} & -\Delta^2_{\geq 1} \\ 0 & -\Delta^2_{\geq 1} & \Delta^2_{\geq 1} + \Delta^2_{\geq 2} \end{pmatrix}$$

relative uncertainties

$$\delta(\sigma_0)^2 = \frac{1}{f_0^2}\,\delta^2_{\text{total}} + \Big(\frac{1}{f_0} - 1\Big)^2 \delta^2_{\geq 1}$$

$$\delta(\sigma_1)^2 = \Big(\frac{1-f_0}{f_1}\Big)^2 \delta^2_{\geq 1} + \Big(\frac{1-f_0}{f_1} - 1\Big)^2 \delta^2_{\geq 2}$$

$$\delta(f_0)^2 = \Big(\frac{1}{f_0} - 1\Big)^2 (\delta^2_{\text{total}} + \delta^2_{\geq 1})\,,$$

$$\delta(f_1)^2 = \delta^2_{\text{total}} + \Big(\frac{1-f_0}{f_1}\Big)^2 \delta^2_{\geq 1} + \Big(\frac{1-f_0}{f_1} - 1\Big)^2 \delta^2_{\geq 2}\,,$$

correlation coefficients

$$\rho(\sigma_0, \sigma_{\text{total}}) = \bigg[1 + \frac{\delta^2_{\geq 1}}{\delta^2_{\text{total}}}(1-f_0)^2\bigg]^{-1/2},$$

$$\rho(\sigma_0, \sigma_1) = -\bigg[1 + \frac{\delta^2_{\text{total}}}{\delta^2_{\geq 1}}\,\frac{1}{(1-f_0)^2}\bigg]^{-1/2} \times \bigg[1 + \frac{\delta^2_{\geq 2}}{\delta^2_{\geq 1}}\Big(1 - \frac{f_1}{1-f_0}\Big)^2\bigg]^{-1/2},$$

$$\rho(\sigma_0, \sigma_{\geq 2}) = 0\,,$$

$$\rho(\sigma_1, \sigma_{\text{total}}) = 0\,,$$

$$\rho(\sigma_1, \sigma_{\geq 2}) = -\bigg[1 + \frac{\delta^2_{\geq 1}}{\delta^2_{\geq 2}}\Big(1 - \frac{f_1}{1-f_0}\Big)^{-2}\bigg]^{-1/2}.$$

$$\rho(f_0, \sigma_{\text{total}}) = \bigg[1 + \frac{\delta^2_{\geq 1}}{\delta^2_{\text{total}}}\bigg]^{-1/2},$$

$$\rho(f_0, f_1) = -\Big(1 + \frac{1-f_0}{f_1}\,\frac{\delta^2_{\geq 1}}{\delta^2_{\text{total}}}\Big)\Big(\frac{1}{f_0} - 1\Big)\frac{\delta^2_{\text{total}}}{\delta(f_0)\delta(f_1)}\,,$$

$$\rho(f_1, \sigma_{\text{total}}) = -\frac{\delta_{\text{total}}}{\delta(f_1)}\,.$$

eg. Numbers for $\{\sigma_{\mathrm{total}}, \sigma_0, \sigma_1\}$

$$p_T^{\mathrm{jet}} \geq 30\,\mathrm{GeV} \qquad p_{T2}^{\mathrm{jet}} \geq 30\,\mathrm{GeV}$$

$$\begin{pmatrix} \Delta^2_{\mathrm{total}} & \Delta^2_{\mathrm{total}} & 0 \\ \Delta^2_{\mathrm{total}} & \Delta^2_{\mathrm{total}} + \Delta^2_{\geq 1} & -\Delta^2_{\geq 1} \\ 0 & -\Delta^2_{\geq 1} & \Delta^2_{\geq 1} + \Delta^2_{\geq 2} \end{pmatrix}$$

start with:

$$\begin{aligned} \sigma_{\mathrm{total}} &= (8.70 \pm 0.75)\,\mathrm{pb} & 8.6\% \\ \sigma_{\geq 1} &= (3.29 \pm 0.62)\,\mathrm{pb} & 18.8\% \\ \sigma_{\geq 2} &= (0.85 \pm 0.49)\,\mathrm{pb} & 57\% \end{aligned}$$

$$\sigma_0 = \sigma_{\mathrm{tot}} - \sigma_{\geq 1}$$

$$\sigma_1 = \sigma_{\geq 1} - \sigma_{\geq 2}$$

propagate to get:

$$\delta(\sigma_0) = 18\% \qquad \delta(\sigma_1) = 32\%$$

$$\rho(\sigma_0, \sigma_{\mathrm{total}}) = 0.77 \qquad \rho(\sigma_1, \sigma_{\geq 2}) = -0.62$$

$$\rho(\sigma_0, \sigma_1) = -0.50$$

or consider jet fractions:

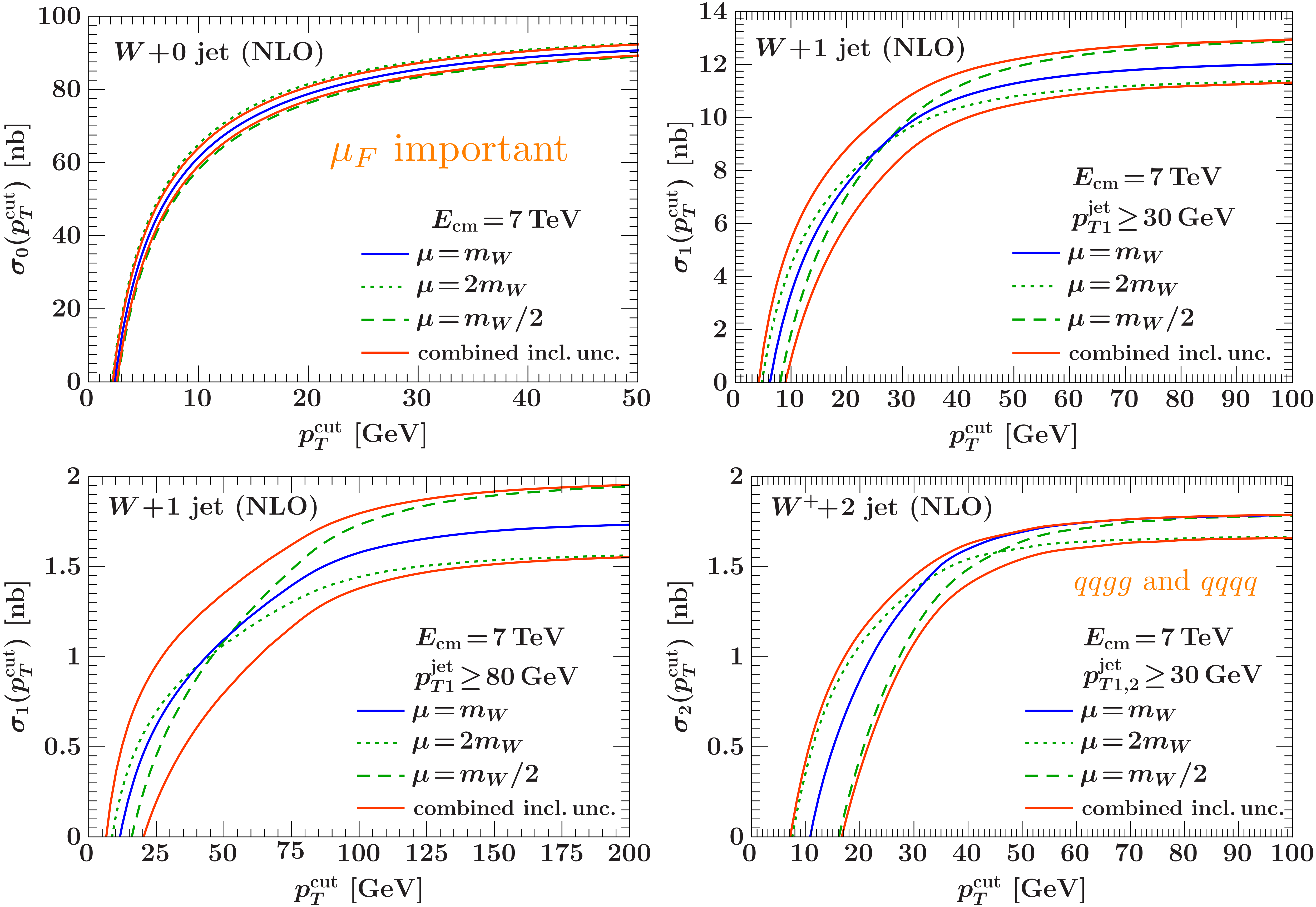
$$f_0 = \frac{\sigma_0}{\sigma_{\mathrm{total}}} \qquad f_1 = \frac{\sigma_1}{\sigma_{\mathrm{total}}}$$

$$\delta(f_0) = 13\% \qquad \delta(f_1) = 33\%$$

$$\rho(f_0, \sigma_{\mathrm{total}}) = 0.42 \qquad \rho(f_1, \sigma_{\mathrm{total}}) = -0.26$$

$$\rho(f_0, f_1) = -0.80$$

W + jets



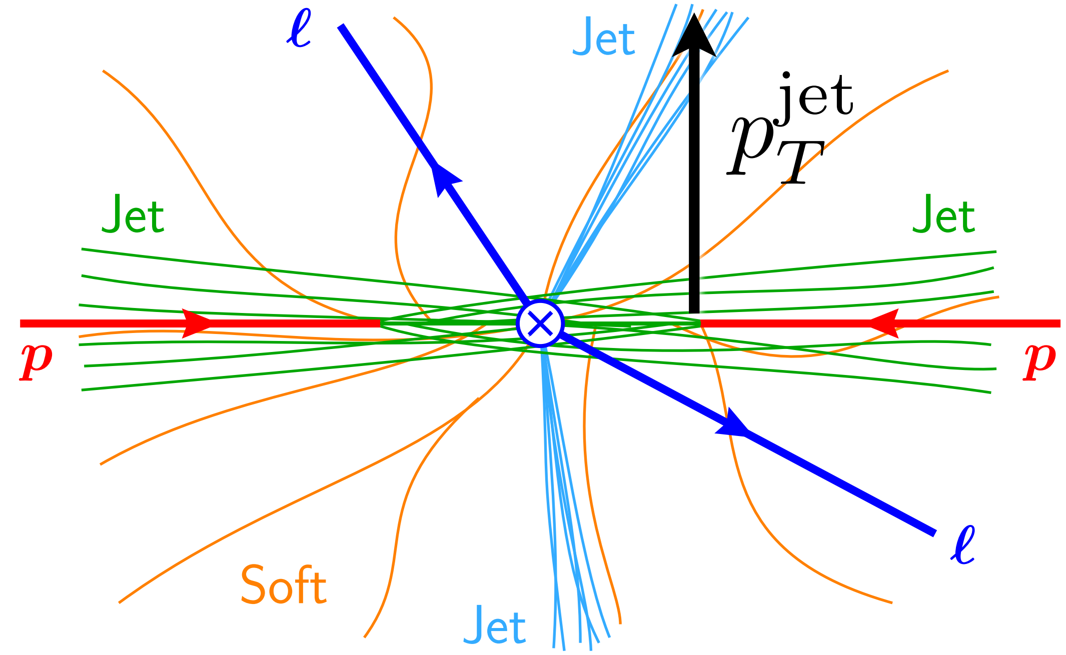
$W+0$ jet (NLO)
μ_F important
$E_{\rm cm} = 7$ TeV
$\mu = m_W$
$\mu = 2m_W$
$\mu = m_W/2$
combined incl. unc.
$\sigma_0(p_T^{\rm cut})$ [nb]
$p_T^{\rm cut}$ [GeV]
$W+1$ jet (NLO)
$E_{\rm cm} = 7$ TeV
$p_{T1}^{\rm jet} \geq 30$ GeV
$\mu = m_W$
$\mu = 2m_W$
$\mu = m_W/2$
combined incl. unc.
$\sigma_1(p_T^{\rm cut})$ [nb]
$p_T^{\rm cut}$ [GeV]
$W+1$ jet (NLO)
$E_{\rm cm} = 7$ TeV
$p_{T1}^{\rm jet} \geq 80$ GeV
$\mu = m_W$
$\mu = 2m_W$
$\mu = m_W/2$
combined incl. unc.
$\sigma_1(p_T^{\rm cut})$ [nb]
$p_T^{\rm cut}$ [GeV]
$W^+ + 2$ jet (NLO)
$qqgg$ and $qqqq$
$E_{\rm cm} = 7$ TeV
$p_{T1,2}^{\rm jet} \geq 30$ GeV
$\mu = m_W$
$\mu = 2m_W$
$\mu = m_W/2$
combined incl. unc.
$\sigma_2(p_T^{\rm cut})$ [nb]
$p_T^{\rm cut}$ [GeV]

(C) Use Resummed Predictions to get Uncertainties

this will allow us to include both types of uncertainties (correlated & uncorrelated) from methods (A) and (B)

- resummed calculation has two sources of uncertainty,
  - one is correlated with $\Delta_{\rm total}$
  - one gives $\Delta_{\rm cut}$

Jet Vetoes



Conventional: Jet Algorithm

- Search for jets and require $p_T^{\rm jet} < p_T^{\rm cut}$

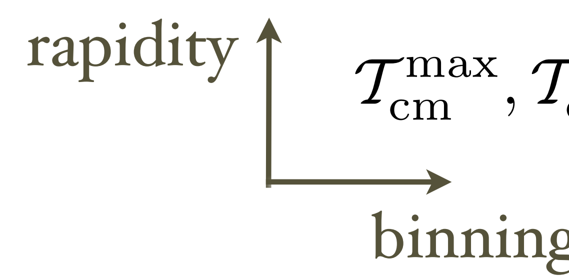
  Tevatron: $p_T^{\rm cut} \simeq 20\,\mathrm{GeV}$

  LHC: $p_T^{\rm cut} \simeq 25\,\mathrm{GeV}$

- Complicated phase-space restrictions

$$p_T^{\max}, p_T^{\max 2}, \ldots, H_T$$

$$\mathcal{T}_{\rm cm}^{\max}, \mathcal{T}_{\rm cm}^{\max 2}, \ldots, \mathcal{T}_{\rm cm}$$



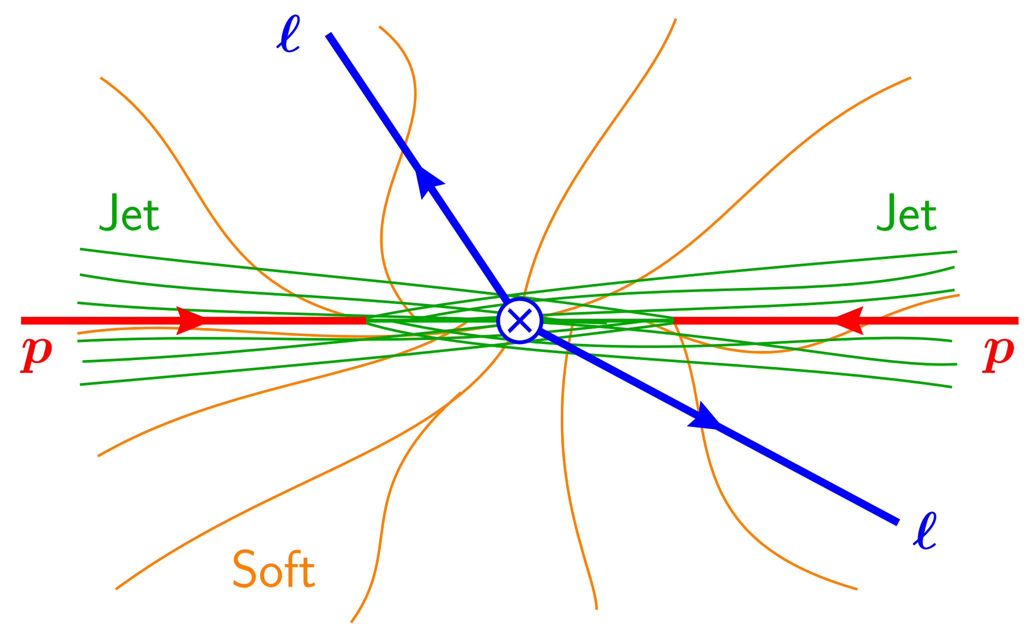
Alternative: Event Shape

- Measure beam thrust for each event

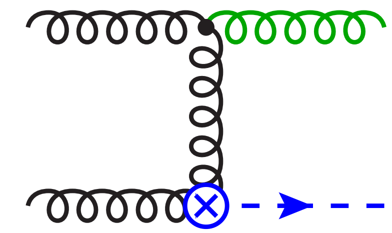
$$\mathcal{T}_{\rm cm} = \sum_k |\vec{p}_{kT}| e^{-|\eta_k|} = \sum_k (E_k - |p_k^z|)$$

  and require $\mathcal{T}_{\rm cm} < \mathcal{T}_{\rm cm}^{\rm cut}$

- Nice for higher order calculations



Jet veto restricts ISR, gives double logs

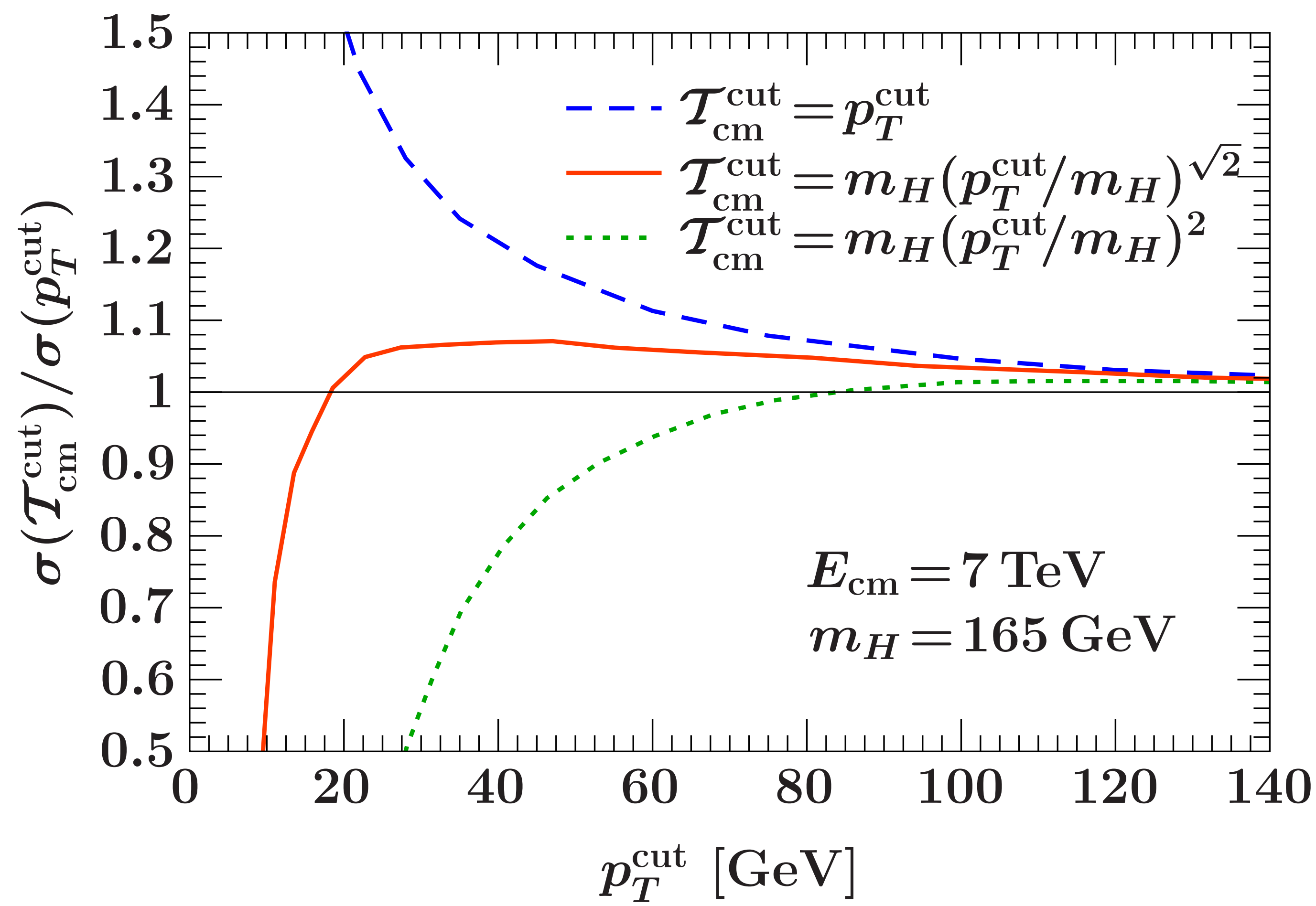


$$L = \ln \frac{p_T^{\mathrm{cut}}}{m_H} \quad \text{or} \quad L = \ln \frac{\mathcal{T}_{\mathrm{cm}}^{\mathrm{cut}}}{m_H}$$

$$\sigma(p_T^{\mathrm{cut}}) \propto 1 - \left(\frac{3\alpha_s}{\pi}\right) 2\ln^2 \frac{p_T^{\mathrm{cut}}}{m_H} + \dots$$

$$\sigma(\mathcal{T}_{\mathrm{cm}}^{\mathrm{cut}}) \propto 1 - \left(\frac{3\alpha_s}{\pi}\right) \ln^2 \frac{\mathcal{T}_{\mathrm{cm}}^{\mathrm{cut}}}{m_H} + \dots$$

NNLO spectra close, agree to 7%

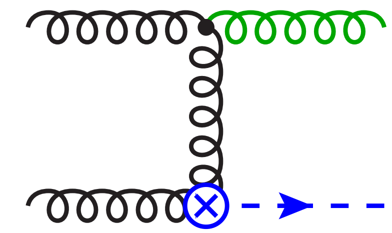


<u>Appropriate correspondence:</u>

$$\mathcal{T}_{\mathrm{cm}}^{\mathrm{cut}} \simeq m_H \left(\frac{p_t^{\mathrm{cut}}}{m_H}\right)^{\sqrt{2}}$$

$\Rightarrow$ $\mathcal{T}_{\mathrm{cm}}^{\mathrm{cut}} \simeq 10\,\mathrm{GeV}$ corresponds to $p_T^{\mathrm{cut}} \simeq 20\,\mathrm{GeV}$ in conventional jet veto

Jet veto restricts ISR, gives double logs



$$L = \ln \frac{p_T^{\rm cut}}{m_H} \qquad \text{or} \qquad L = \ln \frac{\mathcal{T}_{\rm cm}^{\rm cut}}{m_H}$$

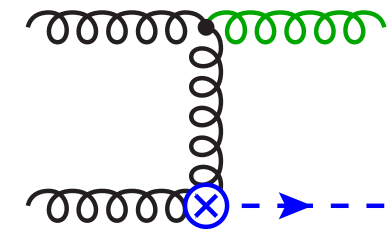
$$\begin{array}{lllllll}
 & \text{LO} & \text{NLO} & & & & \\
\sigma_{0\text{-jet}} = & 1 & + \alpha_s L^2 & + \alpha_s^2 L^4 & + \alpha_s^3 L^6 & + \ldots & \text{LL} \\
 & & + \alpha_s L & + \alpha_s^2 L^3 & + \alpha_s^3 L^5 & + \ldots & \\
 & & + \alpha_s n_1(p_T^{\rm cut}) & + \alpha_s^2 L^2 & + \alpha_s^3 L^4 & + \ldots & \\
 & & & + \alpha_s^2 L & + \alpha_s^3 L^3 & + \ldots & \\
 & & & + \alpha_s^2 n_2(p_T^{\rm cut}) & + \alpha_s^3 L^2 & + \ldots & \\
 & & & & + \alpha_s^3 L & + \ldots & \\
 & & & & + \alpha_s^3 & + \ldots &
\end{array}$$

Parton Shower

eg. Pythia is LL (+ tuning)

eg. MC@NLO is NLO+LL

Jet veto restricts ISR, gives double logs



$$L = \ln \frac{p_T^{\mathrm{cut}}}{m_H} \quad \text{or} \quad L = \ln \frac{\mathcal{T}_{\mathrm{cm}}^{\mathrm{cut}}}{m_H}$$

$$\begin{array}{lllllll}
 & \text{LO} & \text{NLO} & \text{NNLO} & & & \\
\sigma_{0\text{-jet}} = & 1 & + \alpha_s L^2 & + \alpha_s^2 L^4 & + \alpha_s^3 L^6 & + \ldots & \text{LL} \\
 & & + \alpha_s L & + \alpha_s^2 L^3 & + \alpha_s^3 L^5 & + \ldots & \text{NLL} \\
 & & + \alpha_s n_1(p_T^{\mathrm{cut}}) & + \alpha_s^2 L^2 & + \alpha_s^3 L^4 & + \ldots & \\
 & & & + \alpha_s^2 L & + \alpha_s^3 L^3 & + \ldots & \text{NNLL} \\
 & & & + \alpha_s^2 n_2(p_T^{\mathrm{cut}}) & + \alpha_s^3 L^2 & + \ldots & \\
 & & & & + \alpha_s^3 L & + \ldots & \\
 & & & & + \alpha_s^3 & + \ldots &
\end{array}$$

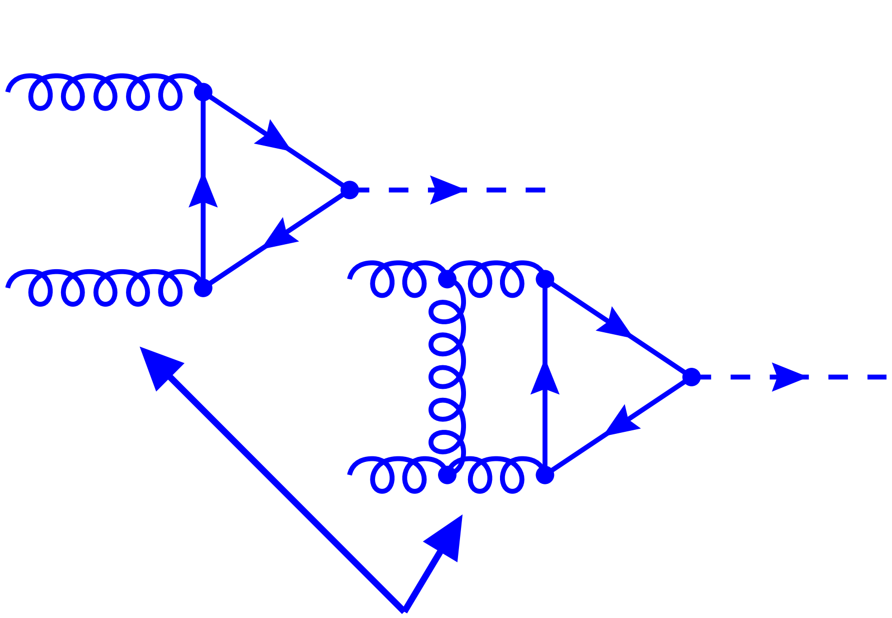
Calculation:

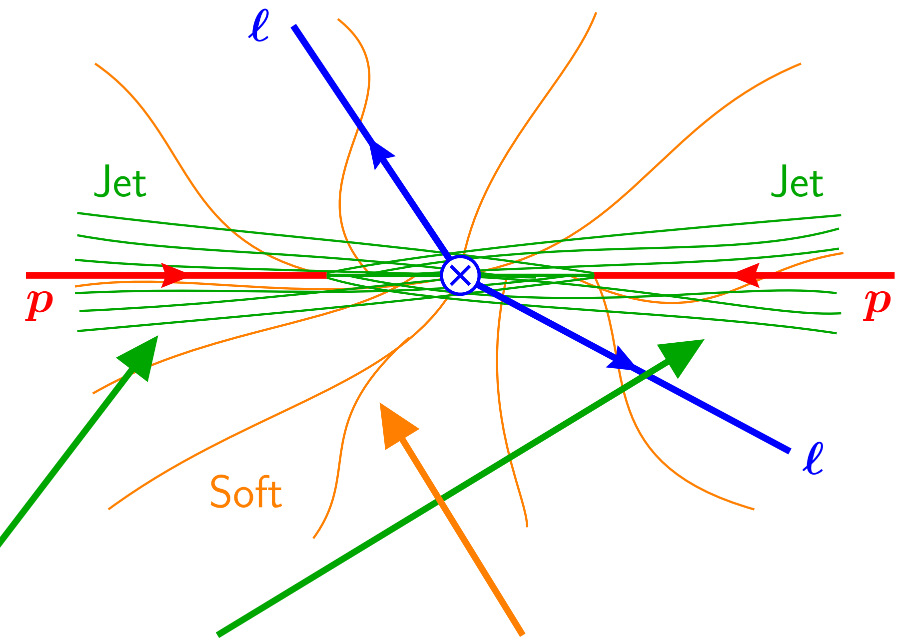
NNLL + NNLO

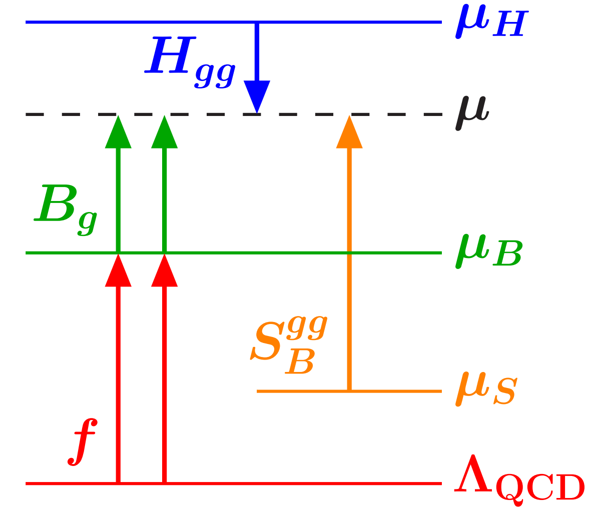
two orders of summation
beyond LL shower programs

arXiv:1012.4480
Berger, Marcantonini, IS, Tackmann, Waalewijn

NNLL + NNLO calculation







$$\frac{d\sigma^{\rm s}}{d\mathcal{T}_{\rm cm}} = H_{gg}(\mu) \int dt_a dt_b \, B_g(t_a,\mu)\, B_g(t_b,\mu)\, S_B^{gg}\Big(\mathcal{T}_{\rm cm} - \frac{t_a+t_b}{m_H}, \mu\Big)$$

$$B_i(t,x) = \int \frac{d\xi}{\xi} \mathcal{I}_{ij}(t, x/\xi)\; f_j(\xi)$$

Function	describes	at the scale
Hard H_{gg}	hard virtual radiation	$\lvert\mu_H\rvert \simeq m_H$
Beam B_g	virtual & real energetic ISR	$\mu_B \simeq \sqrt{\mathcal{T}_{\rm cm} m_H}$
Soft S_B^{gg}	virtual & real soft radiation	$\mu_S \simeq \mathcal{T}_{\rm cm}$

} logs give sensitivity to smaller scales

Perturbation theory at <u>each scale</u> contributes to uncertainties

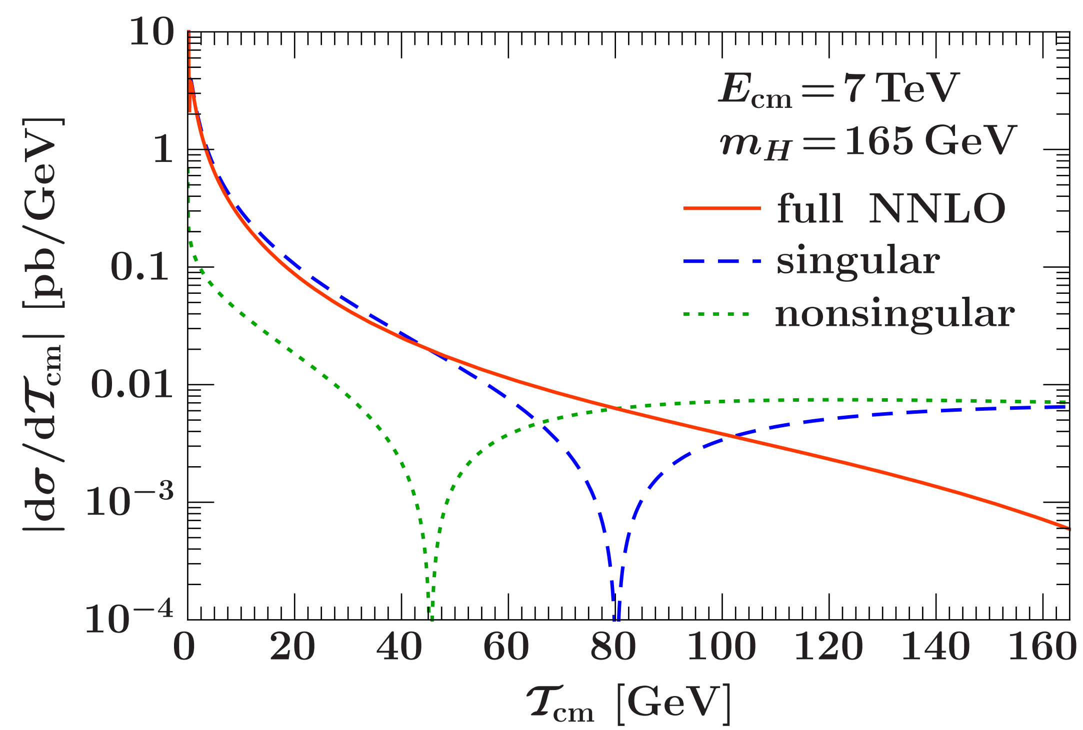
General Structure of the Cross Section

$$\frac{d\sigma}{d\tau} = \underbrace{C^{-1}\delta(\tau) + \sum_k C^k \left[\frac{\ln^k \tau}{\tau}\right]_+}_{\text{singular}} + \underbrace{\frac{d\sigma^{ns}}{d\tau}}_{\text{nonsingular}} \qquad \text{with} \quad \tau = \frac{\mathcal{T}_{cm}}{m_H}$$

Singular (log-enhanced) terms

- Dominant contribution at small τ

$\Rightarrow$ Resummed to NNLL using SCET



Nonsingular terms

- Suppressed by $\mathcal{O}(\tau)$ relative to singular ones
- Required to reproduce full fixed-order cross section at large τ

$\Rightarrow$ Obtained numerically from FEHiP to NNLO

(MCFM)

Summation of Jet-Veto Logarithms

Factorization theorem splits up large logarithms

$$\frac{\mathrm{d}\sigma^{\mathrm{s}}}{\mathrm{d}\mathcal{T}_{\mathrm{cm}}} = H_{gg}(\mu) \int \mathrm{d}t_a \mathrm{d}t_b\, B_g(t_a,\mu)\, B_g(t_b,\mu)\, S_B^{gg}\Big(\mathcal{T}_{\mathrm{cm}} - \frac{t_a+t_b}{m_H}, \mu\Big)$$

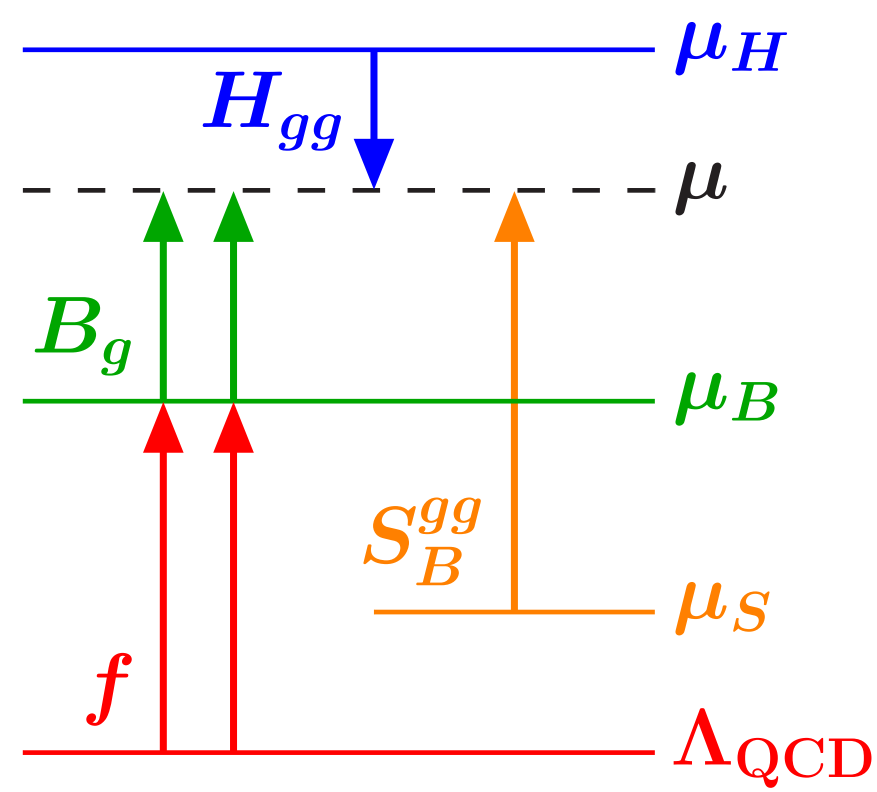
$$\ln^2 \frac{\mathcal{T}_{\mathrm{cm}}}{m_H} = 2\ln^2 \frac{m_H}{\mu} \qquad - \qquad \ln^2 \frac{\mathcal{T}_{\mathrm{cm}} m_H}{\mu^2} \qquad + \qquad 2\ln^2 \frac{\mathcal{T}_{\mathrm{cm}}}{\mu}$$

Logarithms are summed by

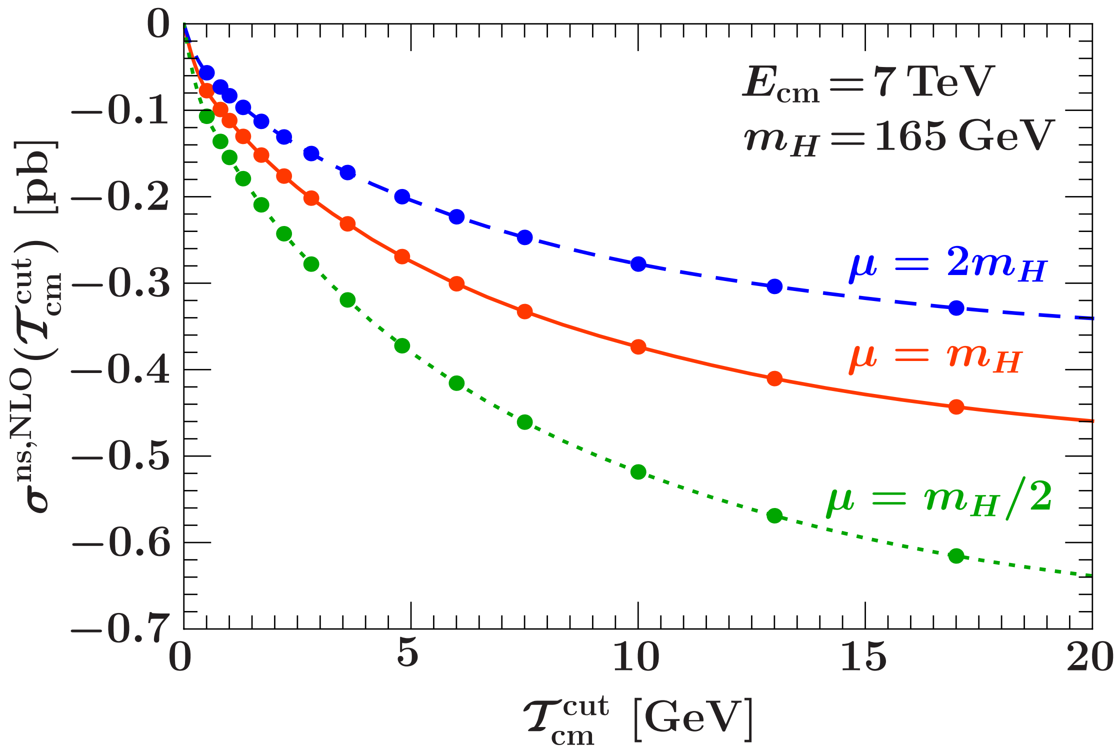
1. Evaluating each function at its natural scale

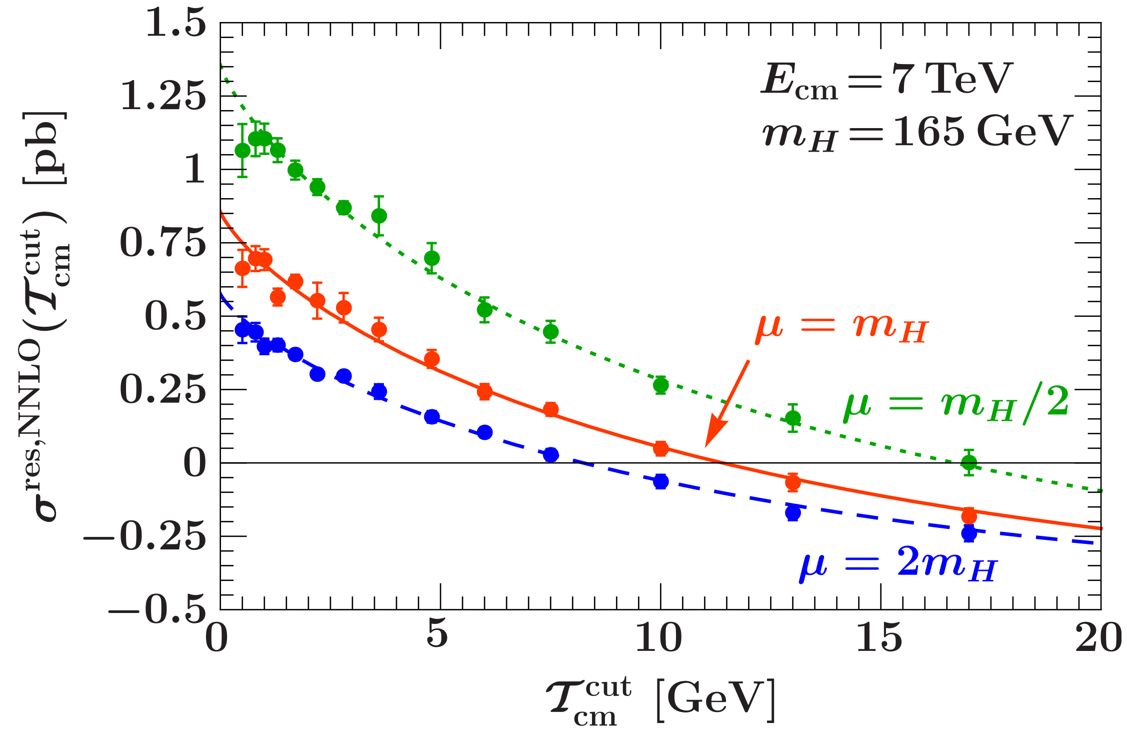
$$|\mu_H| \simeq m_H \gg \mu_B \simeq \sqrt{\mathcal{T}_{\mathrm{cm}} m_H} \gg \mu_S \simeq \mathcal{T}_{\mathrm{cm}}$$

2. RG evolving to common (arbitrary) scale μ
   NNLL requires
   - 1-loop matching
   - 2-loop anomalous dimensions
   - 3-loop cusp anomalous dimension



Nonsingular Corrections





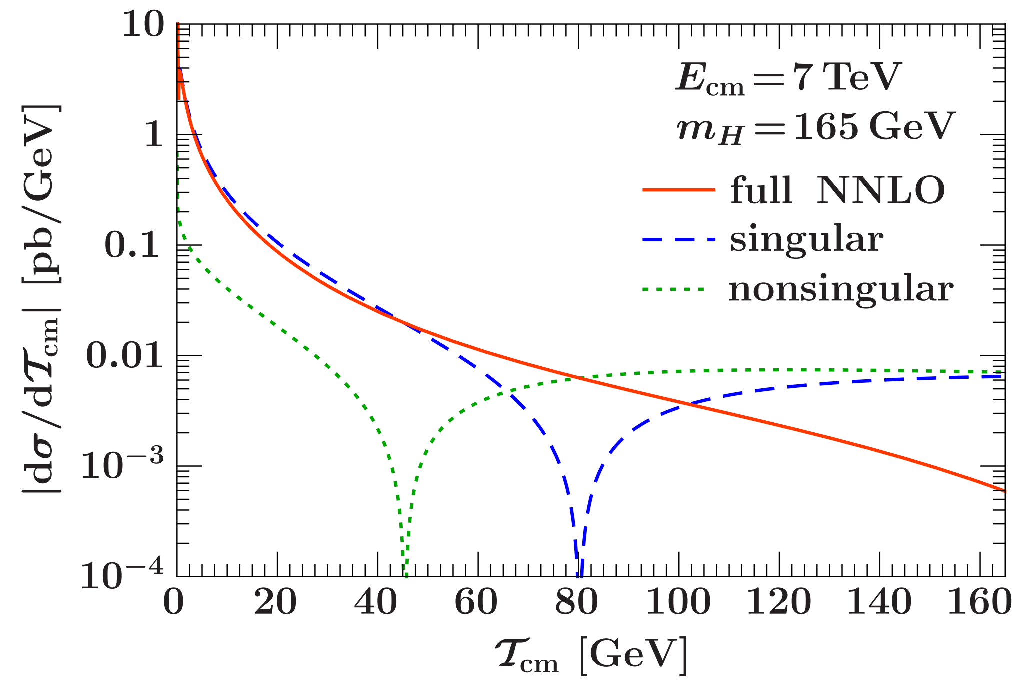
$$\sigma^{\rm ns,NLO}(\tau^{\rm cut}) = \sigma^{\rm NLO}(\tau^{\rm cut}) - \sigma^{\rm s,NNLL}(\tau^{\rm cut})\big|_{\rm NLO}$$
$$\sigma^{\rm res,NNLO}(\tau^{\rm cut}) = \sigma^{\rm NNLO}(\tau^{\rm cut}) - \sigma^{\rm s,NNLL}(\tau^{\rm cut})\big|_{\rm NNLO}$$

- $\sigma^{\rm NLO}$ and $\sigma^{\rm NNLO}$ numerically from FEHiP [Anastasiou, Melnikov, Petriello]
- NNLO $C^{-1}\delta(\tau)$ term is not part of $\sigma^{\rm s,NNLL}$
  - Obtained from intercept at $\tau^{\rm cut} = 0$ and added to singular
  - Proper treatment requires 2-loop hard, beam, soft functions

Scale Profiles

- Nonsingular terms are equally important for $\mathcal{T}_{\mathrm{cm}} \gtrsim m_H/2$

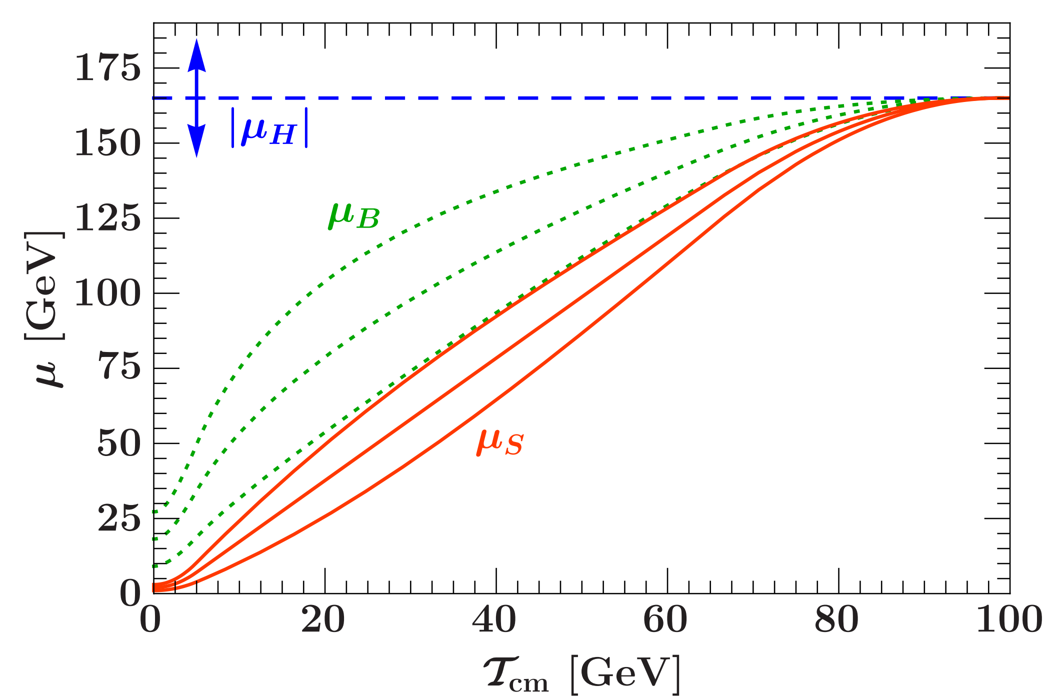
$\Rightarrow$ Resummation in singular terms must be turned off to not spoil large cancellation between singular and nonsingular terms



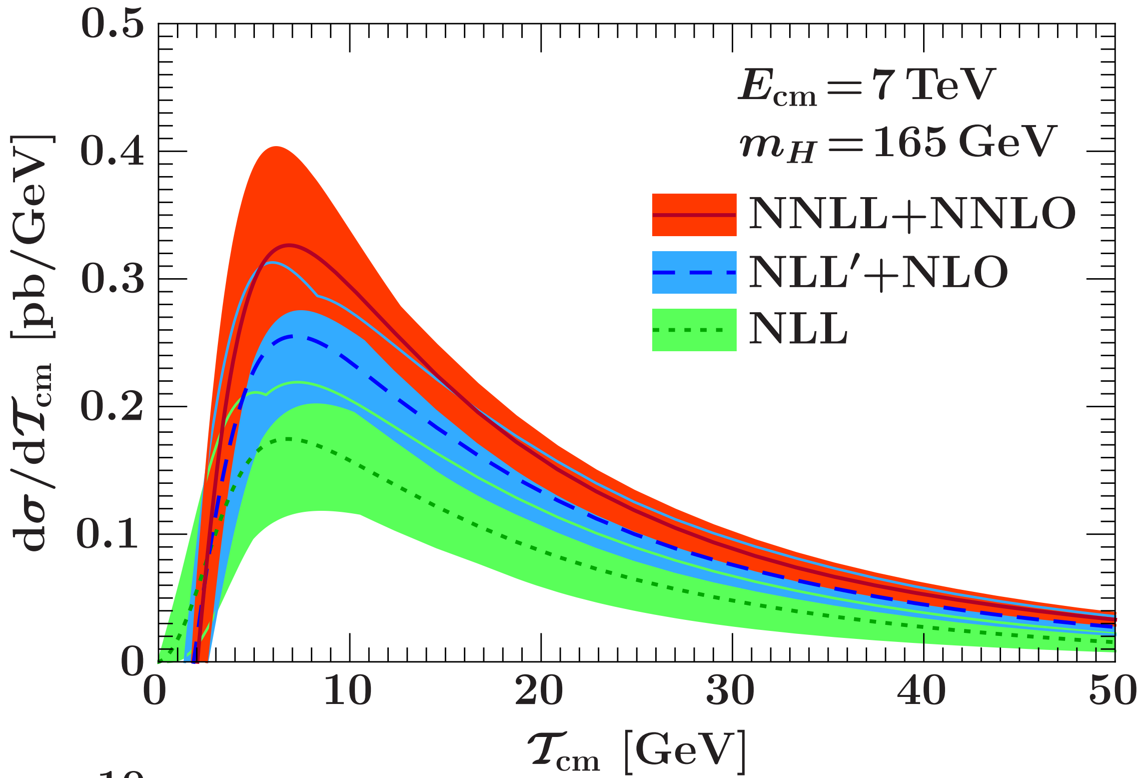
Scale variations

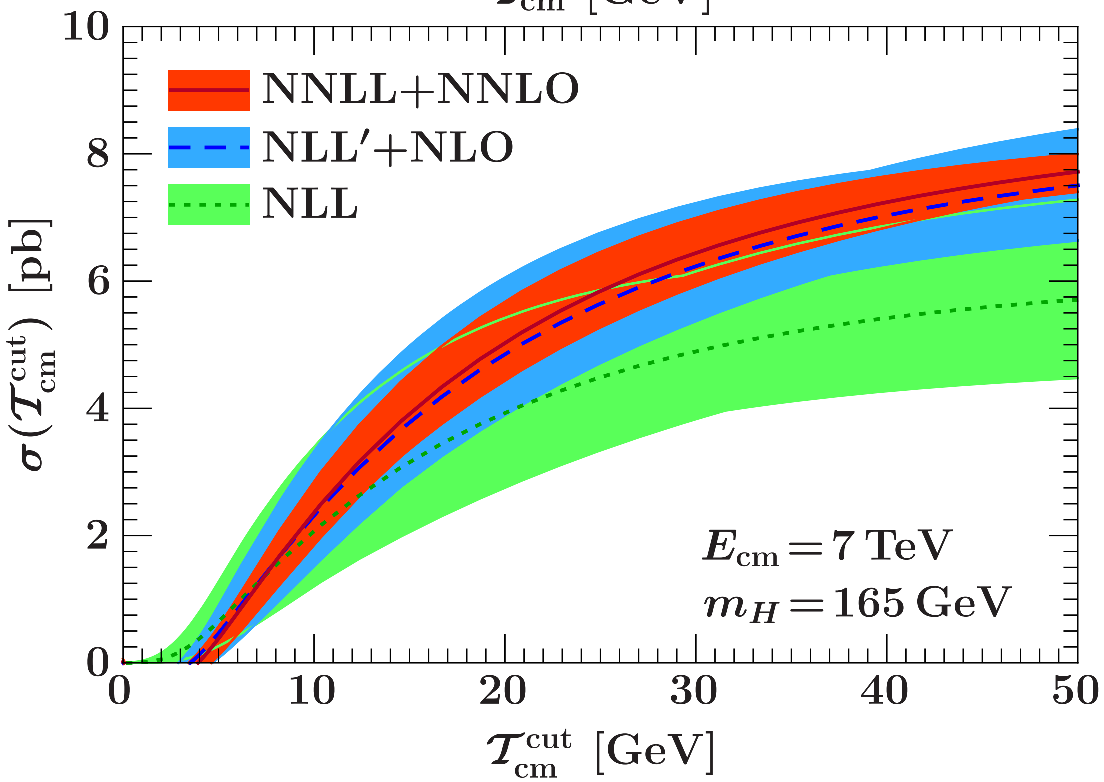
1. Overall scale by factors of 2
2. $\mu_B(\mathcal{T}_{\mathrm{cm}})$ profile
3. $\mu_S(\mathcal{T}_{\mathrm{cm}})$ profile

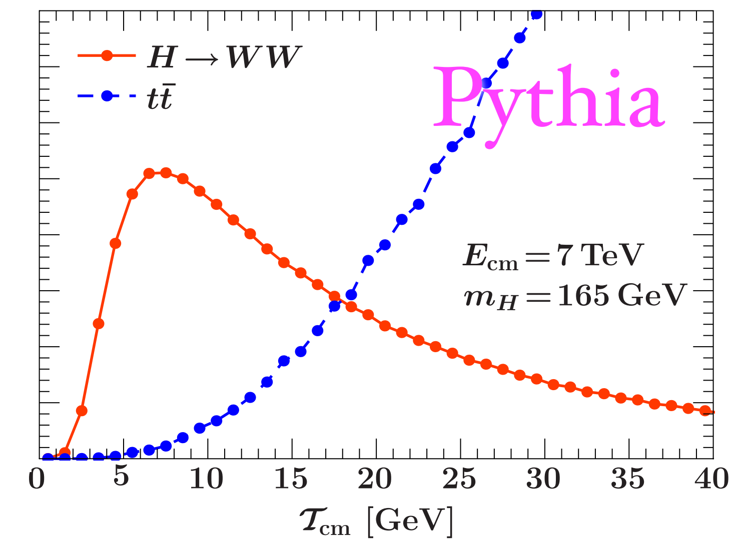
$\Rightarrow$ Perturbative uncertainties estimated by envelope of three variations



Beam Thrust Spectrum and Cumulant







$gg \to H$ production cross section for $m_H = 165\,\mathrm{GeV}$ at the LHC

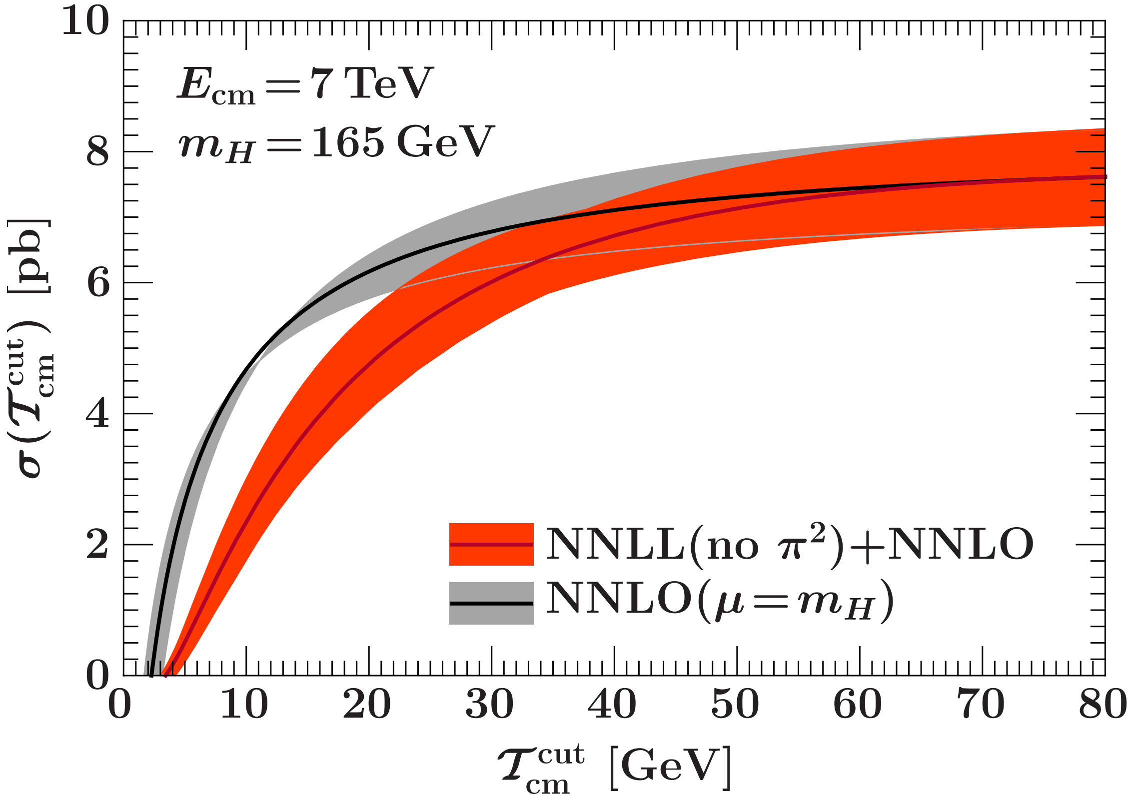
Differential beam-thrust spectrum

- peaks at small $\mathcal{T}_{cm}$
- has rather large tail from ISR

Perturbative corrections are important

- Incoming gluons radiate a lot
- Very large at lower orders
- Good convergence at higher orders

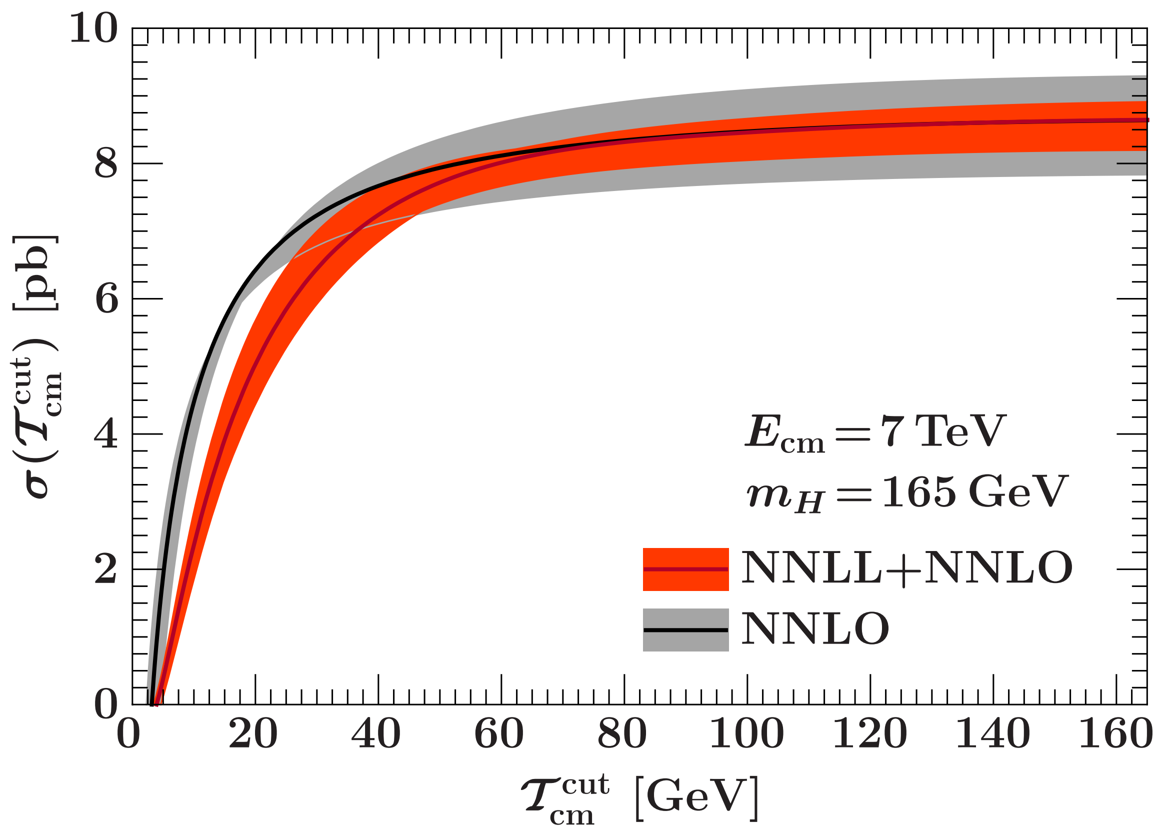
Reproducing Fixed-Order Result at Large $\mathcal{T}_{\mathrm{cm}}$



$\mu \simeq m_H$ in gluon form factor

- Exactly reproduces fixed NNLO at $\boldsymbol{\mu = m_H}$ for large $\boldsymbol{\mathcal{T}_{\mathrm{cm}}}$

  (scale profiles are essential)



$\mu \simeq -i m_H$ in gluon form factor

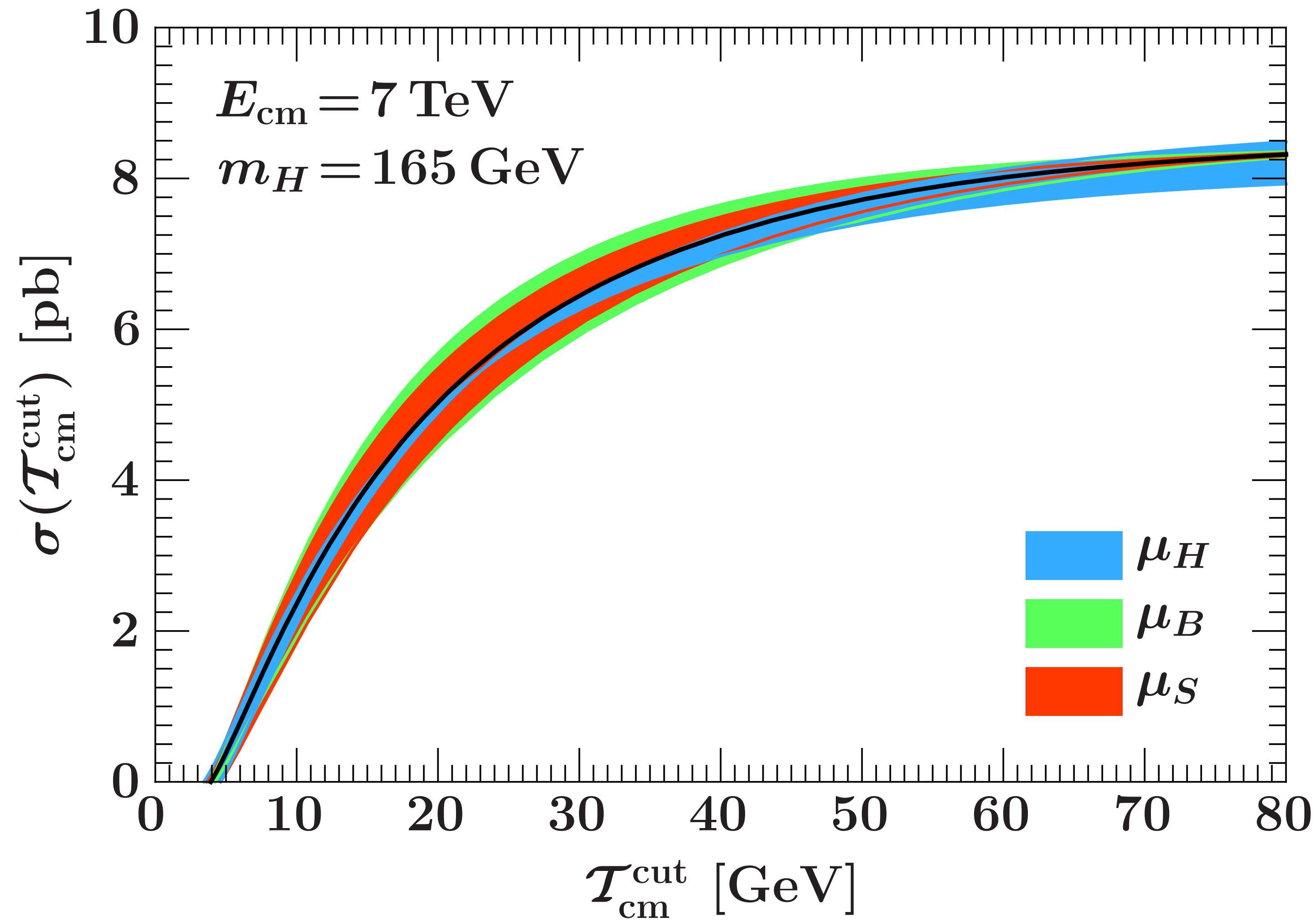
- Increases cross section
- Almost exactly reproduces fixed NNLO at conventional $\boldsymbol{\mu = m_H/2}$
- Smaller incl. uncertainty $+3\%, -5\%$

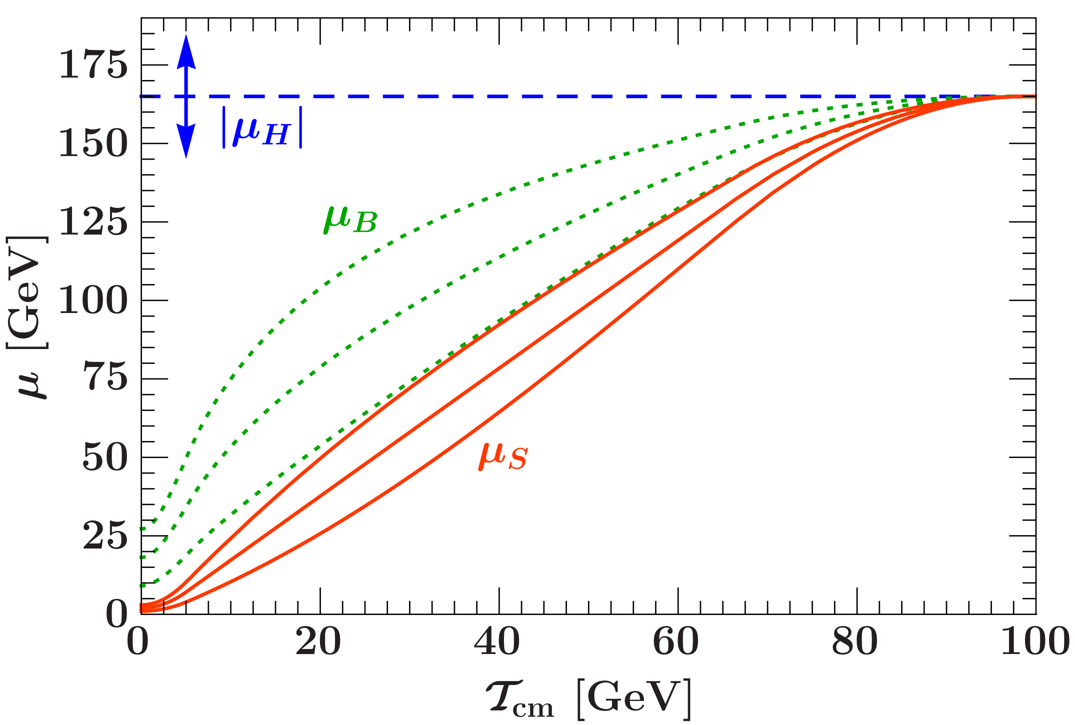
  (same effect as in Becher, Neubert, et al.)

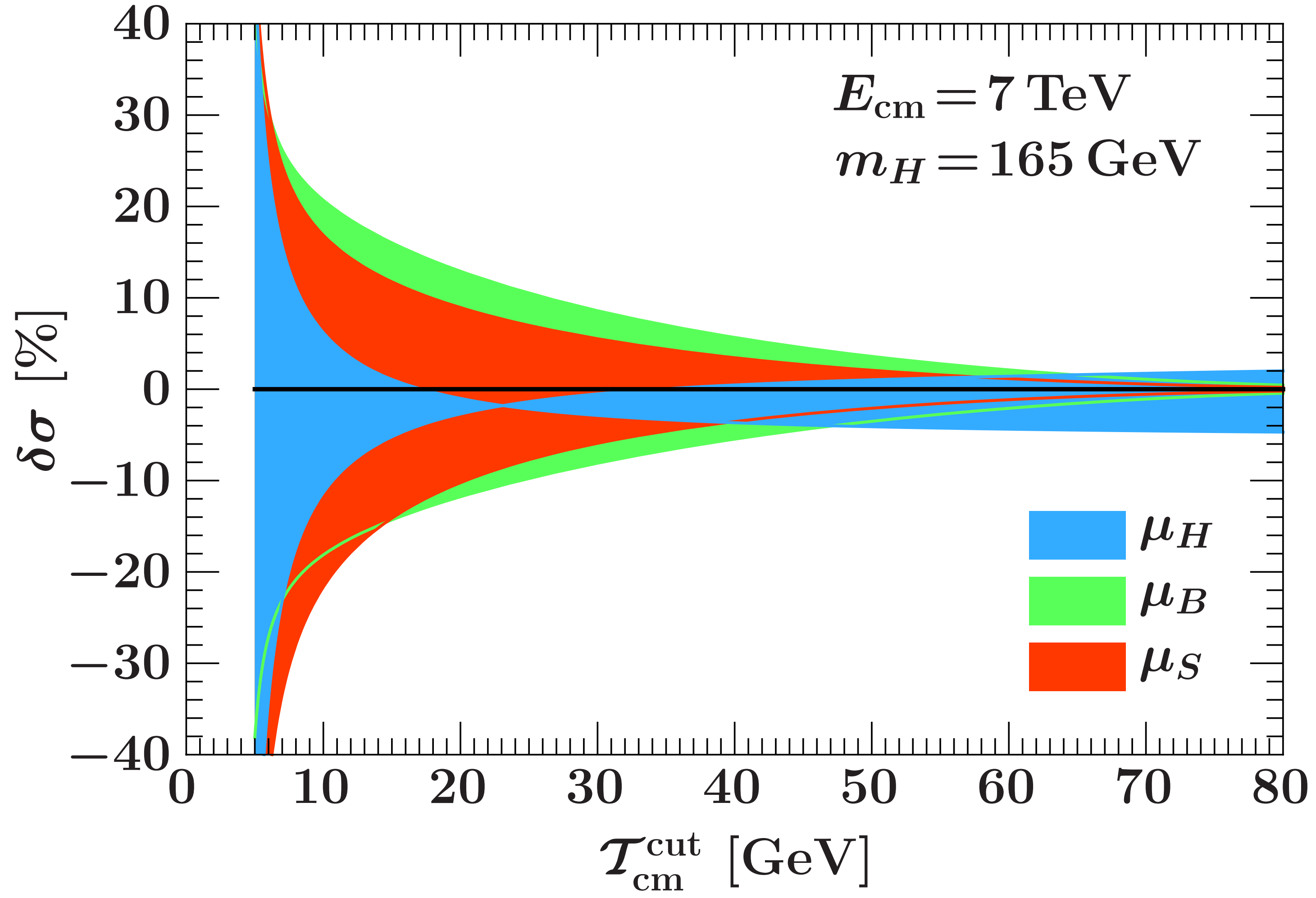
Small $\mathcal{T}_{cm}^{cut}$

individual scale variations

- three separate scale variations
- $\mu_H = \mu_{H0}$ 100% correlated with σ_{total}
- μ_B and μ_S give $\Delta_{cut} = \Delta_{SB}$ (dominate for small $\mathcal{T}_{cm}^{cut}$)







(C) Use Resummed Predictions to get Uncertainties for p_T?

Idea:

- reweigh MC@NLO or POWHEG to NNLO for central values for $p_T^{\rm cut}$ (what you do now)

- resummed calculation has two sources of uncertainty, one is correlated with $\Delta_{\rm total}$, one gives $\Delta_{\rm cut}$

- given these as % errors for spectra in $\mathcal{T}_{\rm cm}$, reweigh a MC sample to apply these errors for $p_T^{\rm cut}$

$$C = C_{SB} + C_H$$

$$C_H = \begin{pmatrix} \Delta^2_{H\mathrm{tot}} & \Delta_{H\mathrm{tot}}\Delta_{H0} & \Delta_{H\mathrm{tot}}\,\Delta_{H\geq 1} \\ \Delta_{H\mathrm{tot}}\,\Delta^2_{H0} & \Delta^2_{H0} & \Delta_{H0}\,\Delta_{H\geq 1} \\ \Delta_{H\mathrm{tot}}\,\Delta_{H\geq 1} & \Delta_{H0}\,\Delta_{H\geq 1} & \Delta^2_{H\geq 1} \end{pmatrix}$$

$$\Delta_{H\mathrm{tot}} = \Delta_{H0} + \Delta_{H\geq 1}$$

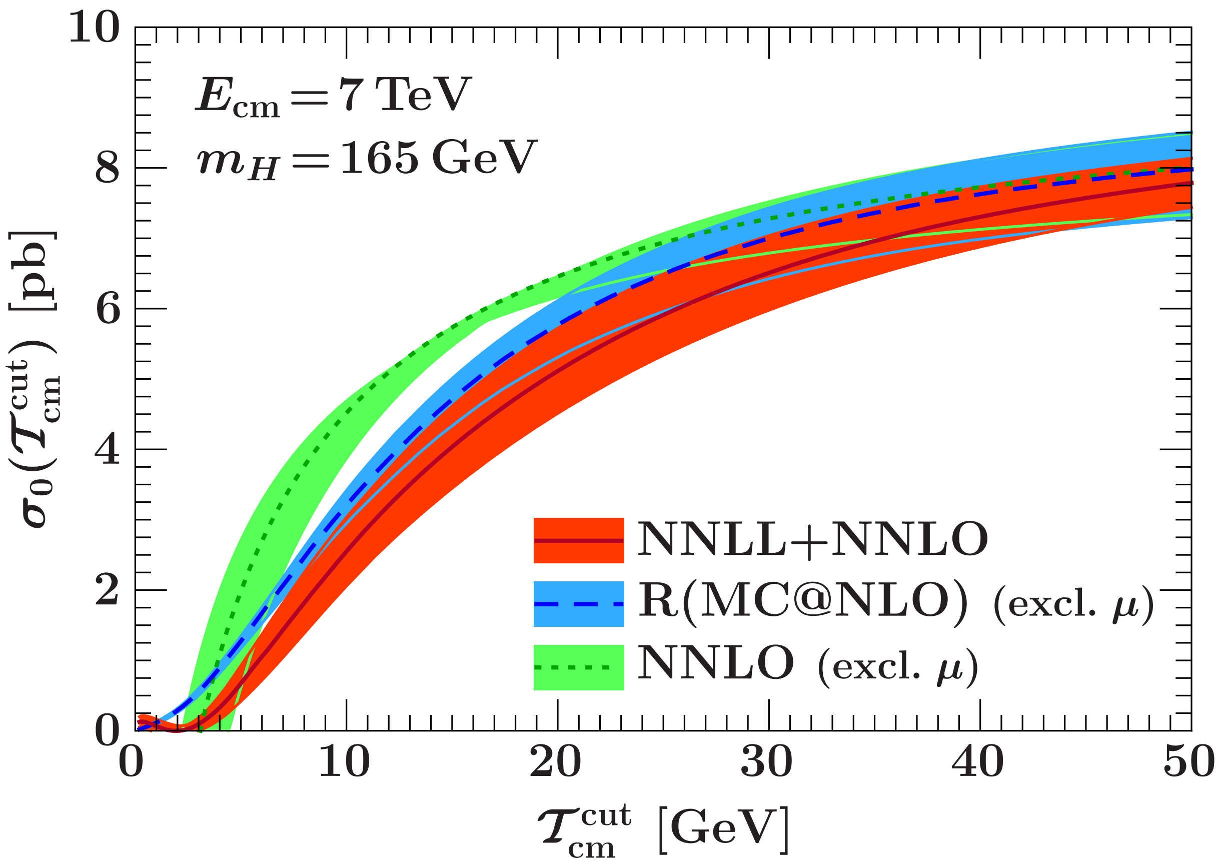
$$C_{SB} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta^2_{SB} & -\Delta^2_{SB} \\ 0 & -\Delta^2_{SB} & \Delta^2_{SB} \end{pmatrix}$$

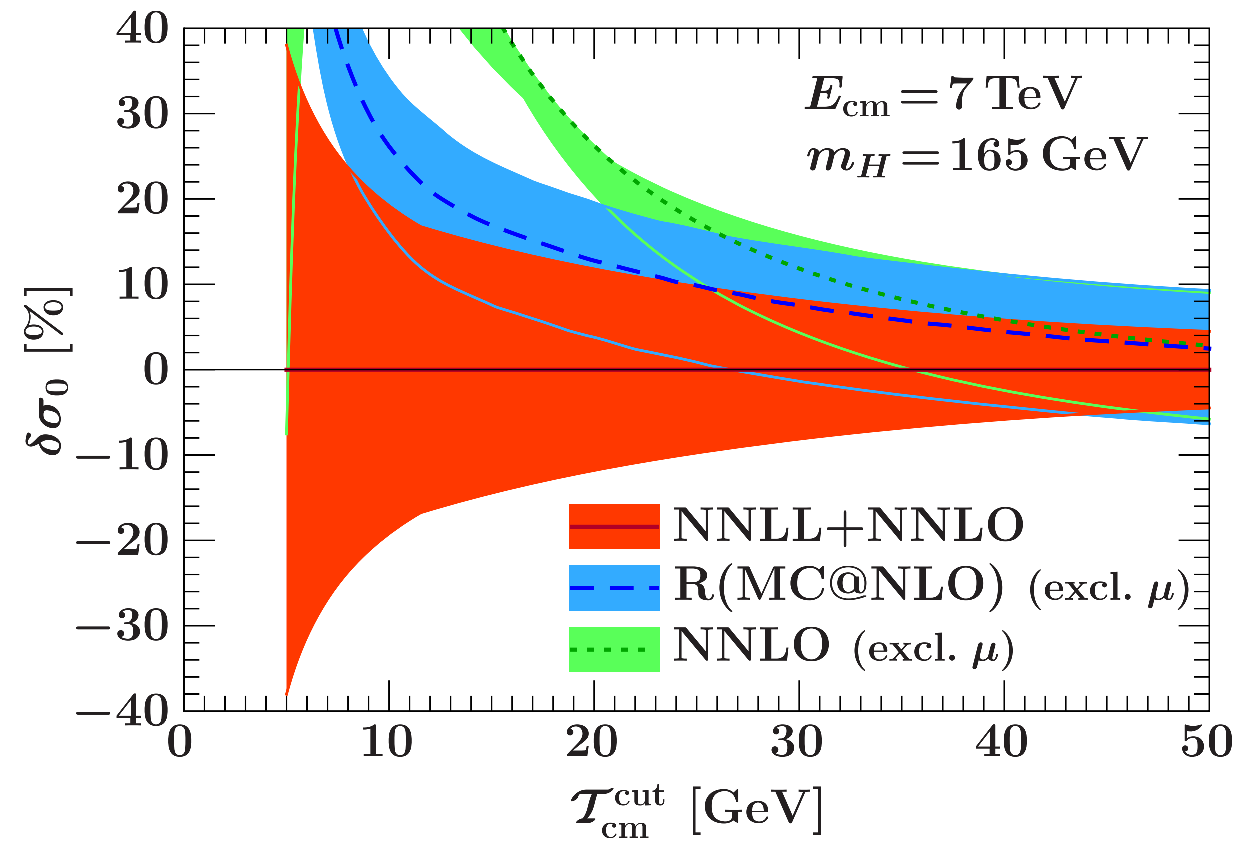
Small $\mathcal{T}_{\rm cm}^{\rm cut}$

like small $p_T^{\rm cut}$

direct exclusive scale variation shown for NNLO & MC@NLO

combined NNLL scale variations shown



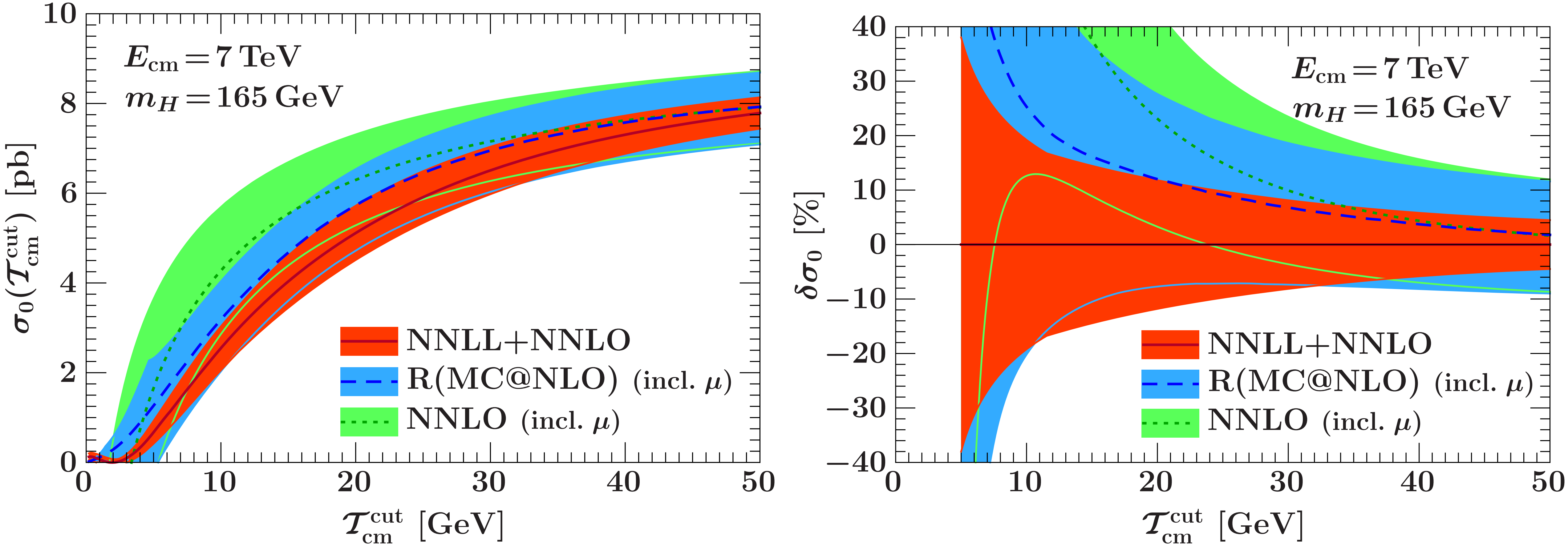


- logs are large, NNLL central value lower than NNLO
- reweigh MC@NLO to match NNLO value/uncertainty at 200GeV
  Central value is nearer NNLL. Uncertainty is only for norm.
- direct exclusive uncertainties here are too small (we discussed that...)

Small $\mathcal{T}_{cm}^{cut}$ like small p_T^{cut}

combined inclusive scale variation shown for NNLO & MC@NLO

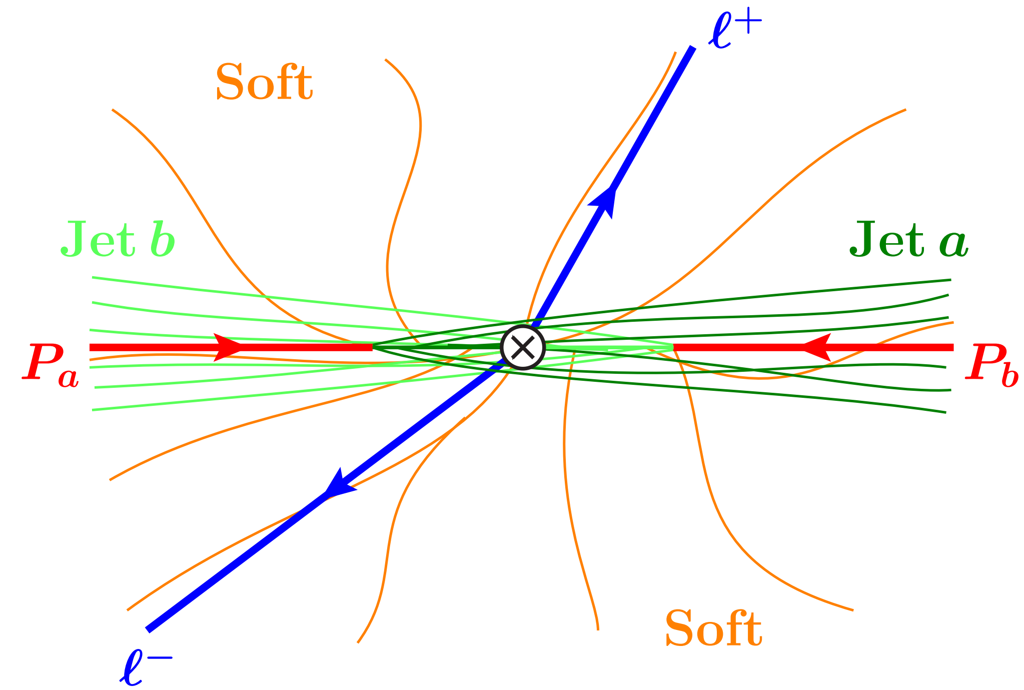
combined NNLL scale variations shown



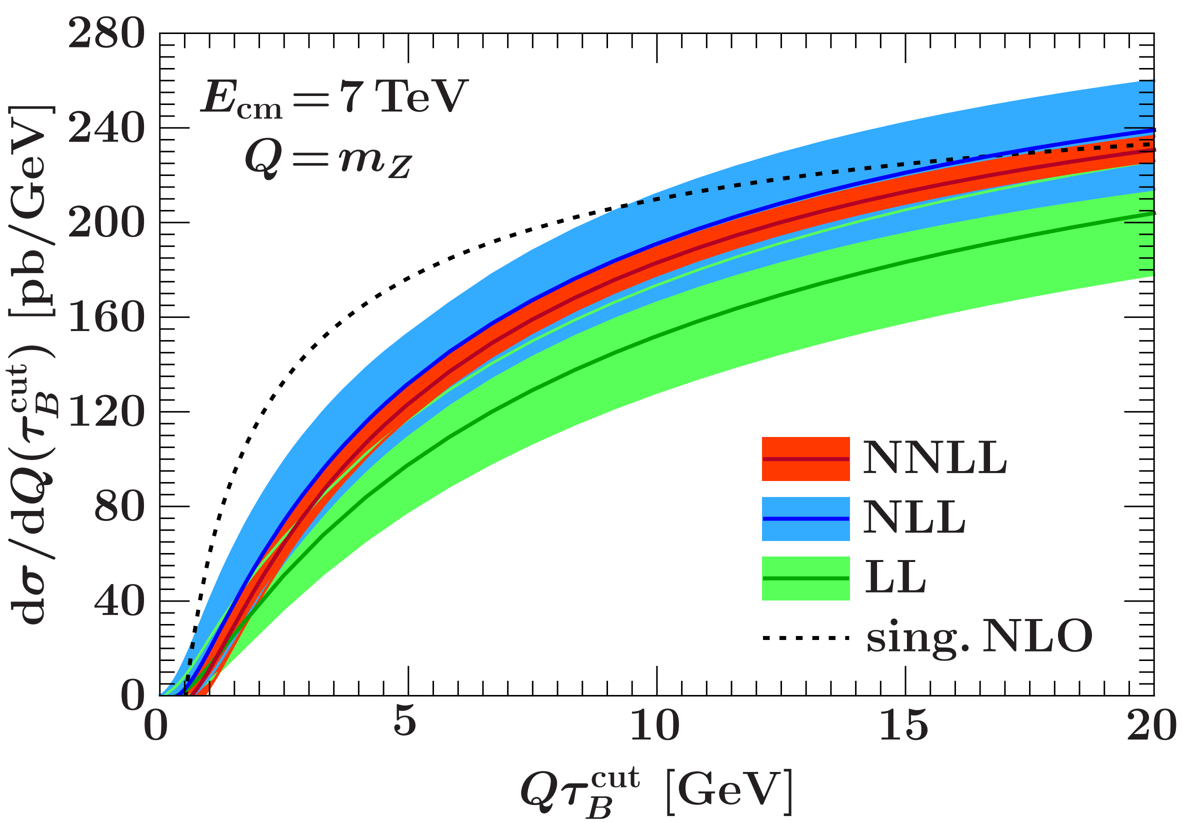
- NNLO band largely overlaps NNLL result
- reweigh MC@NLO to match NNLO incl. relative uncertainties (full spectrum). Overlaps nicely with NNLL.
- This factor of two improvement in uncertainty with NNLL is what one would expect if a similar reweighing exercise is done for p_T^{jet}

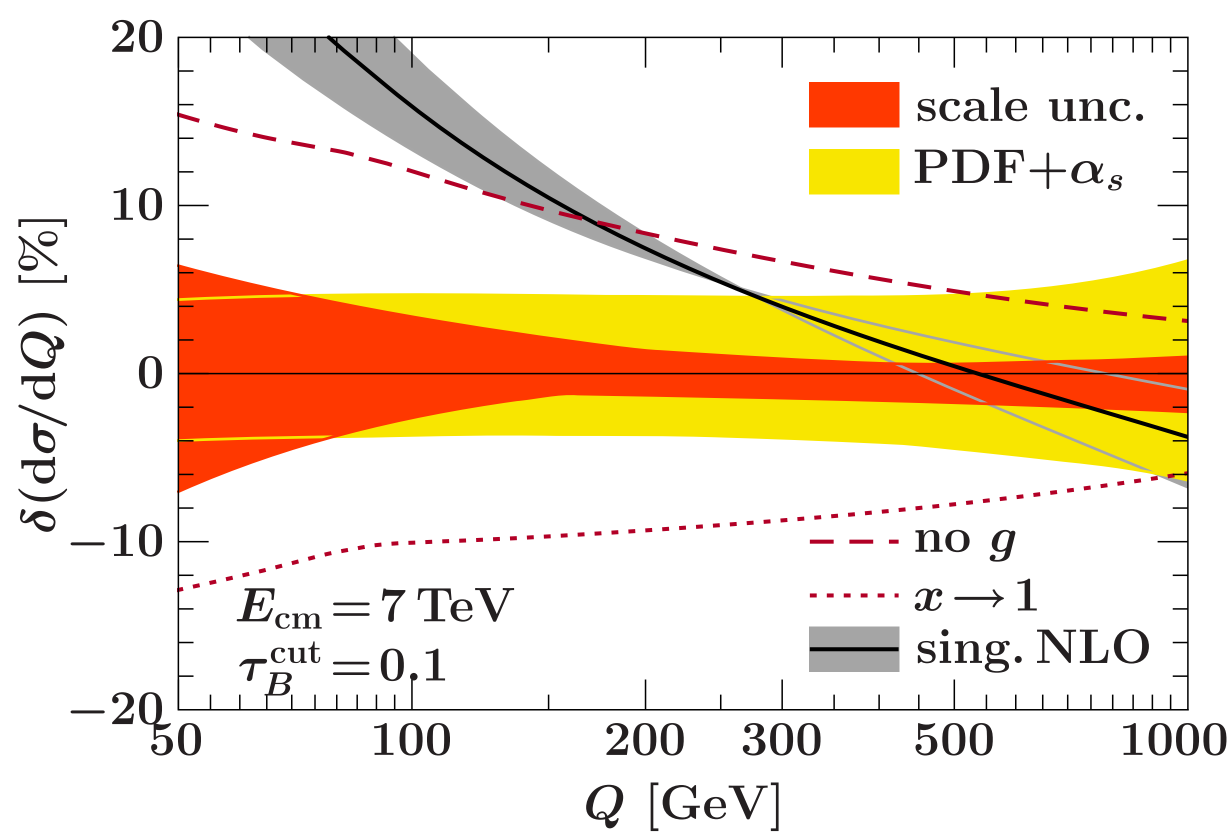
Validation? Other options?

- Drell-Yan pairs from γ^*, Z^* with a jet veto should be used for validation.



- Directly measure beam thrust (important on its own). And UE is no harder than it is for HT.





Friday, October 28, 2011

Theory Plans:

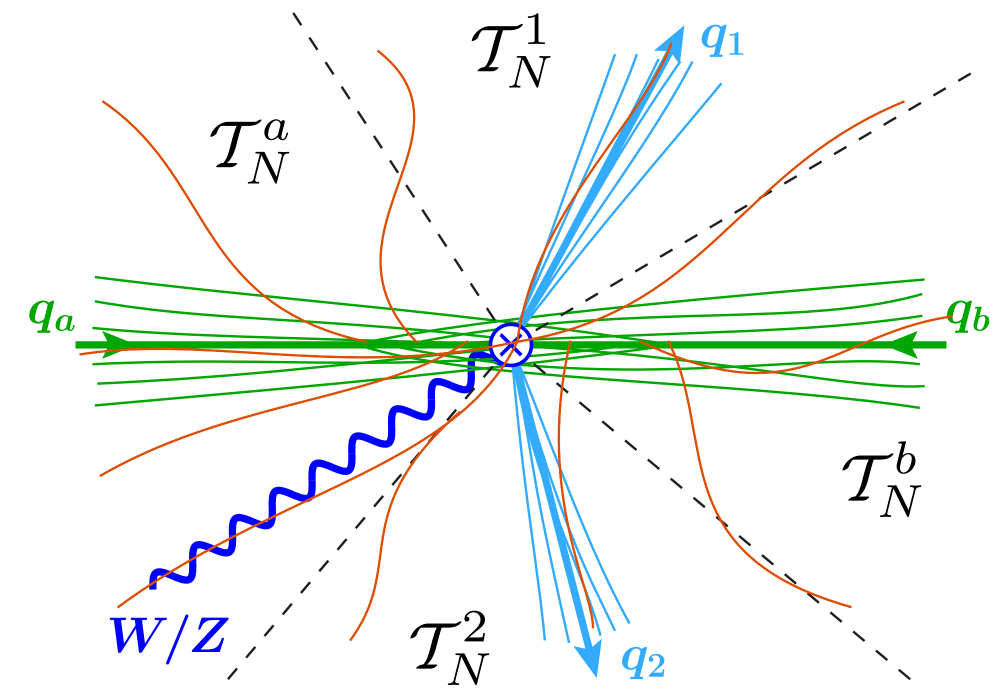
- A calculation of the Higgs + 0-jet cross section at one higher order (N3LL) is feasible. “Only” a missing 2 loop calculation. This will help reduce the perturbative uncertainty.
- Similar resummed calculations for Higgs + 1 jet, H + 2 jets, ...

N-Jettiness Event Shape

IS, Tackmann, Waalewijn
arXiv: 1004.2489

$$\mathcal{T}_N = \mathcal{T}_N(q_a, q_b, q_1, \ldots, q_N)$$

$$\mathcal{T}_N \to 0 \text{ for } N\text{-jets}$$



Factorization Friendly

$$\mathcal{T}_N = \mathcal{T}_N^a + \mathcal{T}_N^b + \mathcal{T}_N^1 + \ldots + \mathcal{T}_N^N$$

Want to calculate N-jet exclusive cross-sections.

eg. differential jet masses

$$\frac{d\sigma}{d\mathcal{T}_N^a \cdots d\mathcal{T}_N^N}$$

Jouttenus, IS, Tackmann, Waalewijn
arXiv: 1102.4344

Why?

- sum logs beyond the parton shower (up to NNLL)
- realistic estimates for theory errors
- test and tune Monte Carlo
- reweight Monte Carlo (eg. Higgs Search)

N-Jettiness $\mathcal{T}_N$

$$pp \to \text{jets},\ pp \to W/Z + \text{jets},\ \ldots$$

consider an inclusive N-jet sample with jet energies E_i & directions $\hat{n}_i$ determined by anti-kT (or any suitable algorithm)

$$q_i^\mu = E_i(1, \hat{n}_i)$$

$$q_a^\mu = \frac{1}{2} x_a\, E_{\rm cm}(1, \hat{z})\,,$$

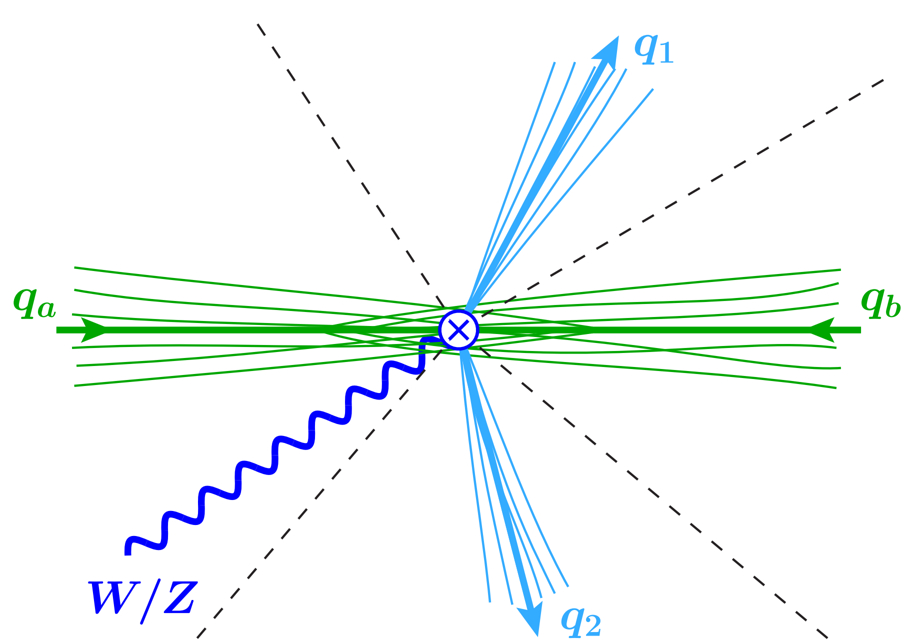
$$q_b^\mu = \frac{1}{2} x_b\, E_{\rm cm}(1, -\hat{z})$$

$$x_a x_b = \frac{Q^2}{E_{\rm cm}^2} = \frac{(q_1 + \ldots + q_N + q)^2}{E_{\rm cm}^2}$$

$$\ln \frac{x_a}{x_b} = Y = \ldots$$

(set $x_a = x_b = 1$ for cases with MET)

measure $\mathcal{T}_N = \sum_k |\vec{p}_{kT}| \min\{d_a(p_k), d_b(p_k), d_1(p_k), d_2(p_k), \ldots, d_N(p_k)\}$



- $d_{a,b}(p_k)$, $d_j(p_k)$: Distance of particle k to beam and jet directions
- Divides phase space into N jet regions and 2 beam regions

N-Jettiness $\mathcal{T}_N$

$$pp \to \text{jets},\ pp \to W/Z + \text{jets},\ \ldots$$

consider an inclusive N-jet sample with jet energies E_i & directions $\hat{n}_i$ determined by anti-kT (or any suitable algorithm)

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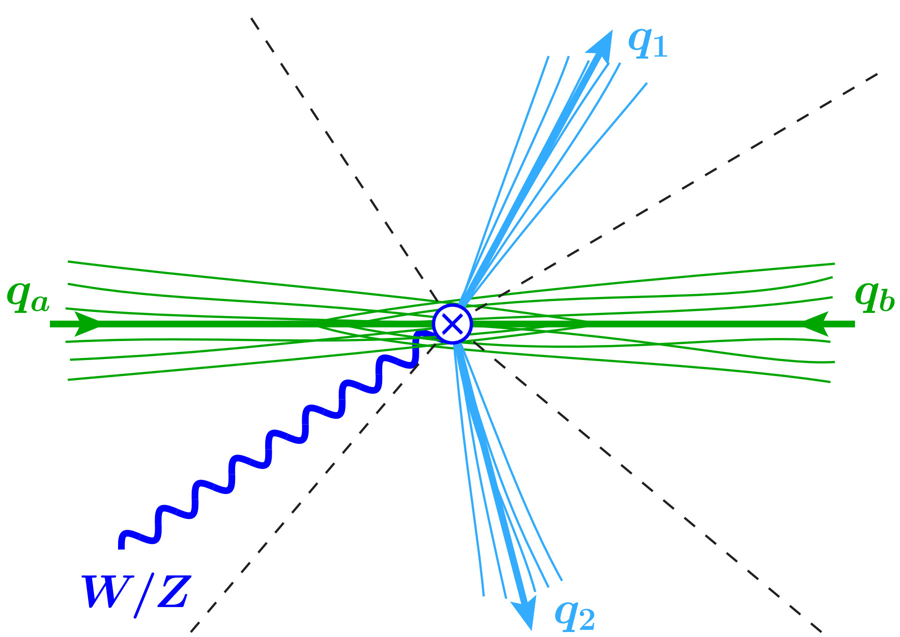
$$x_a x_b = \frac{Q^2}{E_{\rm cm}^2} = \frac{(q_1 + \ldots + q_N + q)^2}{E_{\rm cm}^2}$$

$$\ln \frac{x_a}{x_b} = Y = \ldots$$

(set $x_a = x_b = 1$ for cases with MET)

measure $$\mathcal{T}_N = \sum_k \min\left\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1}, \ldots, \frac{2q_N \cdot p_k}{Q_N} \right\}$$



- Here Q_j determines the measure
- Small $\mathcal{T}_N$ constrains us to N-jets (one added scale)

$$\mathcal{T}_N^{\rm alg.1} = \mathcal{T}_N^{\rm alg.2} + \mathcal{O}[(\mathcal{T}_N^{\rm alg.2})^2]$$

Large $\mathcal{T}_N$ has >N jets

N-Jettiness $\mathcal{T}_N$

$$pp \to \text{jets},\ pp \to W/Z + \text{jets},\ \ldots$$

consider an inclusive N-jet sample with jet energies E_i & directions $\hat{n}_i$ determined by anti-kT (or any suitable algorithm)

$$q_i^\mu = E_i(1, \hat{n}_i)$$

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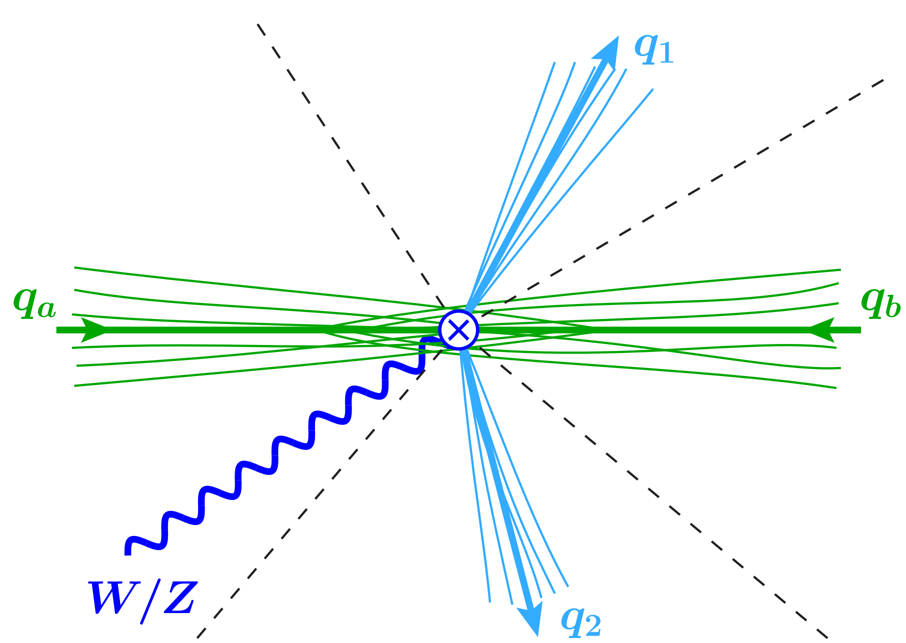
$$q_b^\mu = \frac{1}{2} x_b\, E_{\rm cm}(1, -\hat{z})$$

$$x_a x_b = \frac{Q^2}{E_{\rm cm}^2} = \frac{(q_1 + \ldots + q_N + q)^2}{E_{\rm cm}^2}$$

$$\ln \frac{x_a}{x_b} = Y = \ldots$$

(set $x_a = x_b = 1$ for cases with MET)

measure $$\mathcal{T}_N = \sum_k \min\left\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1}, \ldots, \frac{2q_N \cdot p_k}{Q_N} \right\}$$



"make it a true event shape"

- Determine q_i by minimization

For $Q_i = |\vec{q}_{iT}|$, $\vec{p}^{\,i}_{\rm jet} = \sum_{k \in i} \vec{p}_k$

Thaler, Van Tilburg

- Extension to N-subjettiness

N-Jettiness Factorization

$$\mathcal{T}_N = \sum_k \min\left\{\frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1}, \ldots, \frac{2q_N \cdot p_k}{Q_N}\right\}$$

$$\mathcal{T}_N = \left(\sum_{k\in \mathrm{soft}} \min_m \left\{\frac{2q_m \cdot p_k}{Q_m}\right\}\right) + \sum_{j=a,b,1,\ldots,N} \left(\sum_{k\in \mathrm{coll}_j} \frac{2q_j \cdot p_k}{Q_j}\right)$$

Only soft particles get a nontrivial grouping. Jet boundaries are determined by the q_m

collinear particles all grouped with their q_j

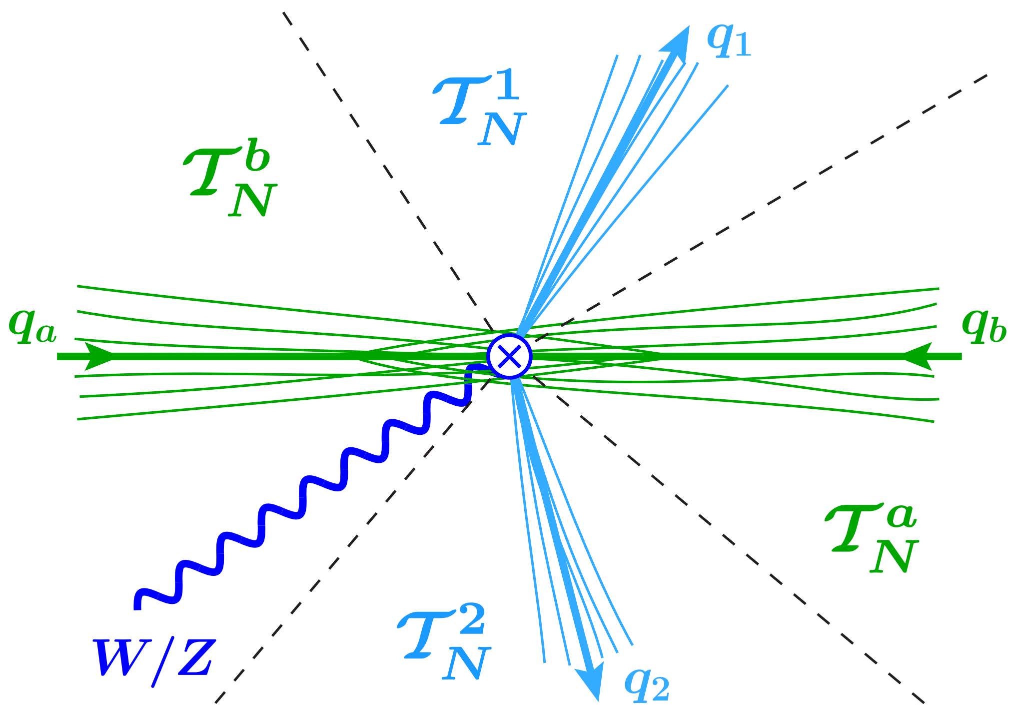
N-Jettiness & Jet Masses

$$\mathcal{T}_N = \sum_k \min\left\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1}, \ldots, \frac{2q_N \cdot p_k}{Q_N} \right\}$$

$$\mathcal{T}_N = \mathcal{T}_a + \mathcal{T}_b + \mathcal{T}_1 + \cdots + \mathcal{T}_N$$

$$\mathcal{T}_N^j = \sum_{k \in j} |\vec{p}_{kT}|\, d_j(p_k)$$

Can measure: $$\frac{d\sigma}{d\mathcal{T}_a d\mathcal{T}_b d\mathcal{T}_1 \cdots d\mathcal{T}_N}$$



with jet axes aligned

These are Jet Masses:

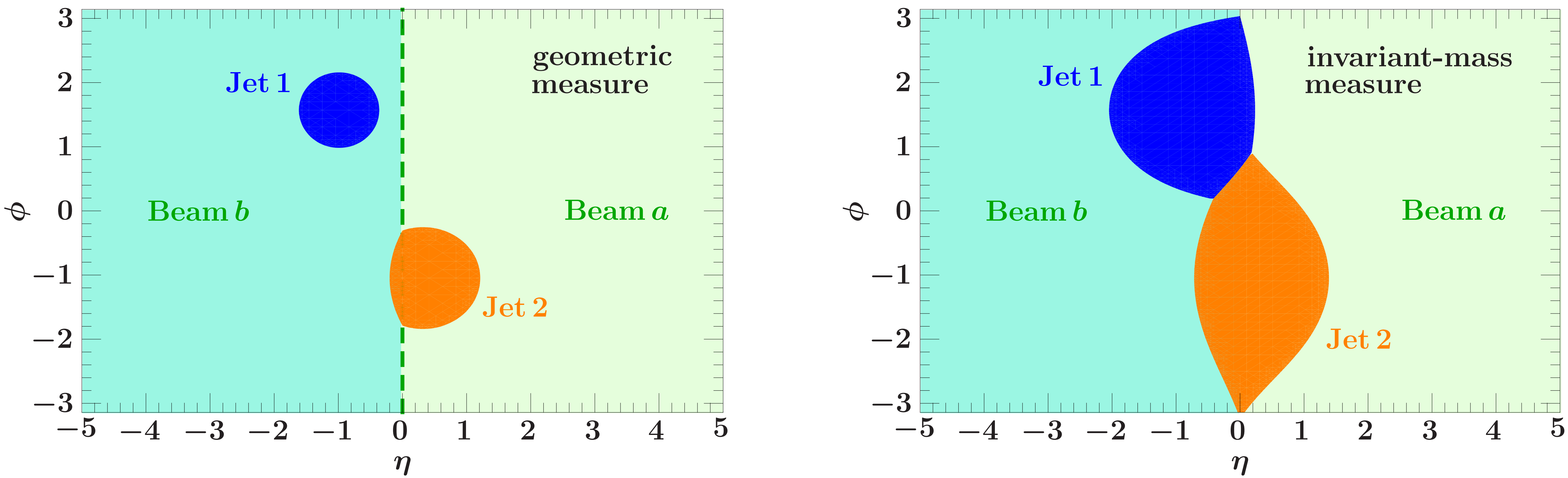
$$M_J^2 = P_J^2 = P_J^- P_J^+ = Q_i \mathcal{T}_N^i$$

So one can study the masses of jets!

Jet definition:

N-jettiness divides particles into jet and beam regions

$$\mathcal{T}_N = \sum_k |\vec{p}_{kT}| \min\{d_a(p_k), d_b(p_k), d_1(p_k), d_2(p_k), \ldots, d_N(p_k)\}$$



$$d_{a,b}(p_k) = e^{\mp\eta_k}$$

$$d_j(p_k) = 2\cosh\Delta\eta_{jk} - 2\cos\Delta\phi_{jk}$$

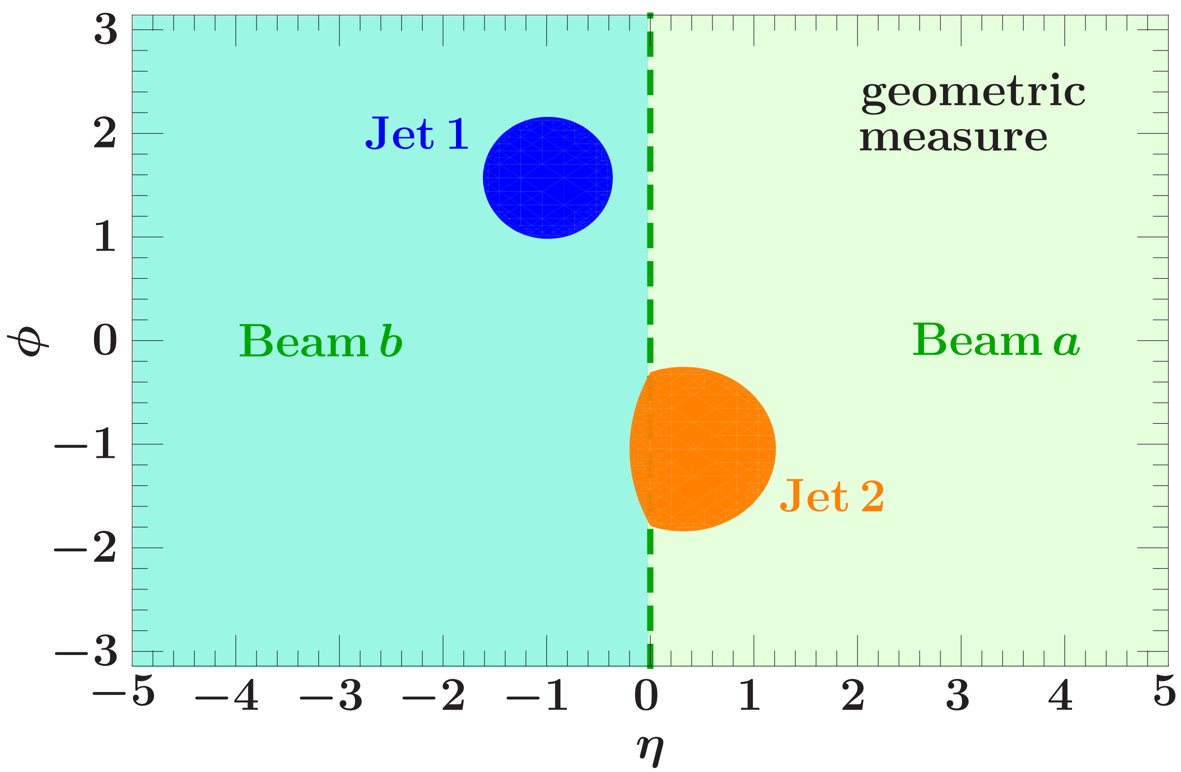
$$\approx (\Delta\eta_{jk})^2 + (\Delta\phi_{jk})^2$$

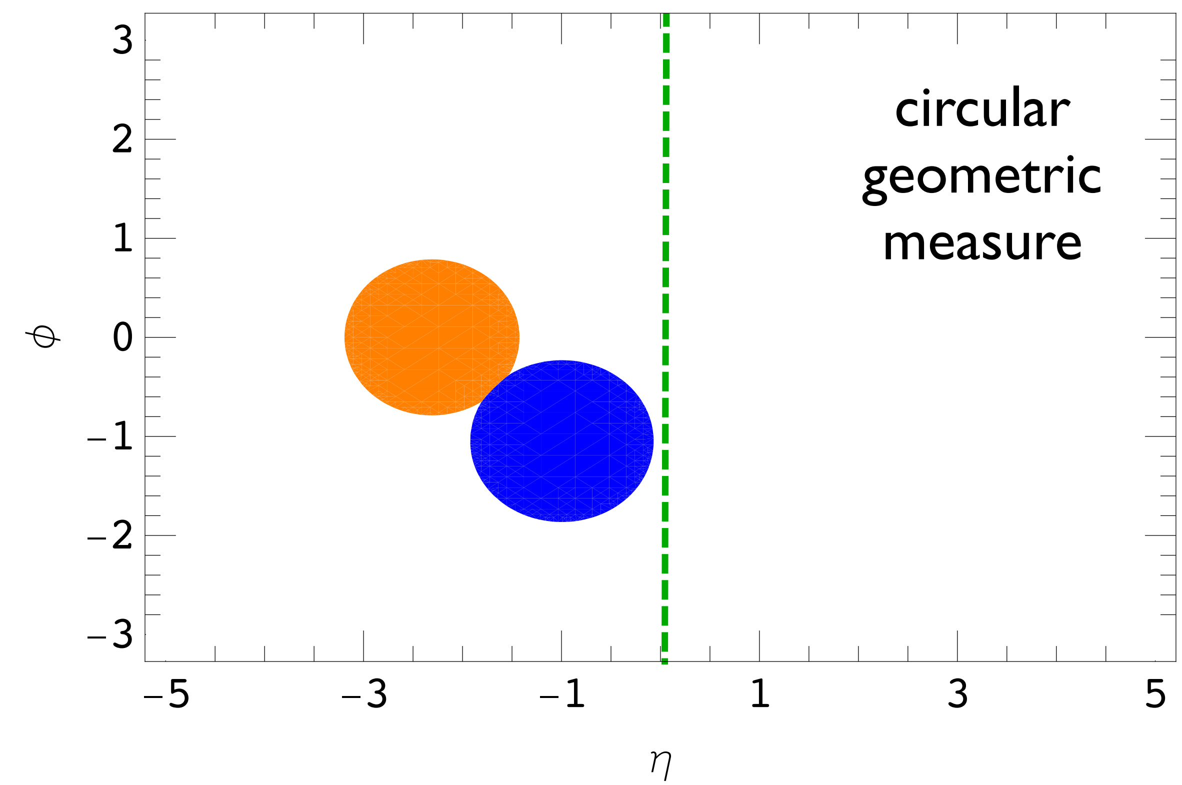
$$d_i(p_k) = \frac{2q_i \cdot p_k}{Q|\vec{p}_{kT}|}$$

Jet definition:

N-jettiness divides particles into jet and beam regions

$$\mathcal{T}_N = \sum_k |\vec{p}_{kT}| \min\{d_a(p_k), d_b(p_k), d_1(p_k), d_2(p_k), \ldots, d_N(p_k)\}$$





$$d_{a,b}(p_k) = e^{\mp\eta_k}$$

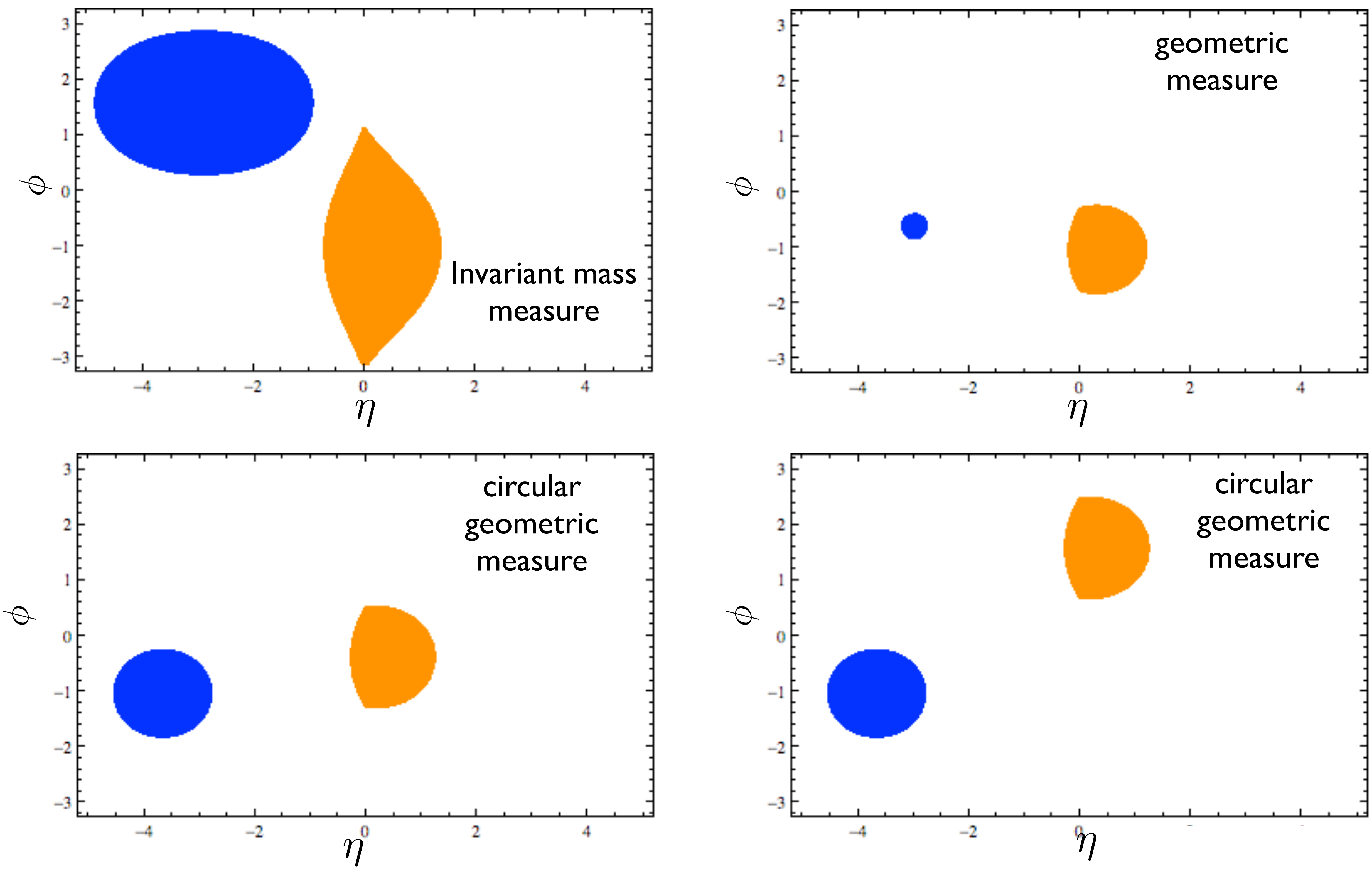
$$d_j(p_k) = 2\cosh\Delta\eta_{jk} - 2\cos\Delta\phi_{jk}$$
$$\approx (\Delta\eta_{jk})^2 + (\Delta\phi_{jk})^2$$

$$d_{a,b}(p_k) = \text{same}$$

$$d_j(p_k) = (\text{same})/\cosh\Delta\eta_{jk}$$

Jets treatment of soft radiation depends on the distance measure

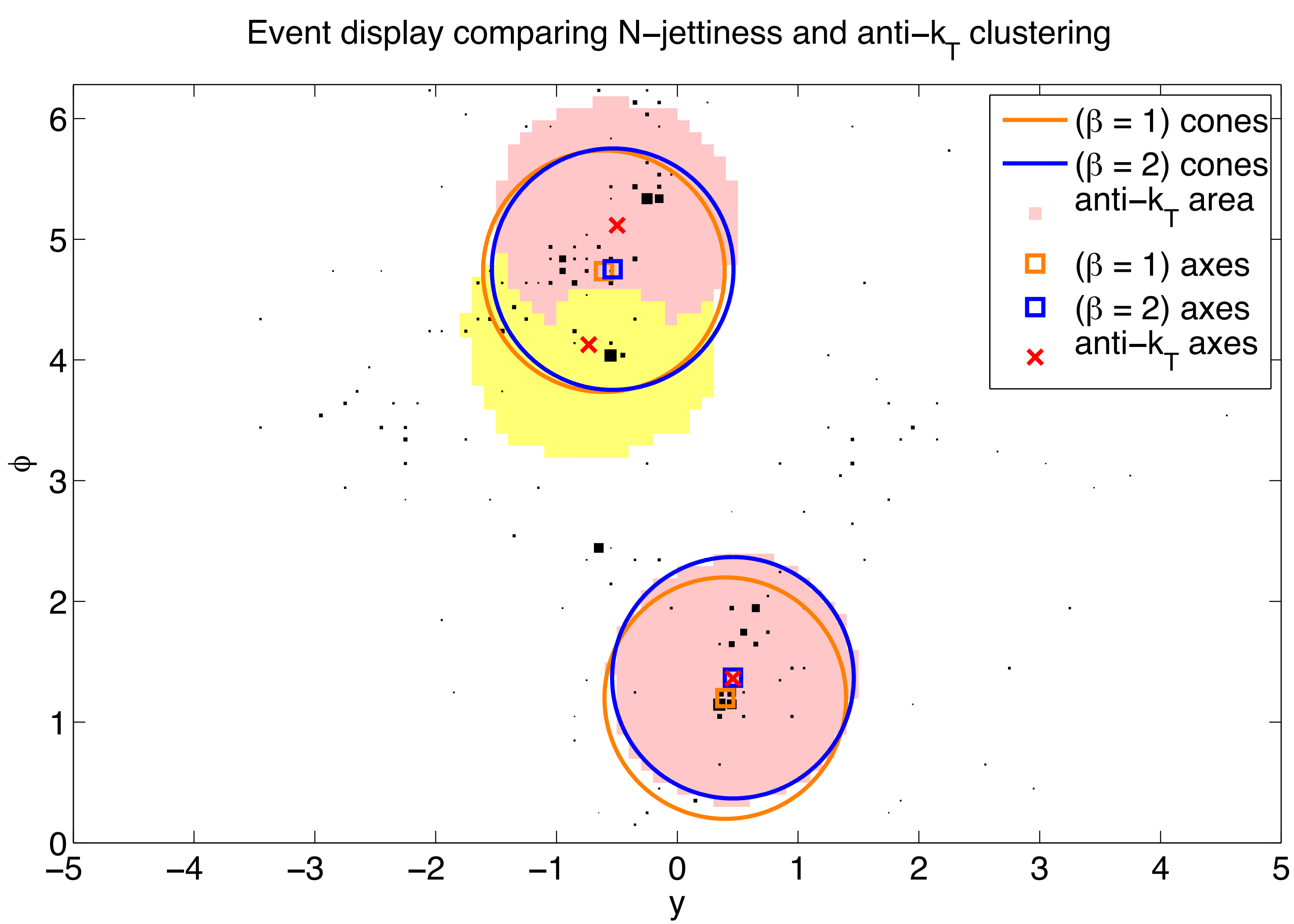
$$\hat{q}_i^\mu \equiv \frac{q_i^\mu}{Q_i}, \qquad \mathcal{T}_N \equiv \sum_k \min_i \{2\hat{q}_i \cdot p_k\}$$



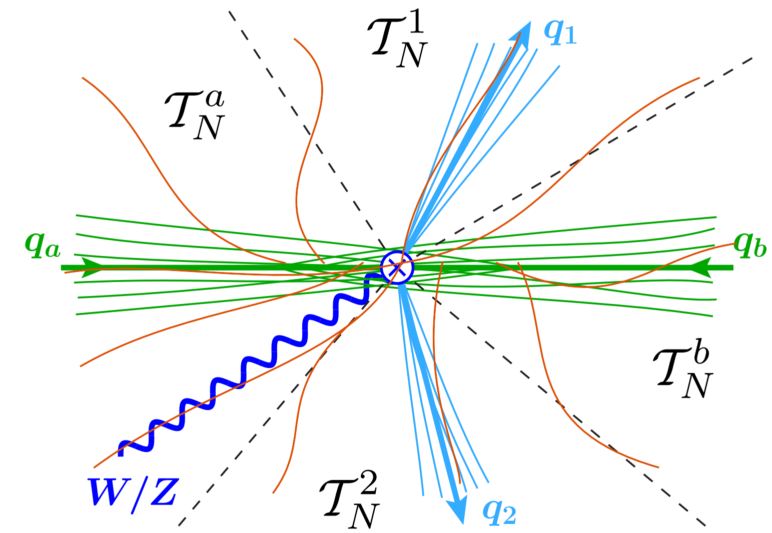
With Minimization

Thaler, Van Tilburg

$$\tau_N^{(\beta,\gamma)}(R_0,\eta_0) = \sum_i p_{T,i} \min\left\{ \left(\exp\frac{-\eta_i}{\eta_0}\right)^{\gamma}, \left(\exp\frac{\eta_i}{\eta_0}\right)^{\gamma}, \left(\frac{\Delta R_{1,i}}{R_0}\right)^{\beta}, \ldots, \left(\frac{\Delta R_{N,i}}{R_0}\right)^{\beta}, 1 \right\}.$$



N-Jettiness Factorization Formula



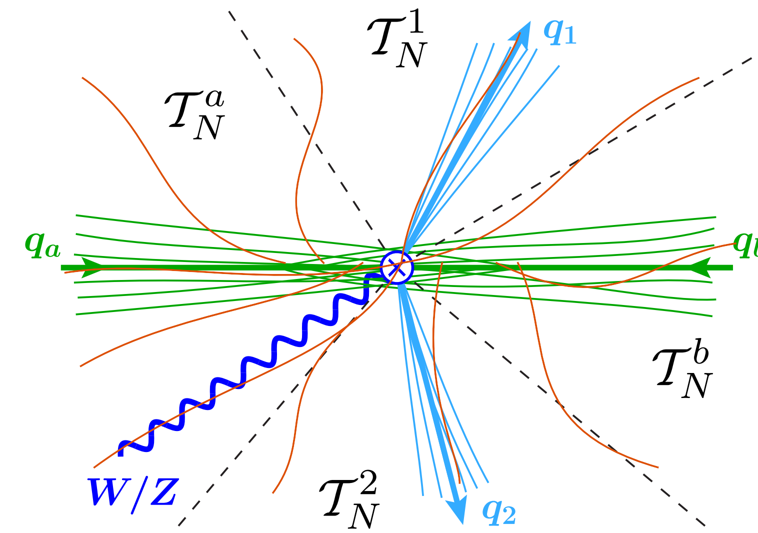
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N^a\,\mathrm{d}\mathcal{T}_N^b\cdots\mathrm{d}\mathcal{T}_N^N} = \int \mathrm{d}x_a \mathrm{d}x_b \int \mathrm{d}(\text{phase space})$$

$$\times \sum_\kappa \int \mathrm{d}t_a\, B_{\kappa_a}(t_a, x_a) \int \mathrm{d}t_b\, B_{\kappa_b}(t_b, x_b) \prod_{J=1}^{N} \int \mathrm{d}s_J\, J_{\kappa_J}(s_J)$$

$$\times\, \mathrm{tr}\Big[H_N^\kappa(\{q_i\cdot q_j\}, x_{a,b})\, \widehat{S}_N^\kappa\Big(\mathcal{T}_N^a - \frac{t_a}{Q_a}, \mathcal{T}_N^b - \frac{t_b}{Q_b}, \mathcal{T}_N^1 - \frac{s_1}{Q_1}, \dots, \mathcal{T}_N^N - \frac{s_N}{Q_N}, \{\hat{q}_i\cdot\hat{q}_j\}\Big)\Big]$$

$$\times\Big[1 + \mathcal{O}(\mathcal{T}_N^j)\Big]$$

N-Jettiness Factorization Formula



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_N^a\,\mathrm{d}\mathcal{T}_N^b\cdots\mathrm{d}\mathcal{T}_N^N} = \int \mathrm{d}x_a \mathrm{d}x_b \int \mathrm{d}(\text{phase space})$$

$$\times \sum_\kappa \int \mathrm{d}t_a\, B_{\kappa_a}(t_a, x_a) \int \mathrm{d}t_b\, B_{\kappa_b}(t_b, x_b) \prod_{J=1}^{N} \int \mathrm{d}s_J\, J_{\kappa_J}(s_J)$$

$$\times\, \mathrm{tr}\Big[H_N^\kappa(\{q_i\cdot q_j\}, x_{a,b})\, \widehat{S}_N^\kappa\Big(\mathcal{T}_N^a - \frac{t_a}{Q_a}, \mathcal{T}_N^b - \frac{t_b}{Q_b}, \mathcal{T}_N^1 - \frac{s_1}{Q_1}, \ldots, \mathcal{T}_N^N - \frac{s_N}{Q_N}, \{\hat{q}_i\cdot\hat{q}_j\}\Big)\Big]$$

hard virtual corrections
$2 \to N + q$

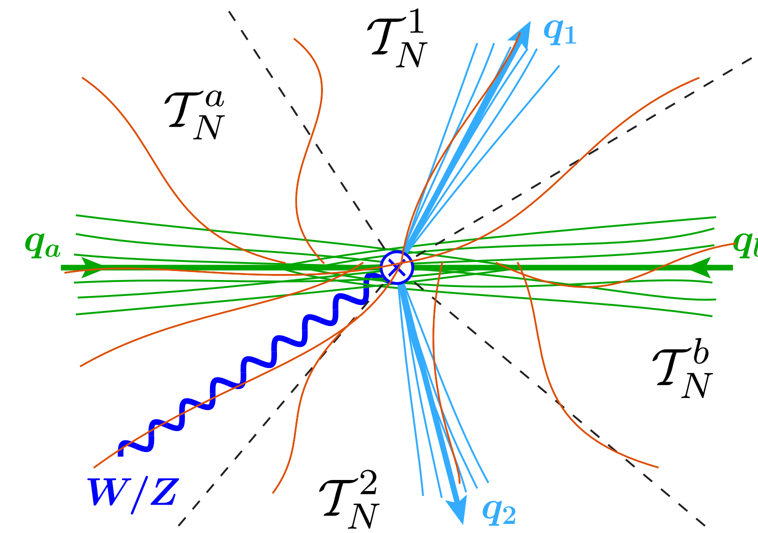
beam function
$B_\kappa = \mathcal{I}_{\kappa\kappa'} \otimes f_{\kappa'}$

N-jettiness soft function

jet function known to $\mathcal{O}(\alpha_s^2)$

$$q_i \cdot q_j = (Q_i Q_j)(\hat{q}_i \cdot \hat{q}_j)$$

N-Jettiness Factorization Formula



$$\frac{d\sigma}{d\mathcal{T}_N^a\, d\mathcal{T}_N^b \cdots d\mathcal{T}_N^N} = \int dx_a dx_b \int d(\text{phase space})$$

$$\times \sum_\kappa \int dt_a\, B_{\kappa_a}(t_a, x_a) \int dt_b\, B_{\kappa_b}(t_b, x_b) \prod_{J=1}^{N} \int ds_J\, J_{\kappa_J}(s_J)$$

$$\times\, \mathrm{tr}\Big[H_N^\kappa(\{q_i \cdot q_j\}, x_{a,b})\, \widehat{S}_N^\kappa\Big(\mathcal{T}_N^a - \frac{t_a}{Q_a}, \mathcal{T}_N^b - \frac{t_b}{Q_b}, \mathcal{T}_N^1 - \frac{s_1}{Q_1}, \ldots, \mathcal{T}_N^N - \frac{s_N}{Q_N}, \{\hat{q}_i \cdot \hat{q}_j\}\Big)\Big]$$

$$q_i \cdot q_j = (Q_i Q_j)(\hat{q}_i \cdot \hat{q}_j)$$

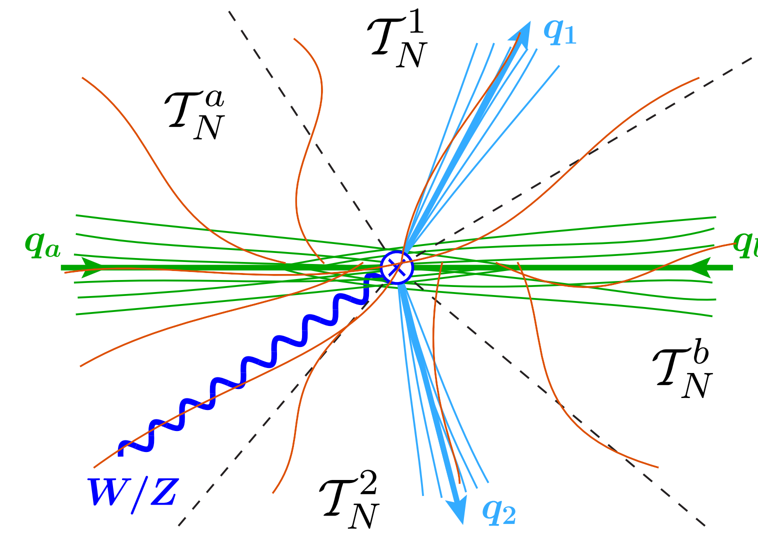
Assumptions needed to sum logs with this formula:

1) $\mathcal{T}_i \sim \mathcal{T}_j$ ($\mathcal{T}_i \ll \mathcal{T}_j$ gives non-global logs of Dasgupta & Salam)

2) $\hat{q}_i \cdot \hat{q}_j \gg \mathcal{T}_i / Q_i$ jets are well separated

3) $Q_i \sim Q_j$

N-Jettiness Factorization Formula



$$\frac{d\sigma}{d\mathcal{T}_N^a\, d\mathcal{T}_N^b \cdots d\mathcal{T}_N^N} = \int dx_a dx_b \int d(\text{phase space})$$

$$\times \sum_\kappa \int dt_a\, B_{\kappa_a}(t_a, x_a) \int dt_b\, B_{\kappa_b}(t_b, x_b) \prod_{J=1}^{N} \int ds_J\, J_{\kappa_J}(s_J)$$

$$\times\, \mathrm{tr}\Big[H_N^\kappa(\{q_i \cdot q_j\}, x_{a,b})\, \widehat{S}_N^\kappa\Big(\mathcal{T}_N^a - \frac{t_a}{Q_a}, \mathcal{T}_N^b - \frac{t_b}{Q_b}, \mathcal{T}_N^1 - \frac{s_1}{Q_1}, \ldots, \mathcal{T}_N^N - \frac{s_N}{Q_N}, \{\hat{q}_i \cdot \hat{q}_j\}\Big)\Big]$$

$$q_i \cdot q_j = (Q_i Q_j)(\hat{q}_i \cdot \hat{q}_j)$$

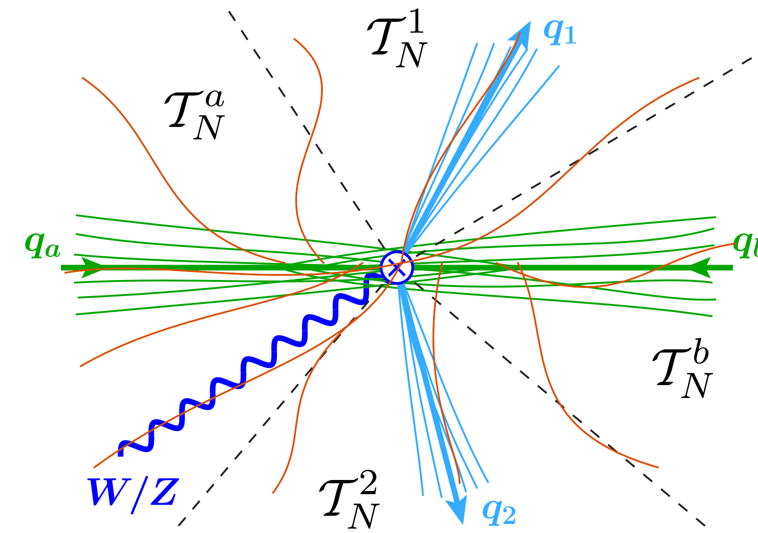
With assumptions: $\mathcal{T}_i \sim \mathcal{T}_j$, $\hat{q}_i \cdot \hat{q}_j \gg \mathcal{T}_i / Q_i$, $Q_i \sim Q_j$

Can explore angular dependence, Q_i dependence

Have Color / Kinematic info. Can look at jet mass in samples with various amounts of quarks vs. gluons.

Again can compute (un)correlated uncertainties.

N-Jettiness Factorization Formula



$$\frac{d\sigma}{d\mathcal{T}_N^a\, d\mathcal{T}_N^b \cdots d\mathcal{T}_N^N} = \int dx_a dx_b \int d(\text{phase space})$$

$$\times \sum_\kappa \int dt_a\, B_{\kappa_a}(t_a, x_a) \int dt_b\, B_{\kappa_b}(t_b, x_b) \prod_{J=1}^{N} \int ds_J\, J_{\kappa_J}(s_J)$$

$$\times \operatorname{tr}\left[H_N^\kappa(\{q_i \cdot q_j\}, x_{a,b})\, \widehat{S}_N^\kappa\left(\mathcal{T}_N^a - \frac{t_a}{Q_a}, \mathcal{T}_N^b - \frac{t_b}{Q_b}, \mathcal{T}_N^1 - \frac{s_1}{Q_1}, \ldots, \mathcal{T}_N^N - \frac{s_N}{Q_N}, \{\hat{q}_i \cdot \hat{q}_j\}\right)\right]$$

$q_i \cdot q_j = (Q_i Q_j)(\hat{q}_i \cdot \hat{q}_j)$

Pieces needed for NNLL are now all in hand:

- Three Loop Cusp Anom. Dim, Two Loop Non Cusp.
  (Note: Beam function has same Logs as Jet Function)
- One Loop Hard functions: when available in QCD literature
  (only part that restricts N)
- Jet & Beam Functions at one loop
- N-jet Soft function

Jouttenus, IS, Tackmann, Waalewijn

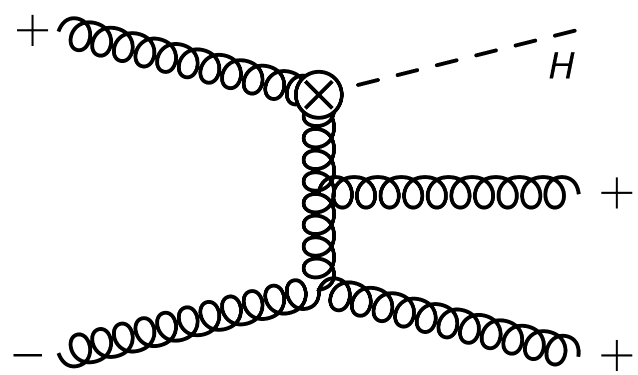
also: Bauer, Hornig, Dunn

If we make use of a helicity basis for SCET operators, then hard matching coefficients are precisely the finite part of the color ordered helicity amplitudes in $\overline{\text{MS}}$.

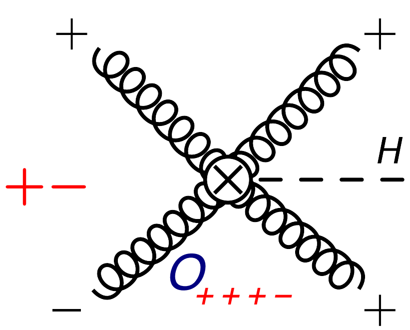
IS, Tackmann, Waalewijn
(work in progress)

eg. $gggg H$: Basis

$$\mathcal{A}(1^+2^+3^+4^-5_H) =$$



$= iC_{+++-}$



- Five helicity operators:

$$O^{abcd}_{++++} = \frac{1}{4!}\mathcal{B}^a_{1+}\mathcal{B}^b_{2+}\mathcal{B}^c_{3+}\mathcal{B}^d_{4+}H_5$$

$$O^{abcd}_{+++-} = \frac{1}{3!}\mathcal{B}^a_{1+}\mathcal{B}^b_{2+}\mathcal{B}^c_{3+}\mathcal{B}^d_{4-}H_5$$

$$\ldots$$

- Tree-level helicity amplitudes calculated by [Kauffmann, Desai, Risal (1997)]

$$A^{(0)}(1^+,2^+,3^-,4^-;5_H)=2\left[\frac{s_{12}^2}{\sqrt{|s_{12}s_{23}s_{34}s_{14}|}}+e^{-2i\phi_2}\frac{s_{34}^2}{\sqrt{|s_{12}s_{23}s_{34}s_{14}|}}\right]e^{i\Phi}$$

Summary

- Experimental measurements require precision jet-bin cross sections with careful assessment of theoretical uncertainties

- Assigning independent uncertainties to inclusive jet cross-sections, and propagating these to exclusive jet cross-sections is a good starting point.

- Theory errors are important for Higgs analyses. Improved precision for exclusive jet cross sections is necessary (through resummation, or full $\mathrm{N}^k\mathrm{LO}$, or approximate $\mathrm{N}^k\mathrm{LO}$, or more realistic analysis corresponding to the experimental measurements)

- Resummation for N jet-bins at NNLL is in sight, but will require coordination between various groups.

The End