SCET - Recent Developments

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Chamonix, 2005

Soft - Collinear Effective Theory

Bauer, Pirjol, Stewart
Fleming, Luke, ...

An effective field theory for energetic hadrons & jets

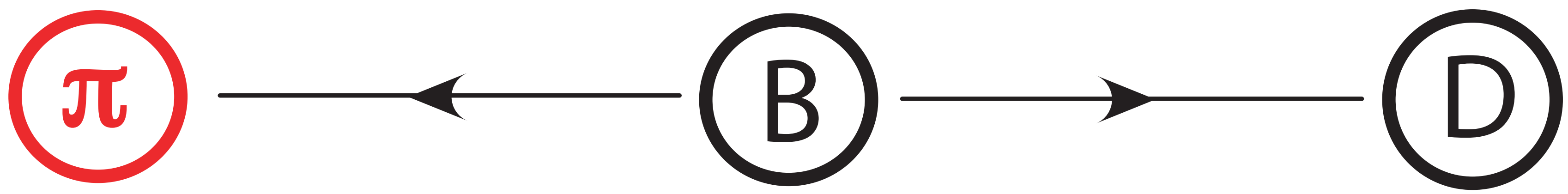
$$E \gg \Lambda_{\mathrm{QCD}}$$

Effective Field Theory

- Separate physics at different momentum scales
- Model independent, systematically improvable
- Power expansion, can estimate uncertainty
- Exploit symmetries
- Resum Sudakov logarithms

Soft Collinear Effective Theory

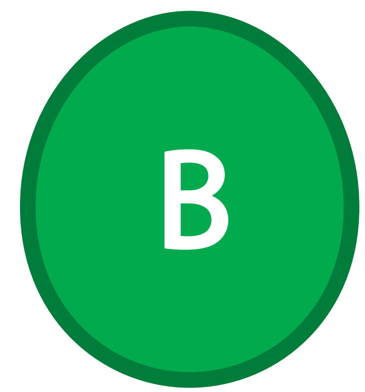
eg.



Pion has: $\quad p_\pi^\mu \quad = \quad (2.3\,\mathrm{GeV})n^\mu = Q\,n^\mu \qquad n^2 = \bar{n}^2 = 0, \;\; (\bar{n}\cdot p = p^-)$

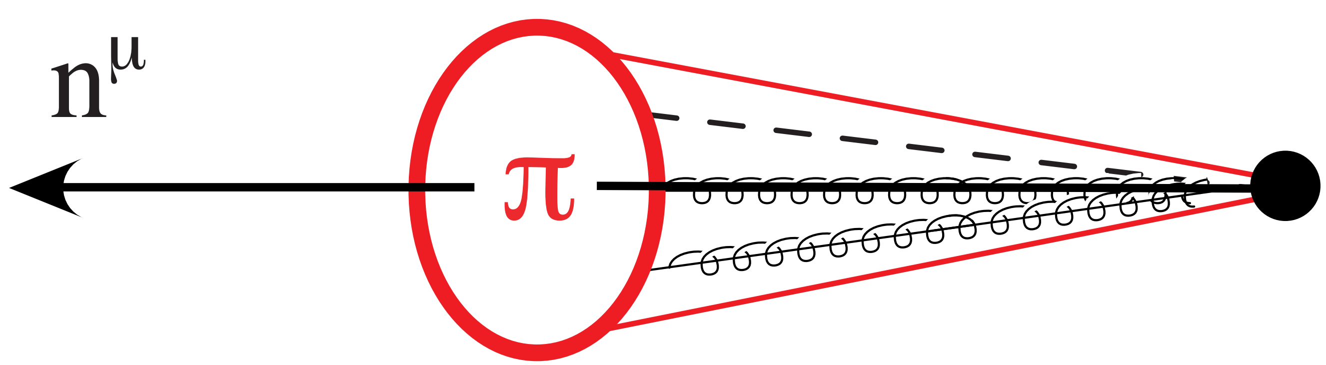
Soft constituents:

$$p_s^\mu = (p^+, p^-, p^\perp) \sim (\Lambda, \Lambda, \Lambda)$$



Collinear constituents:

$$p_c^\mu = (p^+, p^-, p^\perp) \sim \left(\frac{\Lambda^2}{Q}, Q, \Lambda\right) \sim Q(\lambda^2, 1, \lambda) \qquad \lambda = \frac{\Lambda}{Q}$$



Degrees of freedom in SCET

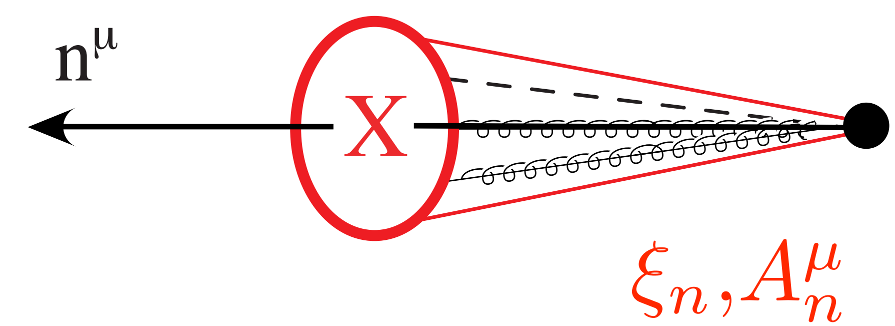
Introduce fields for infrared degrees of freedom (in operators)

modes	$p^\mu = (+, -, \perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us},\ A_{us}^\mu$

$\mathrm{SCET_I}$ $\longrightarrow$ Energetic jets $\quad \Lambda^2 \ll Q\Lambda \ll Q^2$

usoft $\quad p^\mu \sim \Lambda$

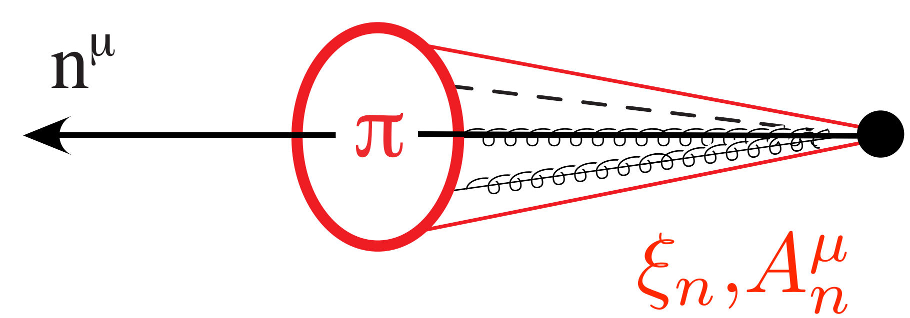
collinear $\quad p_c^2 \sim Q\Lambda, \quad \lambda = \sqrt{\Lambda/Q}$

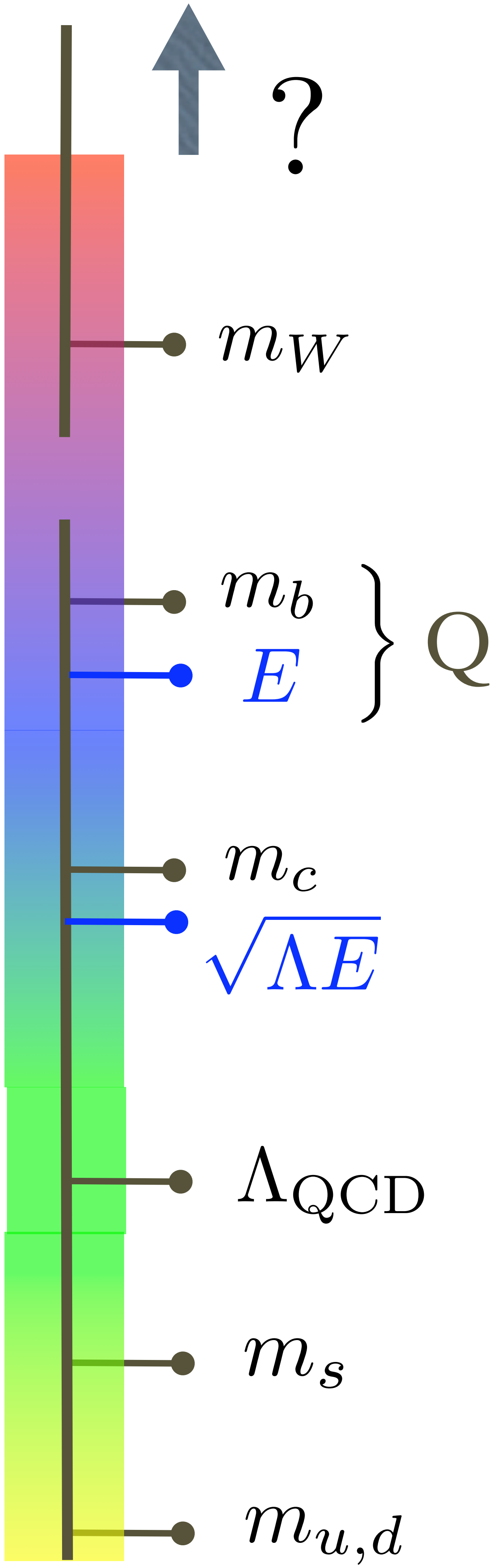


$\mathrm{SCET_{II}}$ $\longrightarrow$ Energetic hadrons

soft $\quad p^\mu \sim \Lambda$

collinear $\quad p_c^2 \sim \Lambda^2, \quad \lambda = \Lambda/Q$





Processes

$B \to D\pi$

$B \to X_u \ell \bar\nu$ $\quad$ $B \to X_s \gamma$ $\quad$ $B \to K^* \gamma$ $\quad$ $B \to \rho\gamma$

$B \to \pi \ell \bar\nu$ $\quad$ $B \to \rho\rho$ $\quad$ $B \to \pi\pi$

$B \to D^* \eta'$ $\quad$ $B \to K\pi$ $\quad$ $B \to \gamma \ell \bar\nu$

$B \to K\pi\gamma$ $\quad$ $B \to K\pi \ell^+ \ell^-$

$B \to \pi\gamma \ell \bar\nu$

$e^+ e^- \to J/\psi\, X$ $\quad$ $\Upsilon \to V\gamma$ $\quad$ $\Upsilon \to X\gamma$

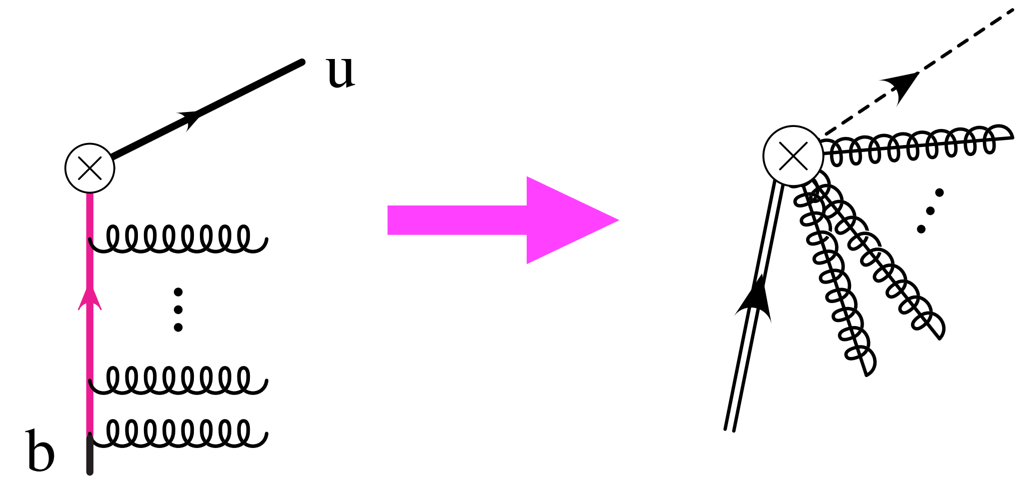
$e^- p \to e^- X$ $\quad$ $p\bar p \to \ell \bar\ell X$ $\quad$ $e^+ e^- \to \text{jets}$

$\gamma^* M_1 \to M_2$ $\quad$ $\gamma^* \gamma \to \pi^0$

Factorization

Factorization

- Separation of scales
  and Decoupling



eg. $\bar{u}\,\Gamma\,b$

$\Rightarrow$ $\bar{\xi}_n W\ \Gamma\ h_v$ — integrate out offshell quarks

$\Rightarrow$ $(\bar{\xi}_n W)\,\Gamma\,(Y^\dagger h_v)$ — usoft-collinear factorization (field redefn.)

$\Rightarrow$ $\int d\omega\, C(\omega)(\bar{\xi}_n W)_\omega\,\Gamma\,(Y^\dagger h_v)$ — hard-collinear factorization

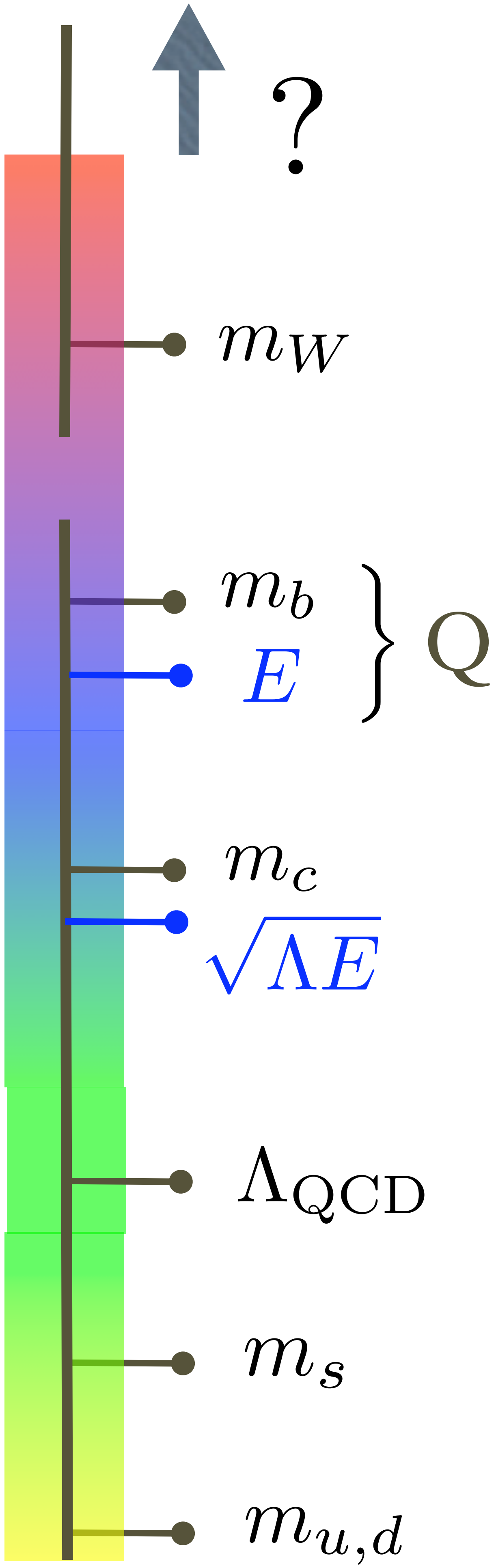
$$\omega \sim p_c^- \sim Q$$

- operators are gauge invariant,
  so factorization is too

$$W = P\exp\left(ig\int_{-\infty}^{y} ds\ \bar{n}\cdot A_n(s\bar{n}^\mu)\right)$$

$$S = P\exp\left(ig\int_{-\infty}^{y} ds\ n\cdot A_s(sn^\mu)\right)$$

$$Y = P\exp\left(ig\int_{-\infty}^{y} ds\ n\cdot A_{us}(sn^\mu)\right)$$

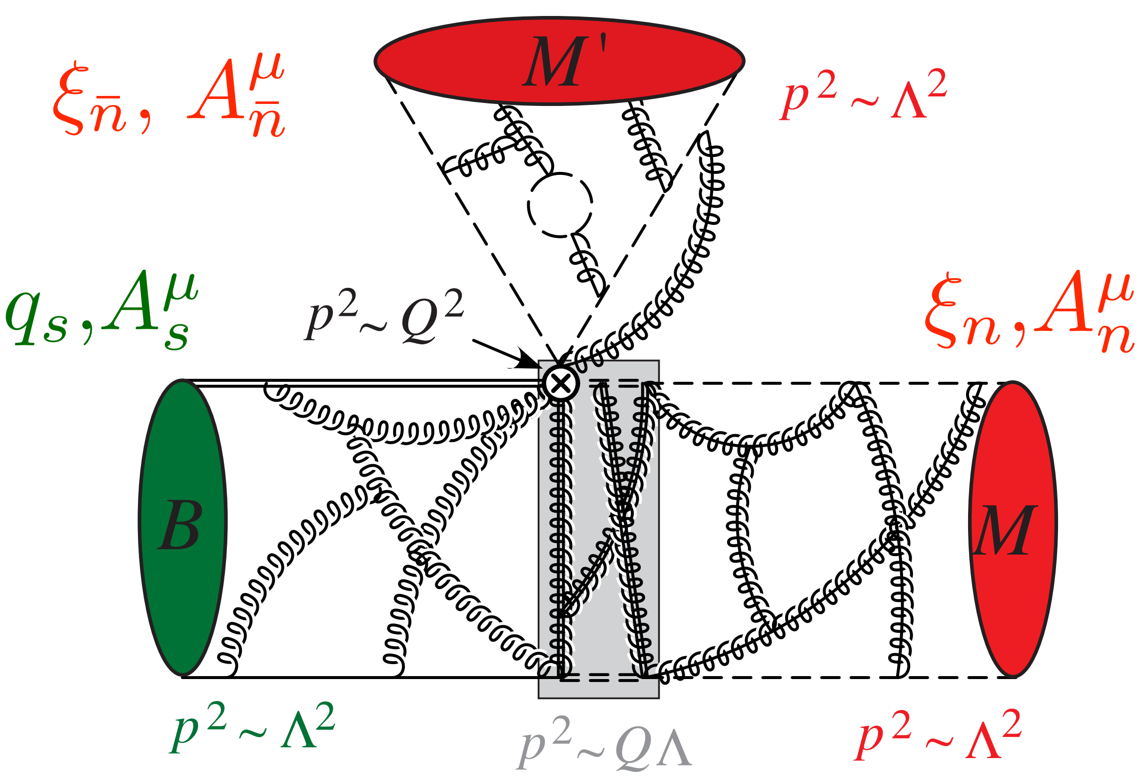


Factorization Theorems

eg. $E_\pi \gg \Lambda_{\rm QCD}$

$$A = \int dz\, dx_i\, dk^+ \underbrace{T(z)}\ \underbrace{J(z, x_i, k^+)}\ \underbrace{\phi_1(x_1)\phi_2(x_2)\phi_B(k^+)} + \ldots$$

$$Q^2 \gg E\Lambda \gg \Lambda^2$$



SCET$_{\mathrm{I}}$ Lagrangians

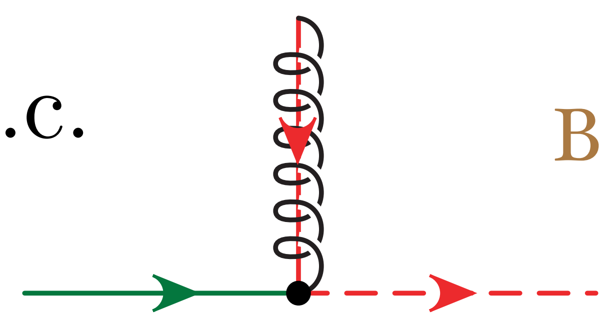
Expansion:

$$\mathcal{L}_c^{(0)} = \bar{\xi}_n \Big\{ n\cdot iD + i\not{D}_c^{\perp} W \frac{1}{\bar{\mathcal{P}}} W^\dagger i\not{D}_c^{\perp} \Big\} \frac{\not{\bar{n}}}{2} \xi_n$$

B.F.P.S.

$$\mathcal{L}_{us,s}^{(0)} = \bar{q}\, i\not{D}\, q$$

$$\mathcal{L}_{\xi q}^{(1)} = \bar{\xi}_n W \frac{1}{\bar{\mathcal{P}}} W^\dagger (ig\not{B}_c^{\perp}) W Y^\dagger q_{us} + \mathrm{h.c.}$$



B.C.D.F.

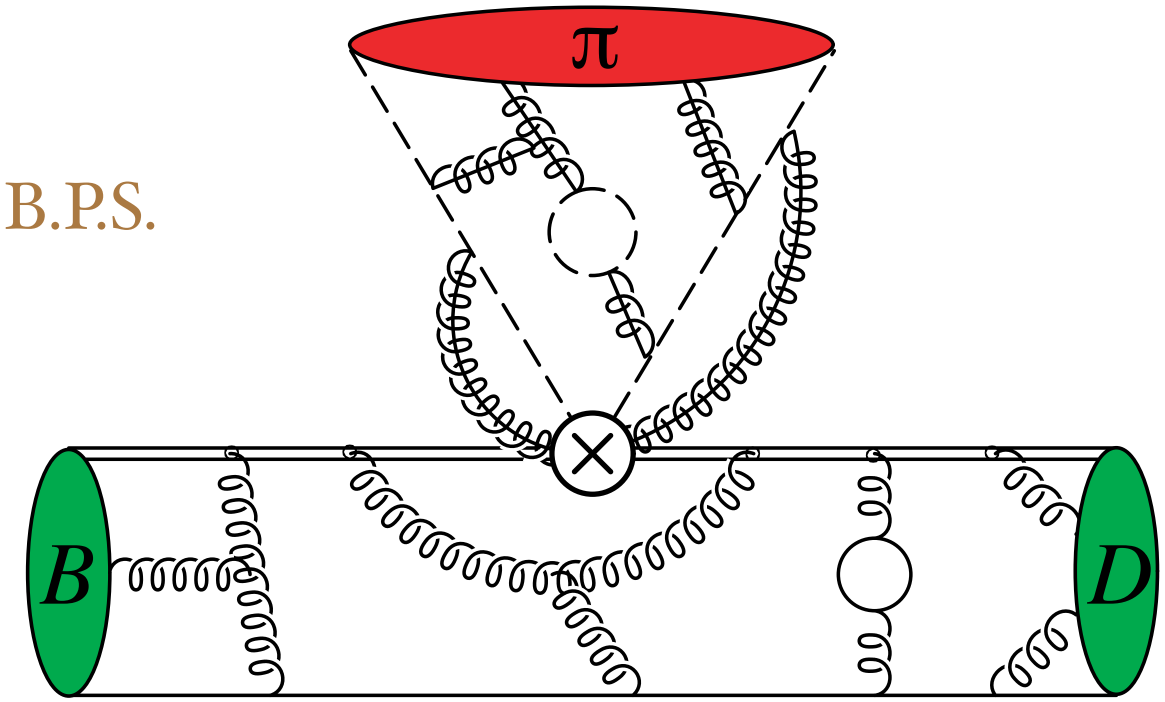
$\mathcal{L}^{(2)}$ known

- Same (subleading!) Lagrangians for all processes
- Many processes require subleading Lagrangians or they vanish

Factorization

- $\bar{B}^0 \to D^+\pi^-$, $B^- \to D^0\pi^-$

B.P.S.



B, D are soft, π collinear

$$\mathcal{L}_{\rm SCET} = \mathcal{L}_s^{(0)} + \mathcal{L}_c^{(0)}$$

Factorization if $\mathcal{O} = O_c \times O_s$

$$\langle D\pi|(\bar{c}b)(\bar{u}d)|B\rangle = N\, \xi(v\cdot v') \int_0^1 dx\, T(x,\mu)\, \phi_\pi(x,\mu)$$

Calculate T

- $\bar{B}^0 \to D^{(*)0}\pi^0$

Mantry, Pirjol, I.S.

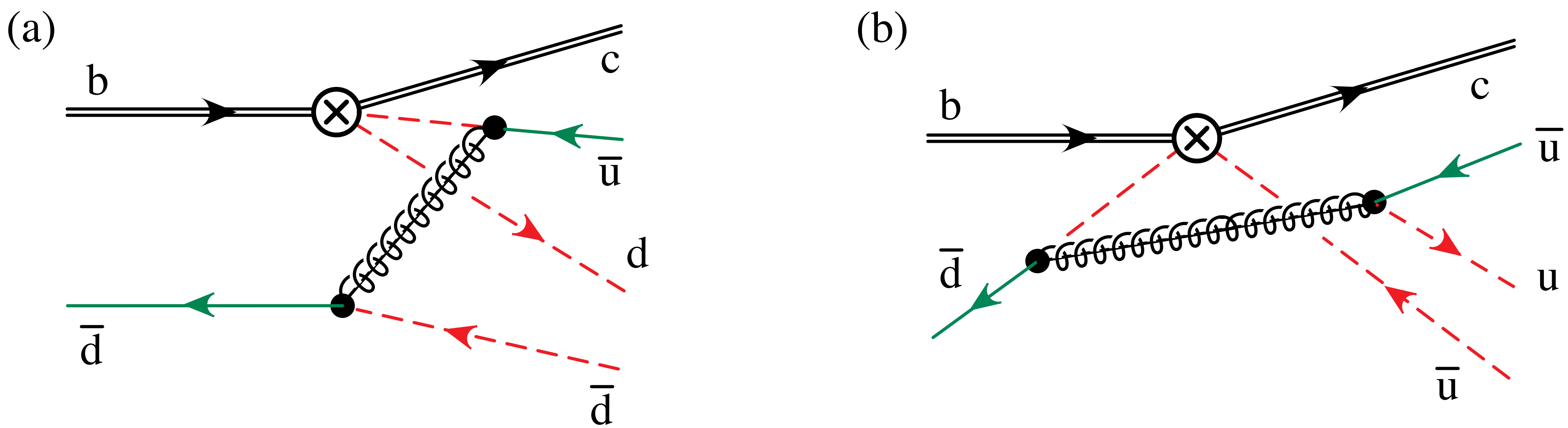
$$A_{00}^{D^{(*)}\pi} = N_0^{(*)} \int dx\, dz\, dk_1^+ dk_2^+\, T^{(i)}(z)\, J^{(i)}(z,x,k_1^+,k_2^+)\, S^{(i)}(k_1^+,k_2^+)\, \phi_\pi(x)$$
$$+ A_{\rm long}^{D^{(*)}\pi}$$

$\dfrac{\Lambda}{E_M}$ & $\dfrac{1}{N_c}$ suppressed

Color Suppressed Decays

- Factorization with SCET

Single class of power suppressed $\mathrm{SCET_I}$ operators $T\{\mathcal{O}^{(0)}, \mathcal{L}^{(1)}_{\xi q}, \mathcal{L}^{(1)}_{\xi q}\}$



Order $\lambda^2 = \left(\sqrt{\Lambda/E}\right)^2 = \Lambda/E$

$$A_{00}^{D^{(*)}\pi} = N_0^{(*)} \int dx\, dz\, dk_1^+ dk_2^+\, T^{(i)}(z)\, J^{(i)}(z, x, k_1^+, k_2^+)\, S^{(i)}(k_1^+, k_2^+)\, \phi_\pi(x)$$
$$+A_{\rm long}^{D^{(*)}\pi}$$

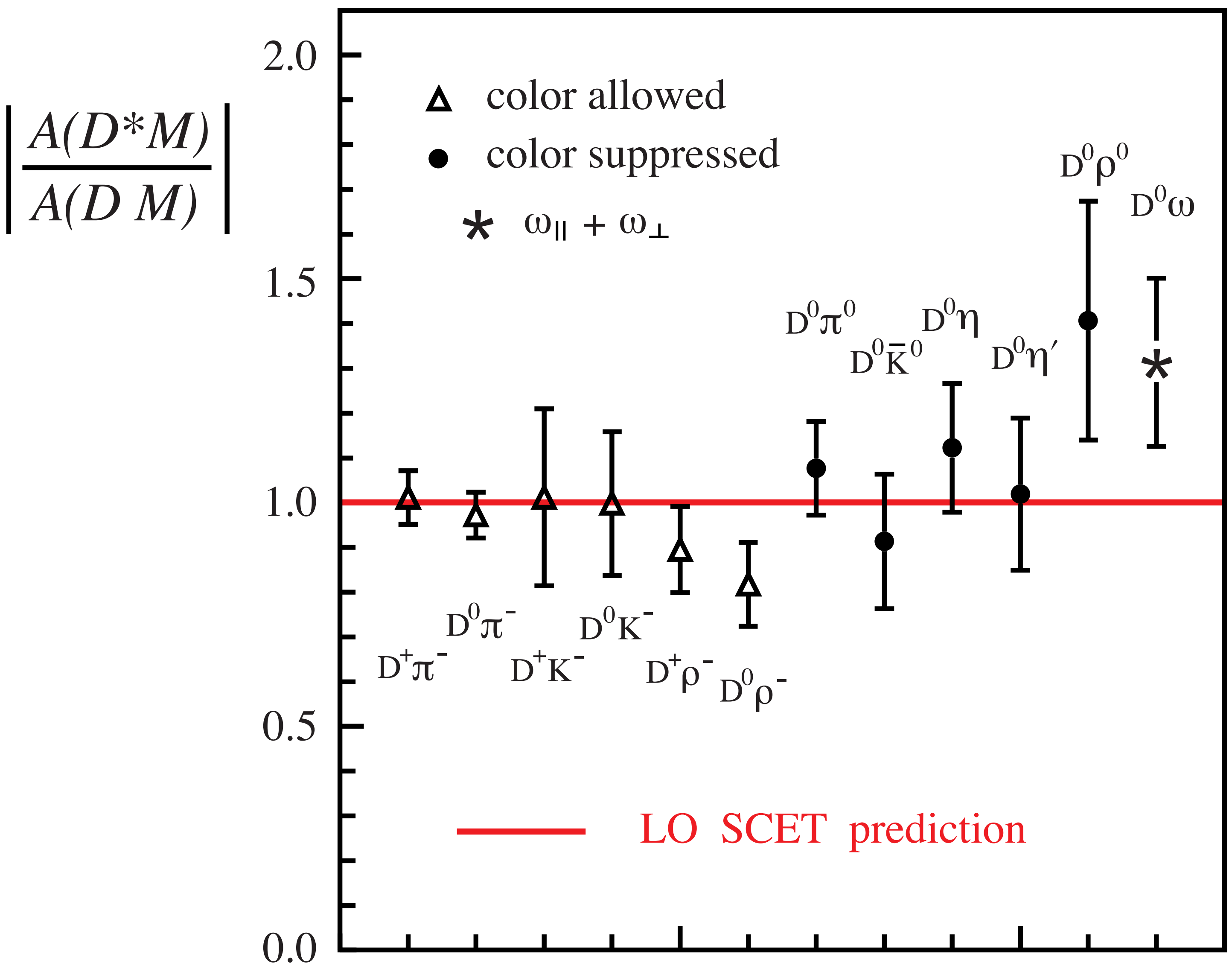
➡ same for D, D* up to $\alpha_s(m_b)$

- with HQET for $\langle D^{(*)0}\pi|(\bar{c}\, b)(\bar{d}\, u)|\bar{B}^0\rangle$ get $\frac{p_\pi^\mu}{m_c} \to \frac{E_\pi}{m_c} = 1.5$

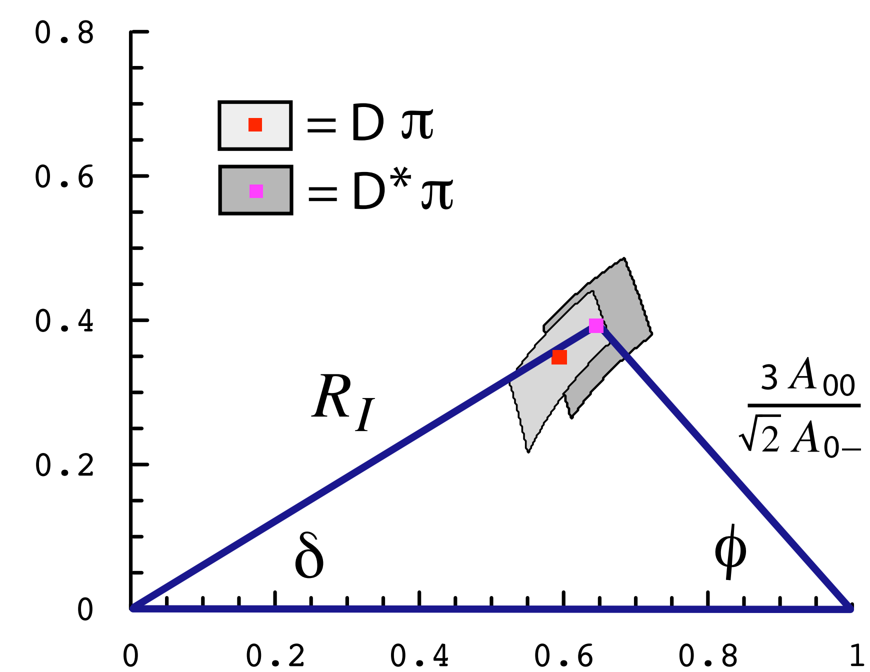
  not a convergent expansion

Expt Average (Cleo, Belle, Babar):

Extension to isosinglets:
Blechman, Mantry, I.S.



isospin triangle



$$\delta(D\pi) \;=\; 30.4 \pm 4.8^\circ$$
$$\delta(D^*\pi) \;=\; 31.0 \pm 5.0^\circ$$

Not yet tested:

- $Br(D^*\rho^0_{\parallel}) \gg Br(D^*\rho^0_{\perp})$, $Br(D^{*0}K^{*0}_{\parallel}) \sim Br(D^{*0}K^{*0}_{\perp})$
- equal ratios $D^{(*)}K^*$, $D_s^{(*)}K$, $D_s^{(*)}K^*$; triangles for $D^{(*)}\rho$, $D^{(*)}K$

Heavy to Light Currents

$b \to u$
$b \to s$

$$
\begin{aligned}
J^{(0)}(\omega) &= \bar{\chi}_{n,\omega} \Gamma \mathcal{H}_v \,, \\
J^{(1a)}(\omega) &= \frac{1}{\omega}\, \bar{\chi}_{n,\omega} \mathcal{P}_\alpha^{\perp\dagger} \Theta^\alpha_{(a)} \mathcal{H}_v \\
J^{(1b)}(\omega_{1,2}) &= \frac{1}{m}\, \bar{\chi}_{n,\omega_1} (ig\mathcal{B}^\perp_\alpha)_{\omega_2} \Theta^\alpha_{(b)} \mathcal{H}_v \,.
\end{aligned}
$$

$$
\begin{aligned}
J^{(2a)}(\omega) &= \frac{1}{2m}\, \bar{\chi}_{n,\omega} \Upsilon^\sigma_{(a)} i\mathcal{D}^T_{us\ \sigma} \mathcal{H}_v \,, \\
J^{(2b)}(\omega) &= -\frac{n\cdot v}{\omega}\, \bar{\chi}_{n,\omega}\, i\bar{n}\cdot \overleftarrow{\mathcal{D}}_{us} \Upsilon_{(b)} \mathcal{H}_v \,, \\
J^{(2c)}(\omega) &= -\frac{1}{\omega}\, \bar{\chi}_{n,\omega} i\overleftarrow{\mathcal{D}}^\perp_{us\,\alpha}\, \Upsilon^\alpha_{(c)} \mathcal{H}_v \,, \\
J^{(2d)}(\omega) &= \frac{1}{\omega^2}\, \bar{\chi}_{n,\omega} \mathcal{P}_\alpha^{\perp\dagger} \mathcal{P}_\beta^{\perp\dagger} \Upsilon^{\alpha\beta}_{(d)} \mathcal{H}_v \,, \\
J^{(2e)}(\omega_{1,2}) &= \frac{1}{m(\omega_1+\omega_2)}\, \bar{\chi}_{n,\omega_1} (ig\mathcal{B}^\perp_\alpha)_{\omega_2} \mathcal{P}_\beta^{\perp\dagger} \Upsilon^{\alpha\beta}_{(e)} \mathcal{H}_v \,,
\end{aligned}
$$

Wilson coefficients and Dirac structures completely determined by RPI

$$
\begin{aligned}
J^{(2f)}(\omega_{1,2}) &= \frac{\omega_2}{m(\omega_1+\omega_2)}\, \bar{\chi}_{n,\omega_1} \left(\frac{\mathcal{P}_\beta^\perp}{\omega_2} + \frac{\mathcal{P}_\beta^{\perp\dagger}}{\omega_1} \right) (ig\mathcal{B}^\perp_\alpha)_{\omega_2} \Upsilon^{\alpha\beta}_{(f)} \mathcal{H}_v \,, \\
J^{(2g)}(\omega_{1,2}) &= \frac{1}{m\, n\cdot v}\, \bar{\chi}_{n,\omega_1} \Big\{ (ign\cdot\mathcal{B})_{\omega_2} + 2(ig\mathcal{B}_\perp)_{\omega_2}\cdot \mathcal{P}_\perp^\dagger \frac{1}{\bar{\mathcal{P}}^\dagger} \Big\} \Upsilon_{(g)} \mathcal{H}_v \,, \\
J^{(2h)}(\omega_{1,2,3}) &= \frac{1}{m(\omega_2+\omega_3)}\, \bar{\chi}_{n,\omega_1} (ig\mathcal{B}^\perp_\beta)_{\omega_2} (ig\mathcal{B}^\perp_\alpha)_{\omega_3} \Upsilon^{\alpha\beta}_{(h)} \mathcal{H}_v \,, \\
J^{(2i)}(\omega_{1,2,3}) &= \frac{1}{m(\omega_2+\omega_3)} \mathrm{Tr}\big[(ig\mathcal{B}^\perp_\beta)_{\omega_2} (ig\mathcal{B}^\perp_\alpha)_{\omega_3}\big]\, \bar{\chi}_{n,\omega_1} \Upsilon^{\alpha\beta}_{(i)} \mathcal{H}_v \,.
\end{aligned}
$$

& four quark operators

What's new?

one-loop matching & running for $J^{(1b)}$

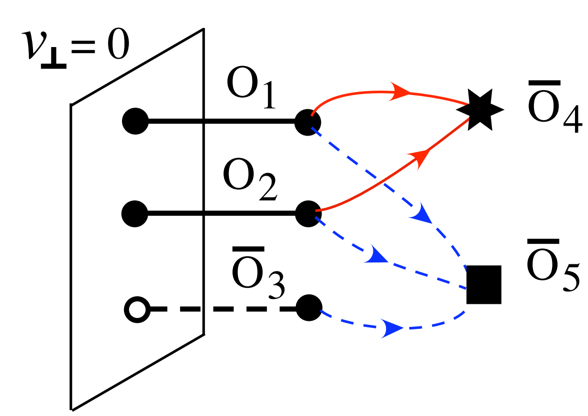
Beneke, Kiyo, Yang
Becher, Hill, Neubert

complete basis of $J^{(2)}$ operators is known

Beneke, Campanario, Mannel, Pecjak

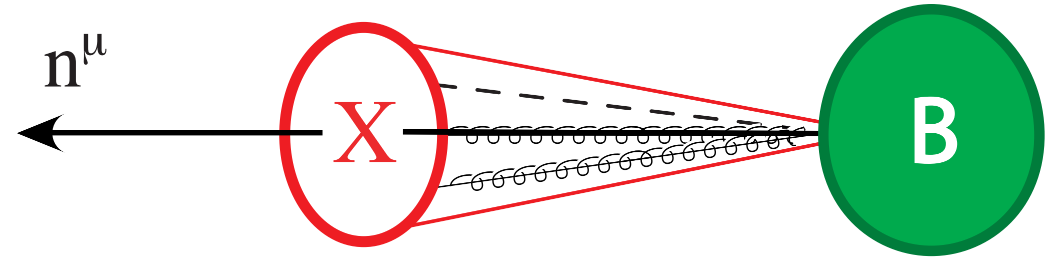
incl. Dirac structures and RPI constraints

Arnesen, Kundu, I.S.



Inclusive B-Decays

Inclusive Decays



$m_X^2 \sim m_b \Lambda$

$P_X^- \gg P_X^+$

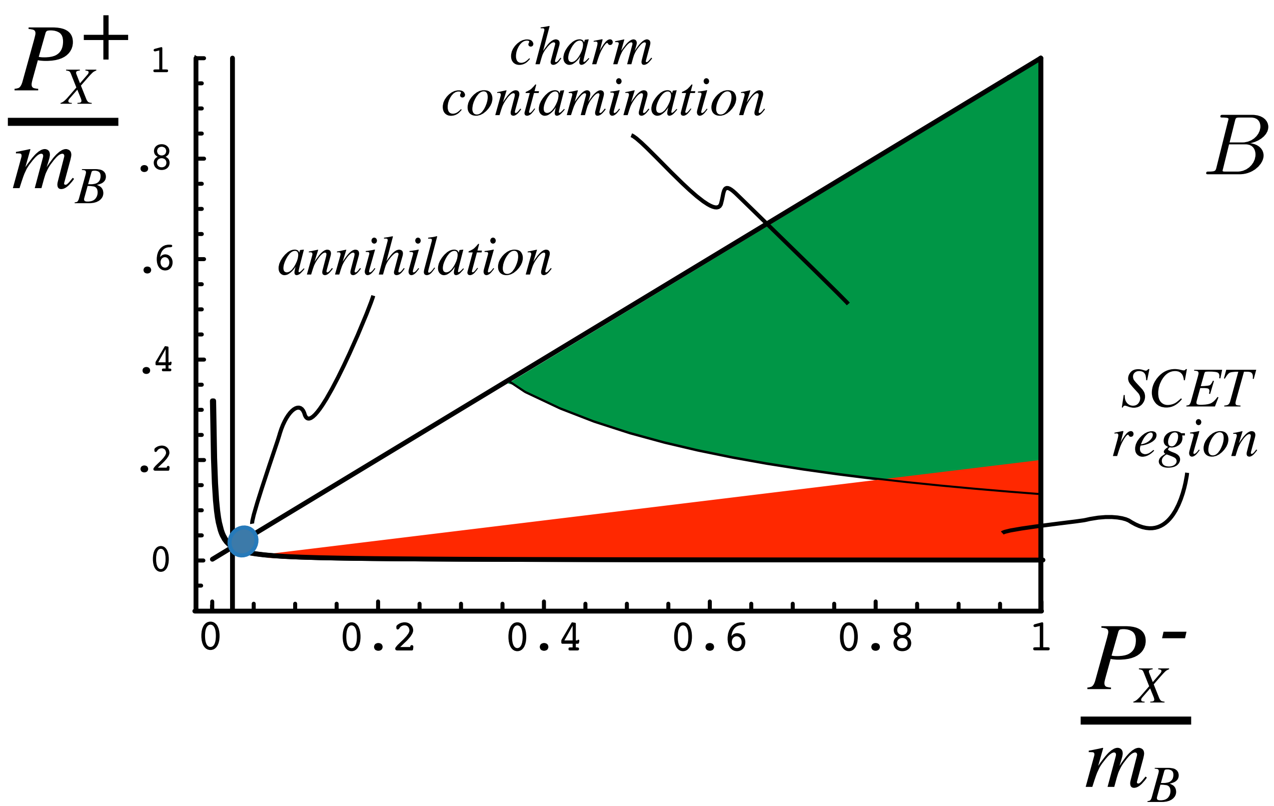
endpt. region

$$d\Gamma = H(m_b, p_X^-) \int dk^+ \, J(p_X^- k^+) \, f(k^+ + \overline{\Lambda} - p_X^+)$$

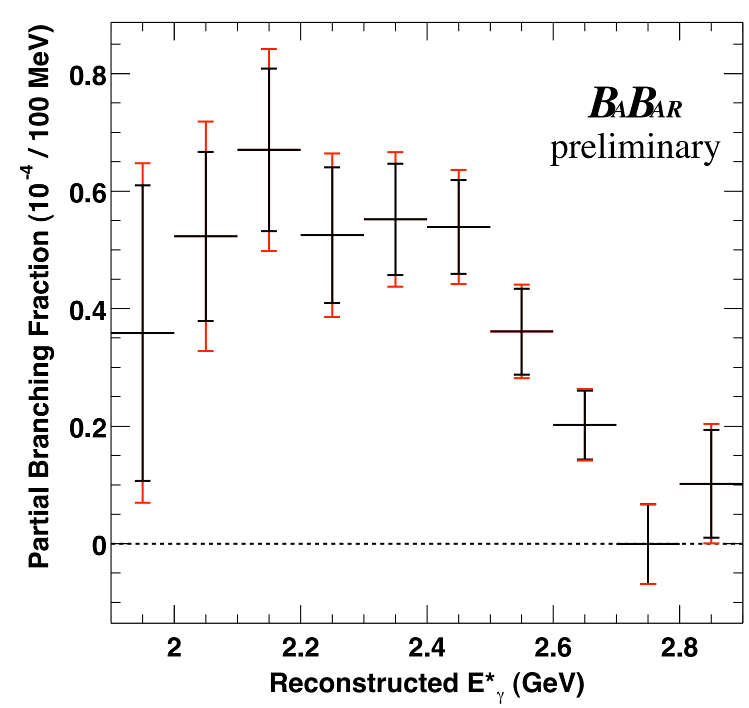
What's new? eg. :

- Event generator — Neubert, Lange, Paz
- Subleading shape functions — Keith Lee, I.S.; Bosch et al.; Beneke et al.

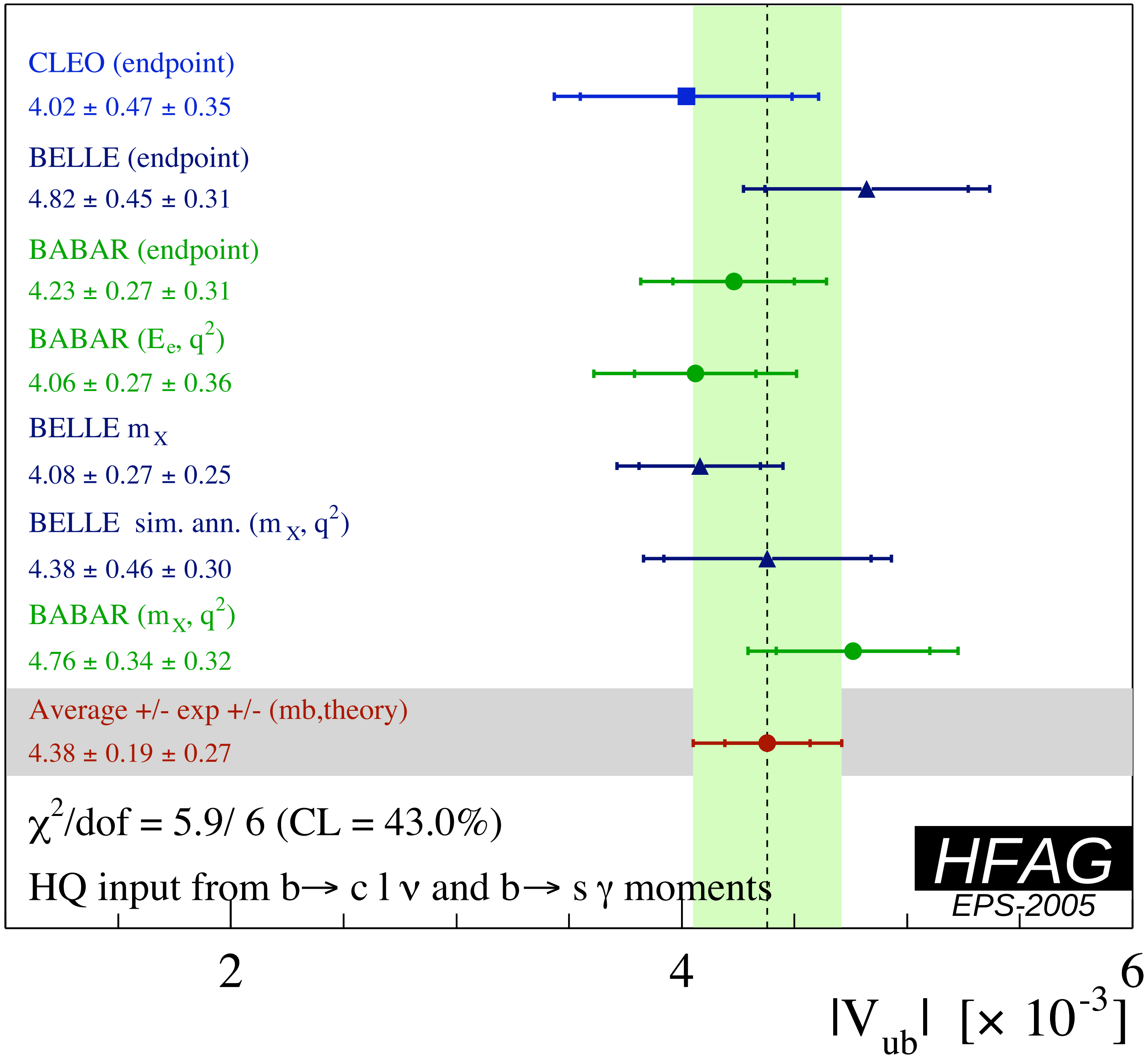
$B \to X_u e \bar{\nu}$



$B \to X_s \gamma$



$$|V_{ub}|^{\mathrm{incl}} = (4.38 \pm 0.33) \times 10^{-3}$$



Factorization at NLO

- derive factorization theorems at subleading order
- complete categorization of all terms at $\dfrac{\Lambda}{m_b}$
- all orders in α_s

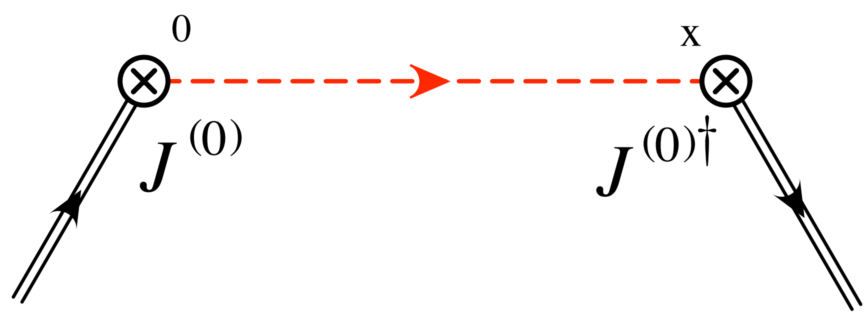
$$J = J^{(0)} + J^{(1)} + J^{(2)} + \dots$$
$$\mathcal{L} = \mathcal{L}_c^{(0)} + \mathcal{L}_{us}^{(0)} + \mathcal{L}_{\xi q}^{(1)} + \mathcal{L}_j^{(1)} + \mathcal{L}_j^{(2)} + \dots$$

LO: $T\{J^{(0)}, J^{(0)\dagger}\}$

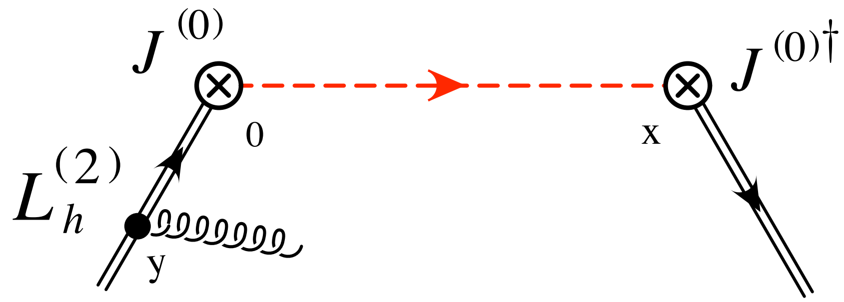
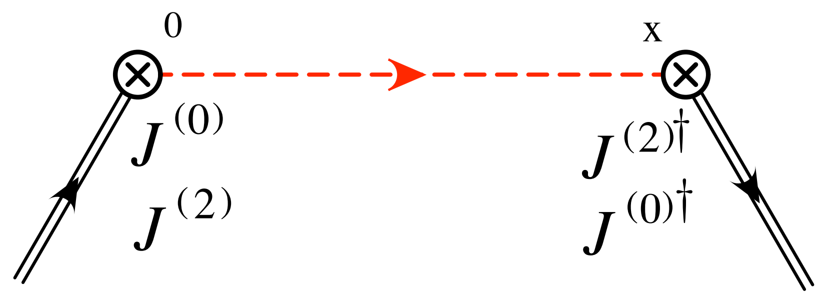
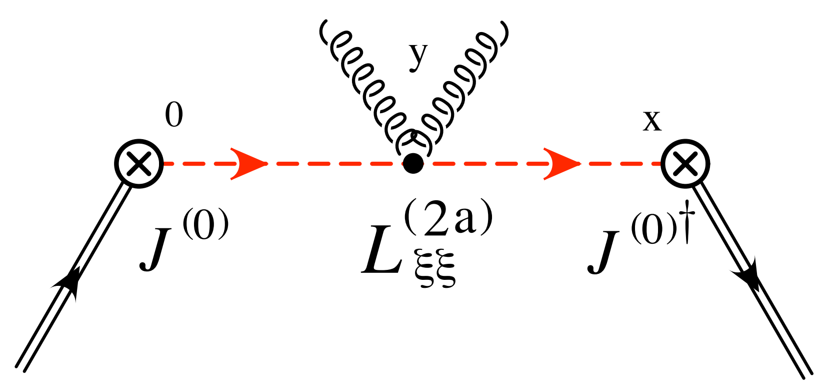
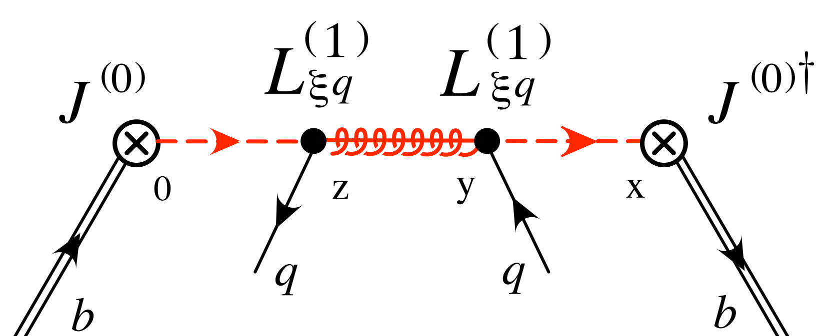
zero: $T\{J^{(0)}, J^{(1)\dagger}\} + \text{h.c.} + T\{J^{(0)}, \mathcal{L}^{(1)}, J^{(0)}\}$

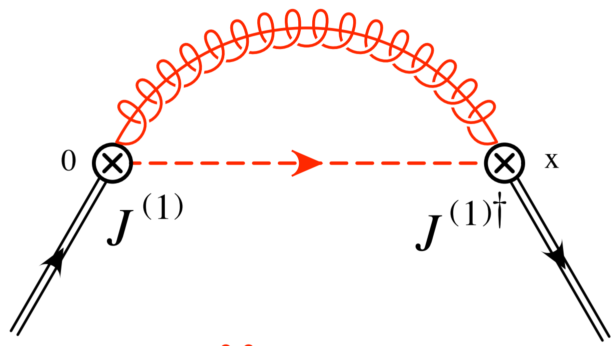
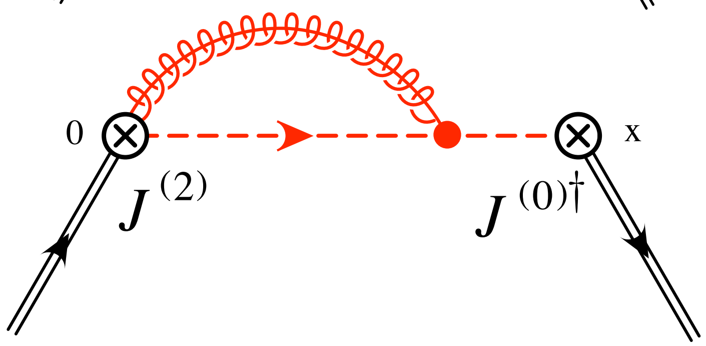
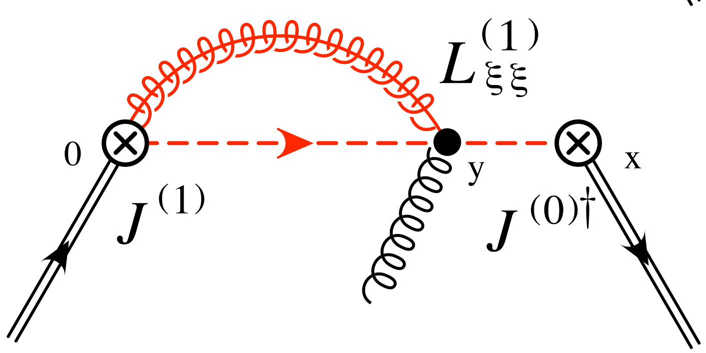
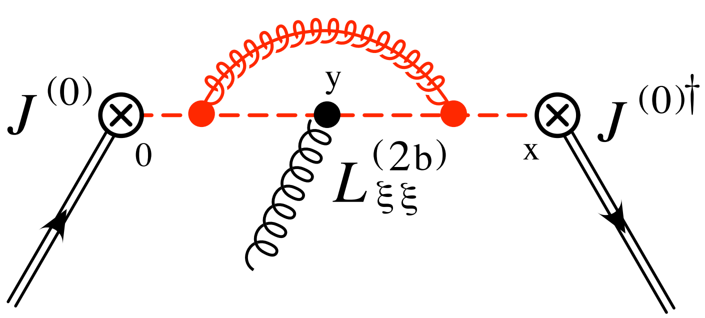
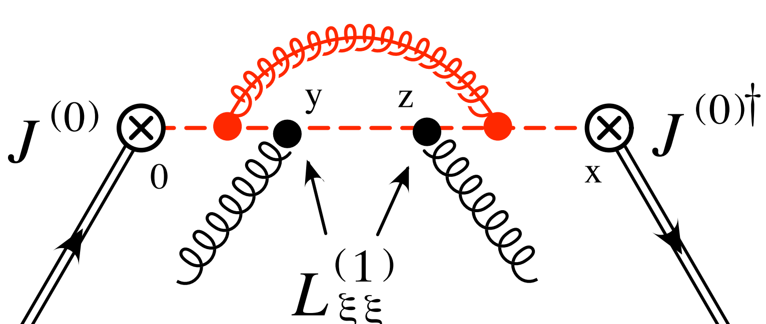
NLO: $T\{J^{(0)}, J^{(2)\dagger}\} + \text{h.c.} + T\{J^{(1)}, J^{(1)\dagger}\}$
$+T\{J^{(0)}, \mathcal{L}^{(1)}, \mathcal{L}^{(1)}, J^{(0)\dagger}\} + \dots$

Leading Order

T-product	Example Diagram	Hard, Jet, and Shape Functions	Usoft Operator
$T^{(0)}$		$h^{[0]}\, \mathcal{J}^{(0)}\, f^{(0)}$	$\bar{h}_v(x) h_v(0)$

Next to Leading Order

T-product	Example Diagram	Hard, Jet, and Shape Functions	Usoft Operator
$\hat{T}^{(2H)}$		$h^0 \mathcal{J}^{(0)} f_0^{(2)}$	$\bar{h}_v(x) h_v(0) i\mathcal{L}_{\rm h}^{(2)}(y)$
$\hat{T}^{(2a)}$		$h^{1,2} \mathcal{J}^{(0)} f_{1,2}^{(2)}$	$\bar{h}_v(x)(D_{T,\perp} h_v)(0)$ $(\bar{h}_v D_{T,\perp})(x) h_v(0)$
$\hat{T}^{(2L)}$		$h^{3,4}\, \mathcal{J}_{1,2}^{(-2)}\, f_{3,4}^{(4)}$ $h^{3,4}\, \mathcal{J}_{3,4}^{(-2)}\, g_{3,4}^{(4)}$	$\bar{h}_v(x)(D_\perp D_\perp)(y) h_v(0)$
$\hat{T}^{(2q)}$		$h^{5,6}\, \mathcal{J}_1^{(-4)}\, f_{5,6}^{(6)}$ $h^{5-8}\, \mathcal{J}_{2-4}^{(-4)}\, g_{5-10}^{(6)}$	$\bar{h}_v(x) q(y) \bar{q}(z) h_v(0)$

T-product	Example Diagram	Hard, Jet, and Shape Functions	Usoft Operator
$\hat{T}^{(2b)}$		$h^{[2b]}\,\mathcal{J}^{(2)}_{1,2}\,f^{(0)}$	$\bar{h}_v(x)h_v(0)$
$\hat{T}^{(2c)}$		$h^{[2c]}\,\mathcal{J}^{(2)}_{3-10}\,f^{(0)}$	$\bar{h}_v(x)h_v(0)$
$\hat{T}^{(2La)}$		$h^{[2La]}\,\mathcal{J}^{(0)}_{j'}\,g^{(2)}_{11,12}$	$\bar{h}_v(x)D_\perp(y)h_v(0)$
$\hat{T}^{(2Lb)}$		$h^{[2Lb]}\,\mathcal{J}^{(0)}_{j'}\,g^{(2)}_{13,14}$	$\bar{h}_v(x)\bar{n}\cdot D(y)h_v(0)$
$\hat{T}^{(2LL)}$		$h^{[2LL]}\,\mathcal{J}^{(-2)}_{j'}\,g^{(4)}_{15-26}$	$\bar{h}_v(x)D_\perp(y)D_\perp(z)h_v(0)$

$$W_i^{(2)} =$$

(triple differential spectra)

$$
\begin{aligned}
&\frac{h_i^{0f}(\bar n\!\cdot\! p)}{2m_b}\int_0^{p_X^+} dk^+\, \mathcal{J}^{(0)}(\bar n\!\cdot\! p\, k^+,\mu)\, f_0^{(2)}(k^+ + r^+,\mu) \\
&+ \sum_{r=1}^{2} \frac{h_i^{rf}(\bar n\!\cdot\! p)}{m_b}\int_0^{p_X^+} dk^+\, \mathcal{J}^{(0)}(\bar n\!\cdot\! p\, k^+,\mu)\, f_r^{(2)}(k^+ + r^+,\mu) \\
&+ \sum_{r=3}^{4} \frac{h_i^{rf}(\bar n\!\cdot\! p)}{m_b}\int dk_1^+\, dk_2^+\, \mathcal{J}_{1\pm 2}^{(-2)}(\bar n\!\cdot\! p\, k_j^+,\mu)\, f_r^{(4)}(k_j^+ + r^+,\mu) \\
&+ \sum_{r=5}^{6} \frac{h_i^{rf}(\bar n\!\cdot\! p)}{\bar n\!\cdot\! p}\int dk_1^+ dk_2^+ dk_3^+\, \mathcal{J}_1^{(-4)}(\bar n\!\cdot\! p\, k_{j'}^+,\mu)\, f_r^{(6)}(k_{j'}^+ + r^+,\mu) \\
&+ \frac{h_i^{00f}(\bar n\!\cdot\! p)}{m_b}\int_0^{p_X^+} dk^+\, \mathcal{J}^{(0)}(\bar n\!\cdot\! p\, k^+,\mu)\, g_0^{(2)}(k^+ + r^+,\mu) \\
&+ \sum_{r=3}^{4} \frac{h_i^{rf}(\bar n\!\cdot\! p)}{m_b}\int dk_1^+\, dk_2^+\, \mathcal{J}_{3\pm 4}^{(-2)}(\bar n\!\cdot\! p\, k_j^+,\mu)\, g_r^{(4)}(k_j^+ + r^+,\mu) \\
&+ \sum_{r=5}^{6} \frac{h_i^{rf}(\bar n\!\cdot\! p)}{\bar n\!\cdot\! p}\int dk_1^+ dk_2^+ dk_3^+\, \mathcal{J}_2^{(-4)}(\bar n\!\cdot\! p\, k_{j'}^+,\mu)\, g_r^{(6)}(k_{j'}^+ + r^+,\mu) \\
&+ \sum_{r=7}^{8} \frac{h_i^{rf}(\bar n\!\cdot\! p)}{\bar n\!\cdot\! p}\int dk_1^+ dk_2^+ dk_3^+\, \Big[\mathcal{J}_3^{(-4)}(\bar n\!\cdot\! p\, k_{j'}^+,\mu)\, g_r^{(6)}(k_{j'}^+ + r^+,\mu) \\
&\qquad\qquad + \mathcal{J}_4^{(-4)}(\bar n\!\cdot\! p\, k_{j'}^+,\mu)\, g_{r+2}^{(6)}(k_{j'}^+ + r^+,\mu)\Big] \\
&+ \sum_{m=1,2}\int dz_1 dz_2\, \frac{h_i^{[2b]m+8}(z_1,z_2,\bar n\!\cdot\! p)}{m_b}\int_0^{p_X^+} dk^+\, \mathcal{J}_m^{(2)}(z_1,z_2,p_X^-\, k^+)\, f^{(0)}(k^+ + \bar\Lambda - p_X^+) \\
&+ \sum_{m=3,4} \frac{h_i^{[2c]m+8}(\bar n\!\cdot\! p)}{m_b}\int_0^{p_X^+} dk^+\, \mathcal{J}_m^{(2)}(p_X^-\, k^+)\, f^{(0)}(k^+ + \bar\Lambda - p_X^+) \\
&+ \sum_{m=5}^{10}\int dz_1\, \frac{h_i^{[2c]m+8}(z_1,\bar n\!\cdot\! p)}{m_b}\int_0^{p_X^+} dk^+\, \mathcal{J}_m^{(2)}(z_1,p_X^-\, k^+)\, f^{(0)}(k^+ + \bar\Lambda - p_X^+) \\
&+ W_i^{[2La]f}[g_{11,12}^{(2)}] + W_i^{[2Lb]f}[g_{13,14}^{(2)}] + W_i^{[2LL]f}[g_{15-26}^{(4)}] + W_i^{[2Ga]f}[f_{3,4}^{(4)}]
\end{aligned}
$$

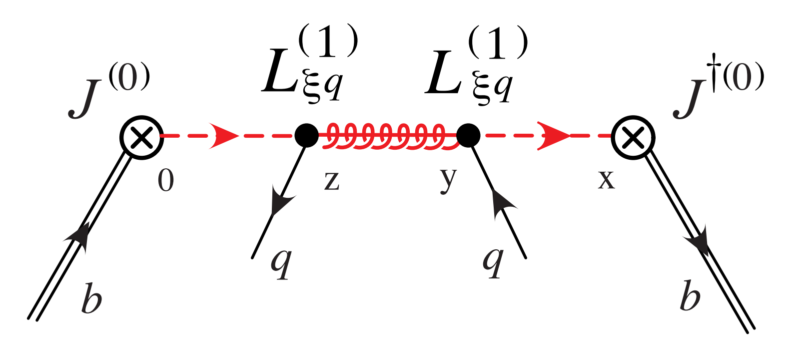
+ phase space & kinematic corrections

$h_i(\bar n\cdot p)$: $\alpha_s(m_b^2)$

$\mathcal{J}(\bar n\cdot p k_j^+)$: $\alpha_s(m_X^2) \sim \alpha_s(m_b\Lambda)$

A brick wall: $\alpha_s \dfrac{\Lambda}{m_b}$

- keep $\dfrac{\Lambda}{m_b}$ and $4\pi\alpha_s \dfrac{\Lambda}{m_b}$



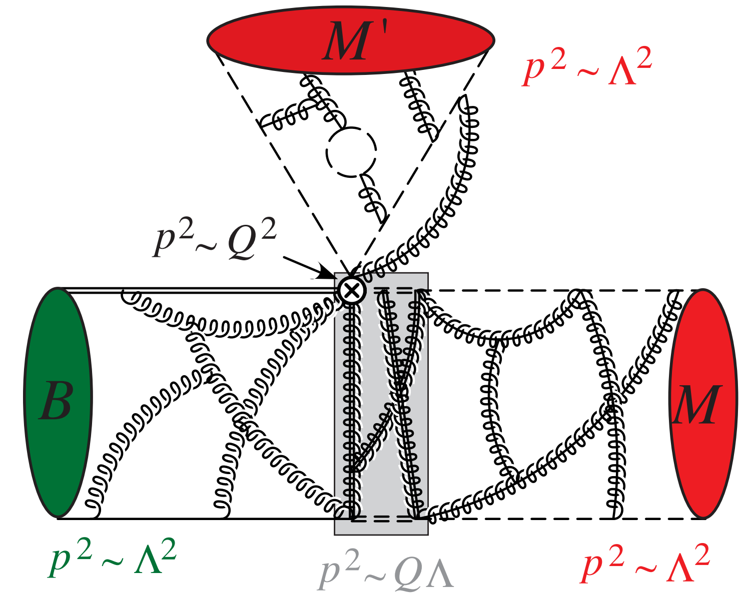
- model these subleading shape functions to get uncertainties

(& interpolate to local OPE)

$$B \to \pi\pi\,,\ B \to \pi\ell\bar{\nu}$$

$$\& \quad |V_{ub}|$$

Factorization (with SCET)



Factorization at m_b

Bauer, Pirjol, Rothstein, I.S.

Nonleptonic $B \to M_1 M_2$

$$A(B \to M_1 M_2) = A^{c\bar{c}} + N\left\{ f_{M_2} \zeta^{BM_1} \int du\, T_{2\zeta}(u) \phi^{M_2}(u) + f_{M_2} \int du dz\, T_{2J}(u,z) \zeta_J^{BM_1}(z) \phi^{M_2}(u) + (1 \leftrightarrow 2) \right\}$$

Form Factors

$B \to$ pseudoscalar: $f_+,\ f_0,\ f_T$

$B \to$ vector: $V,\ A_0,\ A_1,\ A_2,\ T_1,\ T_2,\ T_3$

$$f(E) = \int dz\, T(z,E)\, \zeta_J^{BM}(z,E)$$ } "hard spectator", "factorizable"

$$+\ C(E)\, \zeta^{BM}(E)$$ } "soft form factor", "non-factorizable"

universality at $E\Lambda$

Factorization at $\sqrt{E\Lambda}$

expansion in $\alpha_s(\sqrt{E\Lambda})$

$$\zeta_J^{BM}(z) = f_M f_B \int_0^1 dx \int_0^\infty dk^+\, J(z,x,k^+,E)\, \phi_M(x)\, \phi_B(k^+)$$

$\zeta^{BM} = ?$ (left as a form factor)

Beneke, Feldmann
Bauer, Pirjol, I.S.
Becher, Hill, Lange, Neubert

$Br(B \to \pi^0\pi^0) = 1.45 \pm 0.29$ is large

expected ~ 0.3

NOT a contradiction with factorization.

Why?

$\simeq 3$

- if $\zeta_J^{B\pi} \sim \zeta^{B\pi}$, then a term $\frac{C_1}{N_c}\langle \bar{u}^{-1}\rangle_\pi \, \zeta_J^{B\pi}$ in the factorization theorem ruins color suppression and explains the rate

  if $\zeta^{B\pi} \gg \zeta_J^{B\pi}$ this Br is sensitive to power corrections (small wilson coeffs. at LO could compete with larger ones at subleading order) .

- In the future: determine parameters using improved data on the $B \to \pi\ell\bar{\nu}$ form factor at low q^2 to provide a check.

Use nonleptonic data: $B \to \pi\pi$ determines the parameters

$$|V_{ub}| f_+(0) = F(S_{\pi^+\pi^-}, C_{\pi^+\pi^-}, Br(\pi^+\pi^-), Br(\pi^0\pi^-), \beta, \gamma, V_{ud}) \quad \left[1 + \mathcal{O}\left(\alpha_s(m_b), \frac{\Lambda_{\rm QCD}}{E}\right)\right]$$

- Uses data to remove arbitrary complex penguin amplitude, and color suppressed amplitude. ie. to eliminate the hadronic parameters

Bauer, Pirjol, Rothstein, I.S.

$$|V_{ub}| f_+(0) = \left[\frac{64\pi}{m_B^3 f_\pi^2} \frac{\overline{B}r(B^- \to \pi^0\pi^-)}{\tau_{B^-}|V_{ud}|^2 G_F^2}\right]^{1/2} \times \left[\frac{(C_1+C_2)t_c - C_2}{C_1^2 - C_2^2}\right]\left[1 + \mathcal{O}\left(\alpha_s(m_b), \frac{\Lambda_{\rm QCD}}{m_b}\right)\right],$$

$$t_c = \frac{|T_{\pi\pi}|}{|T_{\pi\pi} + C_{\pi\pi}|}$$

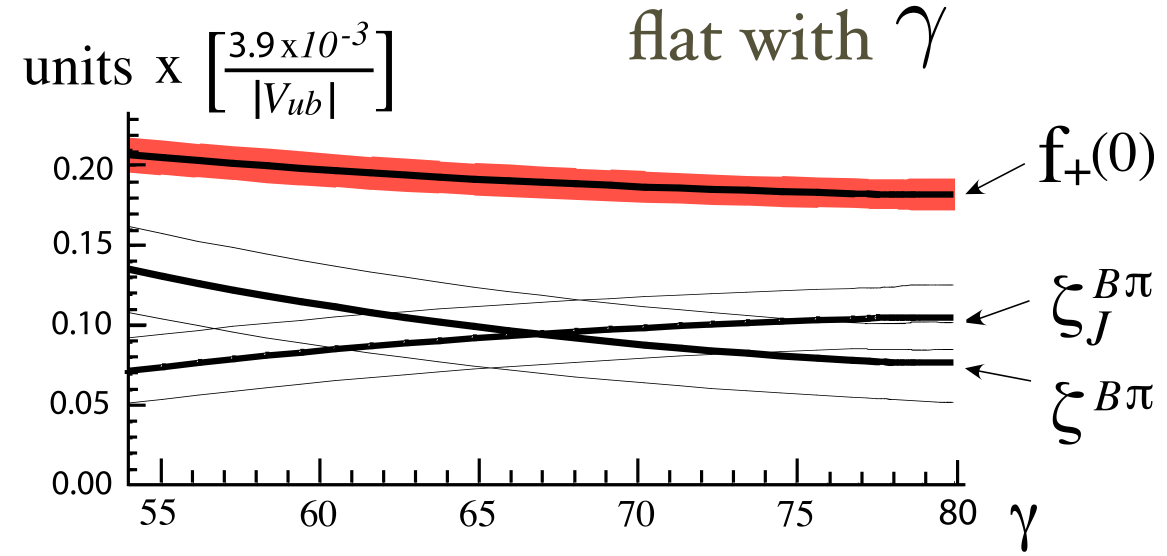
$$t_c = \sqrt{\overline{R}_c \, \frac{(1 + B_{\pi^+\pi^-} \cos 2\beta + S_{\pi^+\pi^-} \sin 2\beta)}{2\sin^2\gamma}}$$

$$\overline{R}_c = \frac{Br(B^0 \to \pi^+\pi^-)\tau_{B^-}}{2Br(B^- \to \pi^0\pi^-)\tau_{B^0}}$$

$$B_{\pi^+\pi^-} = \sqrt{1 - C_{\pi^+\pi^-}^2 - S_{\pi^+\pi^-}^2}$$

$$f_+(0) = \zeta^{B\pi} + \zeta_J^{B\pi}$$

flat with γ



Factorization & $B \to \pi\pi$ determines $|V_{ub}| f_+(0)$

Current Data

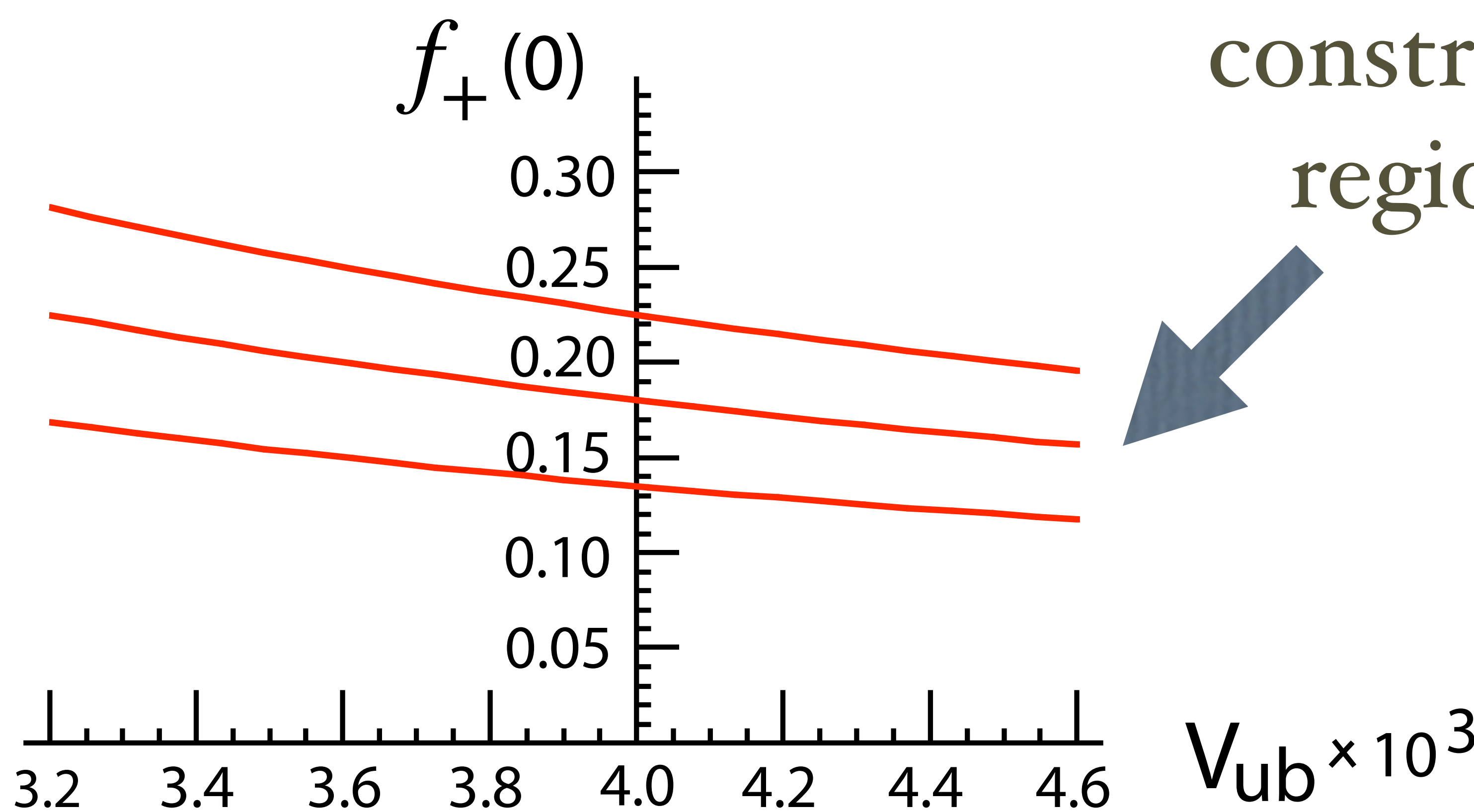
$$f_+(0) = (0.18 \pm 0.01 \pm 0.04)\left(\frac{3.9 \times 10^{-3}}{|V_{ub}|}\right)$$

expt. theory

$$|V_{ub}| f_+(0) = (7.2 \pm 1.8) \times 10^{-4}$$

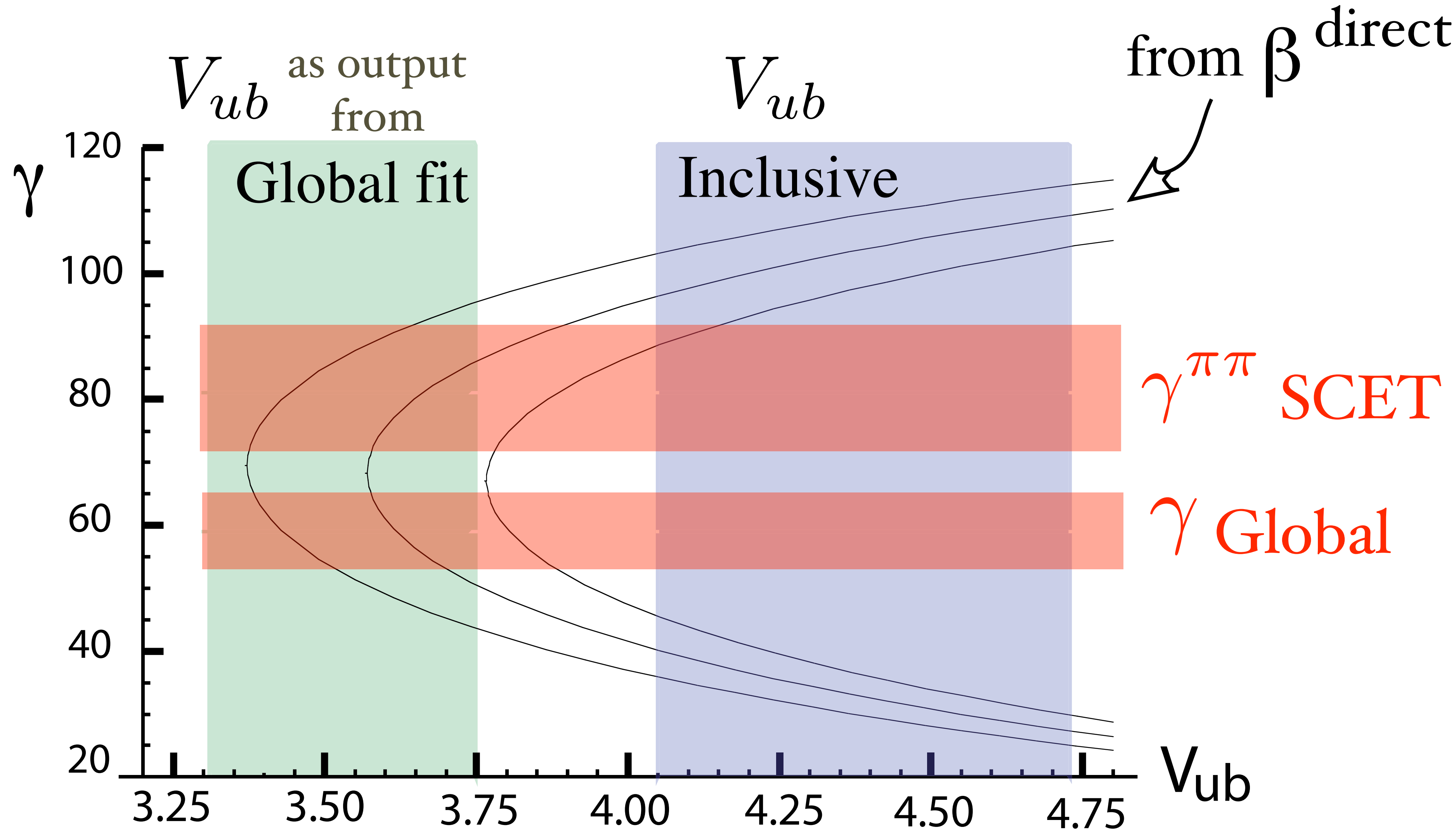
dominated by theory estimate:
$\sim 25\%$ from perturbative
and power corrections

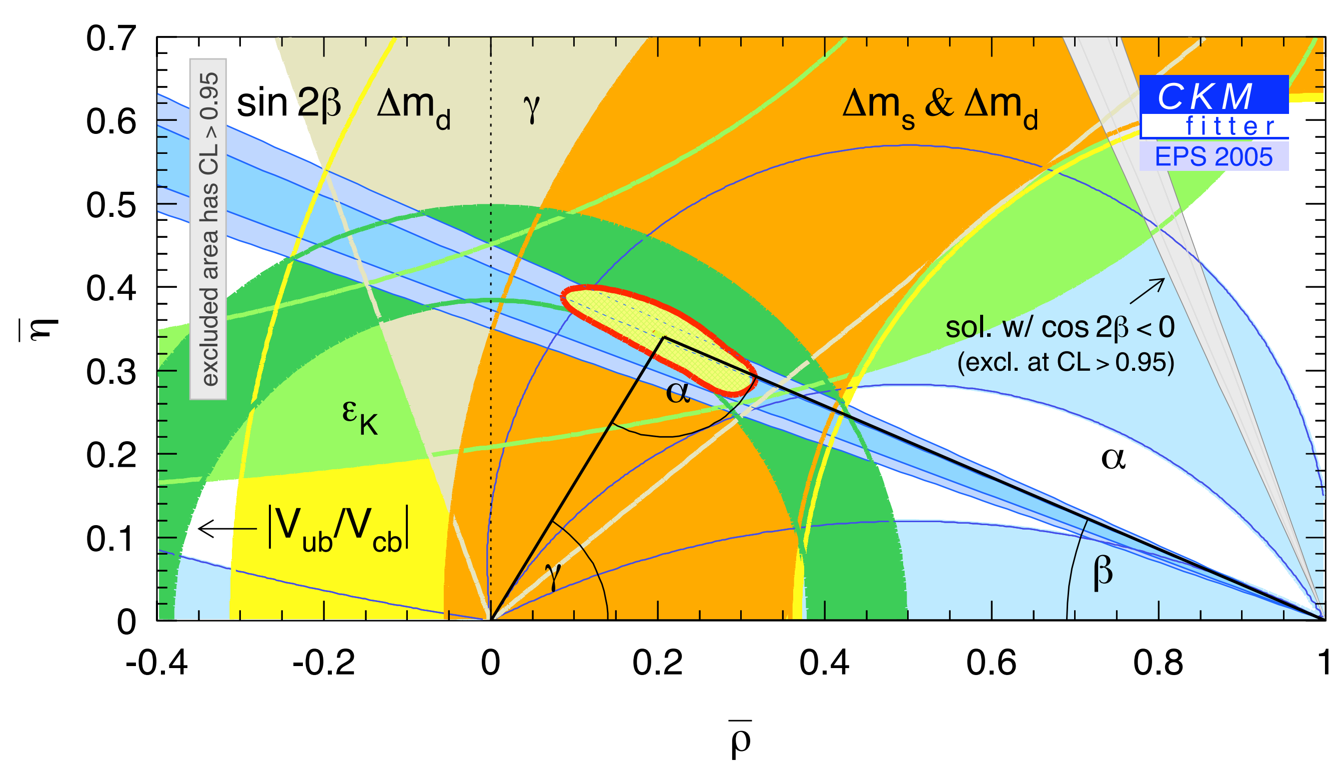
nonleptonic constraint region

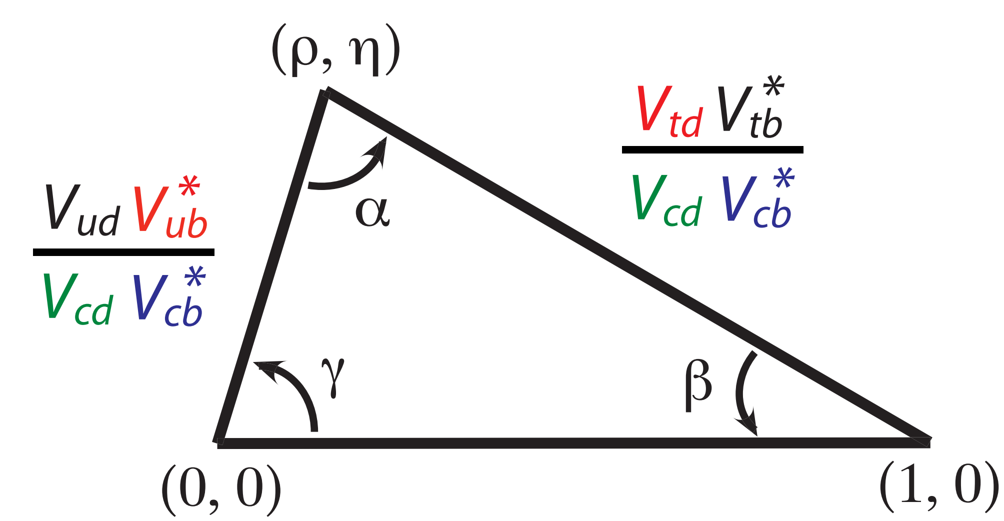


Which Vub?

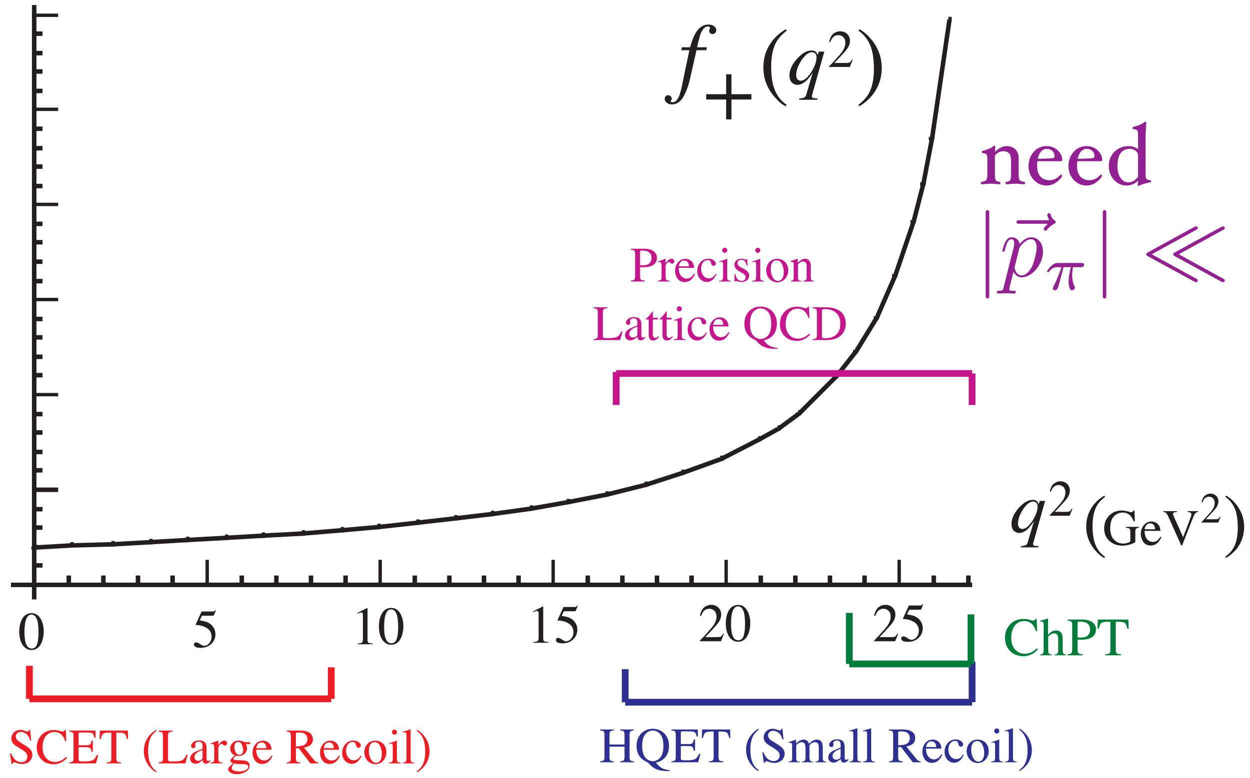
Tension with $\sin(2\beta)$?





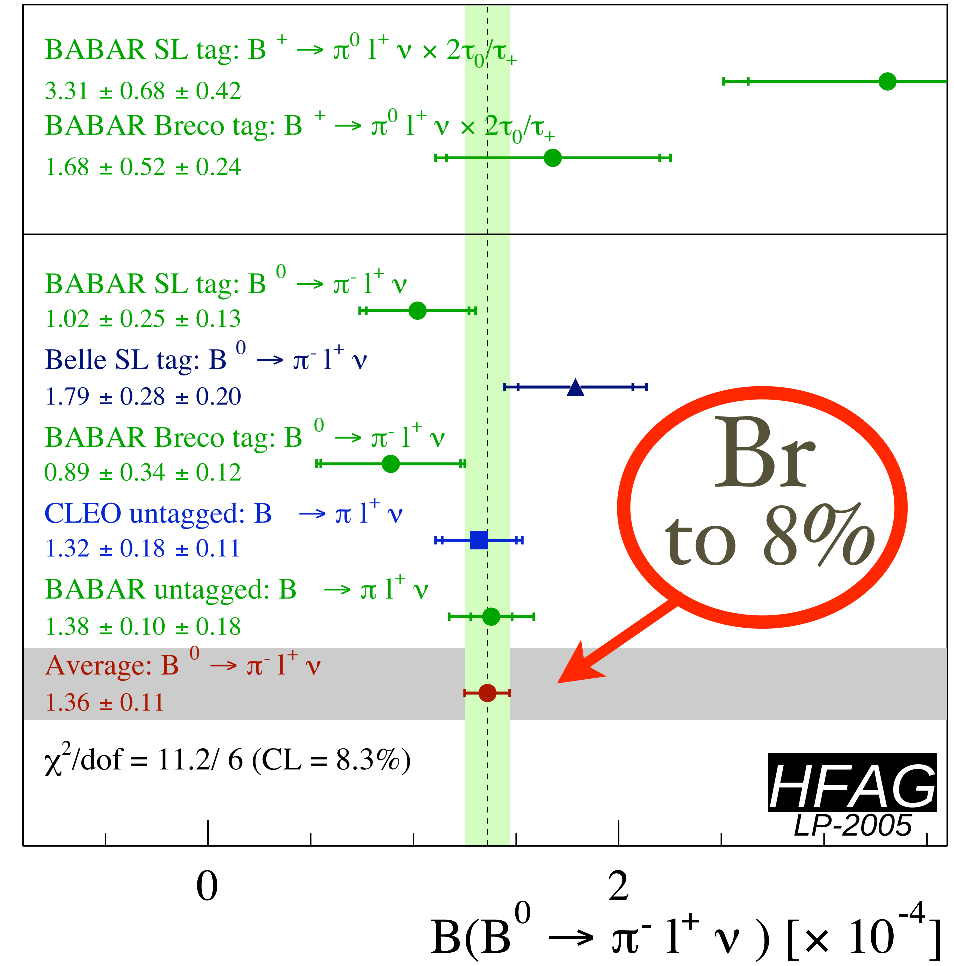


$\boxed{V_{ub}}$ $\quad B \to \pi \ell \bar{\nu}$



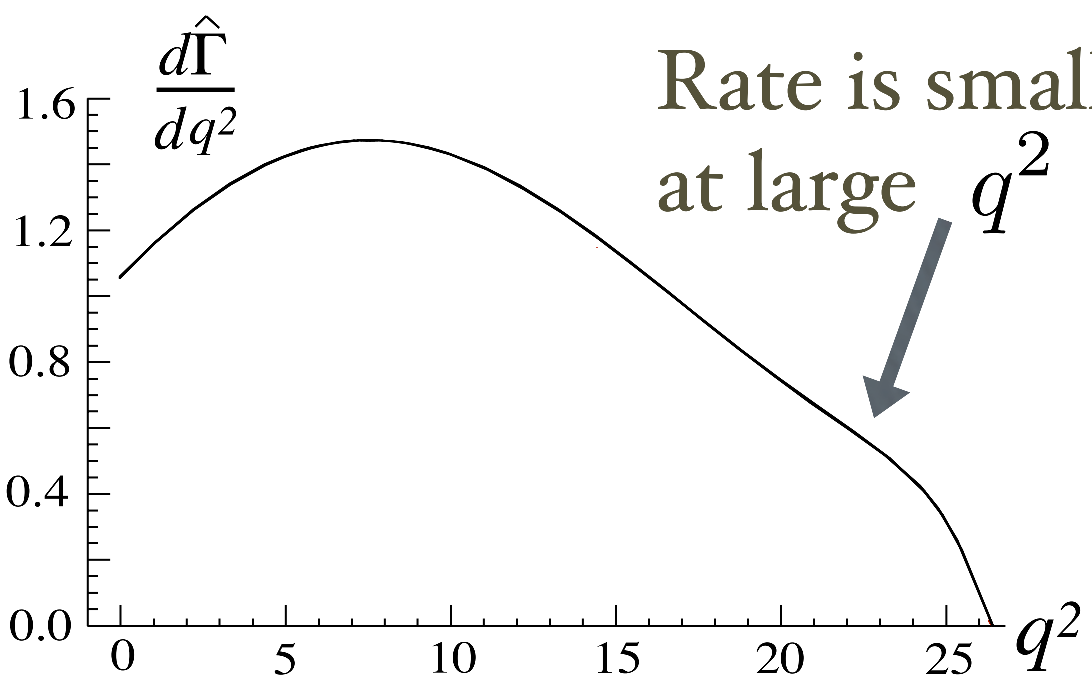
need
$|\vec{p}_\pi| \ll 1/a$

Average from Cleo, Belle, Babar:



$|V_{ub}|$ to 4% !?!

Uncertainty from theory dominates.



Rate is smaller at large q^2

$$\frac{d\Gamma(\bar{B}^0 \to \pi^+ \ell \bar{\nu})}{dq^2} = \frac{G_F^2 |\vec{p}_\pi|^3}{24\pi^3} |V_{ub}|^2 \, |f_+(q^2)|^2$$

Lattice & QCD Dispersion Relations

Arnesen, Grinstein, Rothstein, I.S.

Bourrely et al., Boyd, Grinstein, Lebed, Savage; Lellouch; Fukunaga, Onogi;

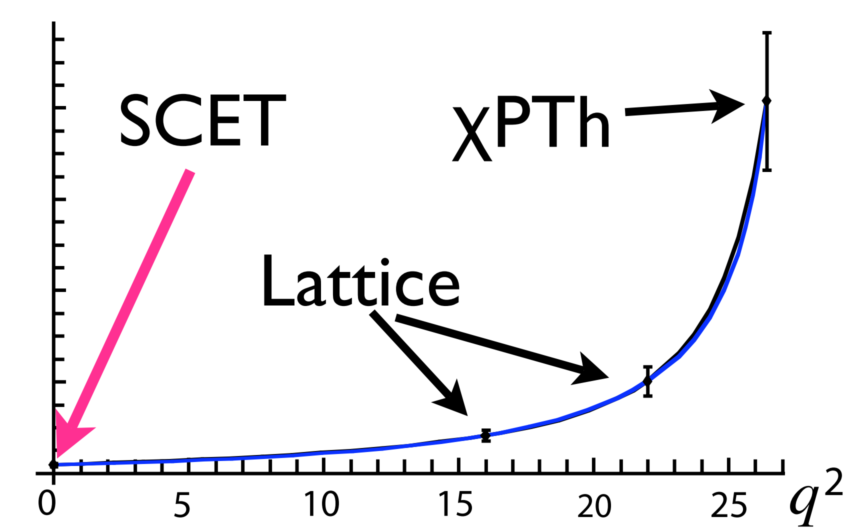
Focus on Vub determination, use:

i) Lattice qcd results at large q^2

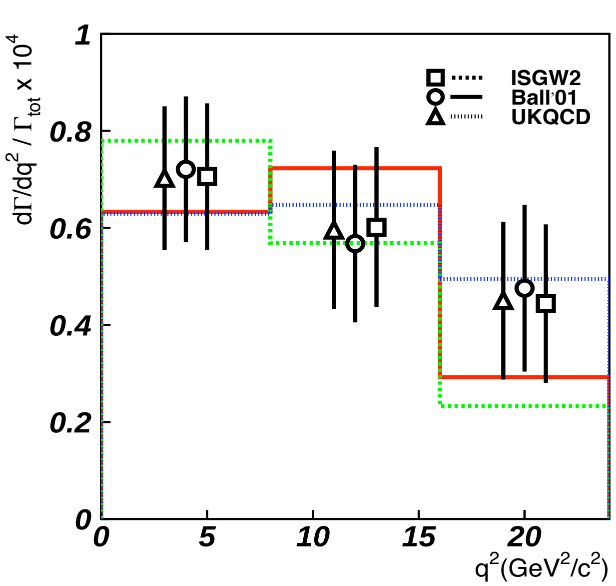
ii) chiral perturbation theory at $q^2_{\max}$

iii) expt. spectra for information at low q^2 & SCET constraint from $B \to \pi\pi$ at $q^2 = 0$

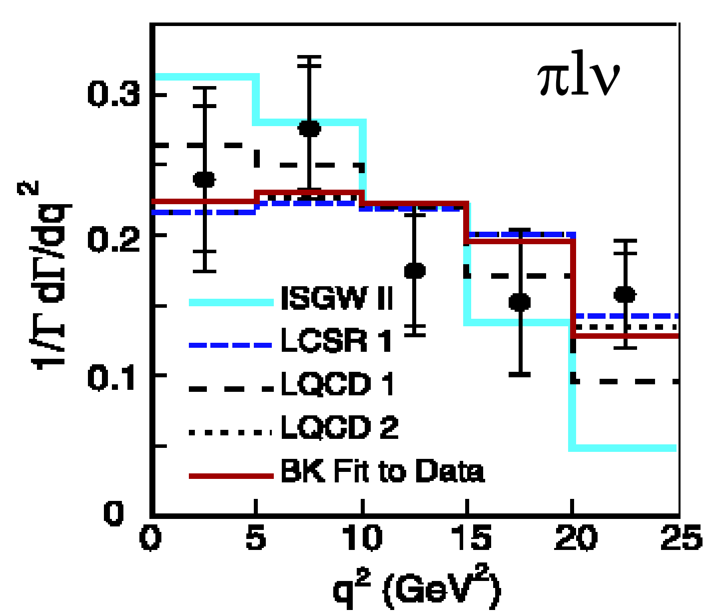
iv) QCD dispersion relations to constrain the form factors shape (model independent)



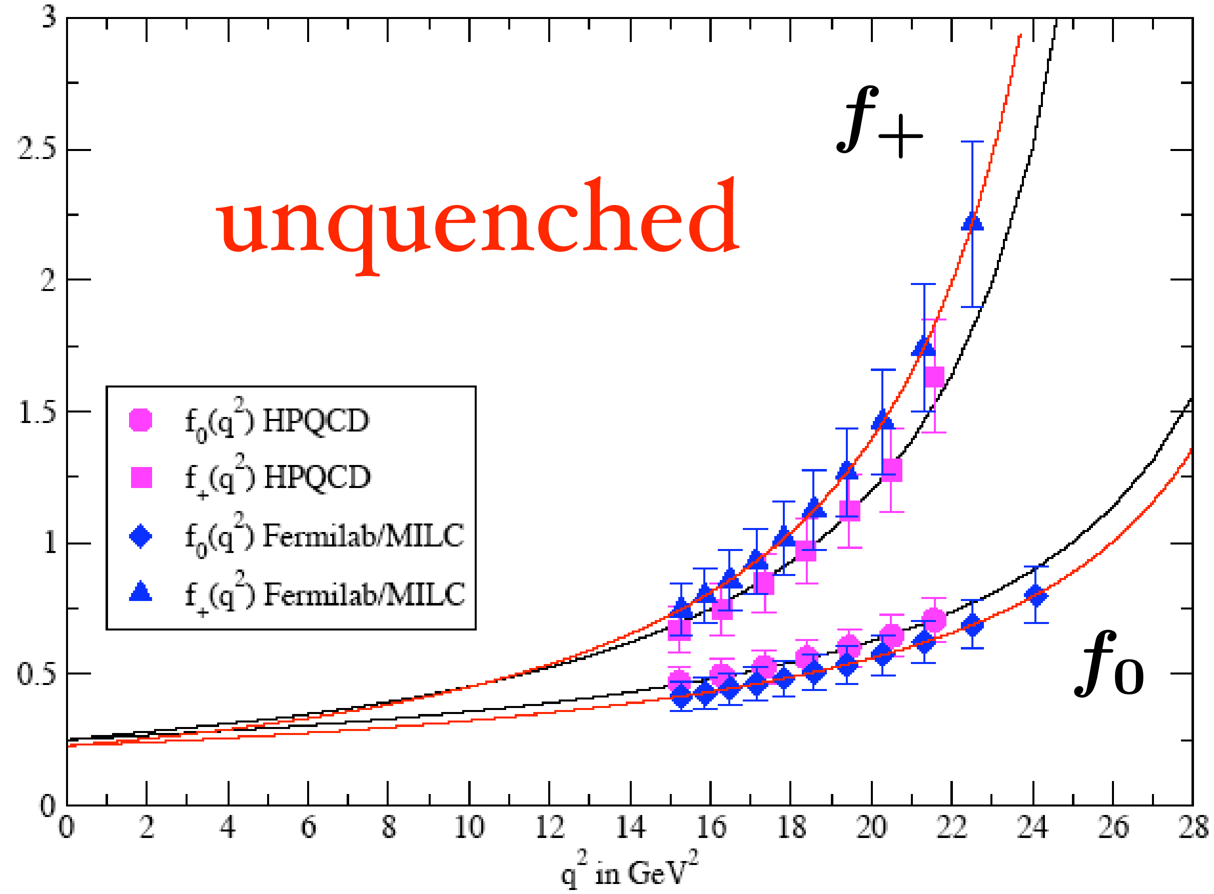
Belle



Babar



More recently, Becher & Hill have studied whether the spectrum data constrains the SCET parameters



$q^2 \geq 16\,\mathrm{GeV}^2$

statistics
4-6%

Systematics	HPQCD errors
perturbative matching	9%
chiral extrapolation	4%
action discretization	2%
matching $a, 1/m_Q$	5%
Total	11%

statistics
$\sim 8\%$

Systematics	Fermilab/ MILC errors
matching	1%
chiral extrapolation	4%
q^2 interp.	4%
finite a	9%
Total	11%

HFAG — expt. — theory

$10^3 \times |V_{ub}| = 3.75 \pm 0.27^{+0.64}_{-0.42}$ FNAL

$10^3 \times |V_{ub}| = 4.45 \pm 0.32^{+0.69}_{-0.47}$ HPQCD

My LP'05 Average for this method:

$10^3 \times |V_{ub}| = 4.1 \pm 0.32^{+0.69}_{-0.47}$

16%
total error

Dispersion Relations

M.N Meiman, '63
S. Okubo, I. Fushih, '71
V. Singh, A. K. Raina, '79
C. Bourrely, B. Machet, E de Rafael, '81
E. de Rafael, J. Taron, '92 & '94
B.Grinstein, P. Mende, '93
C.G.Boyd, B.Grinstein, R. Lebed, '95,'96,'97
L. Lellouch, '96
C.G.Boyd, M. Savage, '97
I. Caprini, L.Lellouch, M. Neubert '97
M. Fukunaga, T. Onogi, '05

Define

$$\Pi_J^{\mu\nu}(q) = \frac{1}{q^2}(q^\mu q^\nu - q^2 g^{\mu\nu})\Pi_J^T(q^2) + \frac{q^\mu q^\nu}{q^2}\Pi_J^L(q^2) \equiv i\int d^4x\, e^{iqx}\langle 0|\mathrm{T}J^\mu(x)J^{\dagger\nu}(0)|0\rangle$$

Dispersion relations

$$\chi^{(0)} = \frac{1}{2}\frac{\partial^2\Pi_J^T}{\partial(q^2)^2}\bigg|_{q^2=0} = \frac{1}{\pi}\int_0^\infty dt\, \frac{\mathrm{Im}\,\Pi_J^T(t)}{t^3}$$

Inequality

$$\mathrm{Im}\Pi_J^{T,L} = \frac{1}{2}\sum_X (2\pi)^4\delta^4(q-p_X)|\langle 0|J|X\rangle|^2 \geq \pi(2\pi)^3\delta^4(q-p_B-p_\pi)|\langle 0|J|B\pi\rangle|^2$$

Perturbative QCD (OPE)

Related by crossing to decay form factor

Bound on Form factor

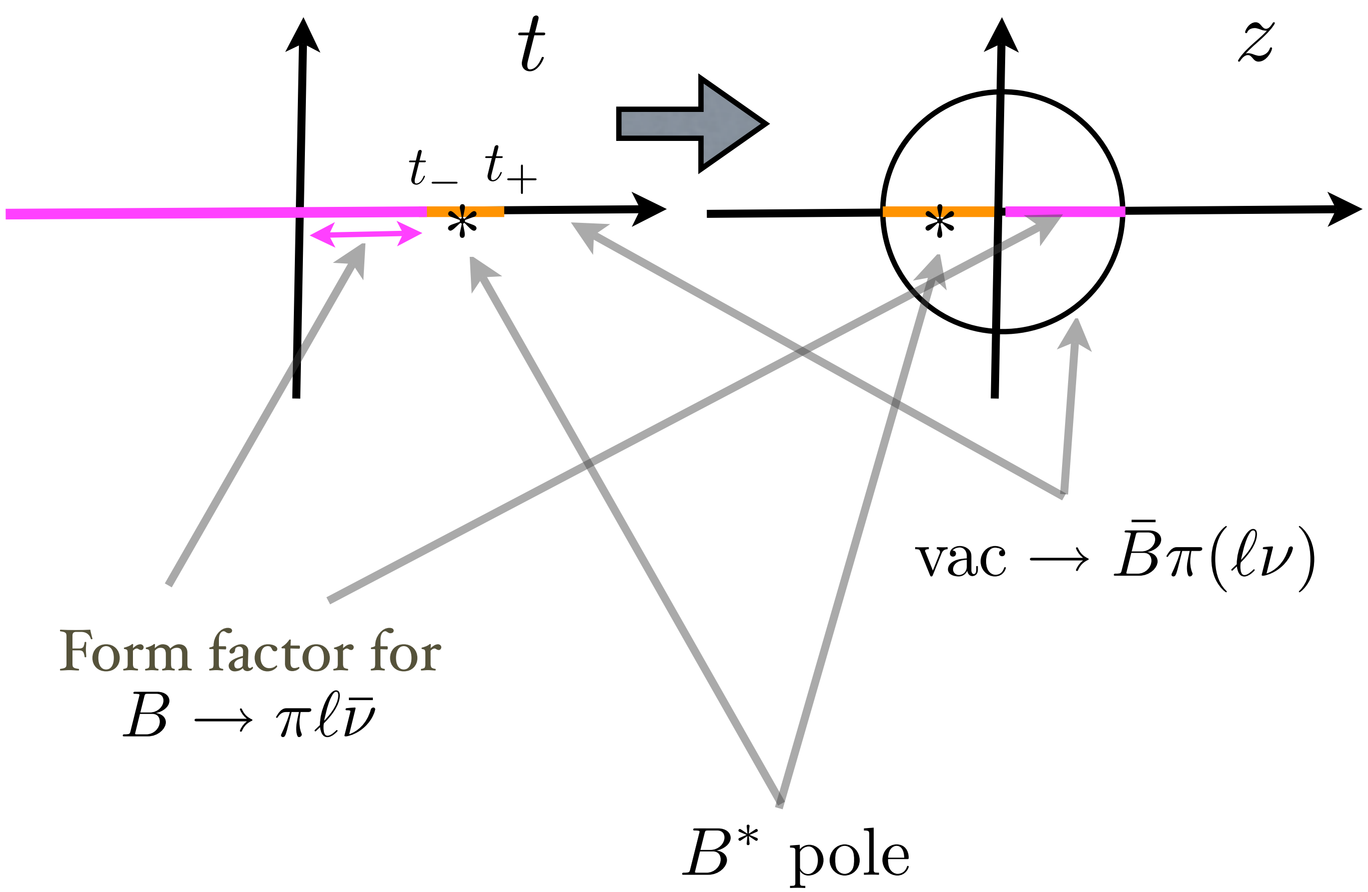
$$\int_{t_+}^\infty dt\, \frac{W(t)|f(t)|^2}{t^3} \leq 1$$

$$t_+ = (m_B + m_\pi)^2$$

Complex Magic

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$



$$P(t)\phi(t)f(t) = \sum_{n=0}^{\infty} a_n\, z^n$$

Blaschke Factor: remove pole at $t = m_{B^*}^2$

Outer function: phase space, Jacobian, $\chi^{(0)}$ in QCD

$t = q^2$

$$\boxed{f_+(t) = \frac{1}{P(t)\phi(t)} \sum_{n=0}^{\infty} a_n\, z^n}$$

$$\sum_n a_n^2 \le 1$$

Pick $t_0 = 0.65\, t_-$ then

$$-0.34 \le z \le 0.22$$

from dispersion

Strategy: use input points to fix first few a's
vary all higher a's to determine uncertainty

Input Points

i) SCET

$$f^0_{\rm in} = |V_{ub}| f_+(0) = (7.2 \pm 1.8) \times 10^{-4}$$

ii) Lattice

$$f^1_{\rm in} = f_+(15.87)$$
$$f^2_{\rm in} = f_+(18.58)$$
$$f^3_{\rm in} = f_+(24.09)$$

FNAL /MILC or HPQCD

take systematic error to be 100% correlated

$E_{ij} = \sigma_i^2 \delta_{ij} + y^2 f^i_{\rm in} f^j_{\rm in}$ (increases uncertainty)

iii) Chiral Pert. Theory

$$f_+(q^2(E_\pi)) = \frac{g f_B m_B}{2 f_\pi (E_\pi + m_{B^*} - m_B)} \Big[1 + \mathcal{O}\Big(\frac{E_\pi}{\Delta}\Big)\Big] \qquad \Delta \sim 600\,\text{MeV}$$

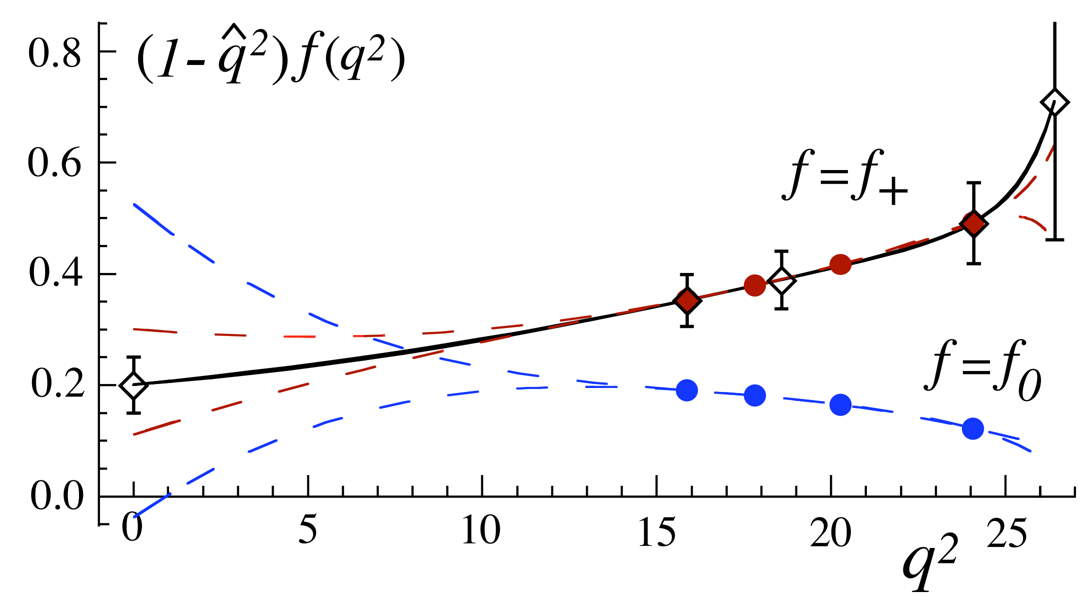
$$f^4_{\rm in} = f_+(26.42) = 10.38 \pm 3.63$$

- solve with $\sum_{n=0}^{5} a_n z^n$, for a_0–a_4
- vary a_5 to get bounds $\sum_n a_n^2 \le 1$

including truncation error from all higher order terms: $a_5 \to \dfrac{a_5}{\sqrt{1-z^2}}$

$$f_+(t) = F_\pm(t, \{f_0/|V_{ub}|, f_1, f_2, f_3, f_4\})$$

$\hat{q}^2 = q^2/m^2_{B^*}$



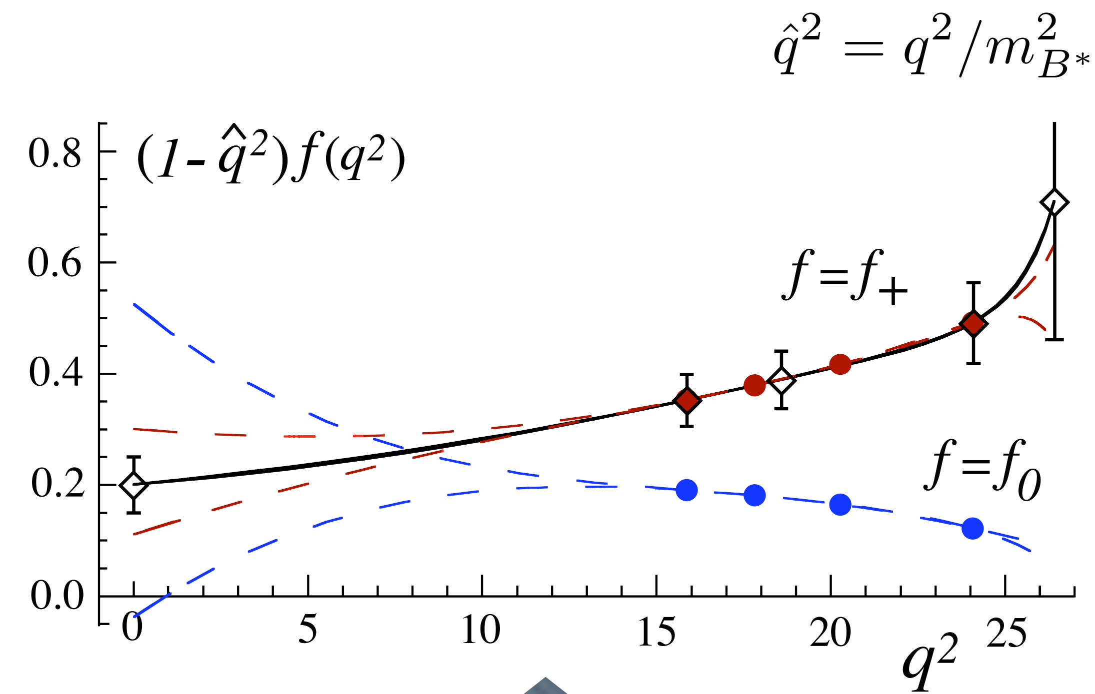
Uncertainties

Bound uncertainty:

- fix $f^i = f^i_{\rm in}, \quad |V_{ub}| = 3.6 \times 10^{-3}$

  ➡ bound uncertainty very small

- compare with 4 lattice points, and constraint $f_0(0) = f_+(0)$



⬆ Dispersion relations show there is a lot of freedom for a pure extrapolation of lattice data

Perturbative uncertainty:

- OPE $\chi^{(0)}$ depends on m_b, order in $\alpha_s(m_b)$, condensates

  ➡ only effects norm., so enters through a_5 , very small

Uncertainty from INPUT POINTS dominates

Method I

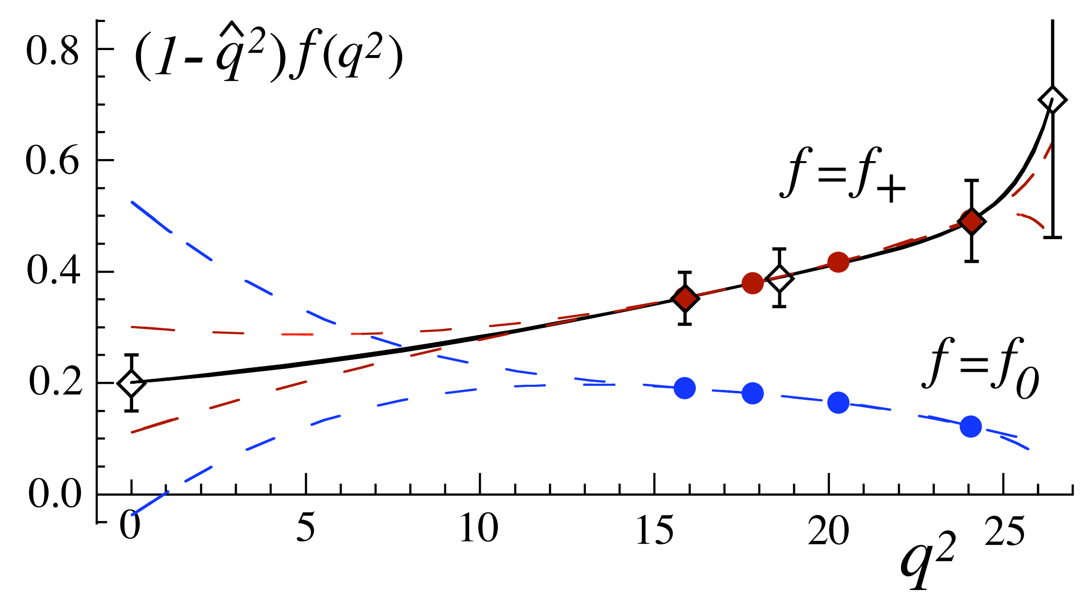
- use only the total Branching ratio
- integrate $\frac{d\Gamma}{dq^2}$ with $f_+(t) = F_\pm(t, \{f_0/|V_{ub}|, f_1, f_2, f_3, f_4\})$
- use Lellouch method to account for theory uncertainty

$$|V_{ub}| = (3.96 \pm \underbrace{0.20}_{\substack{5\% \\ \text{expt}}} \pm \underbrace{0.56}_{\substack{14\% \\ \text{theory}}}) \times 10^{-3}$$

(with $f^i = f^i_{\rm in}$
$|V_{ub}| = 4.13 \times 10^{-3}$)

Type of Error	Variation From	$\boxed{\delta\lvert V_{ub}\rvert^{\rm Br}}$	$\delta\lvert V_{ub}\rvert^{q^2}$
Input Points	1-σ correlated errors	$\pm 14\%$	$\pm 12\%$
Bounds	F_+ versus F_-	$\pm 0.6\%$	$\pm 0.04\%$
$m_b^{\rm pole}$	4.88 ± 0.40	$\pm 0.1\%$	$\pm 0.2\%$
OPE order	2 loop $\to$ 1 loop	-0.2%	$+0.3\%$

without SCET bound error is $\pm 12\%$



Method II

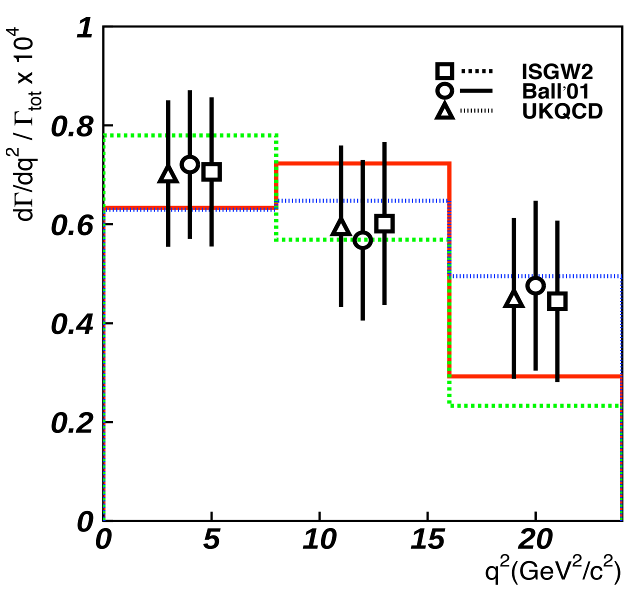
- use q^2 spectra bins: $(\mathrm{Br}_i^{\mathrm{exp}} \pm \delta\mathrm{Br}_i)$, calculate rate in bins
- use Minuit to minimize χ^2 w.r.t. $|V_{ub}|,\ f^{0-4}$

$$\chi^2 = \sum_{i=1}^{17} \frac{[\mathrm{Br}_i^{\mathrm{exp}} - \mathrm{Br}_i(V_{ub}, F_\pm)]^2}{(\delta\mathrm{Br}_i)^2} + \frac{[f_{\mathrm{in}}^0 - f^0]^2}{(\delta f^0)^2} + \frac{[f_{\mathrm{in}}^4 - f^4]^2}{(\delta f^4)^2} + \sum_{i,j=1}^{3} [f_{\mathrm{in}}^i - f^i]\,[f_{\mathrm{in}}^j - f^j]\,(E^{-1})_{ij}\,,$$

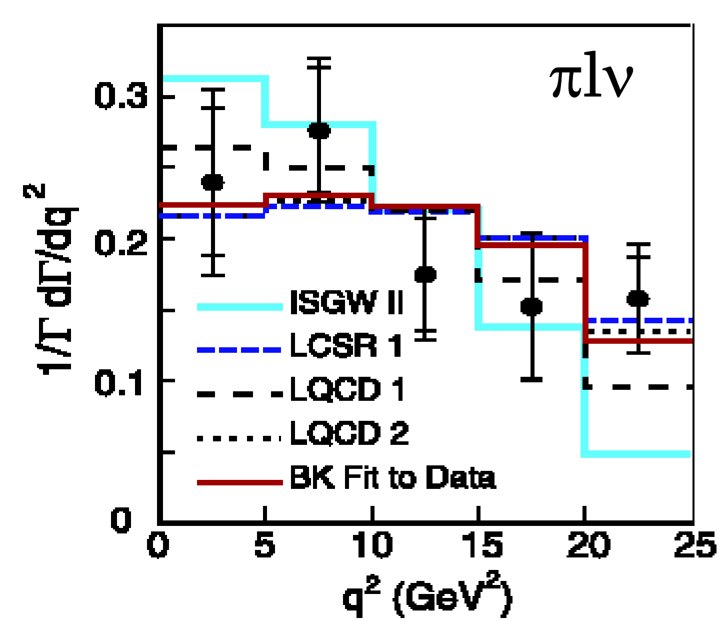
f^{0-4} input points are fit to data & input points (here the spectra constrain the theory error)

can equivalently fit for a's (same answer both ways)

Belle

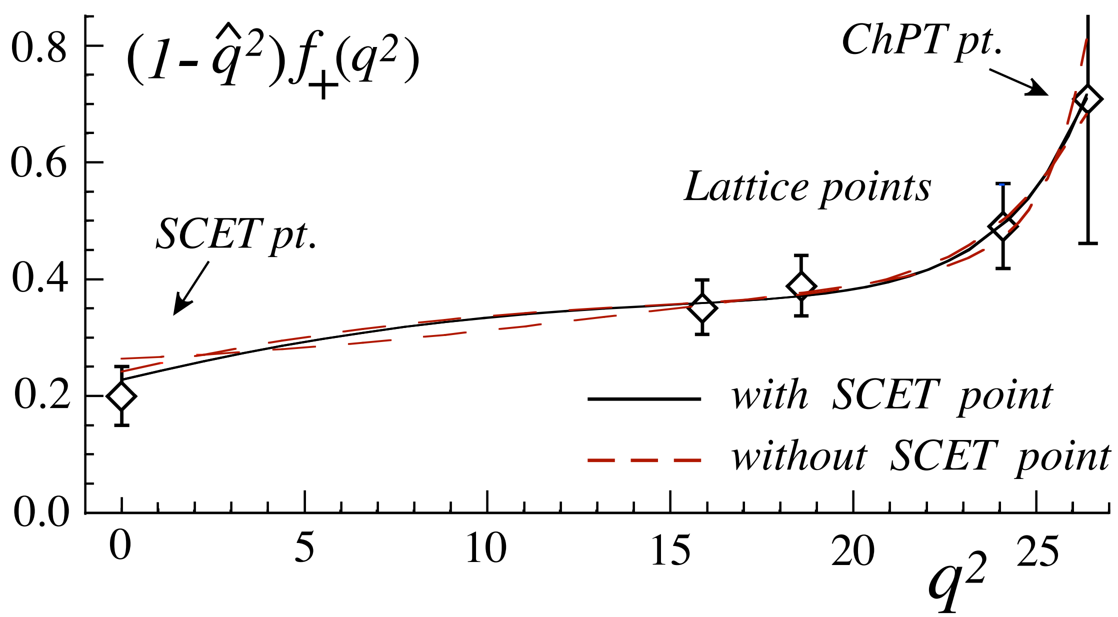


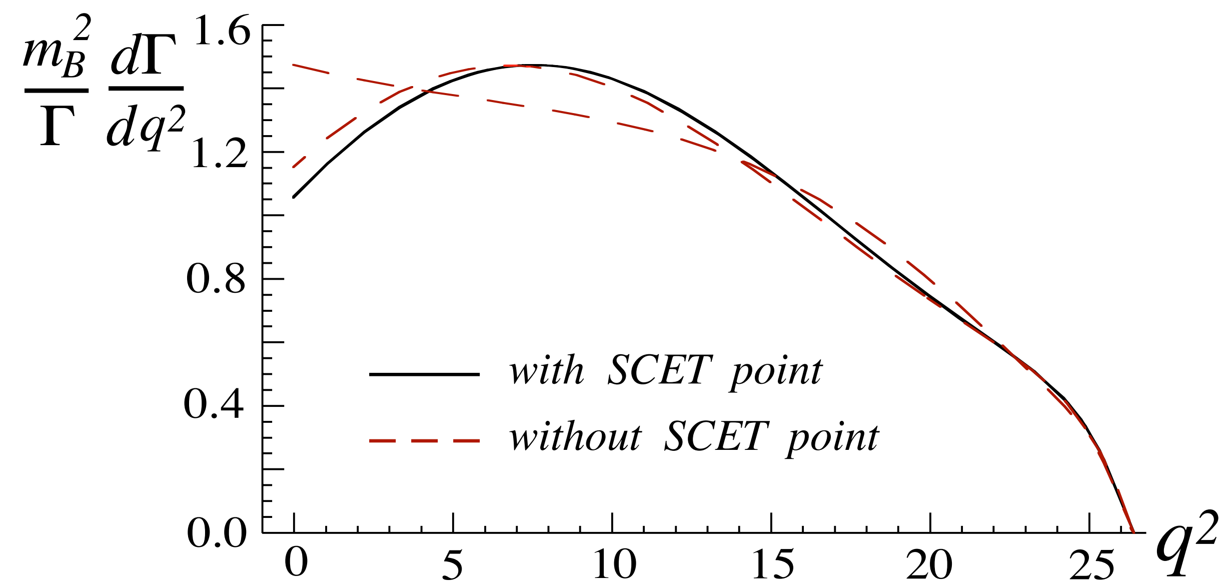
Babar



Method II

Fit to expt. spectra & input points





- expt. spectrum prefers a larger form factor in $\sim 5\text{–}10\,\mathrm{GeV}^2$ region
- Here the SCET point constrains the spectrum, but does not change the determination of Vub

Type of Error	Variation From	$\delta\|V_{ub}\|^{q^2}$
Input Points	1-σ correlated errors	$\pm 13\%$
Bounds	F_+ versus F_-	$< 1\%$
m_b^{pole}	4.88 ± 0.40	$< 1\%$
OPE order	2 loop $\rightarrow$ 1 loop	$< 1\%$

Fit gives:

no SCET: $f_+(0) = 0.25 \pm 0.06$

similar to sum-rules

with SCET: $f_+(0) = 0.23 \pm 0.05$

χ^2 fits to data & input pts. with dispersion relations

Lepton Photon '05

(without SCET point)

$$\chi^2/(dof) \sim 1.0$$

expt. & theory

$$10^3 \times |V_{ub}| = 3.72 \pm 0.52$$ FNAL

$$10^3 \times |V_{ub}| = 4.11 \pm 0.52$$ HPQCD

My Average for this method:

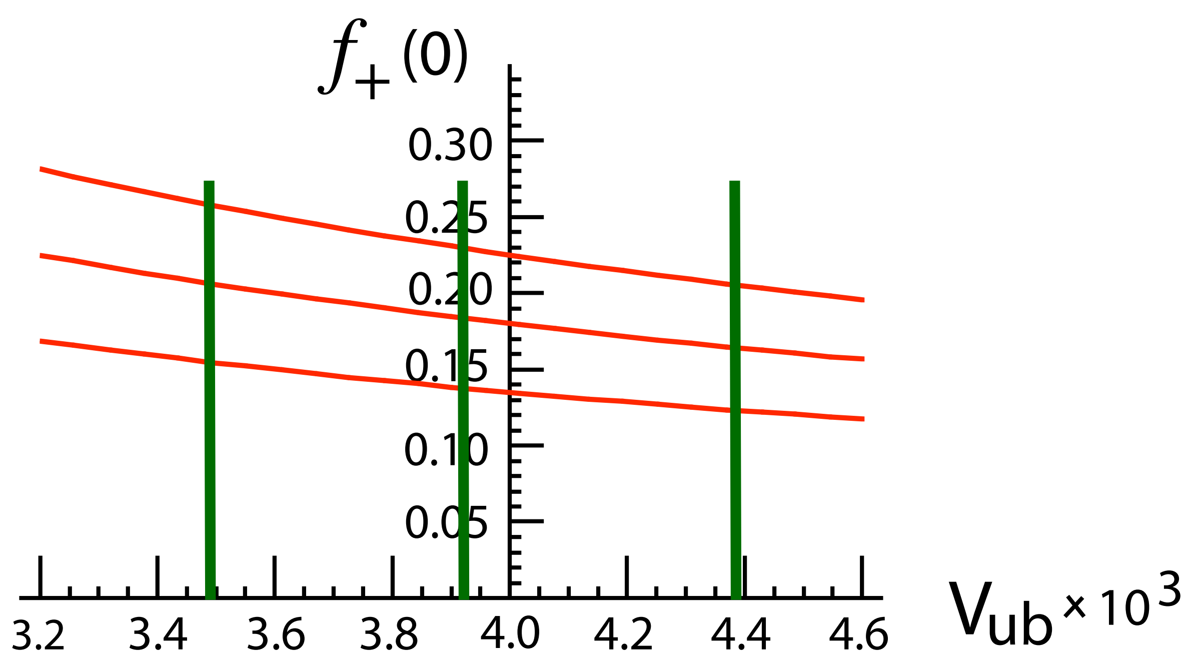
$$\boxed{10^3 \times |V_{ub}| = 3.92 \pm 0.52}$$

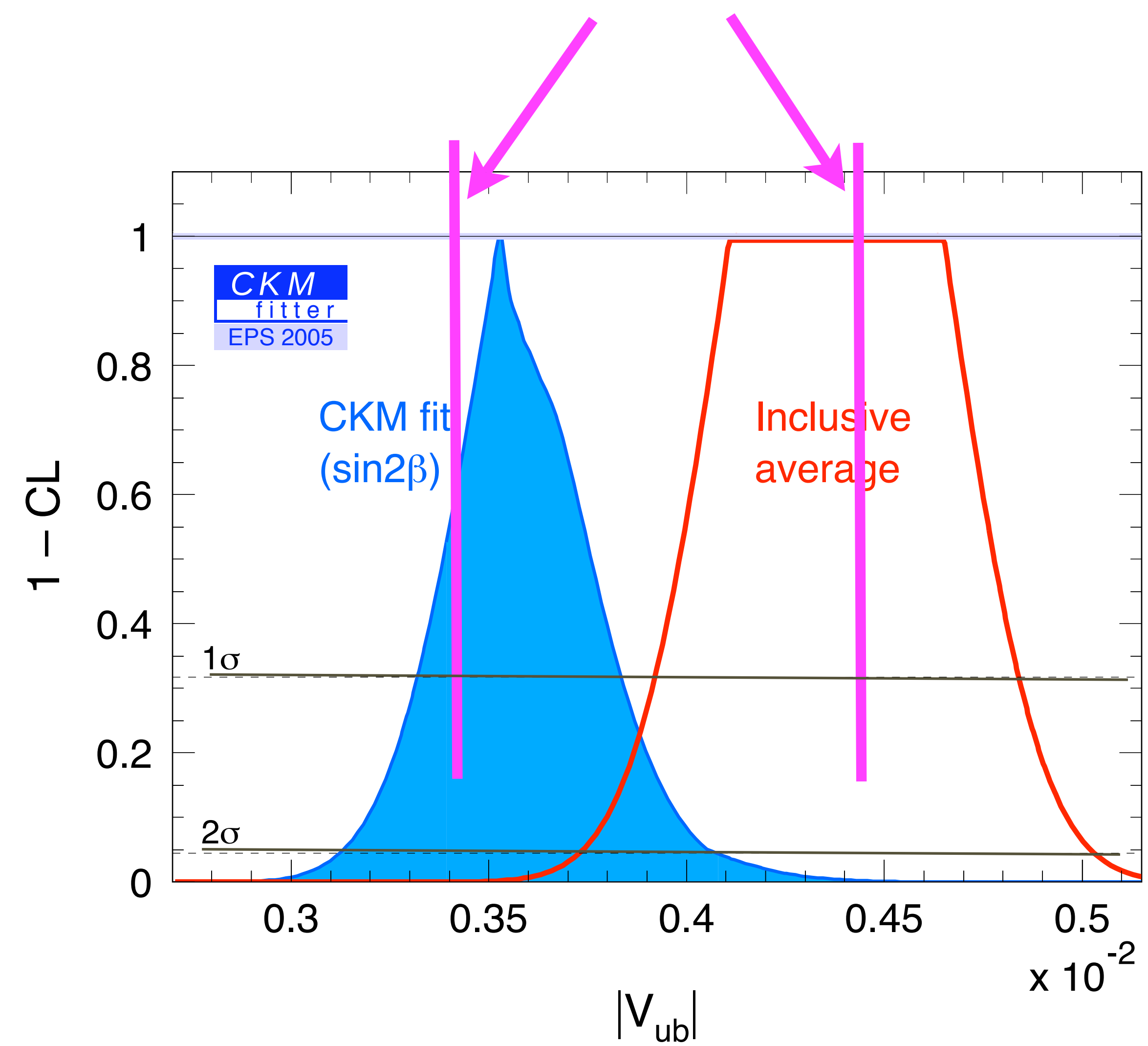
13% total error (4% expt.)

This includes the information in the pure lattice method

Compare Vub's

- $|V_{ub}|^{\rm incl} = (4.38 \pm 0.33) \times 10^{-3}$ (HFAG - EPS'05)
- $|V_{ub}|^{\rm treated\ as\ output}_{\rm in\ global\ CKM} = (3.53^{+0.22}_{-0.21}) \times 10^{-3}$ (CKMfitter)
- $|V_{ub}|^{\rm excl} = (3.92 \pm 0.52) \times 10^{-3}$ (Lattice + Disp. Analysis + Expt. spectrum)





Light-cone sum rules

Babar (LP'05) $q^2 < 16\,{\rm GeV}^2$

(Ball & Zwicky)

$$|V_{ub}| = (3.27 \pm 0.25^{+0.54}_{-0.37}) \times 10^{-3}$$

expt. theory

Outlook

- There is an EFT for processes with energetic jets or hadrons
- We now have the tools to systematically study power corrections
  - ➡ color suppressed decays, inclusive decays
- Exclusive Vub from dispersion + Lattice + spectra
- Nonleptonics
  - ➡ predictions for the size of amplitudes
  - ➡ universal hadronic parameters, strong phases
  - ➡ γ (or α) from individual $B \to M_1 M_2$ channels
- The SCET can be applied to:
  - Nonleptonic decays, Other B decays
  - Jet physics, Exclusive form factors
  - Charmonium, Upsilon physics
  - ... others ?
- A lot of theory and phenomenology left to study ...

Looking into the Future

at B-factories

- improved determination of α, β, γ
- clarify agreement / disagreement between $S_{\eta' K_S}$, $S_{\phi K_s}$, and $\sin(2\beta)$
- precision determination of $|V_{ub}|$
- match theoretical limits for sensitivity in $B \to X_s \gamma$ and $B \to X_s \ell^+ \ell^-$
- observation of $B \to \rho\gamma$ and $B \to \tau\nu$
- Sort out puzzles in $B \to \pi\pi$ and $B \to K\pi$
- .... and of course, the unexpected.

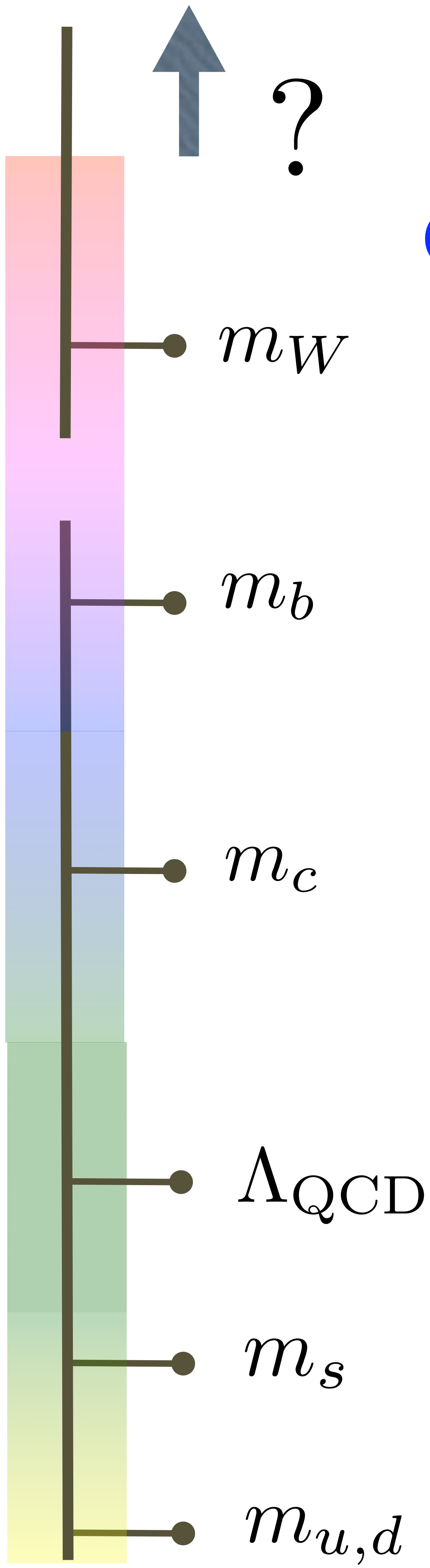
Plan for this talk:

- $B \to X_s\gamma$ $\qquad B \to X_u\ell\bar{\nu}$ $\qquad B \to K^*\gamma \quad B \to \rho\gamma$

$B \to \pi\ell\bar{\nu}$

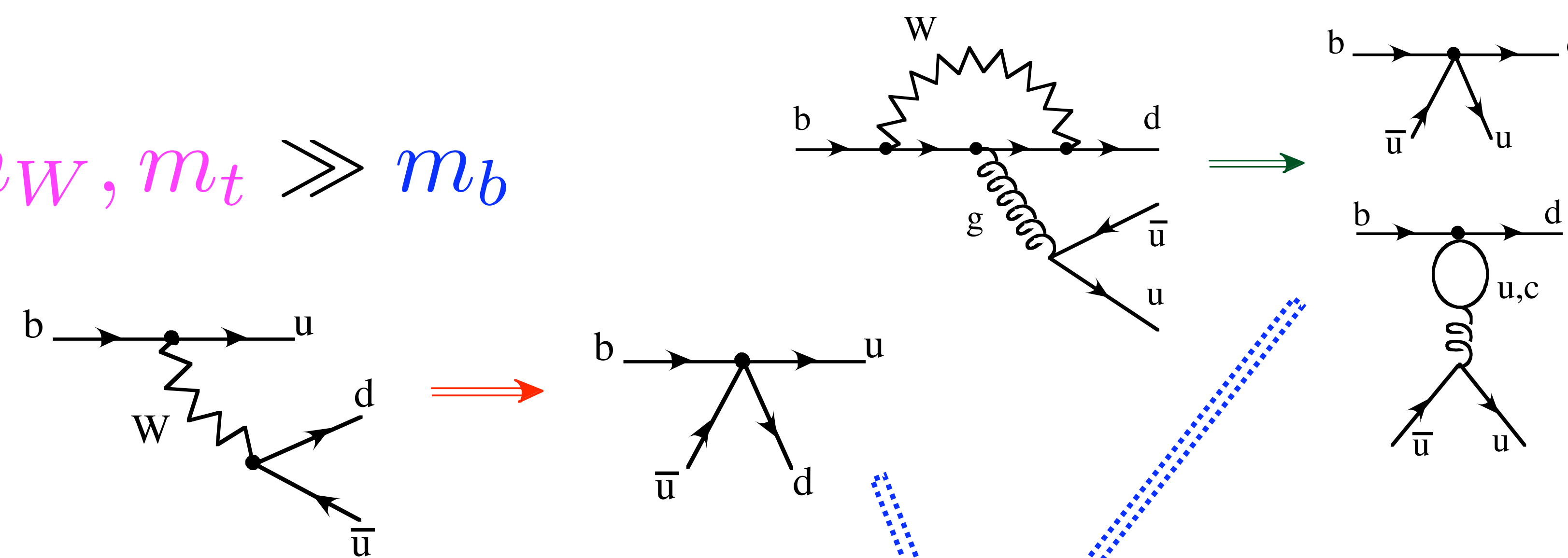
$B \to \rho\rho \quad B \to \pi\pi$

$B \to D\pi$ $\qquad B \to K\pi$



Operator Product Expansion (I)

- $m_W, m_t \gg m_b$



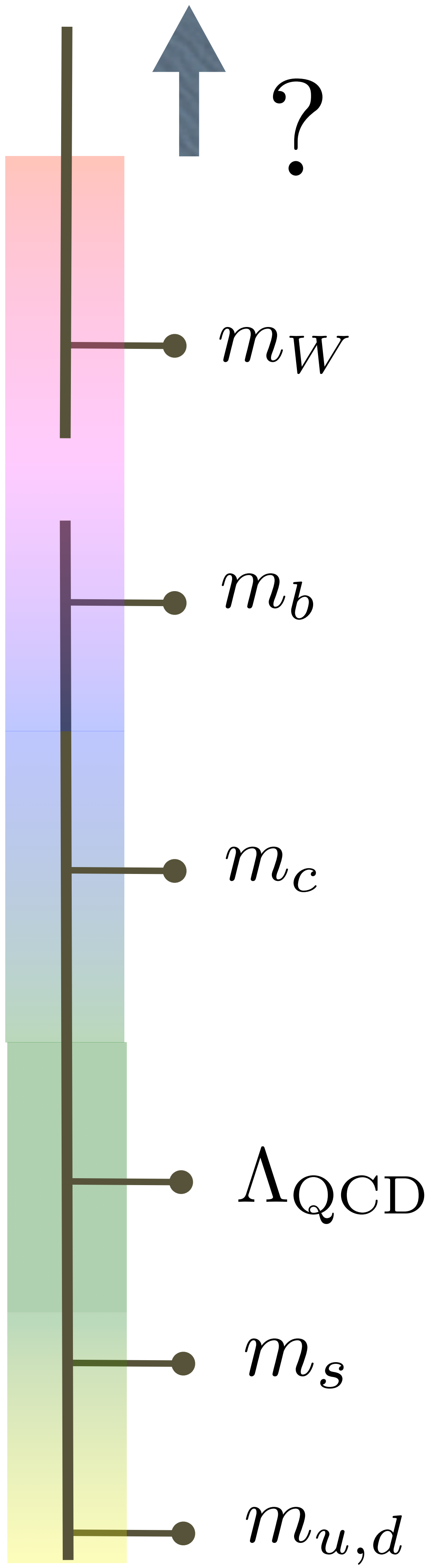
$$\boxed{H_{\rm weak} = \frac{G_F}{\sqrt{2}} \sum_i \lambda^i C_i(\mu) O_i(\mu)}$$

$\lambda^i = \text{CKM},$
$\lambda^1 = V_{ub} V_{ud}^*$

perturbative QCD

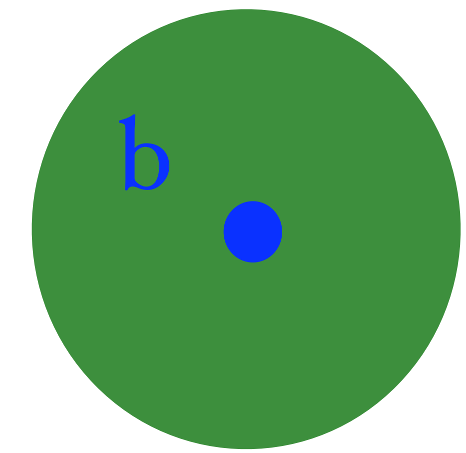
Decays like $B \to X_s \gamma$ & $B \to K\pi$
have contributions from ~ 12 operators

Operator Product Expansion (II)



- $m_b \gg \Lambda_{\mathrm{QCD}}$

B-meson



$$\Gamma = c^{(0)} f^{(0)} + \frac{1}{m_b} c^{(1)} f^{(1)} + \dots$$

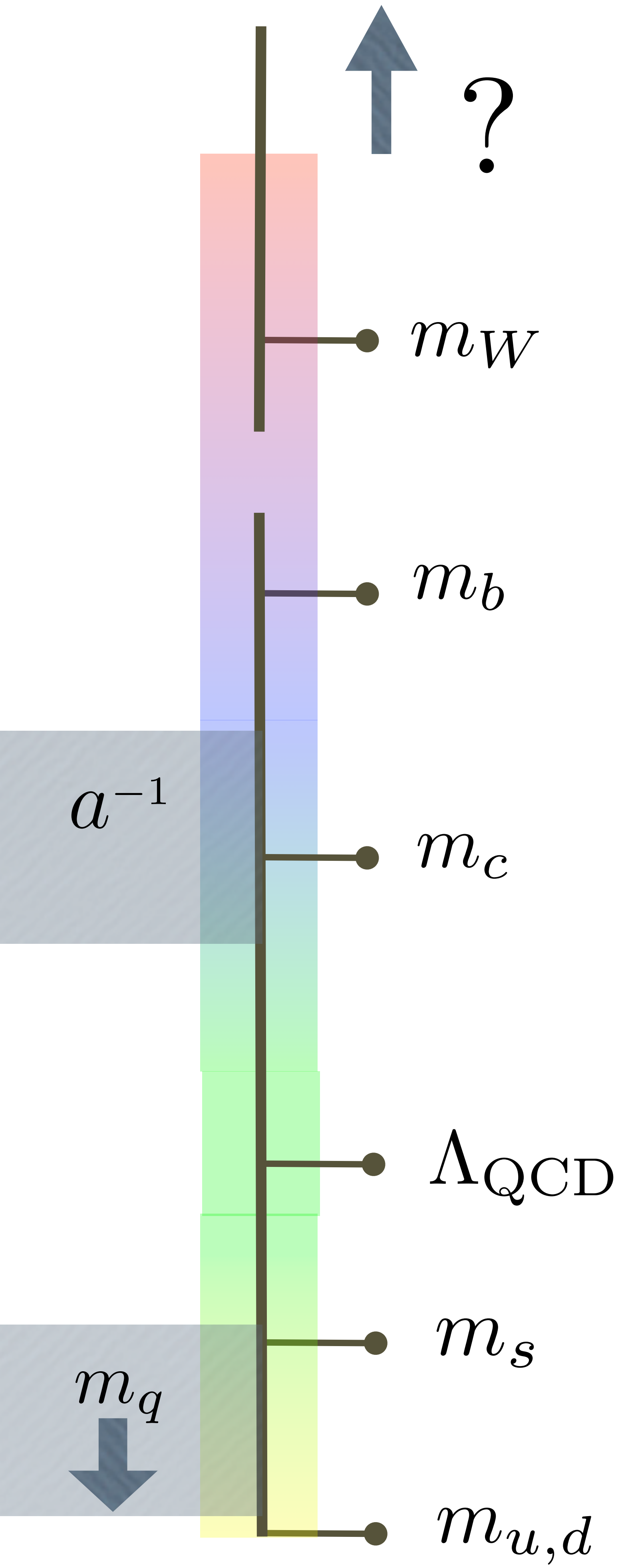
Heavy Quark Effective Theory $\quad h_v, q$

Operator Product Expansion for Inclusive Decays

- Justifies free quark decay as leading approximation

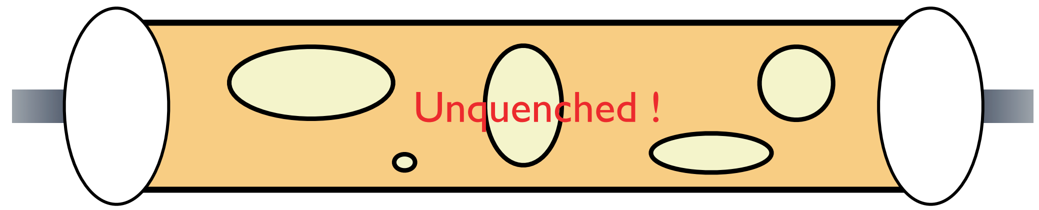
$$\frac{\Lambda}{m_b} \simeq 0.1, \quad \alpha_s(m_b) \simeq 0.2$$

subleading terms are crucial for precision phenomenology



Unquenched Lattice QCD

$$\det(\not{D} + m) \neq 1$$



nonperturbative QCD

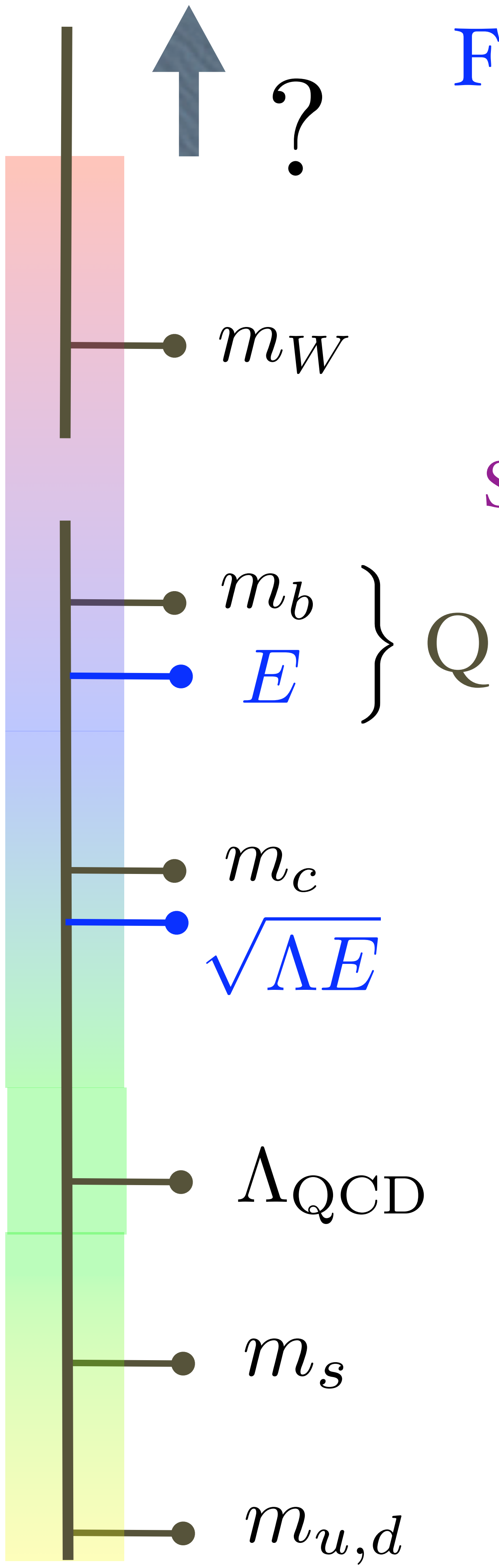
Now:

- Focus on "Gold Plated Observables" for high precision
  - matrix elements with at most one hadron in initial and final state
  - at least 100MeV below threshold, or small widths
- Simulate "real QCD". Use nf=2+1 light flavors, quark masses m_q light enough for extrapolation with chiral perturbation theory (or PQChPT)
- Systematic/parametric estimates of uncertainties using effective field theory methods. eg. heavy quarks:
  - $m_Q \gg \Lambda_{\mathrm{QCD}}$ NRQCD, Fermilab action, RHQ action
- Results for a broad spectrum of observables are obtained using common inputs

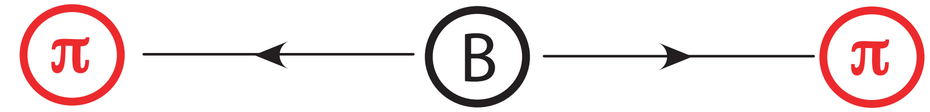
} ChPT, PQChPT

→ tests, predictions, and impact

Factorization Theorems



Energetic Hadrons



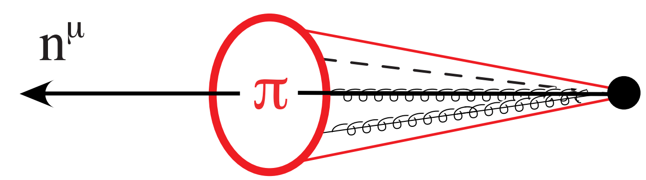
eg. $E_\pi \gg \Lambda_{\mathrm{QCD}}$

Soft-Collinear Effective Theory (SCET)

Bauer, Pirjol, I.S.
Fleming, Luke,
many other authors

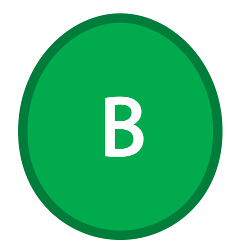
Introduce fields for infrared d.o.f.

collinear:



ξ_n, A_n^μ

soft:

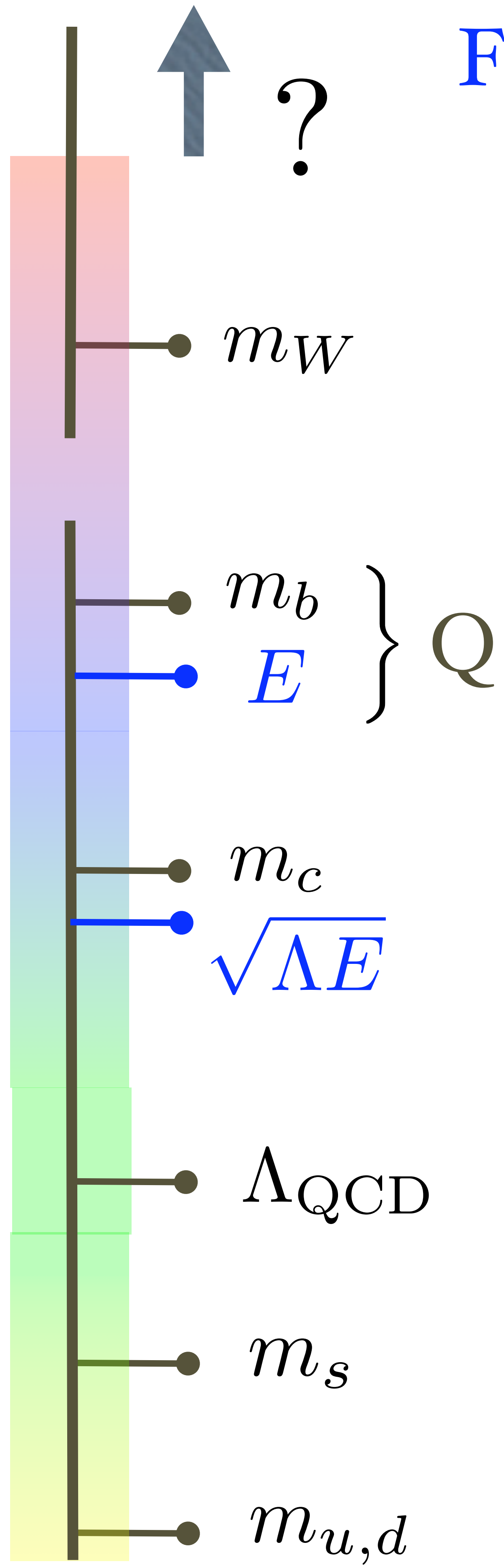


h_v, q_s, A_s^μ

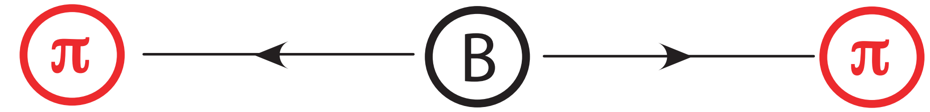
$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots$$

- Separate physics at different momentum scales
- Model independent, systematically improvable

Factorization Theorems

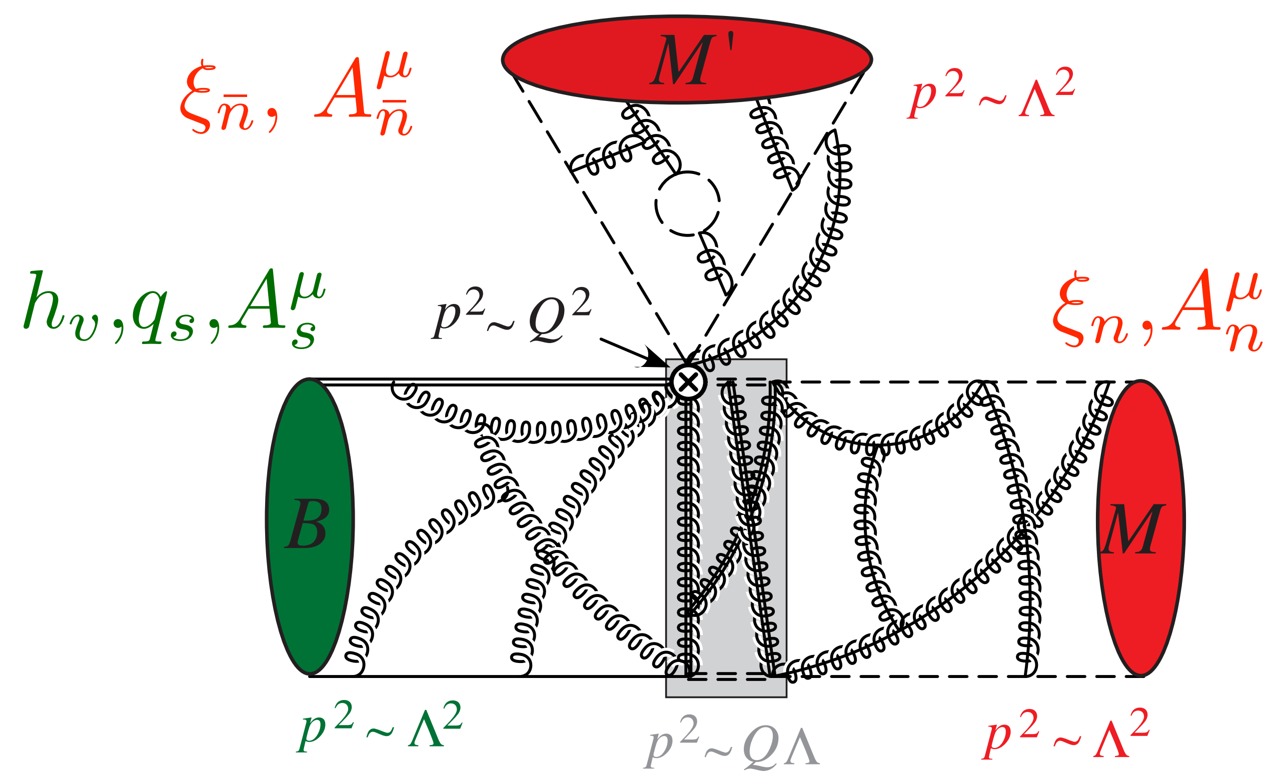


Energetic Hadrons



eg. $E_\pi \gg \Lambda_{\rm QCD}$

$$A = \int dz\, dx_i\, dk^+ \underbrace{T(z)}_{Q^2}\ \gg\ \underbrace{J(z, x_i, k^+)}_{E\Lambda}\ \gg\ \underbrace{\phi_1(x_1)\phi_2(x_2)\phi_B(k^+)}_{\Lambda^2} + \ldots$$



Nonleptonic Decays

$\boxed{B \to K\pi}$

Is there a K-pi CP Puzzle ?

- Br sum rule:

Expand in $\epsilon = \underbrace{\left|\frac{V_{us}^* V_{ub}}{V_{cs}^* V_{cb}}\right|}_{0.02} \frac{T}{P}\,, \left|\frac{V_{us}^* V_{ub}}{V_{cs}^* V_{cb}}\right| \frac{C}{P}\,, \frac{P_{ew}^{(t,c)}}{P}$

Lipkin, many authors

$$R(\pi^0 K^-) - \frac{1}{2} R(\pi^- K^+) + R(\pi^0 K^0) = \mathcal{O}(\epsilon^2)$$

$$R(f) = \frac{\Gamma(B \to f)}{\Gamma(\bar{B}^0 \to \pi^- \bar{K}^0)}$$

$$0.094 \pm 0.073 = \mathcal{O}(\epsilon^2) < .03$$

my estimate
from factorization
in SCET

no puzzle here yet

- Direct-CP sum rule:

Gronau, Rosner

$$\Delta(\bar{K}^0 \pi^0) - \frac{1}{2} \Delta(K^+ \pi^-) + \Delta(K^+ \pi^0) = \mathcal{O}(\epsilon^2)$$

$$\Delta(f) = \frac{A_{CP}(f) \Gamma_{\rm avg}^{\rm CP}(f)}{\Gamma_{\rm avg}^{\rm CP}(\pi^- \bar{K}^0)}$$

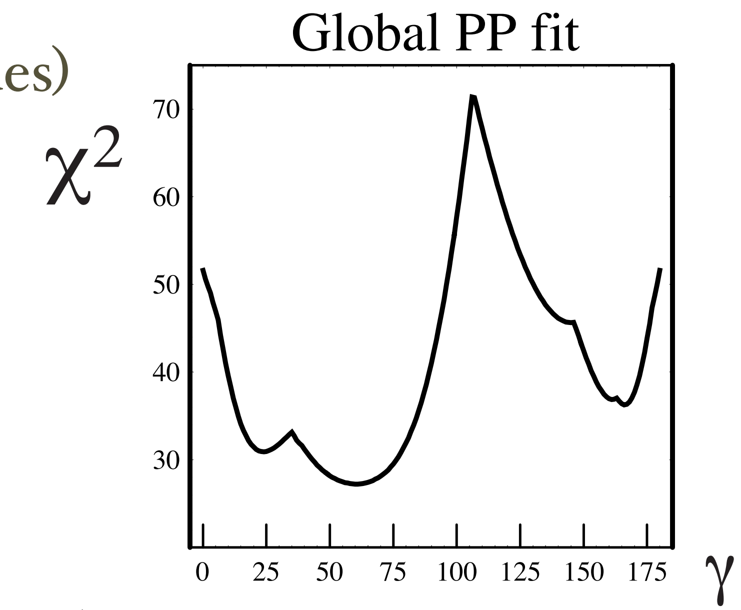
$$0.058 \pm 0.070 = \mathcal{O}(\epsilon^2) < 0.007$$

my estimate
from factorization
in SCET

no puzzle here yet

SU(3), global fits to data

(Neglect E, A, PA amplitudes)



12 parameters, 18 predictions

$\pi\pi, KK, \pi\eta, \pi\eta' K\pi, K\eta, K\eta'$

Chiang, Gronau, Luo, Rosner, Suprun

$\gamma = 61^\circ \pm 11^\circ$ agrees with global fit

better agreement when one adds new Babar data

$Br(K^+\pi^-), Br(K^0\pi^0), A_{CP}(K^0\pi^0)$ give $\Delta\chi^2 = (2.7, 5.9, 2.9)$

pre-LP'05 data

hints of a puzzle?

see also Buras et al.; Kim et al.

SCET based fit

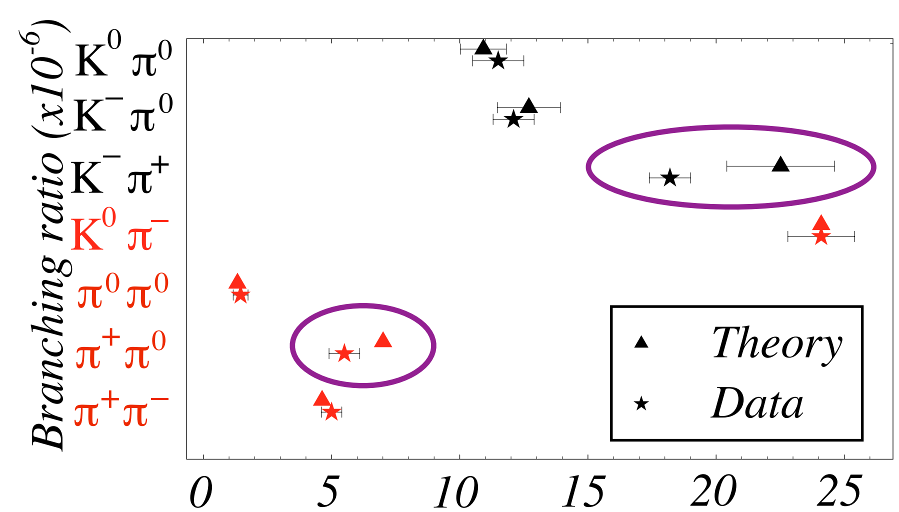
Bauer, Rothstein, I.S. (to appear)

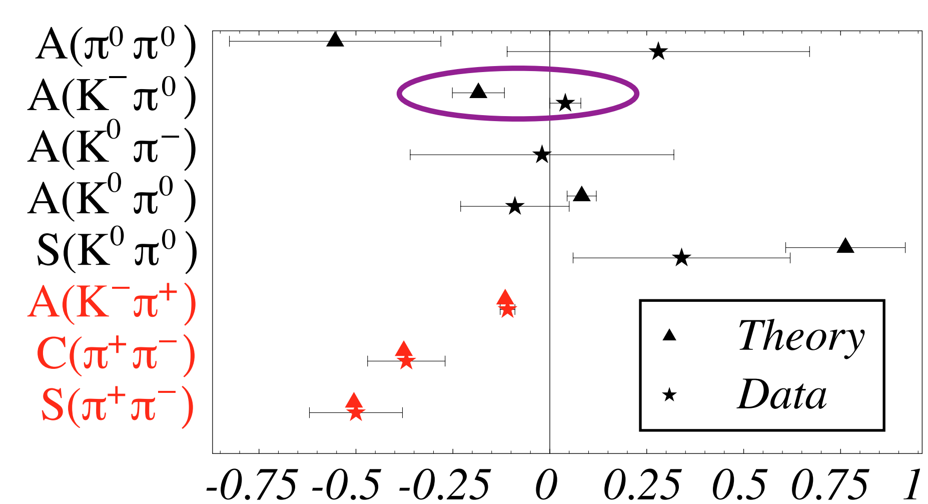
$\gamma = 59^\circ$ fixed

6 parameters + 2 fixed by SU(3)

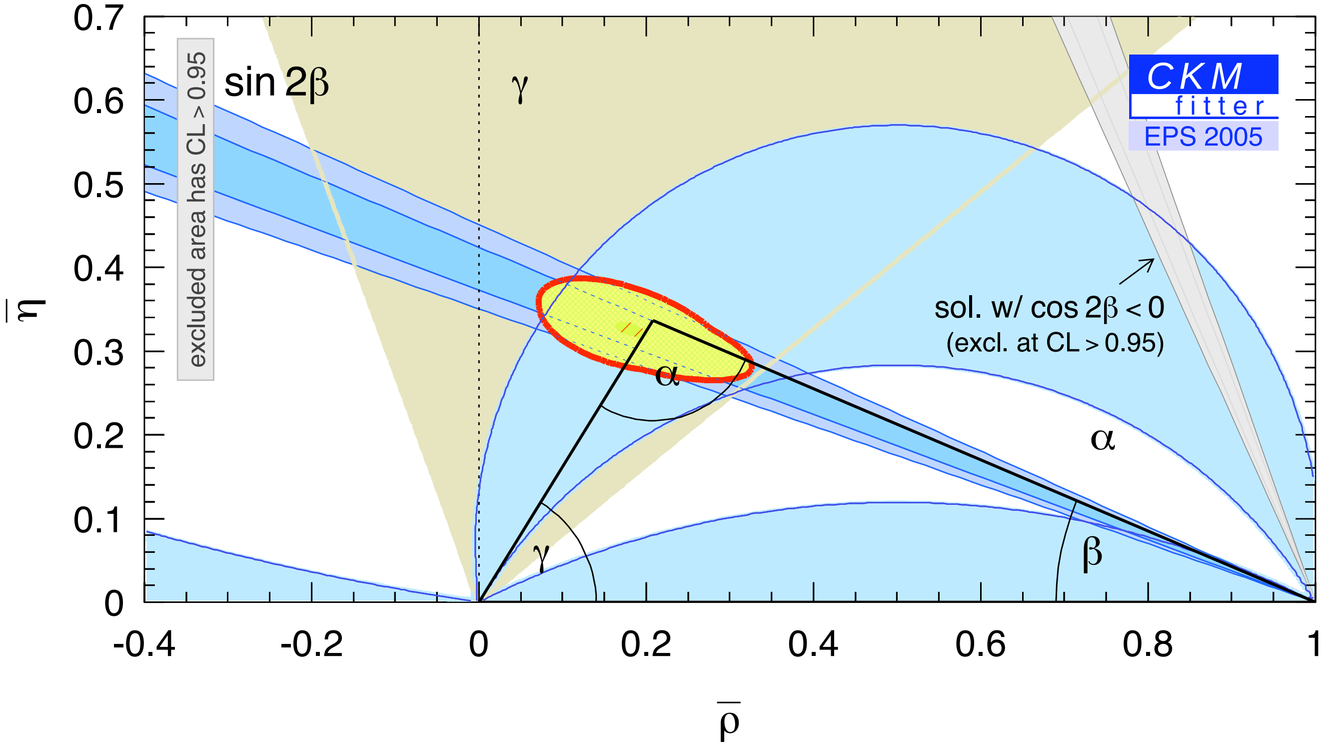
$Br(K^+\pi^-)$ $A_{CP}(K^-\pi^0)$

hints of a puzzle?

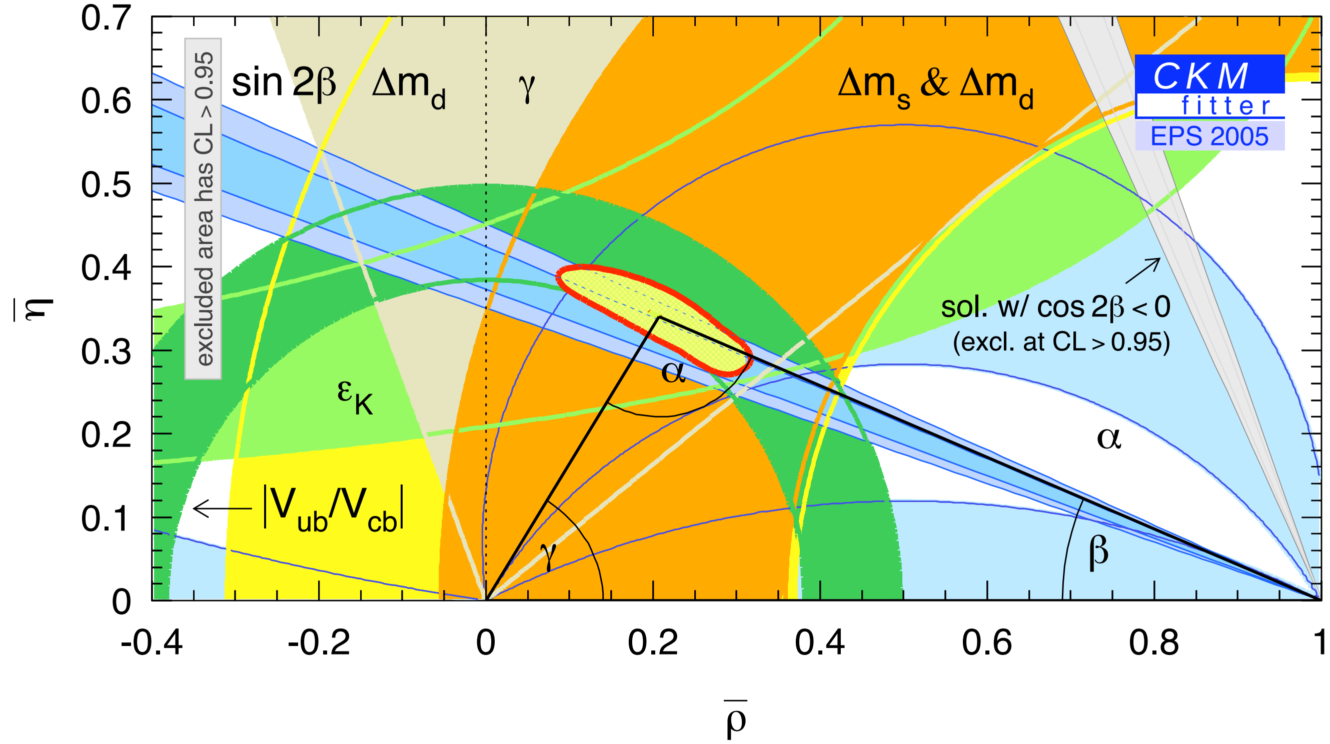




α, β, γ Constraints



All Constraints



α: $B \to \rho\rho$ $\qquad$ β: $B \to \psi K$

γ: $B \to DK$

- constraints from angles dominate, will scale with statistics
- other measurements test the SM, constrain new flavor physics
- Think of $H_{weak} = \sum_{i=1}^{\sim 100} C_i O_i$ where SM relates the C_i and all these connections need to be tested