Ingredients for a Precise Top-Quark Mass Measurement from Jets

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Based on:

A. Hoang, S. Fleming, S. Mantry, & I.S. (hep-ph/0703207)
A. Hoang & I.S. (arXiv:0709.3519)
A. Hoang, S. Fleming, S. Mantry, & I.S. (arXiv:0711.2079)
A. Jain, I. Scimemi, & I.S. (arXiv:0801.0743)

Outline

- Motivation. Why do we want a precision m_t ?
- Top mass measurements. Expt & Theory Issues. Which mass?

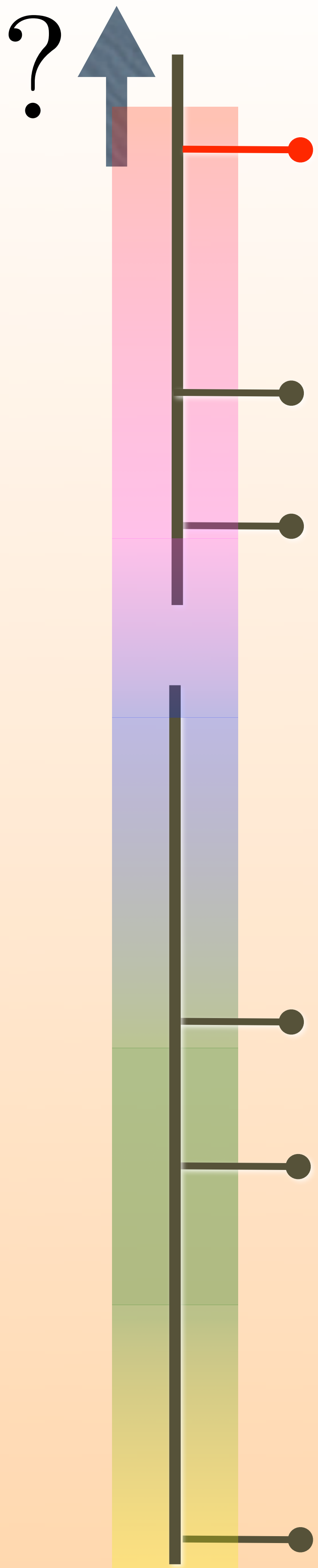
$$M_t^{\rm peak} = m_t + \text{(nonperturbative effects)} + \text{(perturbative effects)}$$

- Factorization theorem for Jet Invariant Masses

$$e^+e^- \to t\bar{t} \qquad\qquad Q \gg m_t \gg \Gamma_t$$

  - Summation of Large Logs
  - Heavy-Quark Jet Function (perturbative shift)
  - Gluon Soft Function (nonperturbative shift)
- Cross Sections Results at NLL order
- Implications

Motivation



Q

- The top mass is a fundamental parameter of the Standard Model

$m_t = 172.6 \pm 1.4\,\text{GeV}$ (a 0.8% experimental error)

m_W

(theory error? what mass is it?)

- Important for precision e.w. constraints
- Top Yukawa coupling is large. Top parameters are important for many new physics models

$\Gamma_t = 1.4\,\text{GeV}$ from $t \to bW$

Λ_{QCD}

- Top is very unstable, it decays before it has a chance to hadronize. How does this effect jet observables involving top-quarks?

$m_{u,d}$

$Q \simeq 1\,\text{TeV}$ Production scale

$m_t = 172.6 \pm 1.4\,\text{GeV}$ Mass scale

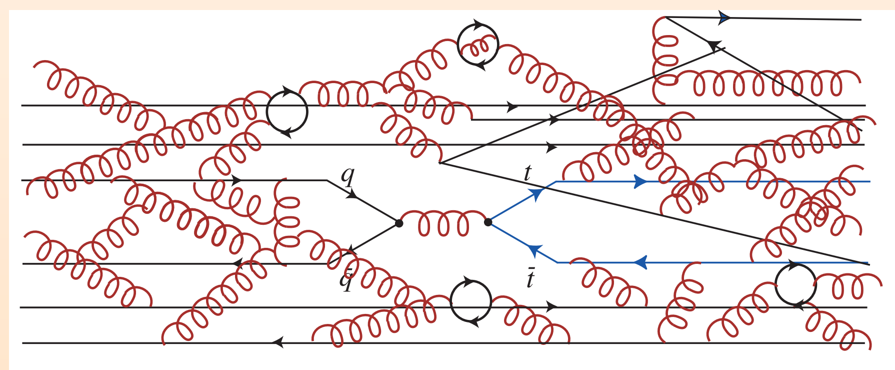
$\Gamma_t \simeq 1.5\,\text{GeV}$ Short Lifetime

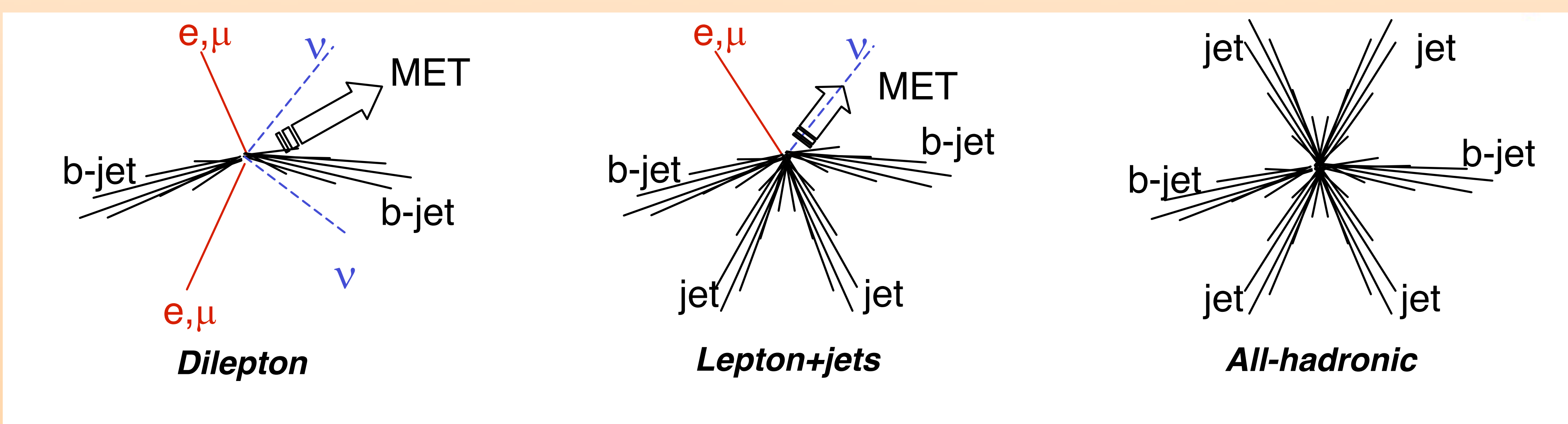
Λ_{QCD}

$$pp \xrightarrow{\sigma} t\bar{t}X$$

Γ: $t \to bW$, $\to \ldots$

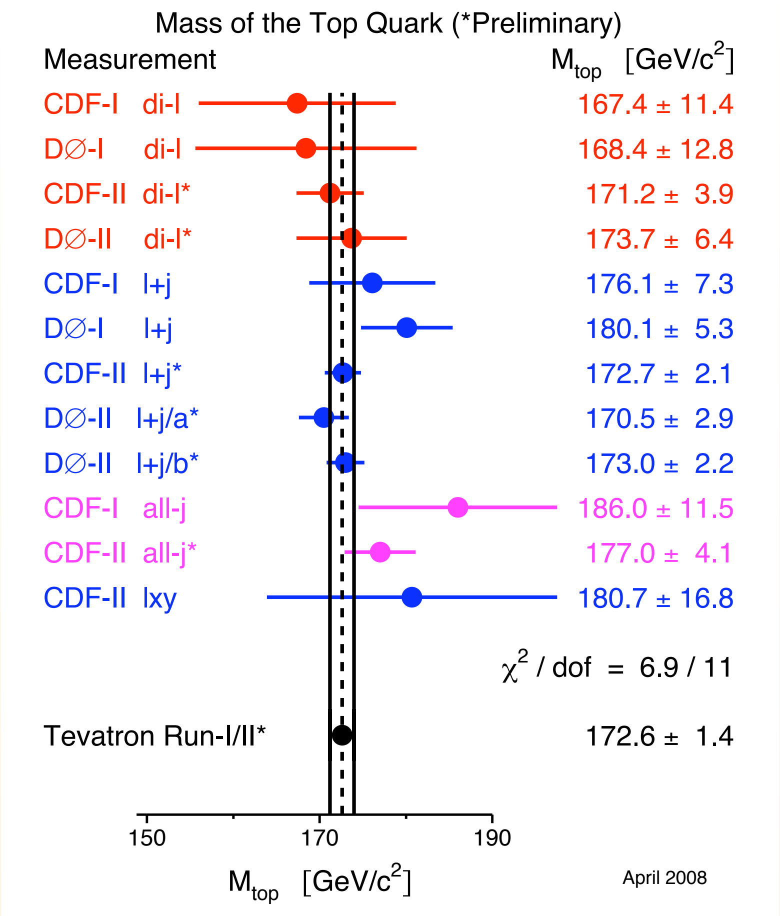
bW: $W \to q\bar{q}'$, $W \to e^+\nu$





World average (2008):

$$m_t = 172.6 \pm 0.8(\text{stat}) \pm 1.1(\text{syst})\,\text{GeV}$$



Why precision m_t?

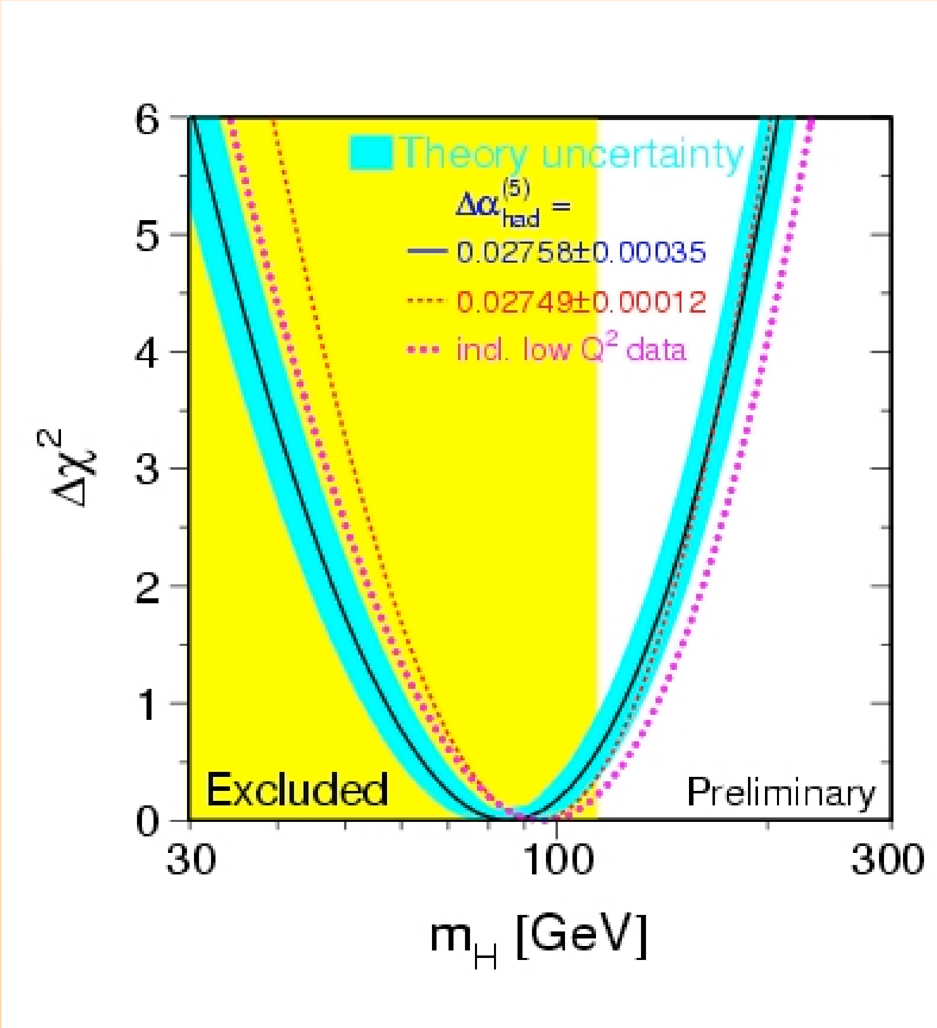
eg. Electroweak precision observables

$$m_H = 76^{+33}_{-24}\,\text{GeV} \rightarrow 87$$

$$m_H < 182\,\text{GeV} \quad (95\%\ \text{CL}) \rightarrow 209$$

A $2\,\text{GeV}$ shift in m_t changes these central values by 15%.

Gruenewald, EPS(2007)

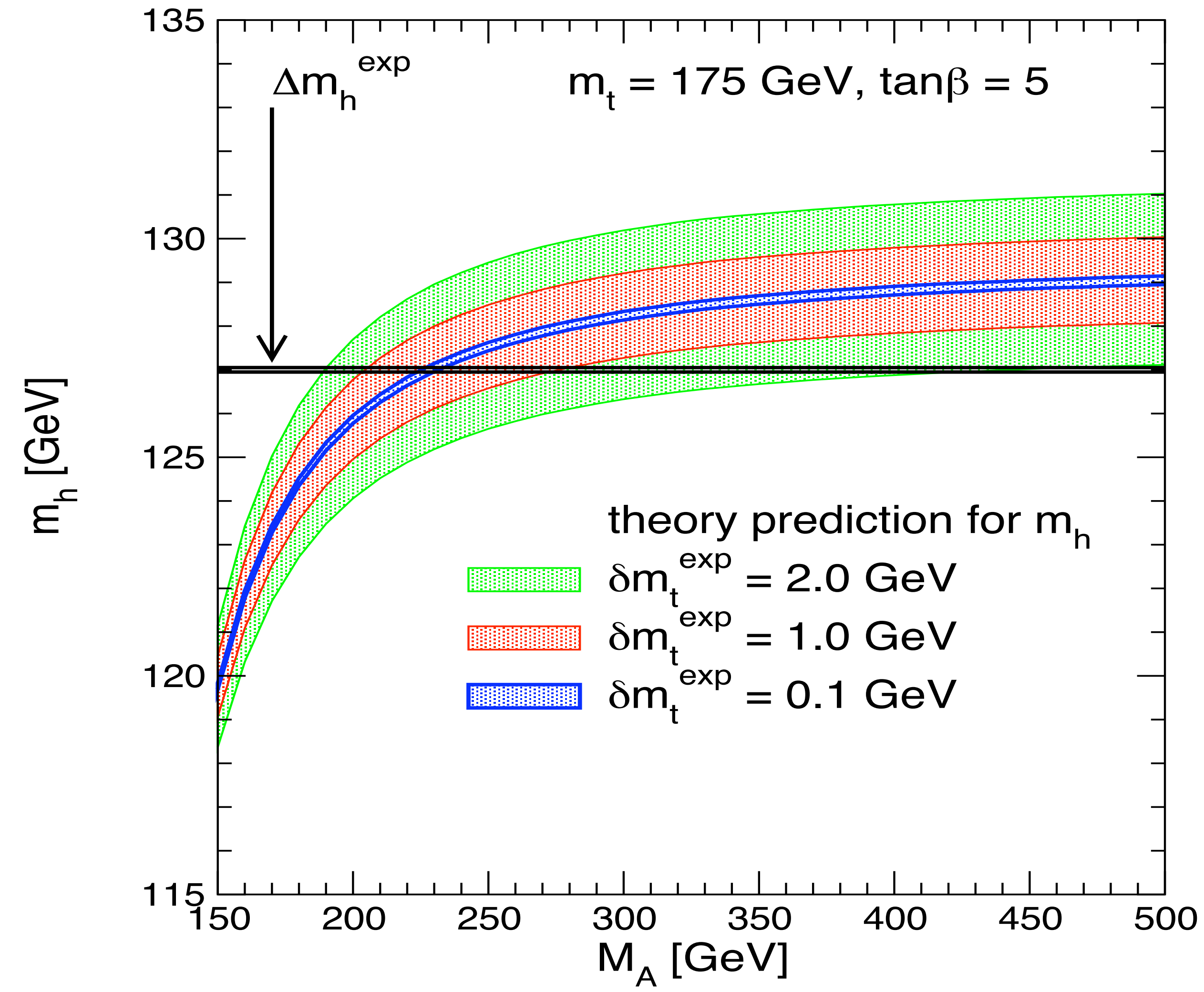


Mass of Lightest MSSM Higgs Boson

$$m_h^2 \simeq M_Z^2 + \frac{G_F\, m_t^4}{\pi^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right)$$

rule of thumb, want:

$$\delta m_t \sim \delta m_h$$



Heinemeyer et.al.('03)

How is the top-mass measured?

from A.Juste

Template Method (CDF II)

- Principle: perform kinematic fit and reconstruct top mass event by event. E.g. in lepton+jets channel:

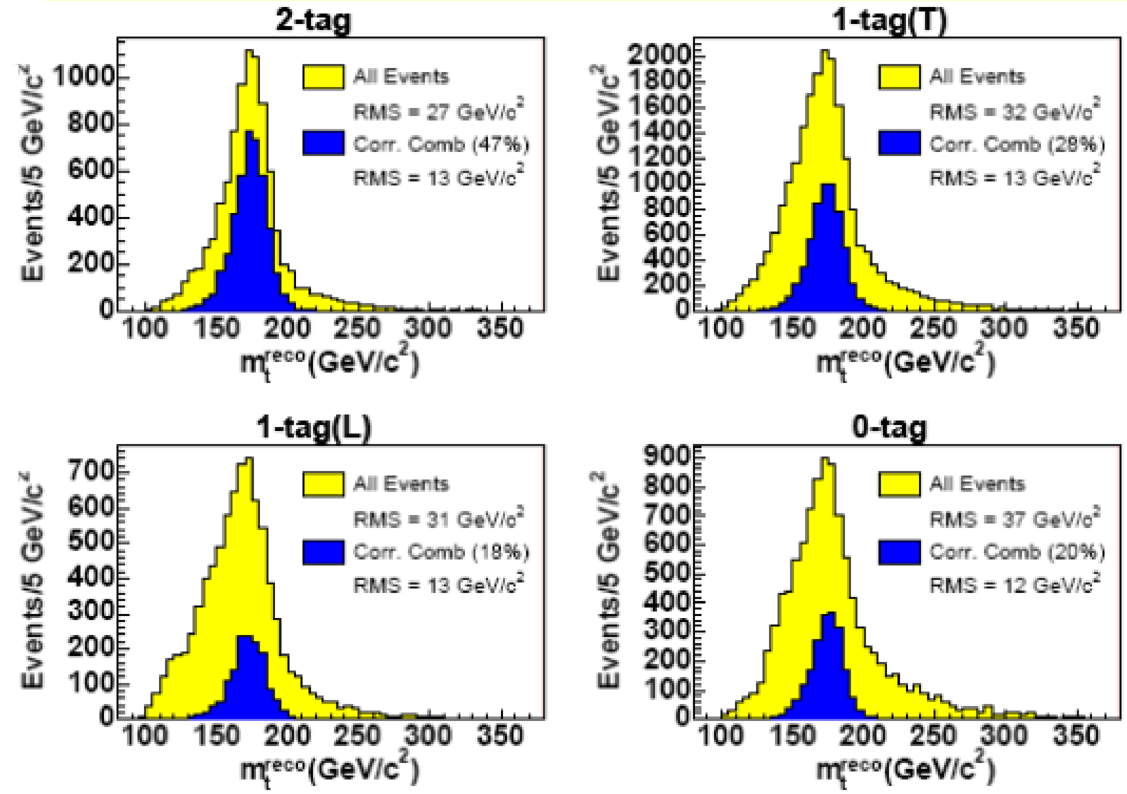
$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit} - p_T^{i,meas})^2}{\sigma_i^2} + \sum_{j=x,y} \frac{(p_j^{UE,fit} - p_j^{UE,meas})^2}{\sigma_j^2}$$

$$+ \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2} + \frac{(M_{jj} - M_W)^2}{\Gamma_W^2} + \frac{(M_{b\ell\nu} - m_t^{reco})^2}{\Gamma_t^2} + \frac{(M_{bjj} - m_t^{reco})^2}{\Gamma_t^2}$$

Usually pick solution with lowest χ^2.

- Build templates from MC for signal and background and compare to data.

Lepton+jets (≥1 b-tag); Signal-only templates



Dynamics Method (D0 II)

- Principle: compute event-by-event probability as a function of m_t making use of all reconstructed objects in the events (integrate over unknowns). Maximize sensitivity by:

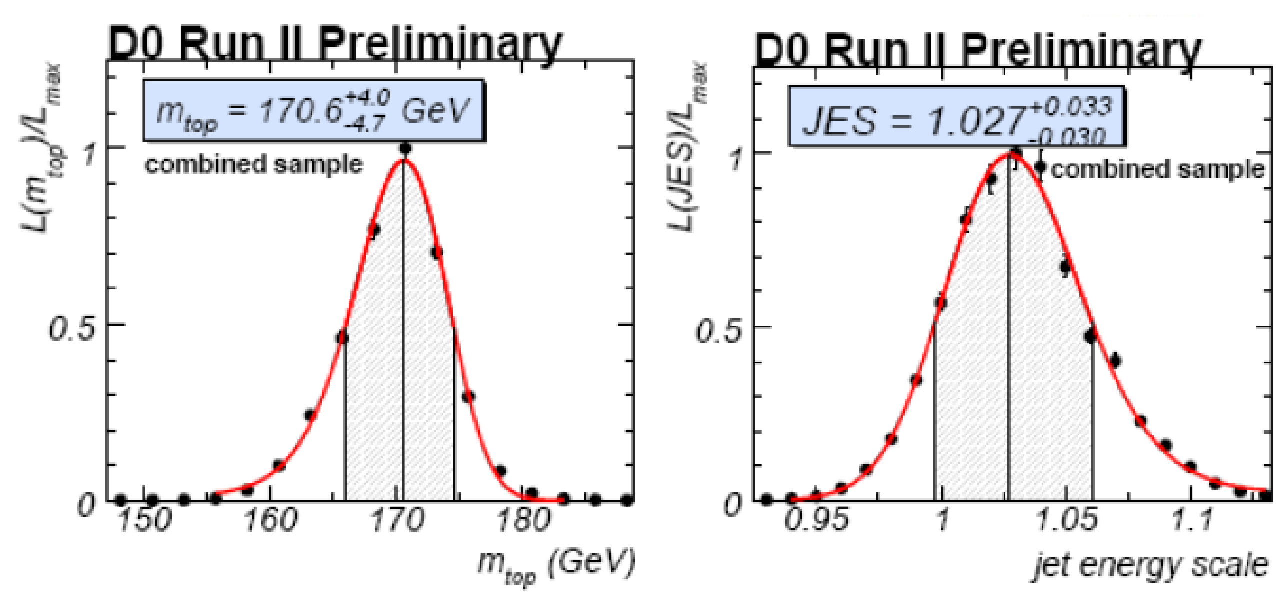
$$P(x;m_t) = \frac{1}{\sigma} \int d^n\sigma(y;m_t)\, dq_1 dq_2\, f(q_1)\, f(q_2)\, W(x \mid y)$$

parton distribution functions

differential cross section (LO matrix element)

transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)

Lepton+jets (370 pb^{-1})



Uncertainties

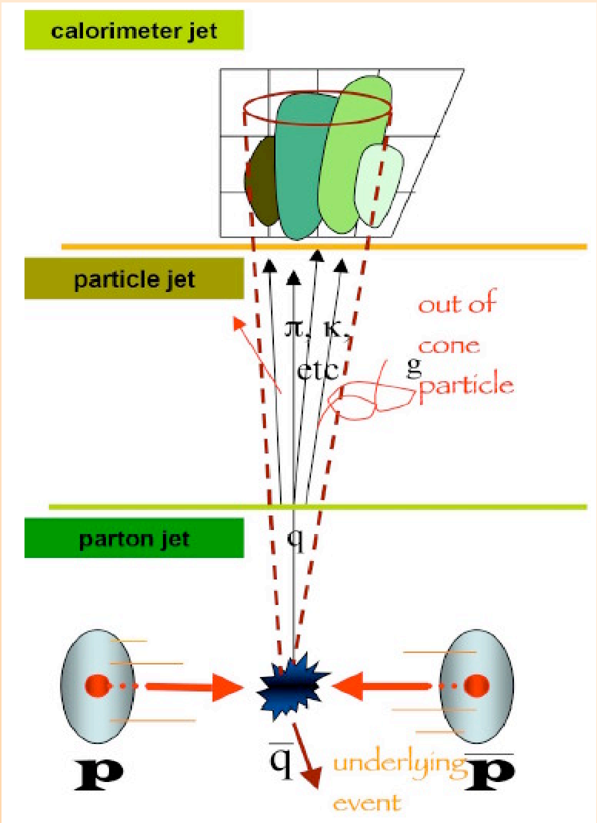
$$m_t = 172.6 \pm 0.8(\mathrm{stat}) \pm 1.1(\mathrm{syst})\ \mathrm{GeV}$$

(eg. reconstruction)

- determine parton momentum of daughters, combinatorics
- jet-energy scale: calorimeter response, uninstrumented zones, multiple hard interactions, energy outside the jet “cone”, underlying event (spectator partons)

W-mass helps

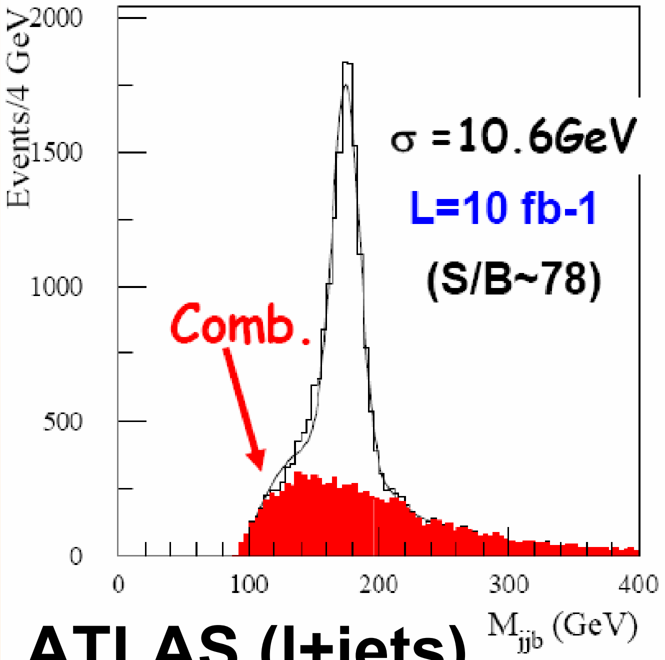
- initial & final state radiation, parton distribution functions, b-fragmentation
- which jet algorithm? which Monte-Carlo?
- background (W+jets), b-tagging efficiency
- Statistics



Future -LHC: $pp \to t\bar{t}X$

top factory, 8 million $t\bar{t}$ / year

$\delta m_t \sim 1\,\mathrm{GeV}$ systematics dominated



ATLAS (l+jets)

Future -ILC: $e^+e^- \to t\bar{t}$

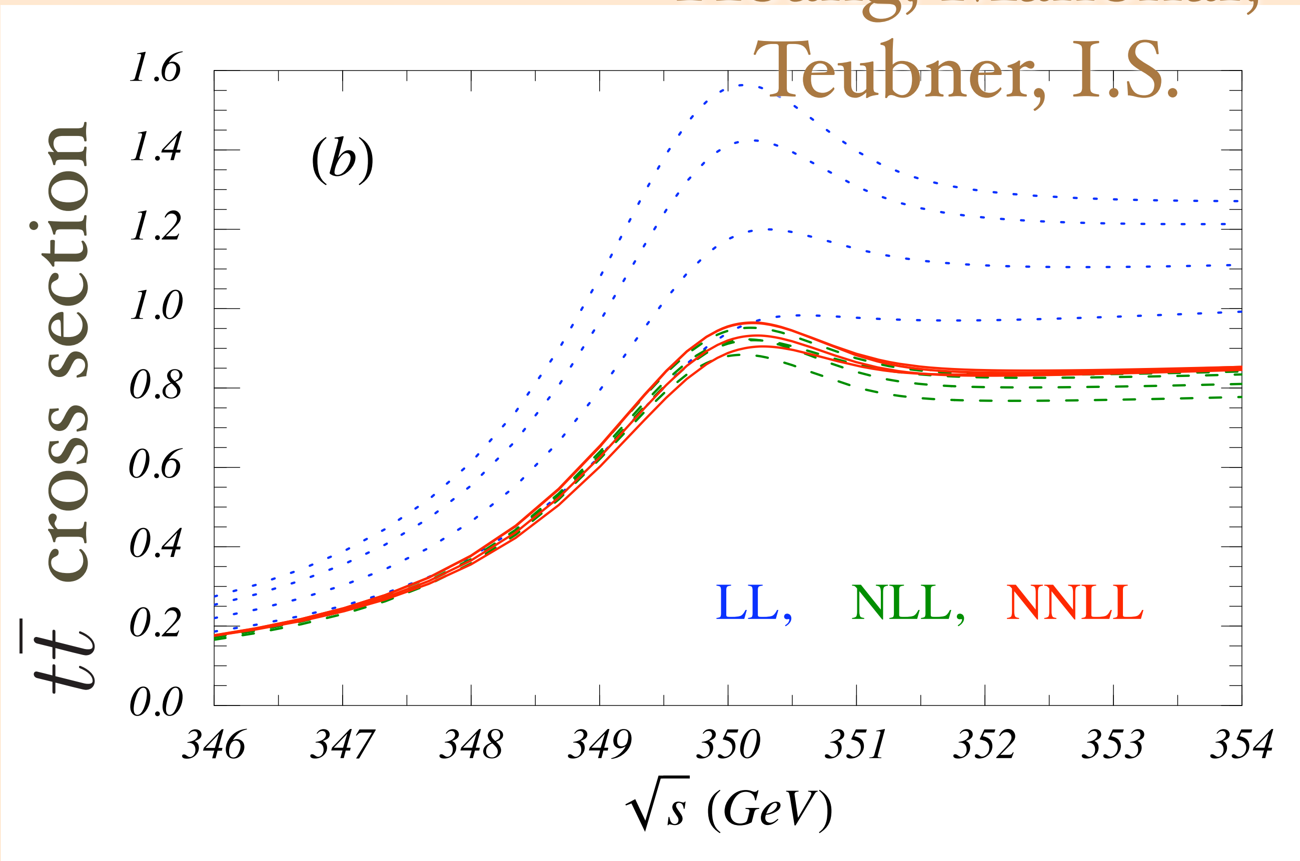
exploit threshold region

$$\sqrt{s} \simeq 2m_t$$

with high precision theory calculations

$$\delta m_t \sim 0.1\,\mathrm{GeV}$$

Hoang, Manohar, Teubner, I.S.

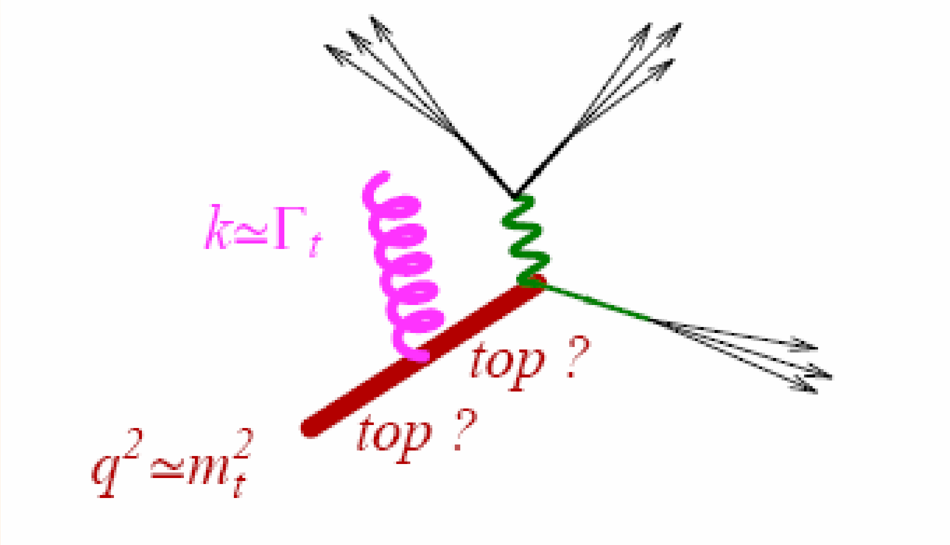


What mass is it?

$m_t = 172.6 \pm 0.8(\mathrm{stat}) \pm 1.1(\mathrm{syst})\ \mathrm{GeV}$

- pole mass?
  - ambiguity $\delta m \sim \Lambda_{\mathrm{QCD}}$, linear sensitivity to IR momenta
  - poor behavior of α_s expansion
  - not used anymore for m_b, m_c

    e.g. $m_b^{1S} = (4.70 \pm 0.04)\,\mathrm{GeV}$
  - Monte Carlo has cutoff on shower / hadronization model



$\delta m \sim \alpha_s(\Gamma)\Gamma$

quark masses are Lagrangian parameters, use a suitable scheme

$$m_t^{\mathrm{pole}} = m_t^{\mathrm{schemeA}}(1 + \alpha_s + \alpha_s^2 + \ldots)$$

or

$$m_t^{\mathrm{pole}} = m_t^{\mathrm{schemeB}} + R\,(\alpha_s + \alpha_s^2 + \ldots)$$

- top $\overline{\text{MS}}$ mass? $\delta\overline{m} = m^{\text{pole}} - m^{\overline{\text{MS}}}(m) \sim 8\,\text{GeV}$

If top-decay is described by Breit-Wigner, the answer is NO

When we switch to a short-distance mass scheme we must expand in α_s

$$\delta\overline{m} \sim \alpha_s \overline{m} \gg \Gamma$$

$$\frac{\Gamma}{\left[\frac{(M_t^2 - m^{\text{pole}\,2})^2}{m^{\text{pole}\,2}} + \Gamma^2\right]} = \frac{\Gamma}{\left[\frac{(M_t^2 - \overline{m}^2)^2}{\overline{m}^2} + \Gamma^2\right]} + \frac{(4\,\hat{s}\,\Gamma)\,\delta\overline{m}}{\left[\frac{(M_t^2 - \overline{m}^2)^2}{\overline{m}^2} + \Gamma^2\right]^2}$$

$$\sim 1/\Gamma \qquad\qquad \sim \alpha_s \overline{m}/\Gamma^2$$

not a correction! it swamps the 1st term

- must be a "top-resonance mass scheme" $R \sim \Gamma$

$$m^{\text{pole}} - m \sim \alpha_s \Gamma$$

Lesson: some schemes are more appropriate than others

Theory Issues for $pp \to t\bar{t}X$

- jet observable ★★
- suitable top mass for jets ★
- initial state radiation
- final state radiation ★
- underlying events
- color reconnection ★
- beam remnant
- parton distributions
- sum large logs ★

Here we'll study
$e^+e^- \to t\bar{t}X$
and the issues ★

Top Mass from Jets far above threshold at the ILC

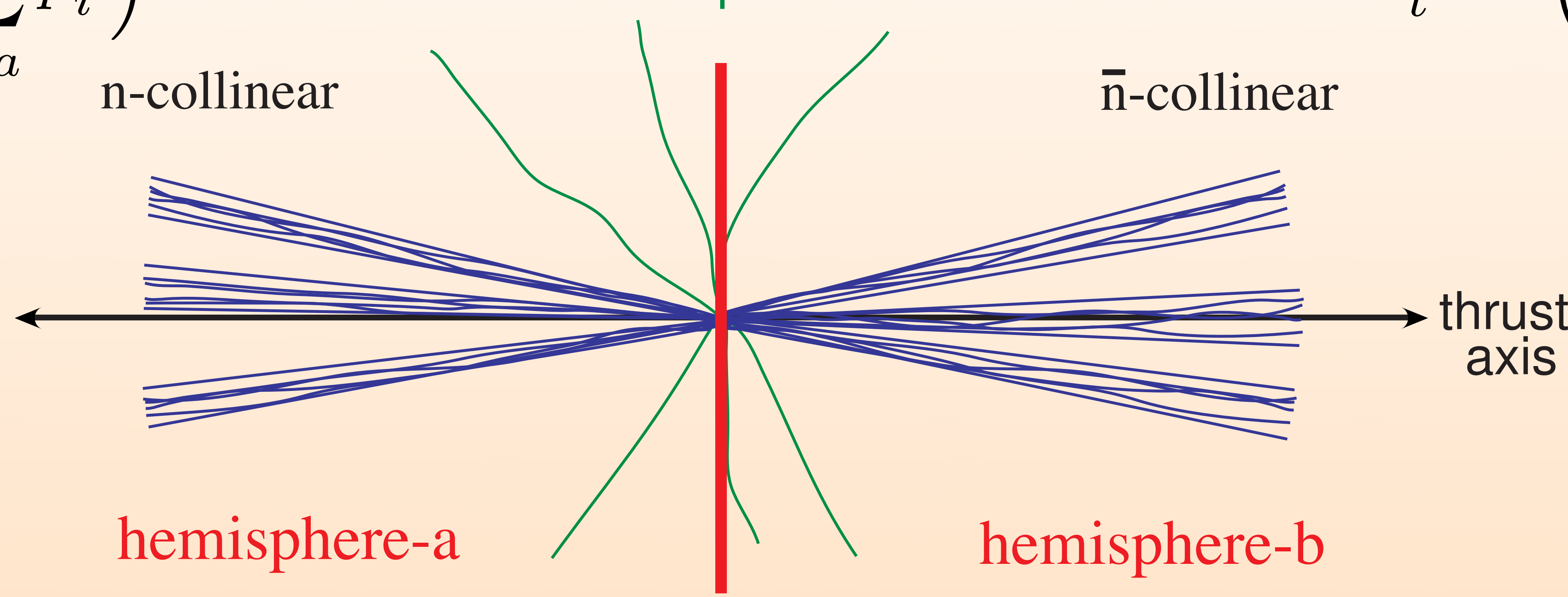
$$Q \gg m_t \gg \Gamma_t$$

Measure what observable?

$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}$$

Hemisphere Invariant Masses

$$M_t^2 = \Big(\sum_{i\in a} p_i^\mu\Big)^2 \qquad M_{\bar t}^2 = \Big(\sum_{i\in b} p_i^\mu\Big)^2$$

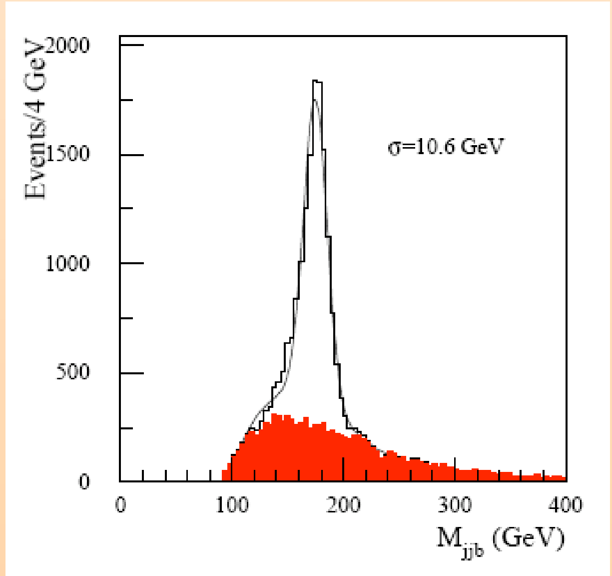


Peak region:

$$s_t \equiv M_t^2 - m^2 \sim m\Gamma \ll m^2$$

$$\boxed{\hat s_t \equiv \frac{M_t^2 - m^2}{m} \sim \Gamma \ll m}$$

Breit Wigner: $\dfrac{m\Gamma}{s_t^2 + (m\Gamma)^2} = \left(\dfrac{\Gamma}{m}\right)\dfrac{1}{\hat s_t^2 + \Gamma^2}$



- $Q \gg m$ "dijets" dominate, inclusive in decay products

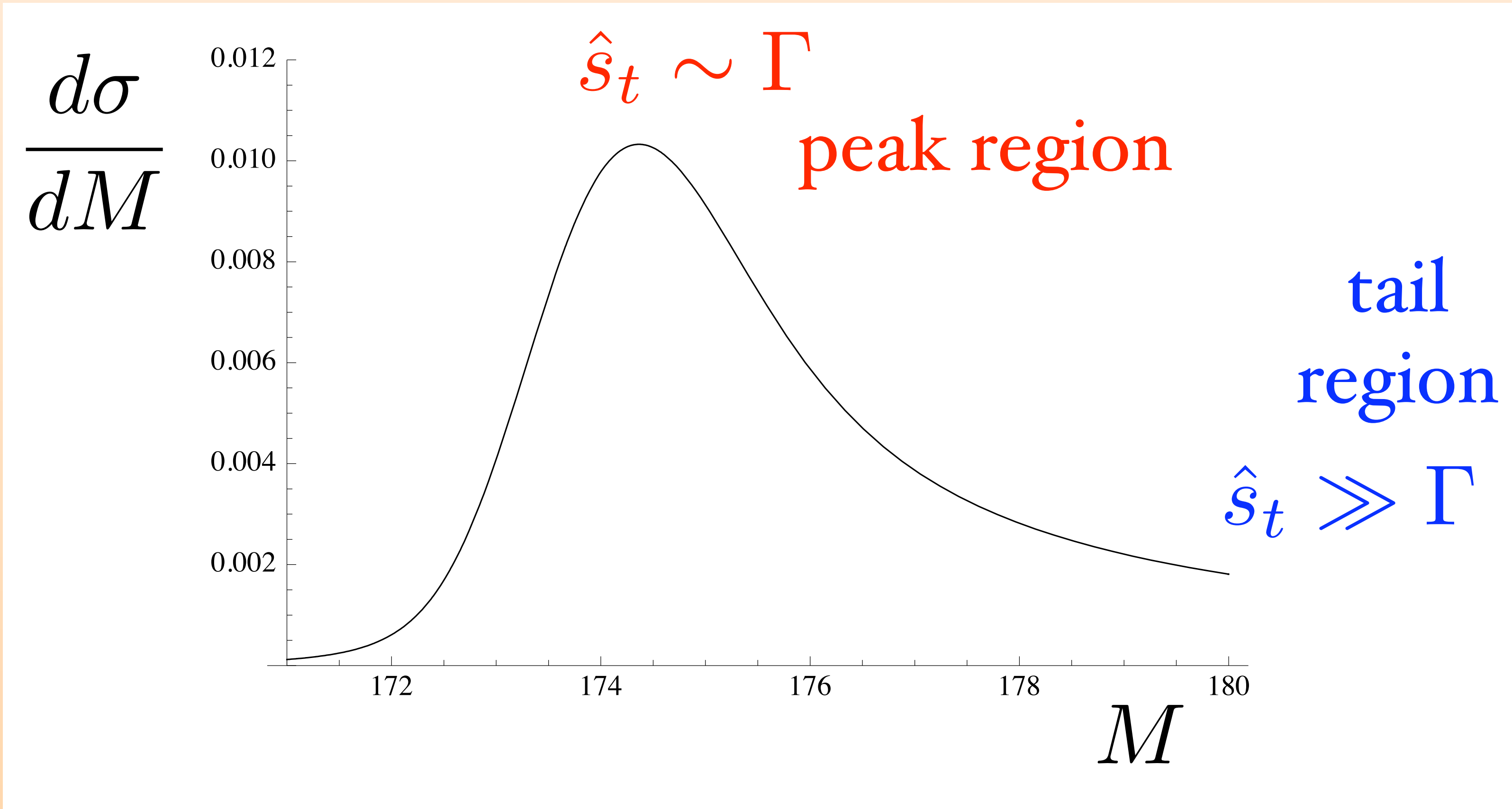
- $m \gg \Gamma$ = physical width

$$\Gamma = \Gamma_t + \ldots$$

- $m \gg \hat{s}_t$

- $\Gamma > \Lambda_{\rm QCD}$

$$\hat{s}_t \equiv \frac{M_t^2 - m^2}{m}$$



$$Q \gg m \gg \Gamma \sim \hat{s}_{t,\bar{t}}$$

Disparate Scales → Effective Field Theory

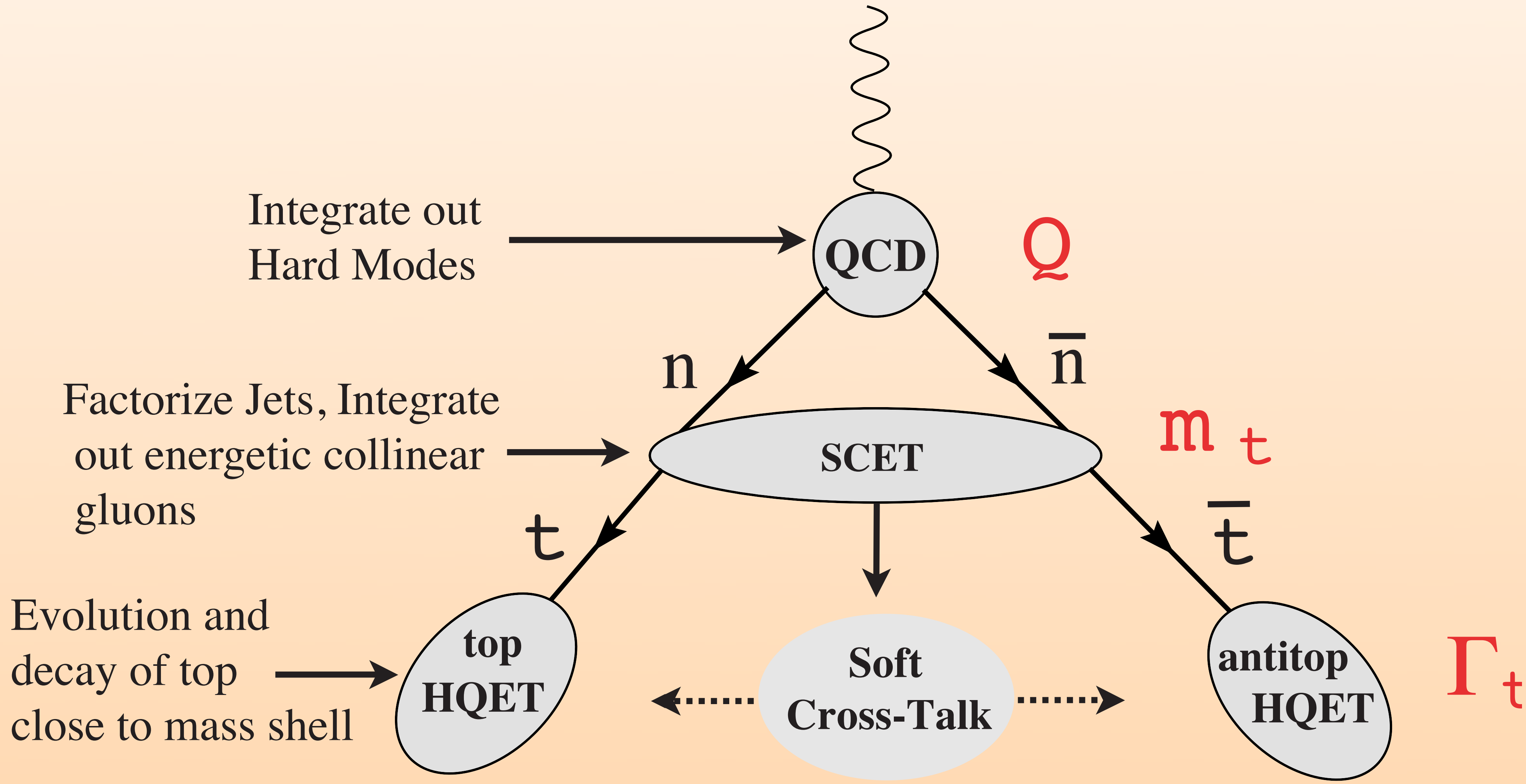
QCD

↓

SCET

↓

HQET



Derive a Factorization Theorem:

Fleming, Hoang, Mantry, I.S.

Answer

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\text{hemi}}(\ell^+, \ell^-, \mu).$$

$$+\,\mathcal{O}\Big(\frac{m\alpha_s(m)}{Q}\Big) + \mathcal{O}\Big(\frac{m^2}{Q^2}\Big) + \mathcal{O}\Big(\frac{\Gamma_t}{m}\Big) + \mathcal{O}\Big(\frac{s_t, s_{\bar t}}{m^2}\Big)$$

Valid to all orders in α_s

Compare to factorization theorem for massless dijets:

$$\left(\frac{d^2\sigma}{dM_a^2 dM_b^2}\right) = \sigma_0 H(Q,\mu) \int d\ell^+ d\ell^-\, J_+(M_a^2 - Q\ell^+, \mu)\, J_-(M_b^2 - Q\ell^-, \mu)\, S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Korchemsky & Sterman

Hard Production modes integrated out

"Hard" collinear gluons integrated out

Answer

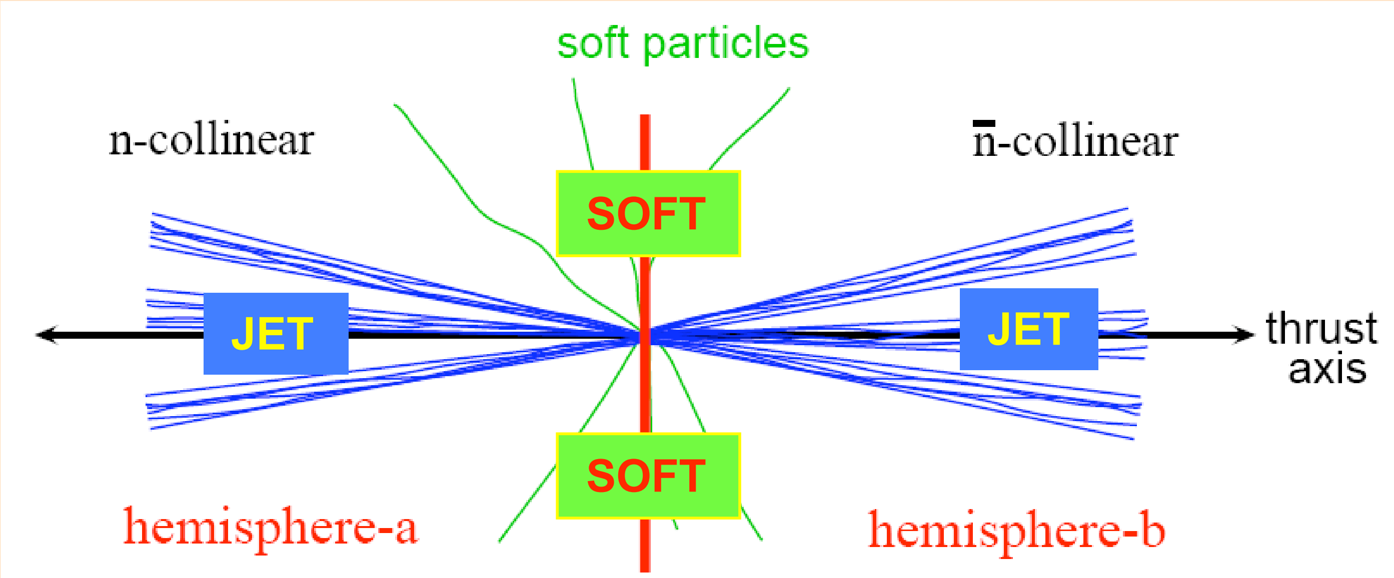
$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu).$$

Jet Functions

Evolution and decay of top quark close to mass shell

Soft Function

Non-perturbative Cross talk



Hard Production modes integrated out

"Hard" collinear gluons integrated out

Answer

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu).$$

Evolution and decay of top quark close to mass shell

Non-perturbative Cross talk

At tree level in α_s expansion this is a Breit-Wigner.

- B.W. receives calculable perturbative corrections

$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu).$$

Answer

- cross-section depends on a hadronic soft function, not just B.W.'s
  ** the B.W. is only a good approx. for collinear top & gluons **

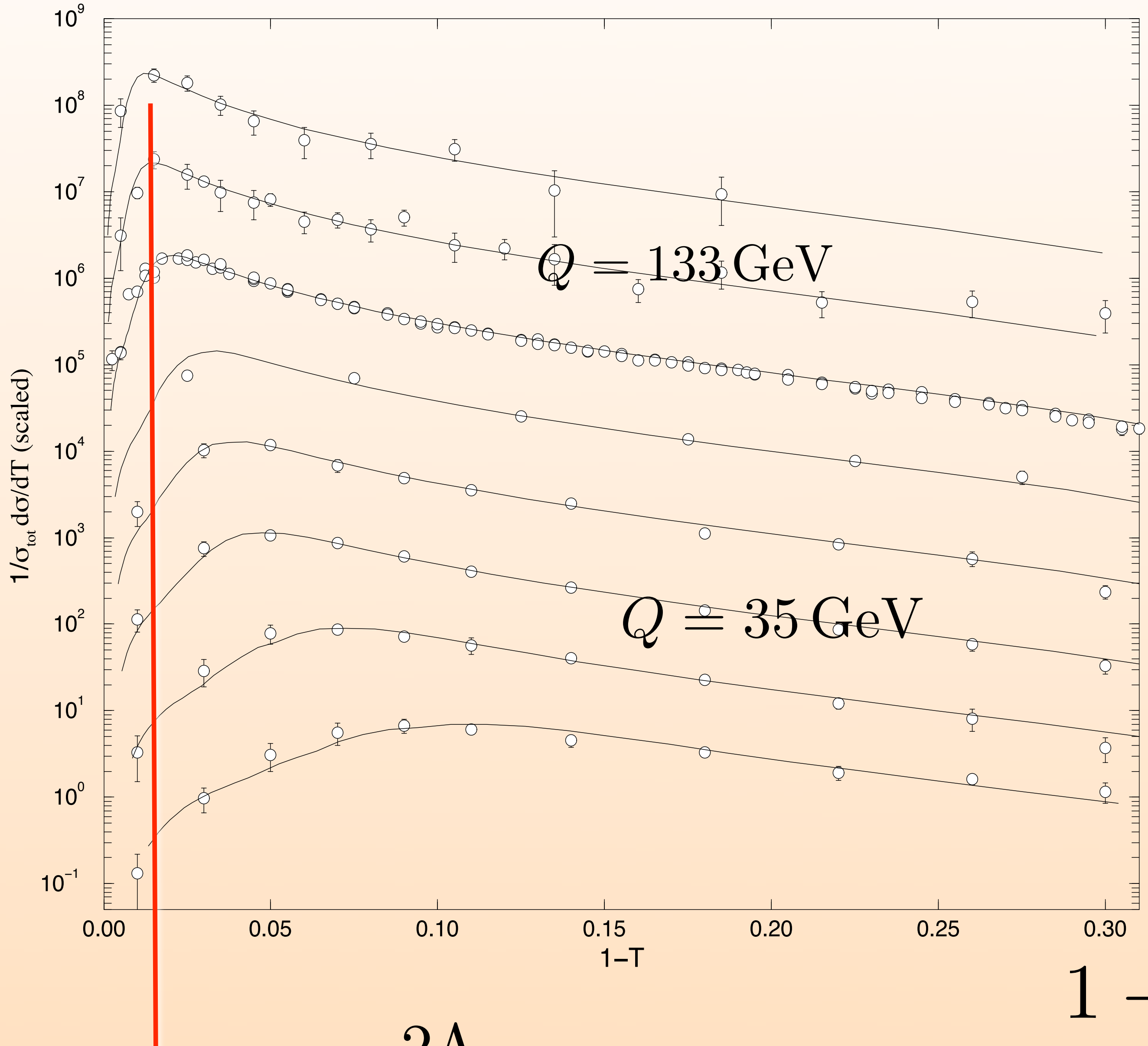
- the formula removes the largest component of soft momentum to get the correct argument for evaluating the B.W. functions

$$\hat s_t = \frac{M_t^2 - m^2}{m}$$

Everything but the soft function is calculable in perturbation theory.

S_hemi is universal, & measured in massless jet event shapes (at LEP!)

Eg. Thrust data from massless quark jets at LEP



$$T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \mathbf{p}_i|}{Q}$$

Korchemsky & Sterman

$1 - T$

peaks at $\dfrac{2\Lambda_{\mathrm{QCD}}}{Q}$

$$1 = \int dT\ \delta\Big(1 - T - \frac{M_a^2 + M_b^2}{Q^2}\Big)$$

$$\left(\frac{d^2\sigma}{dM_a^2 dM_b^2}\right) = \sigma_0 H(Q,\mu) \int d\ell^+ d\ell^-\ J_+(M_a^2 - Q\ell^+, \mu)\ J_-(M_b^2 - Q\ell^-, \mu)\ S_{\mathrm{hemi}}(\ell^+, \ell^-, \mu)$$

For our event shape for massive quarks:

Answer

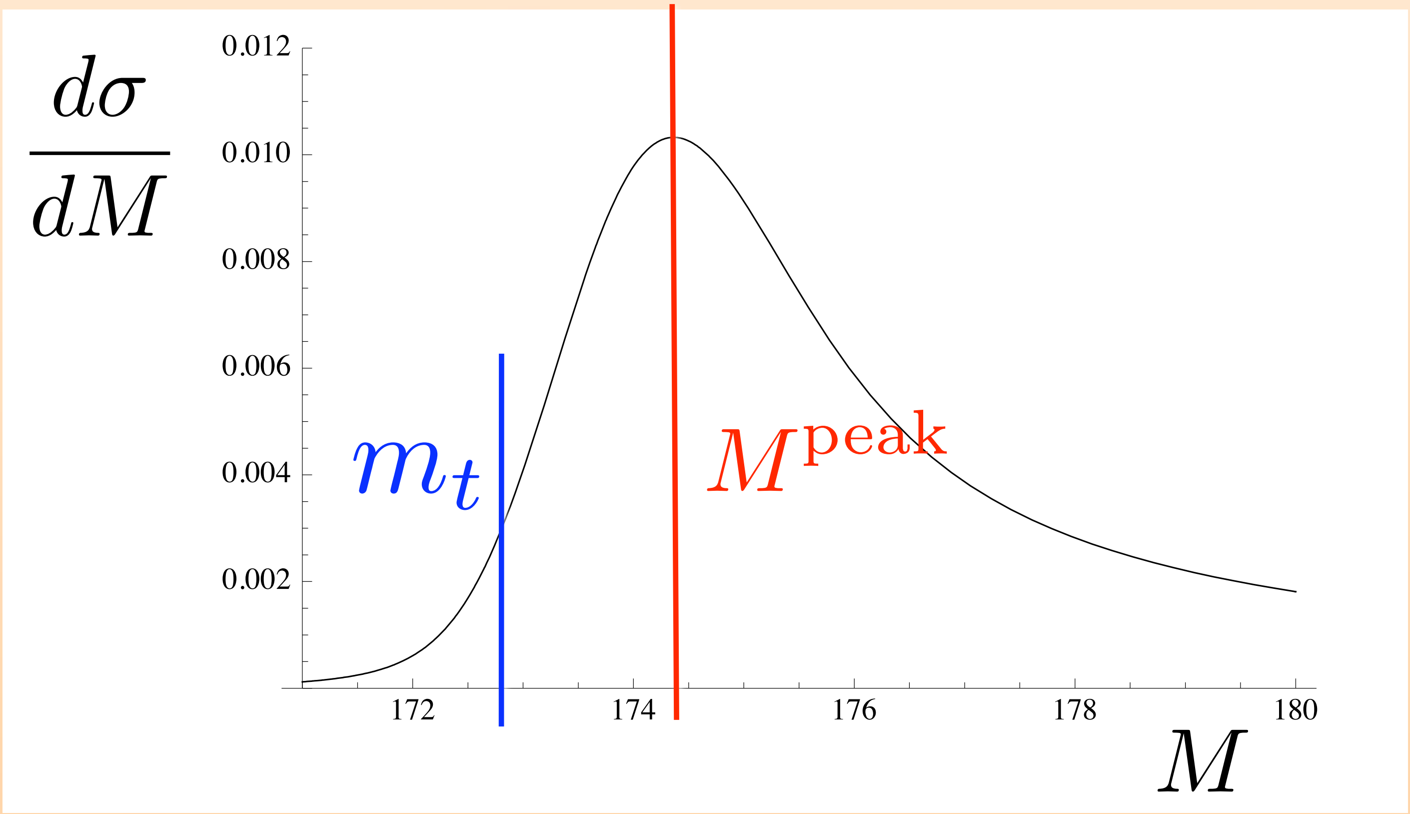
$$\left(\frac{d^2\sigma}{dM_t^2\, dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m, \frac{Q}{m}, \mu_m, \mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\Big)\, S_{\rm hemi}(\ell^+, \ell^-, \mu).$$

$$M^{\rm peak} = m_t + \Gamma_t(\alpha_s + \alpha_s^2 + \ldots) + \frac{Q\Lambda_{\rm QCD}}{m_t}$$

measure this ($M^{\rm peak}$)

extract this (m_t)

Short distance m_t can (in principle) be determined to better than $\Lambda_{\rm QCD}$

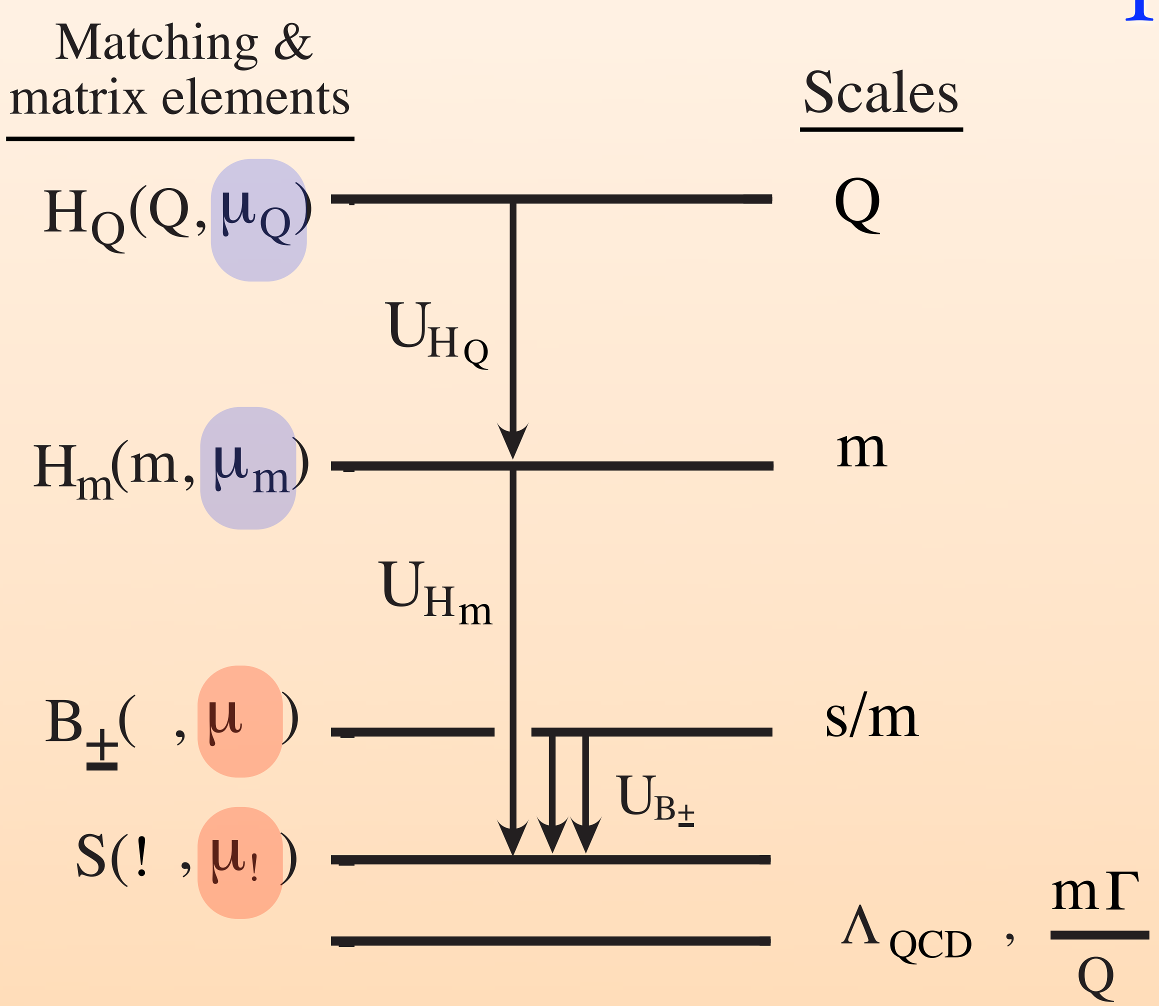


Summing the Large Logs

$$\frac{d\sigma}{dM_t^2\, dM_{\bar t}^2} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m_J, \frac{Q}{m_J}, \mu_m, \mu\Big)$$
$$\times \int d\ell^+ d\ell^- B_+\Big(\hat{s}_t - \frac{Q\ell^+}{m_J}, \quad {}_t, \mu\Big)\, B_-\Big(\hat{s}_{\bar t} - \frac{Q\ell^-}{m_J}, \quad {}_t, \mu\Big) S(\ell^+, \ell^-, \mu)$$

The various functions are sensitive to different scales

To minimize the logs we need several stages of matching and running



$$\mu_Q \simeq Q$$
$$\mu_m \simeq m$$
$$\mu_\Gamma \simeq \mathcal{O}\Big(\Gamma_t + \frac{Q\Lambda}{m} + \frac{s_{t,\bar t}}{m}\Big),$$
$$\mu_\Lambda \simeq \mathcal{O}\Big(\Lambda + \frac{m\Gamma_t}{Q} + \frac{s_{t,\bar t}}{Q}\Big).$$

so typically $\frac{\mu_\Gamma}{\mu_\Delta} \sim \frac{Q}{m}$

Result with resummation:

$$\frac{d^2\sigma}{dM_t^2 dM_{\bar t}^2} = \sigma_0 \;\; H_Q(Q,\mu_h)\, U_{H_Q}(Q,\mu_h,\mu_m)\, H_m(m,\mu_m)\, U_{H_m}\Big(\frac{Q}{m_J},\mu_m,\mu_\Lambda\Big)$$
$$\times \int_{-\infty}^{\infty} d\hat s_t'\, d\hat s_{\bar t}'\; U_{B_+}(\hat s_t - \hat s_t', \mu_\Lambda,\mu_\Gamma)\, U_{B_-}(\hat s_{\bar t} - \hat s_{\bar t}', \mu_\Lambda,\mu_\Gamma)$$
$$\times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\; B_+\Big(\hat s_t' - \frac{Q\ell^+}{m},\Gamma,\mu_\Gamma\Big)\, B_-\Big(\hat s_{\bar t}' - \frac{Q\ell^-}{m},\Gamma,\mu_\Gamma\Big)\, S(\ell^+,\ell^-,\mu_\Lambda)$$

Here: sum double logs $\mathrm{LL} = \sum_k [\alpha_s \ln^2]^k$

$$\mu\frac{d}{d\mu} H_m\Big(m,\frac{Q}{m},\mu\Big) = \gamma_{H_m}\Big(\frac{Q}{m},\mu\Big)\, H_m\Big(m,\frac{Q}{m},\mu\Big)$$

$$\mu\frac{d}{d\mu} B_\pm(\hat s,\mu) = \int d\hat s'\; \gamma_{B_\pm}(\hat s - \hat s',\mu)\, B_\pm(\hat s',\mu)$$

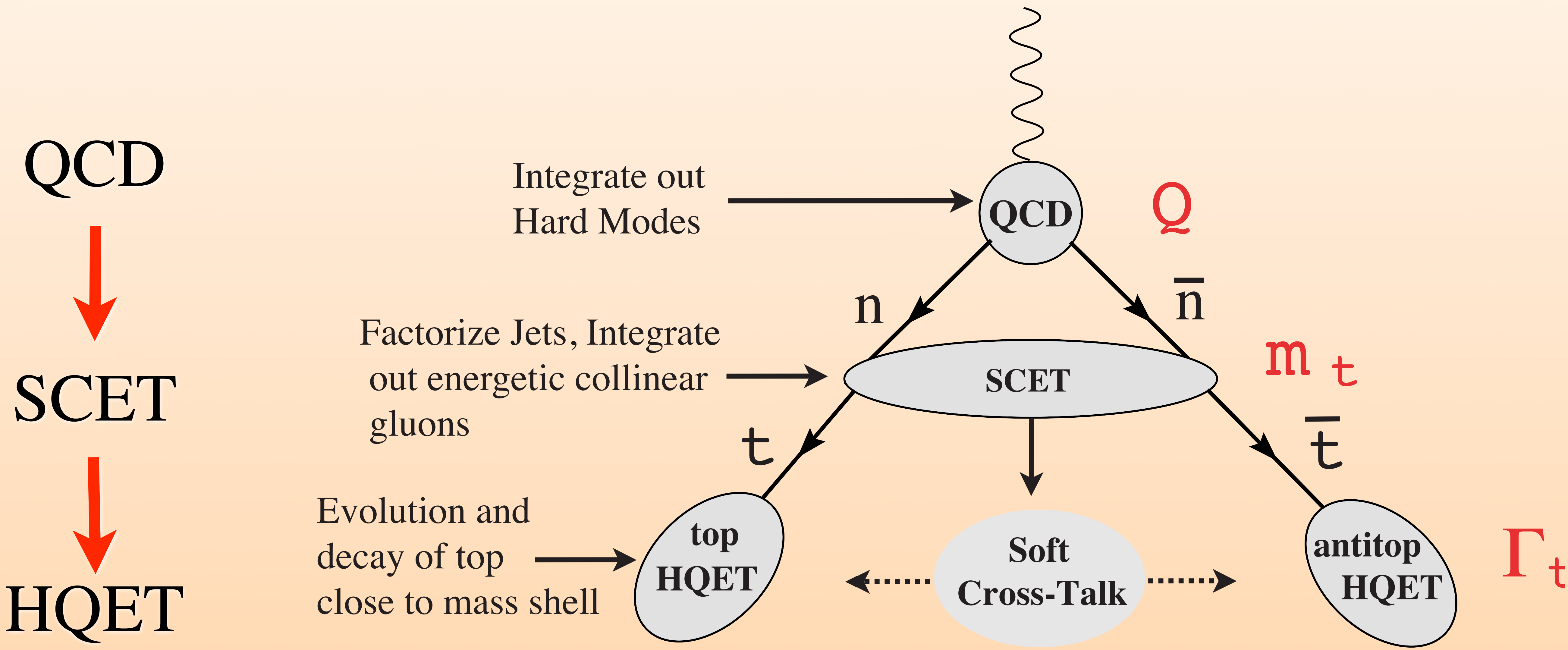
$$H_m\Big(m,\frac{Q}{m},\mu_m,\mu\Big) = H_m(m,\mu_m)\, U_{H_m}\Big(\frac{Q}{m},\mu_m,\mu\Big)$$

$$B_\pm(\hat s,\mu) = \int d\hat s'\; U_B(\hat s - \hat s',\mu,\mu_\Gamma)\; B_\pm(\hat s',\mu_\Gamma)$$

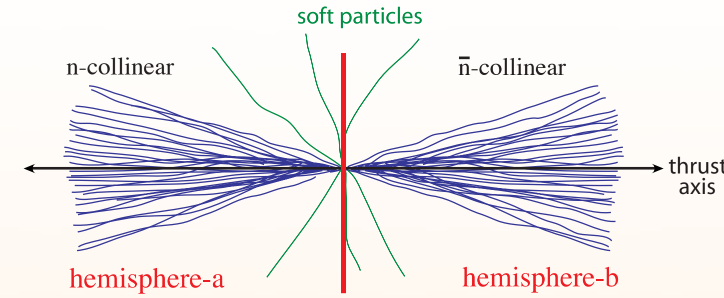
Only the logs between μ_Γ and μ_Λ can modify the shape of the invariant mass distribution (the rest just modify normalization)

All objects are defined in field theory.

Lets study the soft & jet functions in more detail



SCET



Degrees of Freedom

SCET $[\lambda \sim m/Q \ll 1]$		
	n-collinear $(\xi_n,\ A_n^\mu)$	$p_n^\mu \sim Q(\lambda^2, 1, \lambda)$
	$\bar{n}$-collinear $(\xi_{\bar{n}},\ A_{\bar{n}}^\mu)$	$p_{\bar{n}}^\mu \sim Q(1, \lambda^2, \lambda)$
Crosstalk:	soft $(q_s,\ A_s^\mu)$	$p_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

quark fields

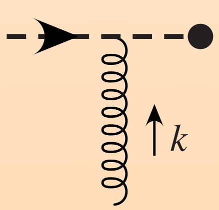
gluon fields

$(+, -, \perp)$

light-cone coordinates

LO collinear Lagrangian:

$$\mathcal{L}_{qn}^{(0)} = \bar{\xi}_n \left[i n \cdot D_s + g n \cdot A_n + (i \not{D}_c^\perp - m) W_n \frac{1}{\bar{n} \cdot \mathcal{P}} W_n^\dagger (i \not{D}_c^\perp + m) \right] \frac{\not{\bar{n}}}{2} \xi_n$$



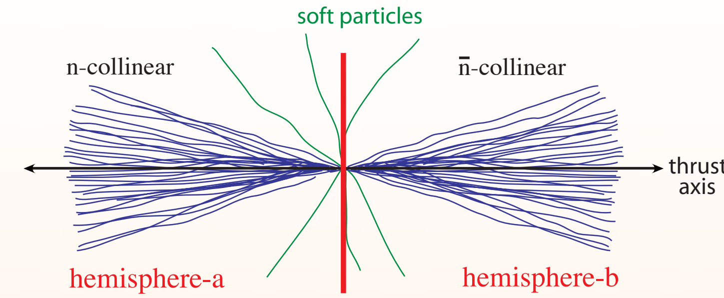
$$\frac{i}{n \cdot k + i\epsilon}$$

eikonal soft couplings

collinear Wilson line

$$W_n = P \exp\left(ig \int_0^\infty ds\, \bar{n} \cdot A_n(s\bar{n}) \right)$$

SCET



Degrees of Freedom

SCET $[\lambda \sim m/Q \ll 1]$		
	n-collinear $(\xi_n,\ A_n^\mu)$	$p_n^\mu \sim Q(\lambda^2, 1, \lambda)$
	$\bar{n}$-collinear $(\xi_{\bar{n}},\ A_{\bar{n}}^\mu)$	$p_{\bar{n}}^\mu \sim Q(1, \lambda^2, \lambda)$
Crosstalk:	soft $(q_s,\ A_s^\mu)$	$p_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

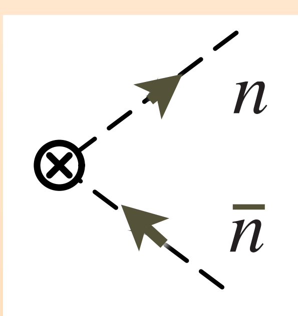
quark fields

gluon fields

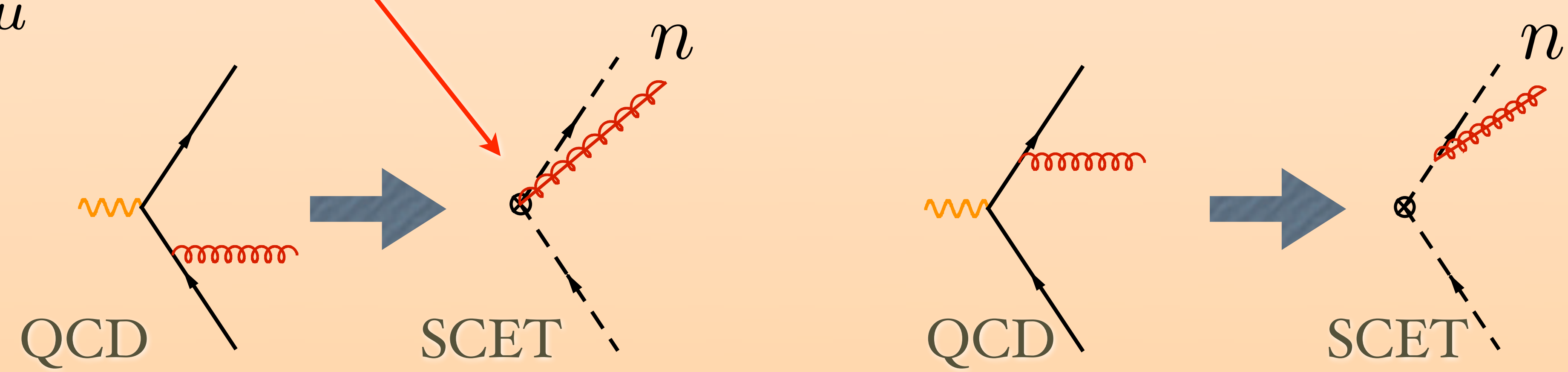
$(+, -, \perp)$

light-cone coordinates

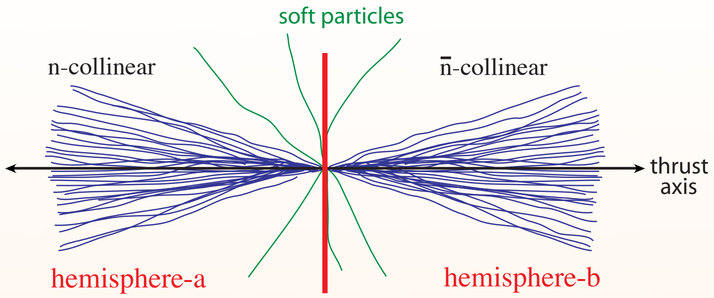
Production Current: $\quad Q \gg m$



$$\underbrace{\bar{\psi}\,\Gamma^\mu \psi}_{\mathcal{J}_i^\mu} \to (\bar{\xi}_n W_n)_\omega\, \Gamma^\mu (W_{\bar{n}}^\dagger \xi_{\bar{n}})_{\bar{\omega}} = (\bar{\xi}_n W_n)_\omega\, Y_n^\dagger \Gamma^\mu Y_{\bar{n}} (W_{\bar{n}}^\dagger \xi_{\bar{n}})_{\bar{\omega}}$$



SCET



Degrees of Freedom

SCET $[\lambda \sim m/Q \ll 1]$			
	n-collinear	$(\xi_n,\ A_n^\mu)$	$p_n^\mu \sim Q(\lambda^2, 1, \lambda)$
	$\bar{n}$-collinear	$(\xi_{\bar{n}},\ A_{\bar{n}}^\mu)$	$p_{\bar{n}}^\mu \sim Q(1, \lambda^2, \lambda)$
Crosstalk:	soft	$(q_s,\ A_s^\mu)$	$p_s^\mu \sim Q(\lambda^2, \lambda^2, \lambda^2)$

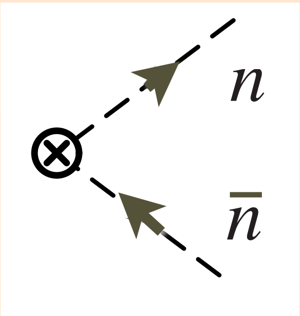
quark fields

gluon fields

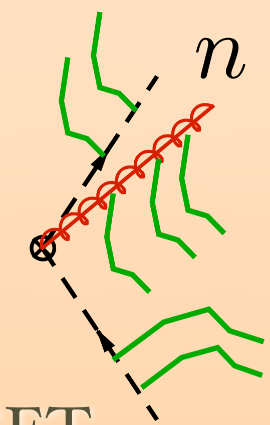
$(+, -, \perp)$

light-cone coordinates

Production Current: $\quad Q \gg m$



$$\underbrace{\bar{\psi}\,\Gamma^\mu \psi}_{\mathcal{J}_i^\mu} \to (\bar{\xi}_n W_n)_\omega\, \Gamma^\mu (W_{\bar{n}}^\dagger \xi_{\bar{n}})_{\bar{\omega}} \rightarrow (\bar{\xi}_n W_n)_\omega\, \underbrace{Y_n^\dagger \Gamma^\mu Y_{\bar{n}}}\, (W_{\bar{n}}^\dagger \xi_{\bar{n}})_{\bar{\omega}}$$

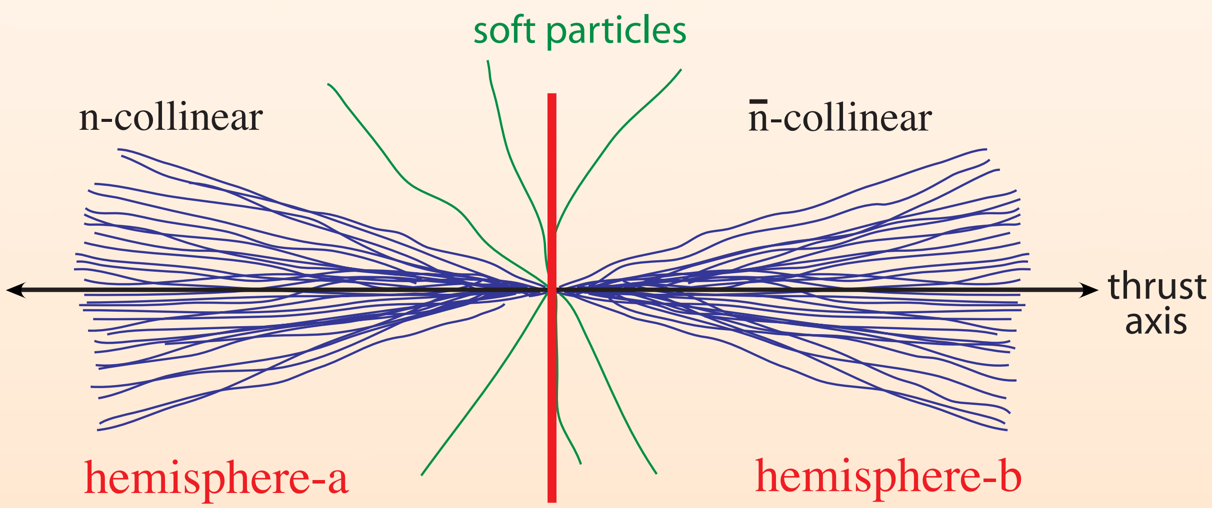


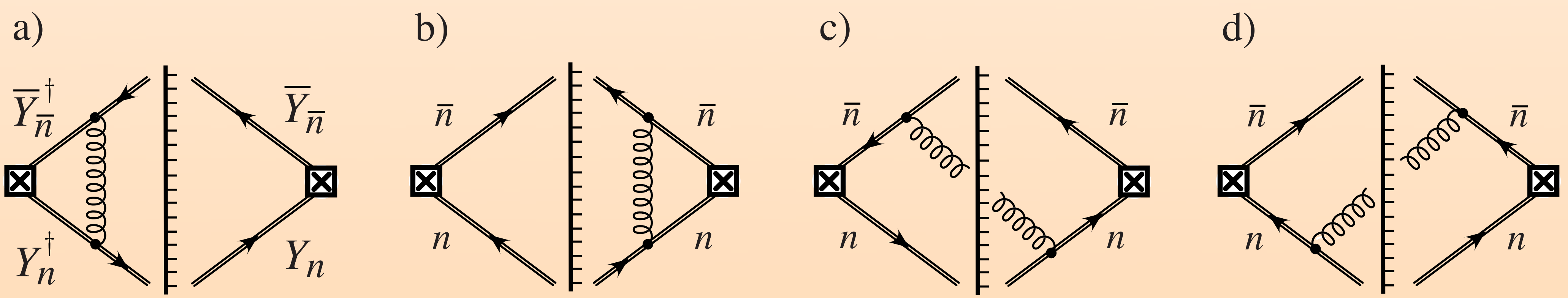
$$Y(x) = P \exp\left(ig \int_0^\infty ds\, n\cdot A_{us}(x+ns)\right)$$

Soft Function

$$S_{\rm hemi}(\ ^{+},\ ^{-},\mu) = \frac{1}{N_c}\sum_{X_s} (\ ^{+} - k_s^{+a})\ (\ ^{-} - k_s^{-b})\langle 0|\underbrace{\overline{Y}_{\bar n}\, Y_n}(0)|X_s\rangle\langle X_s|\underbrace{Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger}(0)|0\rangle$$

soft Wilson lines





$$S_{\rm part}(\ell^+,\ell^-,\mu) = \delta(\ell^+)\delta(\ell^-) + \delta(\ell^+)S^1_{\rm part}(\ell^-,\mu) + \delta(\ell^-)S^1_{\rm part}(\ell^+,\mu)\,,$$

$$S^1_{\rm part}(\ell,\mu) = \frac{C_F\alpha_s(\mu)}{\pi}\Big[\frac{\pi^2}{24}\delta(\ell) - 2\mathcal{L}^1(\ell)\Big]$$

$$\mathcal{L}^1(\ell) = \frac{1}{\mu}\Big[\frac{\theta(\ell)\ln(\ell/\mu)}{\ell/\mu}\Big]_+$$

$S(\ell^+, \ell^-, \mu)$

- Anomalous dimension determined by partonic calculation.

  it has cusp anom.dim.

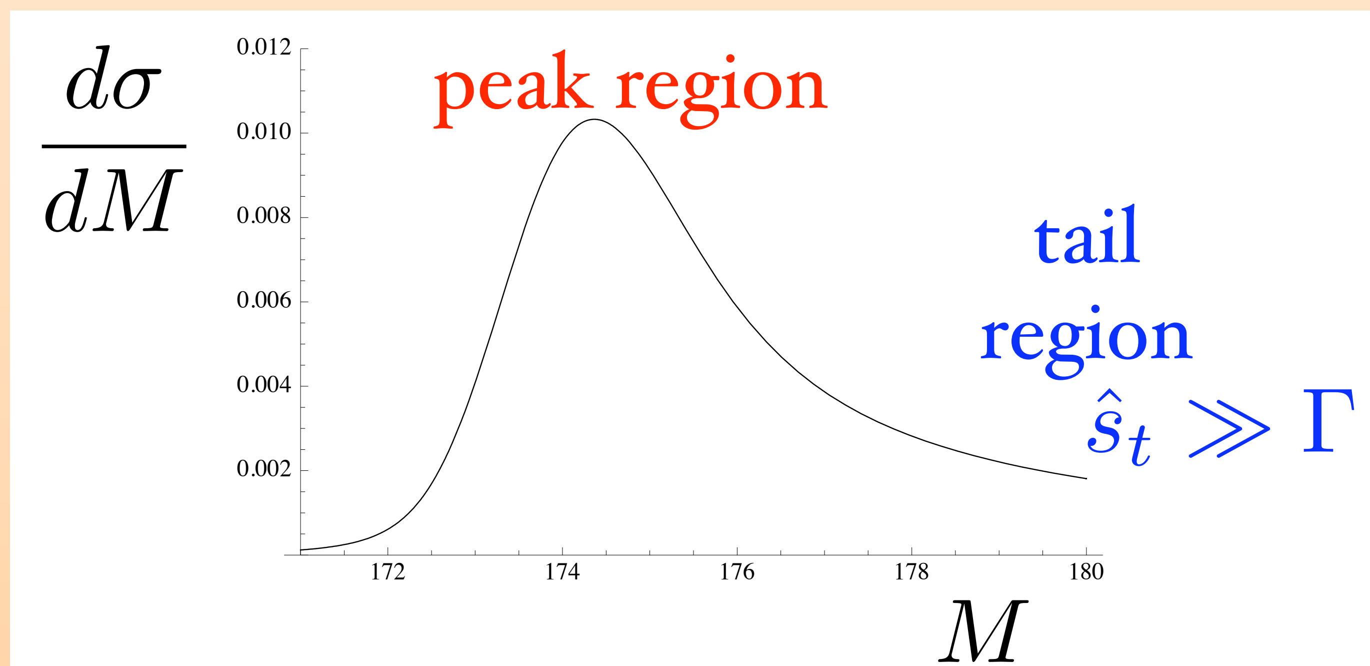
$$\int_{-\infty}^{L} d\ell^+ \int_{-\infty}^{L} d\ell^- \; S(\ell^+, \ell^-, \mu) \; = 1 + \frac{C_F \alpha_s(\mu)}{\pi} \left\{ \frac{\pi^2}{12} - 2 \ln^2\left(\frac{L}{\mu}\right) \right\} + \ldots$$

- Cross-section in the tail region has $^{\pm} \sim \dfrac{\hat{s}\, m}{Q} \gg \Lambda_{\rm QCD}$ and the

  soft function becomes perturbatively calculable

- In the peak region $\ell^{\pm} \sim \Lambda_{\rm QCD}$ $\longrightarrow$ nonperturbative soft function

these features should be built into S



A Convolution Formula does this

Hoang & I.S.

$$S(\ell^+,\ell^-,\mu) = \int_{-\infty}^{+\infty} d\tilde{\ell}^+ \int_{-\infty}^{+\infty} d\tilde{\ell}^- \; S_{\rm part}(\ell^+-\tilde{\ell}^+,\ell^--\tilde{\ell}^-,\mu)\, S_{\rm mod}(\tilde{\ell}^+,\tilde{\ell}^-)$$

partonic soft function calculated at fixed order

normalized model function (exponential fall off)

$$\int_{-\infty}^{+\infty} d\ell^+ d\ell^- \; \mathsf{S}_{\rm mod}(\ell^+,\ell^-) \;=\; 1$$

- Soft-function has a $(u = 1/2)$ renormalon ambiguity

  implying that the partonic and model parts are sensitively tied together

- This is removed by introducing a minimum energy gap for the soft radiation

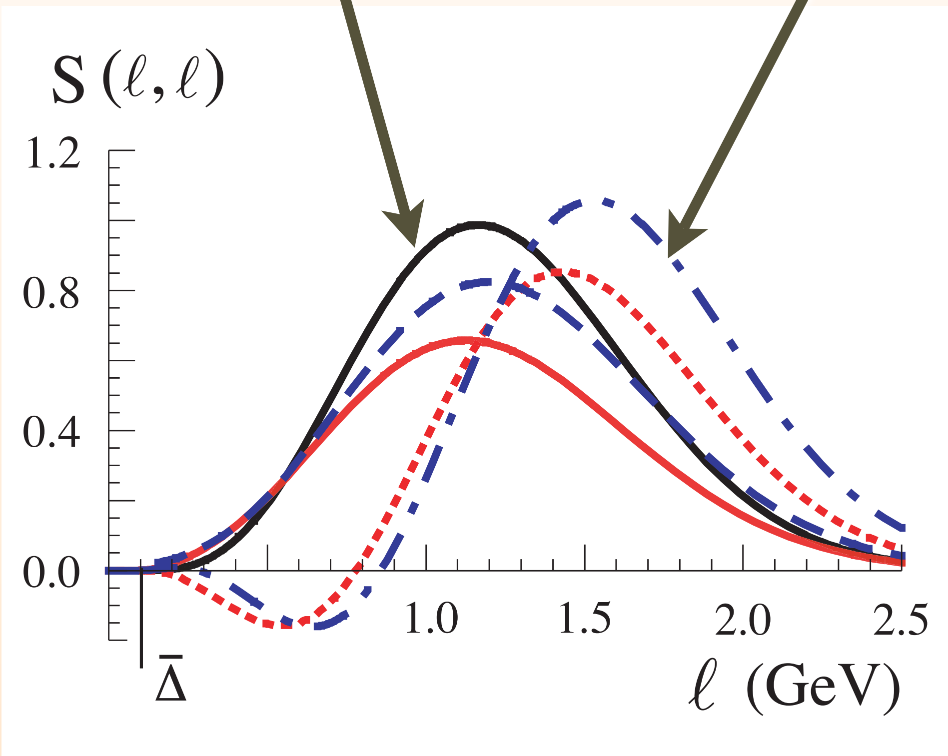
$$S(\ell^+,\ell^-,\mu) = \int_{-\infty}^{+\infty} d\tilde{\ell}^+ \int_{-\infty}^{+\infty} d\tilde{\ell}^- \; S_{\rm part}(\ell^+-\tilde{\ell}^+,\ell^--\tilde{\ell}^-,\mu)\, f_{\rm exp}(\tilde{\ell}^+-\boldsymbol{\Delta},\tilde{\ell}^--\boldsymbol{\Delta})$$

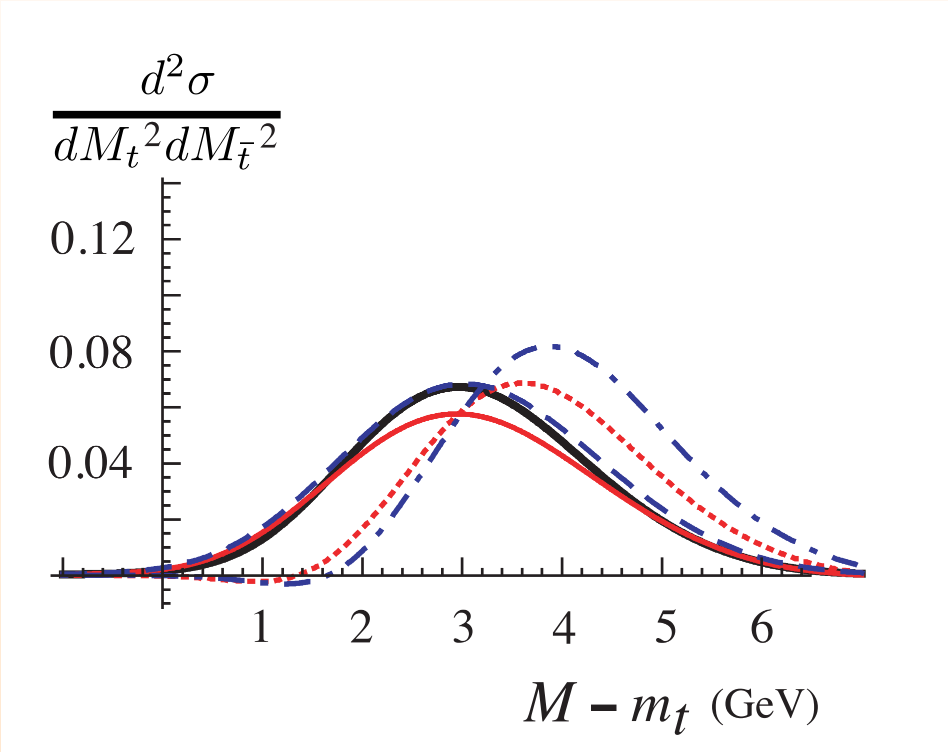
$$= \int_{-\infty}^{+\infty} d\tilde{\ell}^+ \int_{-\infty}^{+\infty} d\tilde{\ell}^- \; S_{\rm part}(\ell^+-\tilde{\ell}^+-\delta,\ell^--\tilde{\ell}^--\delta,\mu)\, f_{\rm exp}(\tilde{\ell}^+-\bar{\Delta},\tilde{\ell}^--\bar{\Delta})$$

$$\Delta = \bar{\Delta} + \delta = \bar{\Delta} + (\alpha_s + \alpha_s^2 + \ldots) \qquad\qquad \bar{\Delta} = \text{renormalon free}$$

with renormalon subtraction

without renormalon subtraction

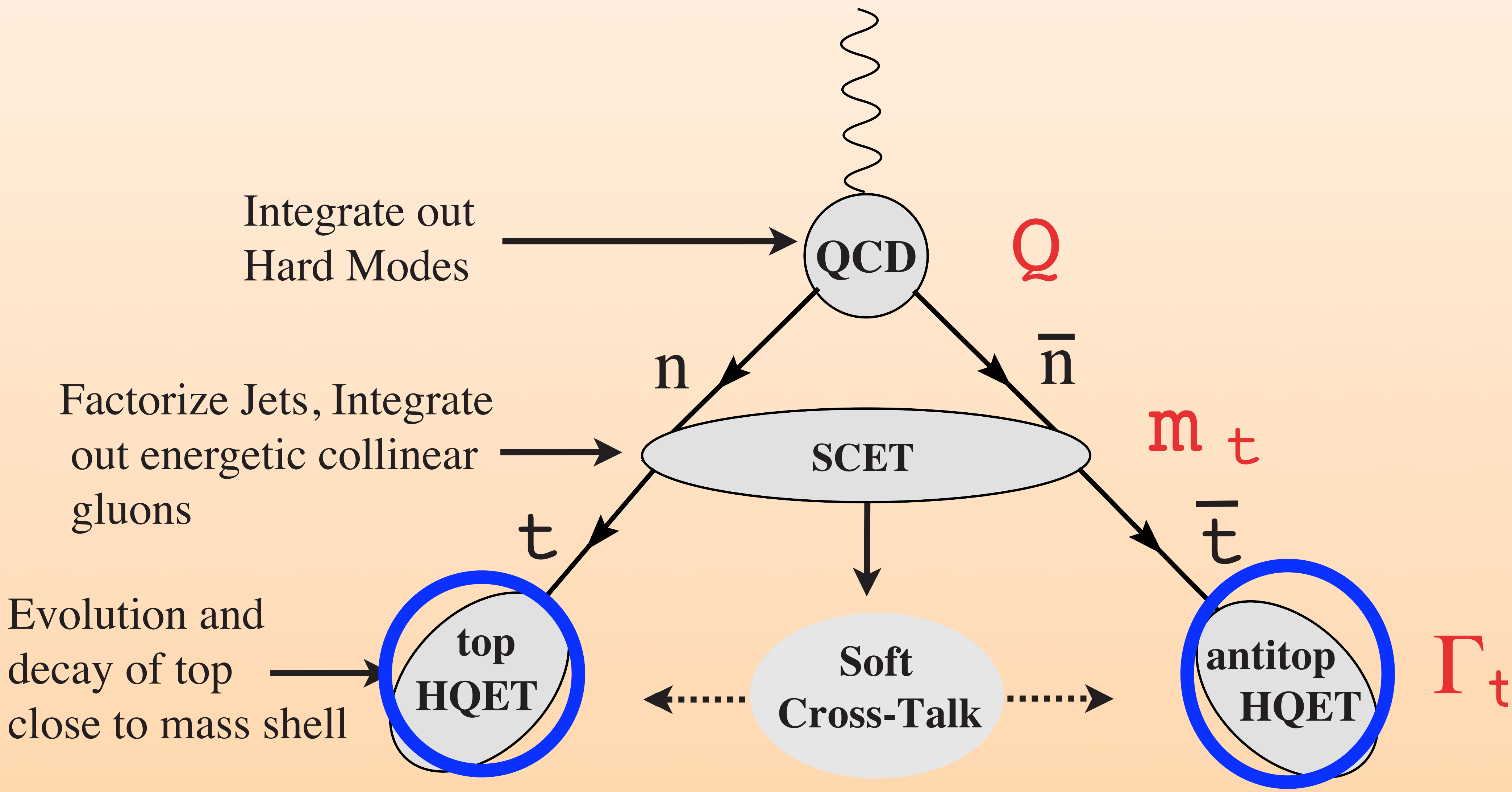




Gives soft function that:

- has correct μ dependence for MS-bar scheme
- has model parameters that are stable & not sensitive to μ
- has correct large momentum behavior

Heavy Quark Jet Function



unstable boosted HQET

$$v_+^\mu = \left(\frac{m}{Q}, \frac{Q}{m}, \mathbf{0}_\perp\right) \sim (\lambda, \lambda^{-1}, 0)$$

fluctuations beneath the mass

$$p^\mu = m v_+^\mu + k^\mu$$

collinear, but with smaller overall scale

one HQET for top

one HQET for antitop

bHQET $[\Gamma/m \ll 1]$	
n-ucollinear $(h_{v_+},\ A^\mu_{v_+})$	$k^\mu \sim \Gamma(\lambda, \lambda^{-1}, 1)$
$\bar{n}$-ucollinear $(h_{v_-},\ A^\mu_{v_-})$	$k^\mu \sim \Gamma(\lambda^{-1}, \lambda, 1)$
same soft $(q_s,\ A^\mu_s)$	$p^\mu_s \sim (\Delta, \Delta, \Delta)$

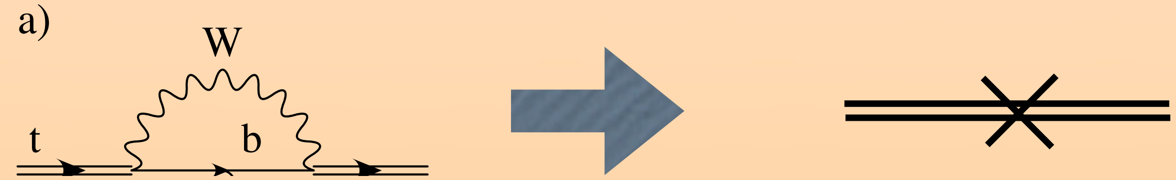
$$\mathcal{L}_+ = \bar{h}_{v_+}\Big(i v_+ \cdot D_+ - \delta m + \frac{i}{2}\Gamma_t\Big) h_{v_+}, \qquad \mathcal{L}_- = \bar{h}_{v_-}\Big(i v_- \cdot D_- - \delta m + \frac{i}{2}\Gamma_t\Big) h_{v_-}$$

mass scheme choice

$$\delta m = m^{\text{pole}} - m$$

our observable is inclusive in top decay products

a)

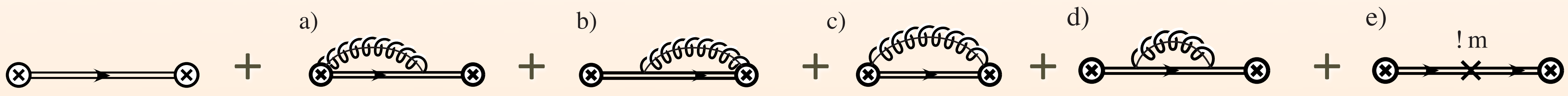


Heavy Quark Jet Function

Can be computed perturbatively

$$B(\hat{s}, \delta m, \Gamma_t, \mu) = \mathrm{Im}\big[\mathcal{B}(\hat{s}, \delta m, \Gamma_t, \mu)\big]$$

$$= \mathrm{Im}\Big[$$



$$+ \dots \Big]$$

$$\mathcal{B}(2v_+ \cdot r, \delta m, \Gamma_t, \mu) = \frac{-i}{4\pi N_c m} \int d^4x\, e^{ir\cdot x} \left\langle 0 \left| T\{\bar{h}_{v_+}(0) W_n(0) W_n^\dagger(x) h_{v_+}(x)\} \right| 0 \right\rangle$$

shift property $\mathcal{B}(\hat{s}, \delta m, \Gamma_t, \mu) = \mathcal{B}(\hat{s} - 2\delta m + i\Gamma_t, \mu)$

Renormalization and RGE:

convolutions $$\mathcal{B}(\hat{s},\mu) = \int d\hat{s}'\ Z_B^{-1}(\hat{s}-\hat{s}',\mu)\ \mathcal{B}^{\text{bare}}(\hat{s}')$$

$$\mu\frac{d}{d\mu}\mathcal{B}(\hat{s},\mu) = \int d\hat{s}'\ \gamma_B(\hat{s}-\hat{s}',\mu)\,\mathcal{B}(\hat{s}',\mu)$$

$$\gamma_B(\hat{s},\mu) = -2\Gamma^c[\alpha_s]\frac{1}{\mu}\left[\frac{\mu\,\theta(\hat{s})}{\hat{s}}\right]_+ + \gamma[\alpha_s]\delta(\hat{s})$$

cusp anom.dim. — known to 3 loops

non-cusp term — now known to 2 loops

Position space:

$$\tilde{\gamma}_B(y,\mu) = 2\Gamma^c[\alpha_s]\ \ln\left(ie^{\gamma_E}y\,\mu\right) + \gamma[\alpha_s]$$

solution: $$\tilde{B}(y,\mu) = e^{K(\mu,\mu_0)}\left(ie^{\gamma_E}y\,\mu_0\right)^{\omega(\mu,\mu_0)}\ \tilde{B}(y,\mu_0)$$

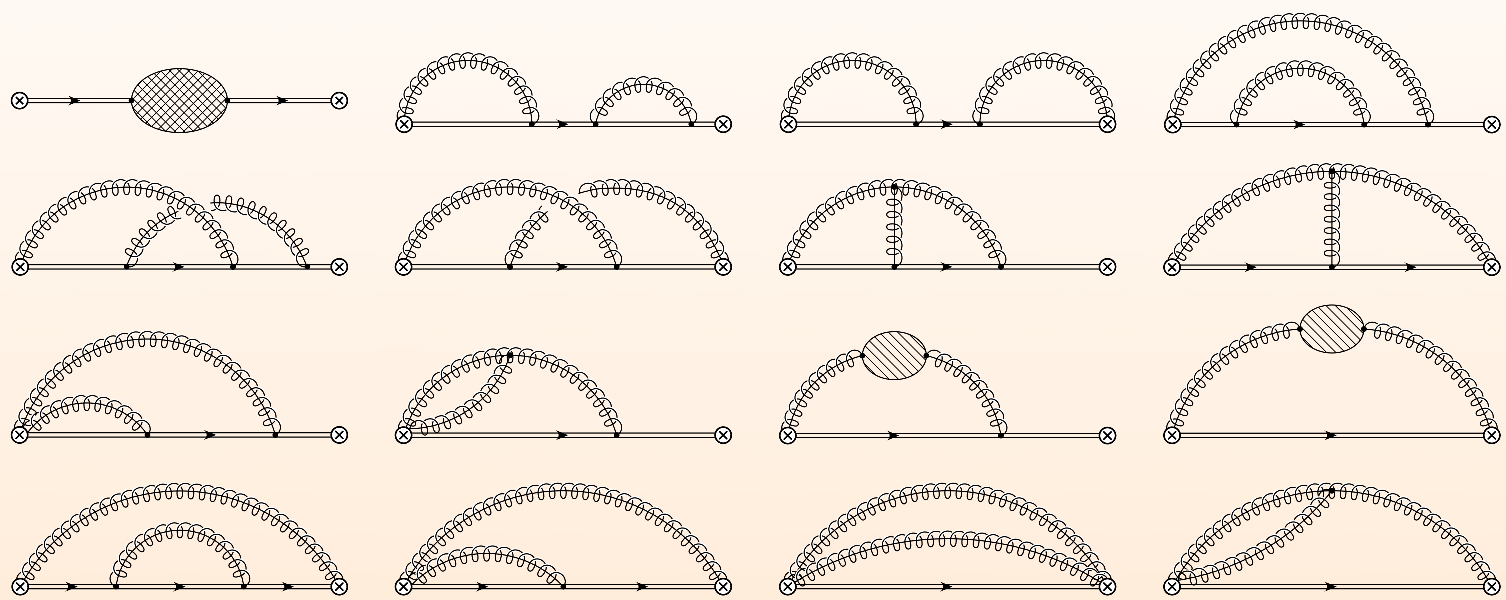
$$\omega(\mu,\mu_0) = 2\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)}\frac{d\alpha}{\beta[\alpha]}\Gamma^c[\alpha]\quad,\quad K(\mu,\mu_0) = \ldots$$

Momentum space:

$$B(\hat{s},\mu) = \int_{-\infty}^{+\infty} d\hat{s}'\ U_B(\hat{s}-\hat{s}',\mu,\mu_0)\ B(\hat{s}',\mu_0)\,,\qquad U_B(\hat{s}-\hat{s}',\mu,\mu_0) = \frac{e^K\left(e^{\gamma_E}\right)^\omega}{\mu_0\,\Gamma(-\omega)}\left[\frac{\mu_0^{1+\omega}\,\theta(\hat{s}-\hat{s}')}{(\hat{s}-\hat{s}')^{1+\omega}}\right]_+$$

Two-Loop Result

Jain, Scimemi, I.S.



$$m\,\mathcal{B}_2(\hat{s},\delta m,\mu) = C_F^2\Big[\frac{1}{2}L^4 + L^3 + \Big(\frac{3}{2}+\frac{13\pi^2}{24}\Big)L^2 + \Big(1+\frac{13\pi^2}{24}-4\zeta_3\Big)L^1 + \Big(\frac{1}{2}+\frac{7\pi^2}{24}+\frac{53\pi^4}{640}-2\zeta_3\Big)L^0\Big]$$
$$+ C_F C_A\Big[\Big(\frac{1}{3}-\frac{\pi^2}{12}\Big)L^2 + \Big(\frac{5}{18}-\frac{\pi^2}{12}-\frac{5\zeta_3}{4}\Big)L^1 + \Big(-\frac{11}{54}+\frac{5\pi^2}{48}-\frac{19\pi^4}{960}-\frac{5\zeta_3}{8}\Big)L^0\Big]$$
$$+ C_F\beta_0\Big[\frac{1}{6}L^3 + \frac{2}{3}L^2 + \Big(\frac{47}{36}+\frac{\pi^2}{12}\Big)L^1 + \Big(\frac{281}{216}+\frac{23\pi^2}{192}-\frac{17\zeta_3}{48}\Big)L^0\Big]$$
$$-2\delta m_2(L^0)' + 2(\delta m_1)^2(L^0)'' - 2\delta m_1\, C_F\Big[L^2 + L^1 + \Big(1+\frac{5\pi^2}{24}\Big)L^0\Big]'.$$

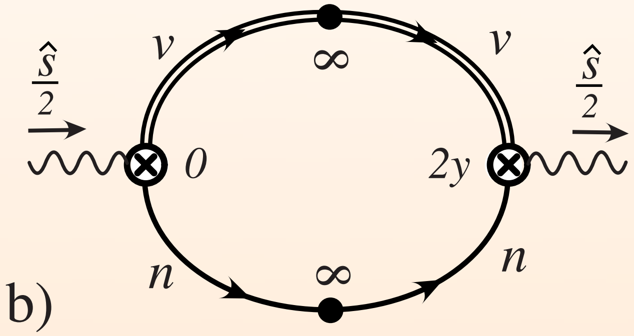
$$L^k = \frac{1}{\pi(-\hat{s}-i0)}\ \ln^k\Big(\frac{\mu}{-\hat{s}-i0}\Big)$$

Still need to find a suitable mass scheme

$$\delta m = \frac{\alpha_s(\mu)}{\pi}\,\delta m_1(\mu) + \frac{\alpha_s^2(\mu)}{\pi^2}\,\delta m_2(\mu) + \ldots .$$

Wilson Loop Definition:

$$B(\hat{s},\mu) = \frac{1}{2\pi}\int dy\, e^{i\hat{s}\,y}\,\tilde{B}(y,\mu)\,, \qquad \tilde{B}(y,\mu) = \frac{1}{m\,N_c}\,\langle 0|\mathrm{tr}\left[\overline{T}\,W_n^\dagger(2y)W_v(2y)\right]\left[T\,W_v^\dagger(0)W_n(0)\right]|0\rangle\,.$$



Satisfies criteria for non-abelian exponentiation Theorem

Gatheral, Frenkel & Taylor

non-abelian: $m\tilde{B}(y,\mu) = e^{K(\mu,\mu_y)+T[\alpha_s(\mu_y)]}$

abelian: $m\tilde{B}(y,\mu)^{\mathrm{abelian}} = \exp\left[\frac{\alpha_s}{4\pi}\left(\Gamma_0^c\tilde{L}^2 + \gamma_0\tilde{L} + T_0\right)\right]$

$\tilde{L} \equiv \ln\left(ie^{\gamma_E}y\,\mu\right)$

A convenient result for testing mass-schemes

Mass Scheme should:

- be renormalon free (not m^{pole})
- be a top-resonance mass scheme $\delta m \sim \alpha_s \Gamma_t$ (not $\overline{\text{MS}}$)
- have a RGE in μ

$\delta m = m_{pole} - m$

3 possibilites for scheme with stable peak position:

"peak" a) $$\frac{d}{d\hat{s}}\, B(\hat{s}, \delta m^{\text{peak}}, \Gamma_t, \mu)\Big|_{\hat{s}=0} = 0\,,$$

"moment" b) $$\int_{-\infty}^{R} d\hat{s}\;\hat{s}\;B(\hat{s}, \delta m^{\text{mom}}, \mu) = 0\,,$$ $R \sim \Gamma_t$

"position" c) $$\delta m_J = \frac{-i}{2\,\tilde{B}(y,\mu)}\,\frac{d}{dy}\,\tilde{B}(y,\mu)\Big|_{y=-ie^{-\gamma_E}/R} = e^{\gamma_E}\,\frac{R}{2}\,\frac{d}{d\ln(iy)}\ln\tilde{B}(y,\mu)\Big|_{iye^{\gamma_E}=1/R}\,.$$

Only c) has a consistent anomalous dimension equation, for the others the anom.dim. does not have a consistent pert. expn.

"top jet mass scheme"

(two loop conversion to $\overline{\text{MS}}$ is now known)

This scheme is nice:

$$\frac{dm_J(\mu)}{d\ln\mu} = -e^{\gamma_E} R\, \Gamma^c[\alpha_s(\mu)]$$

anom.dim. is determined by cusp term, and therefore is known to 3 loops

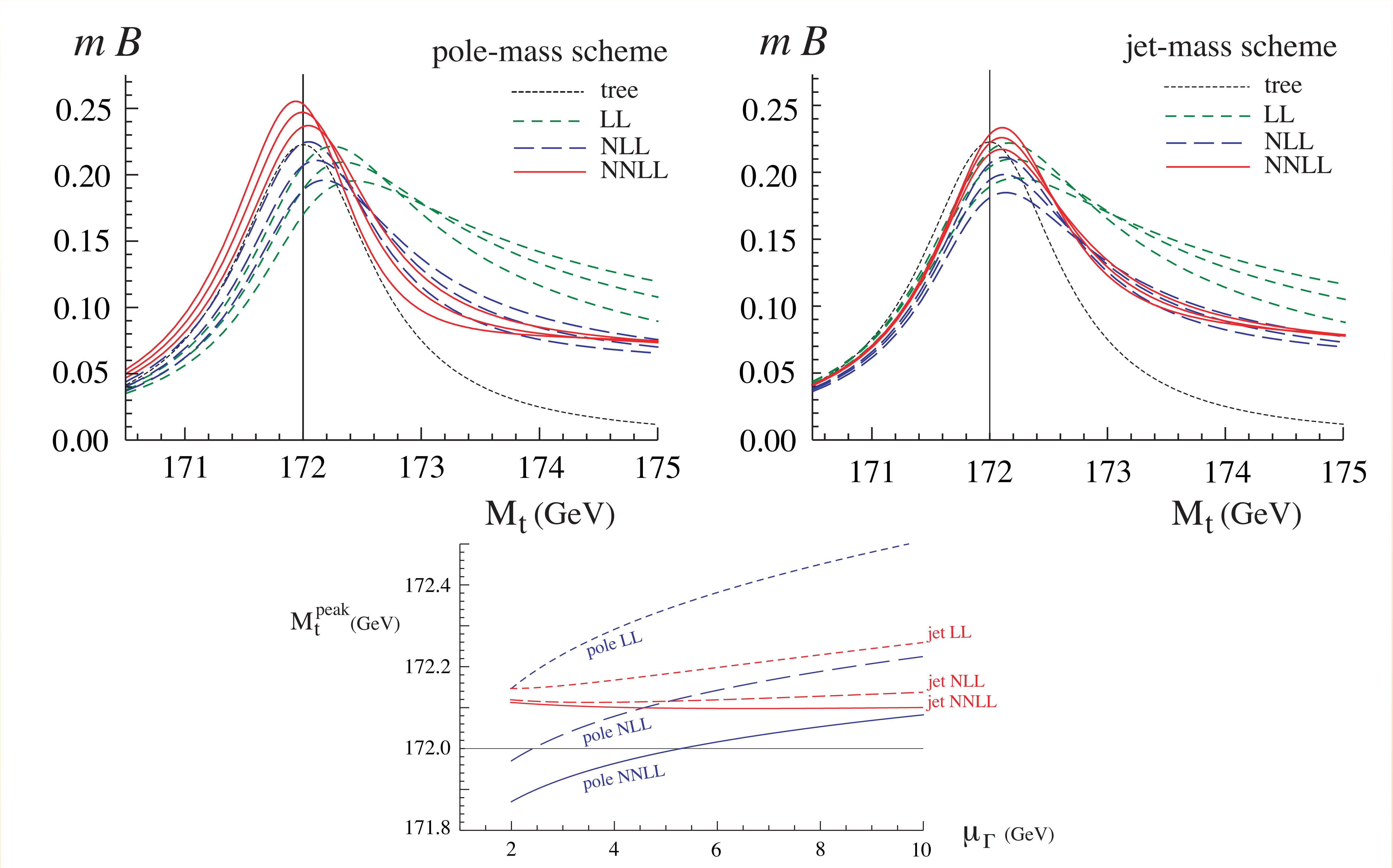
Result is jet-function with resummation:

$$B(\hat{s}, \delta m_J, \Gamma_t, \mu_\Lambda, \mu_\Gamma) \equiv \int d\hat{s}'\ U_B(\hat{s}-\hat{s}', \mu_\Lambda, \mu_\Gamma)\ B(\hat{s}', \delta m_J, \Gamma_t, \mu_\Gamma)$$

$$= \int d\hat{s}'\, d\hat{s}''\ U_B(\hat{s}-\hat{s}', \mu_\Lambda, \mu_\Gamma)\ \underbrace{B(\hat{s}'-\hat{s}'', \delta m_J, \mu_\Gamma)}\ \frac{\Gamma_t}{\pi(\hat{s}''^{\,2} + \Gamma_t^2)}\,.$$

convolute result from the previous page to sum logs and include width effects

Jet Function Results up to NNLL:



Fleming, Hoang,
Mantry, I.S.

Analysis at NLL order

(Next-to-Leading-Order with resummation to all orders of next-to-leading logarithms)

Analysis to NLL order

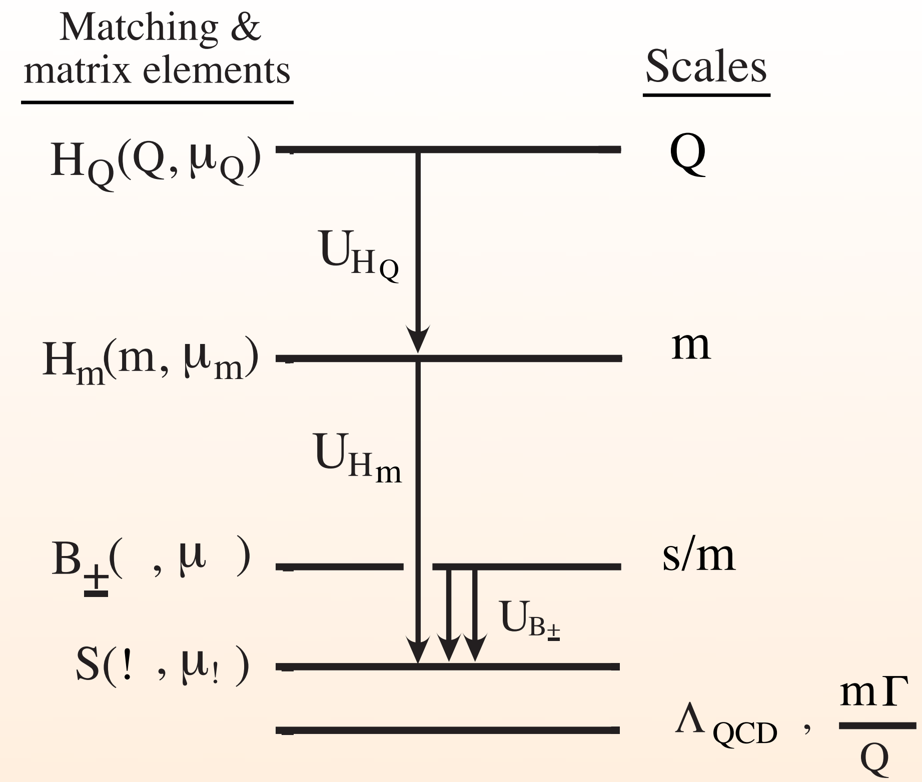
- One-loop matching
- One-loop matrix element for B+, and for the soft function:

$$S(\ell^+,\ell^-,\mu) = \int_{-\infty}^{+\infty} d\tilde{\ell}^+ \int_{-\infty}^{+\infty} d\tilde{\ell}^- \; S_{\rm part}(\ell^+ - \tilde{\ell}^+, \ell^- - \tilde{\ell}^-, \mu, \delta_i)\, S_{\rm mod}(\tilde{\ell}^+, \tilde{\ell}^-)$$

- Renormalon Free Schemes for Jet and Soft functions
- RGE evolution, sum large logs $Q \gg m \gg \Gamma \sim \hat{s}_{t,\bar{t}}$

  (Two-loop cusp anom.dims. & One-loop non-cusp)
- Proper choice for the scales

NLL Cross-Section Results



normalized cross-section

$$\frac{d^2\sigma}{dM_t\, dM_{\bar t}} = \frac{\sigma_0}{\Gamma_t^2}\, F\Big(M_t, M_{\bar t}, m_J, \frac{Q}{m_J}\Big)$$

numerical

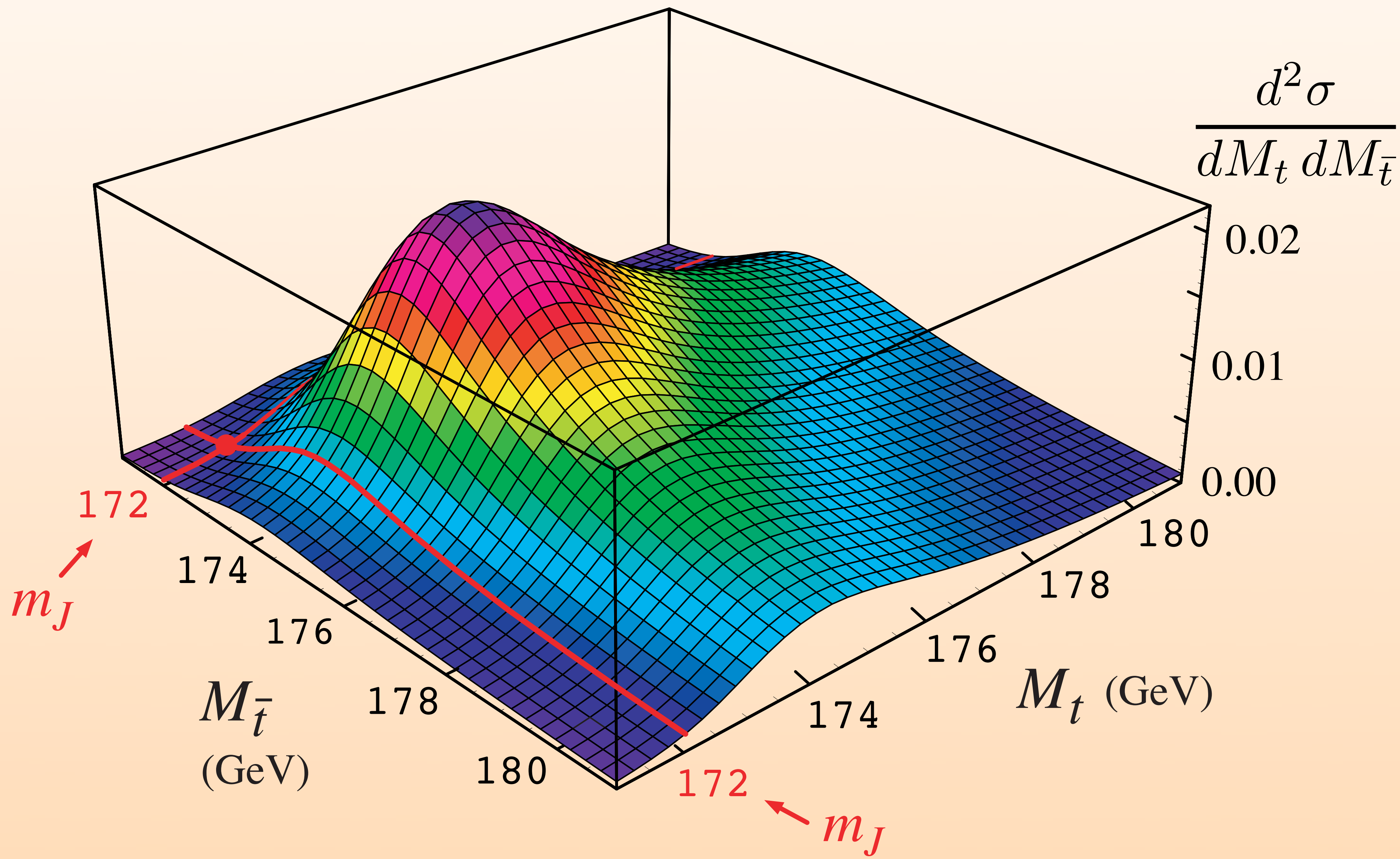
$$F\Big(M_t, M_{\bar t}, m_J, \frac{Q}{m_J}\Big) = \int_{-\infty}^{\infty} d\ell^+\, d\ell^-\, P\Big(\hat s_t - \frac{Q\ell^+}{m_J} - \frac{Q\bar\Delta(\mu_\Lambda)}{m_J}, \hat s_{\bar t} - \frac{Q\ell^-}{m_J} - \frac{Q\bar\Delta(\mu_\Lambda)}{m_J}, \mu_\Lambda\Big)\, S^{\rm mod}(\ell^+, \ell^-, 0)$$

perturbative part is analytic

$$P\big(\hat s_t, \hat s_{\bar t}, \mu_\Lambda\big) = 4M_t M_{\bar t}\Gamma_t^2\, H_Q(Q, \mu_h)\, U_{H_Q}(Q, \mu_h, \mu_m)\, H_m(m, \mu_m)\, U_{H_m}\Big(\frac{Q}{m_J}, \mu_m, \mu_\Lambda\Big) \times G_+\Big(\hat s_t, \frac{Q}{m_J}, \Gamma_t, \mu_\Lambda\Big) G_-\Big(\hat s_{\bar t}, \frac{Q}{m_J}, \Gamma_t, \mu_\Lambda\Big)\,.$$

$$G_\pm\ \hat s, \frac{Q}{m_J}, \Gamma_t, \mu_\Lambda\ \equiv \int_{-\infty}^{+\infty} d\hat s'\, d\hat s''\, d\ell'\ \ U_B(\hat s - \hat s', \mu_\Lambda, \mu_\Gamma) \times B_\pm^{\Gamma=0}\ \ \hat s' - \hat s'' - \frac{Q}{m_J}\ell', \mu_\Gamma, \delta m\ \ \tilde S_{\rm part}(\ell', \mu_\Lambda, \delta_1)\, \frac{\Gamma_t}{\pi(\hat s''^2 + \Gamma_t^2)}$$

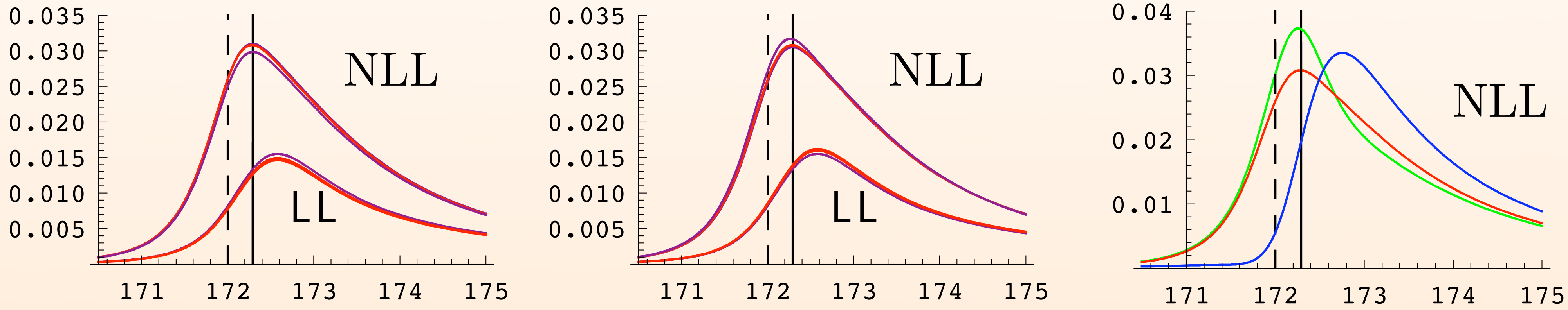
NLL Cross-Section Results



Perturbative corrections

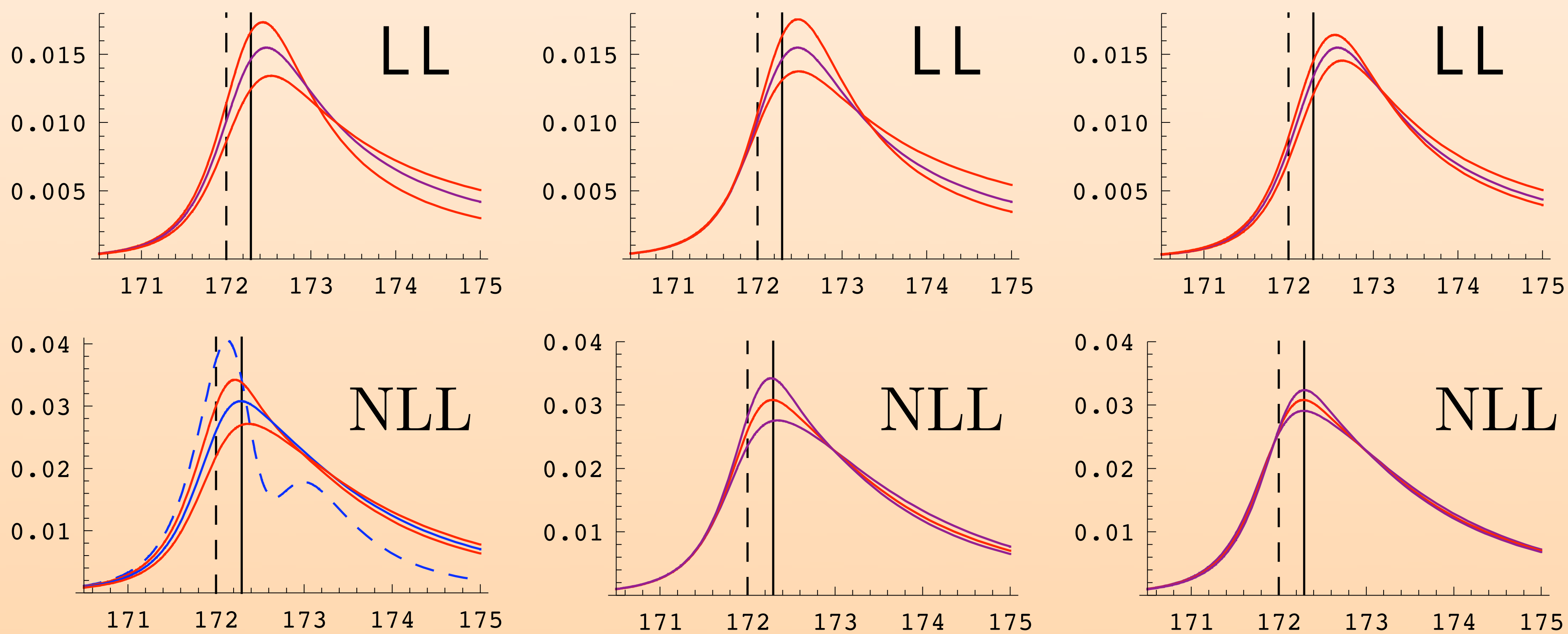
$P(M_t, M_t)$ versus M_t

$\mu_m = 86, 172, 344\,\text{GeV}$ $\mu_Q = 430, 860, 1720\,\text{GeV}$.



$\mu_\Gamma = 3.3, 5, 7.5\,\text{GeV}$ $\mu_\Delta = 0.8, 1.0, 1.2\,\text{GeV}$ $\mu_\Delta = 0.8, 1.0, 1.2\,\text{GeV}$ &

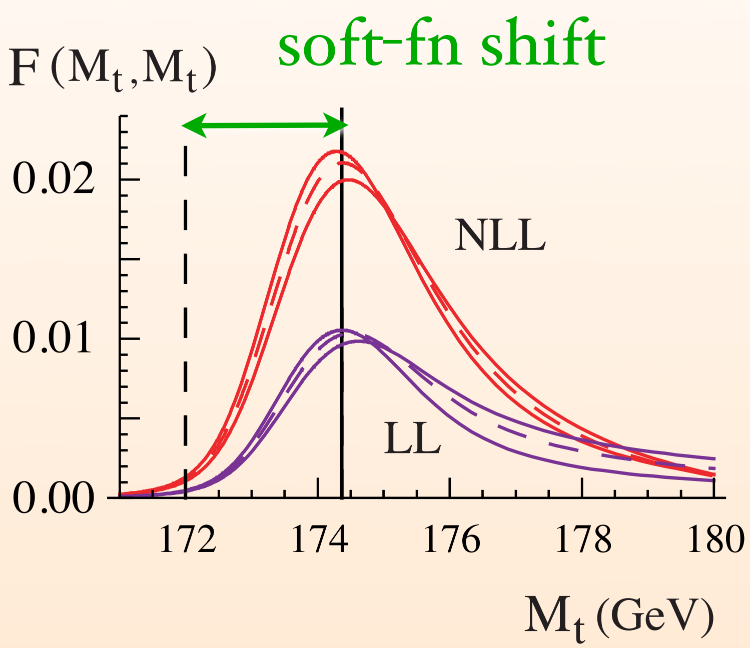
$\mu_\Gamma/\mu_\Lambda = Q/m$

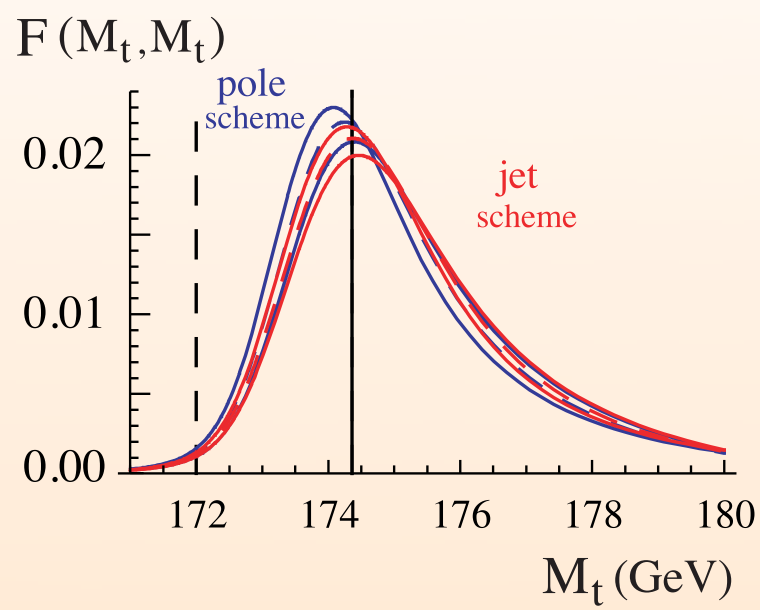


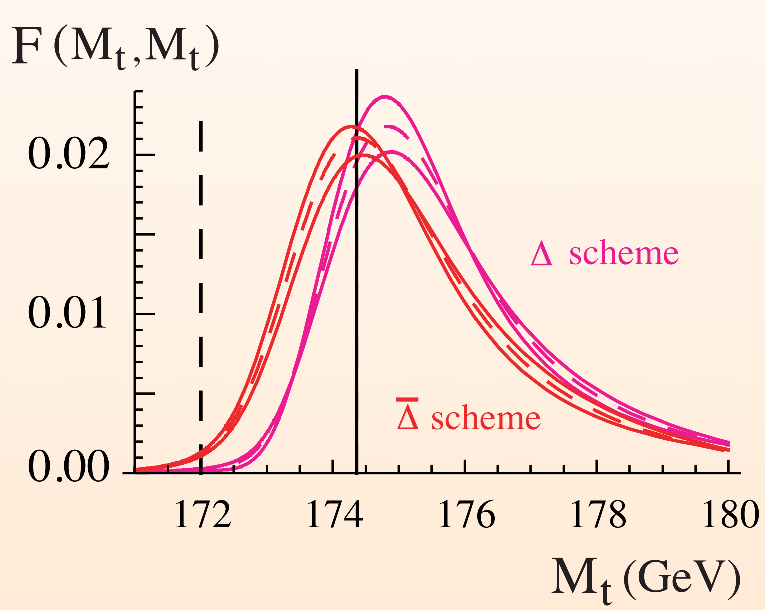
Normalized Cross-Section

$F(M_t, M_t)$ versus M_t.

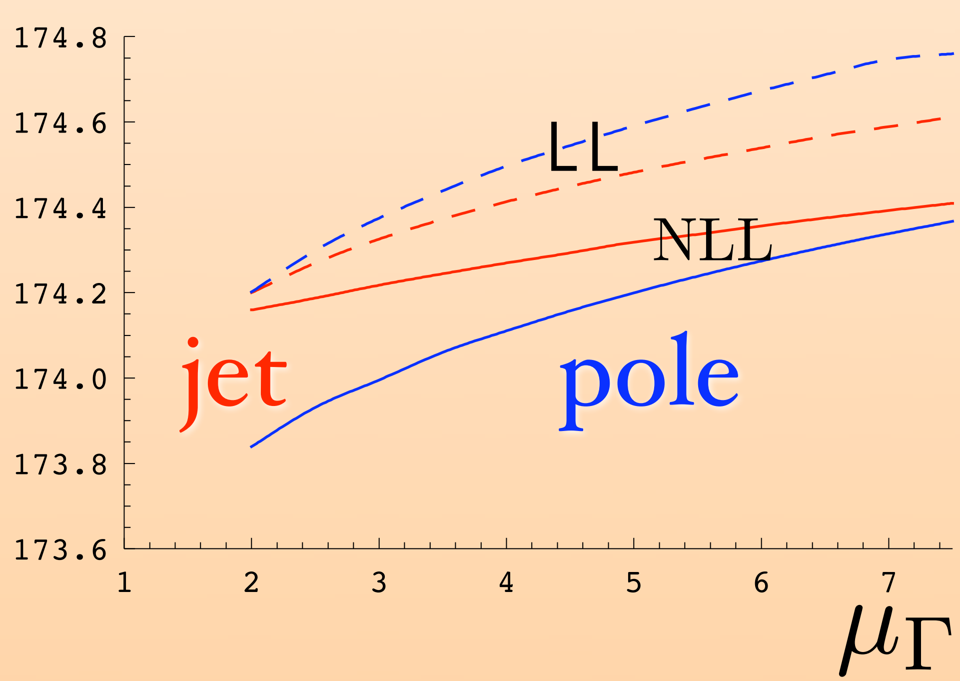
$\mu_\Gamma = 3.3, 5, 7.5\,\text{GeV}$



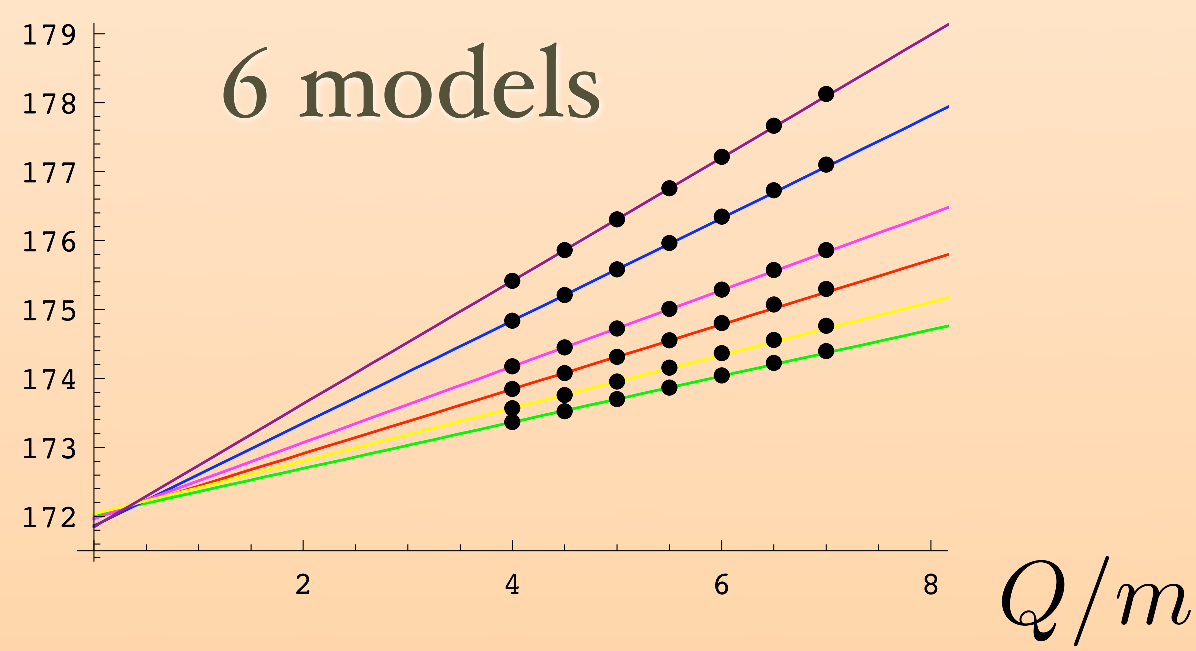




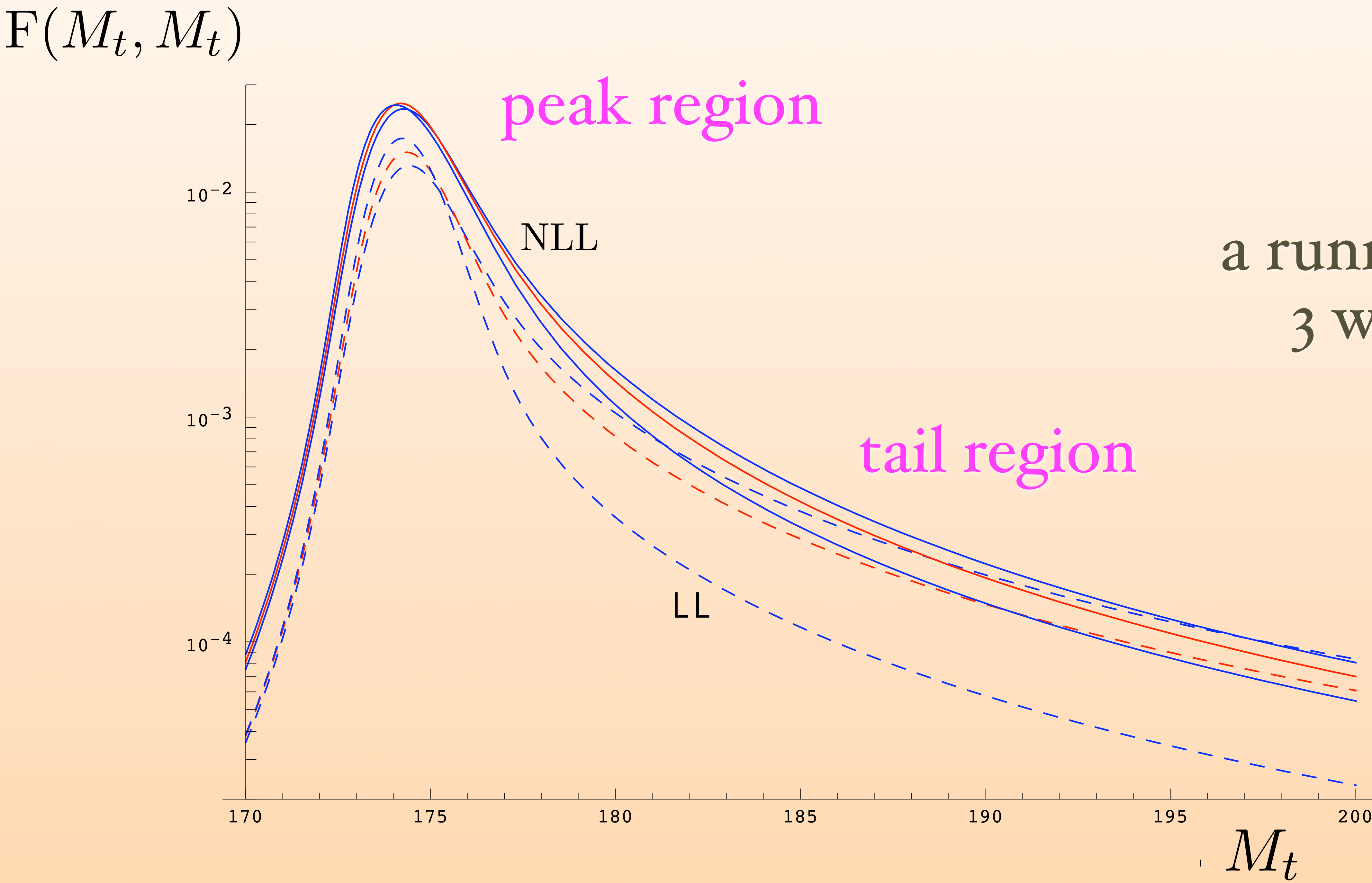
Peak Positions vs. μ_Γ



Peak Positions vs. Q/m



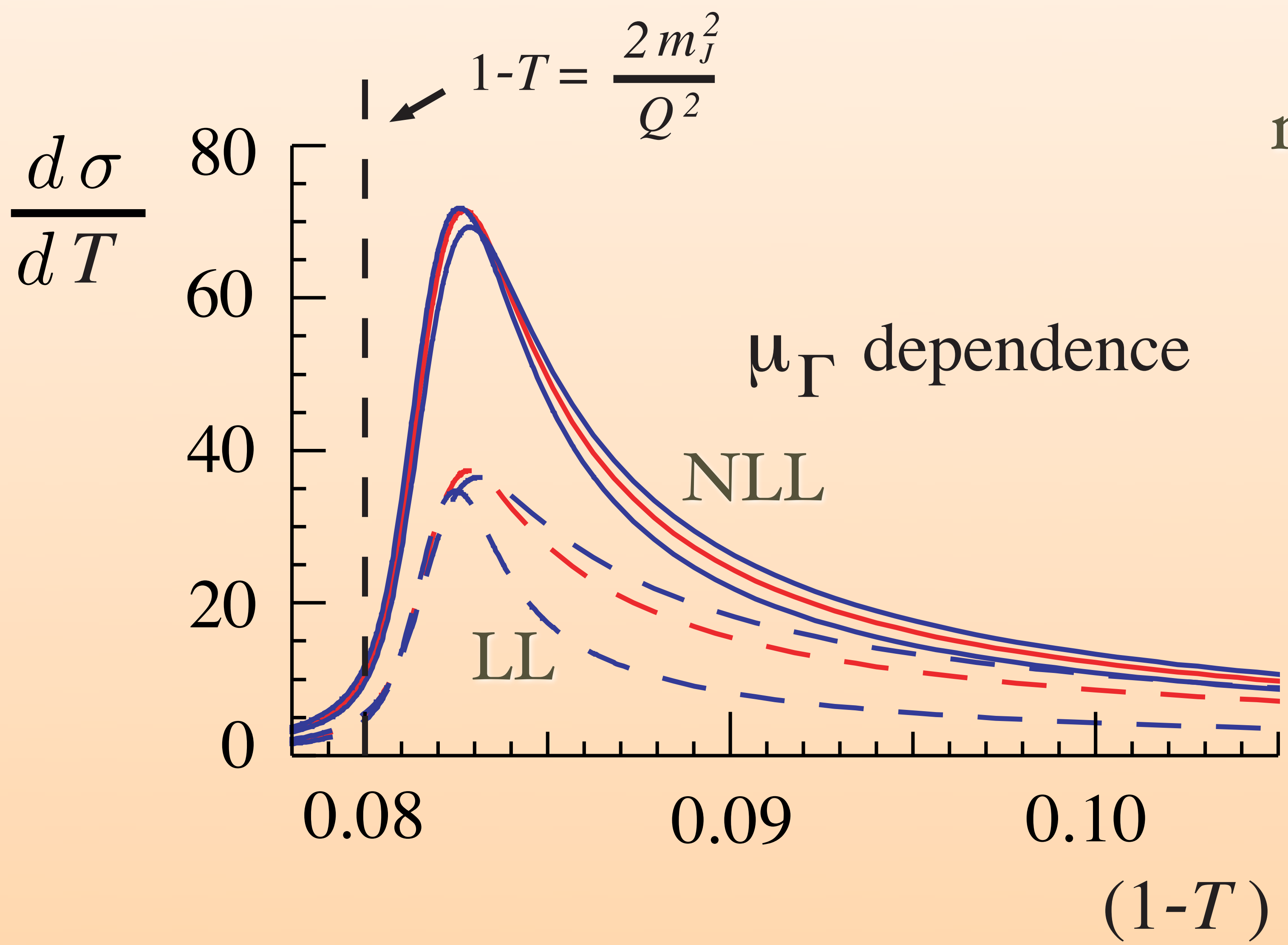
Beyond the peak region



a running μ_Γ
3 ways

This observable contains other event shapes as projections, like thrust

$$\frac{1}{\sigma_0}\frac{d\sigma}{dT} = \int_0^\infty dM_t^2 \int_0^\infty dM_{\bar{t}}^2 \; \delta\left(\tau - \frac{M_t^2 + M_{\bar{t}}^2}{Q^2}\right) \frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2}$$



Thrust peak position vs. Q can also be used to measure the short-distance top-mass

$$T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \mathbf{p}_i|}{Q}$$

What (if anything) can be said about the Tevatron mass?

- Given that top decay is described by a Breit-Wigner, we know that the mass should be close to a pole mass (top-resonance mass scheme)

$$m_{\rm pole} = m(R,\mu) + \delta m(R,\mu)\,, \qquad \delta m(R,\mu) = R \sum_{n=1}^{\infty} \sum_{k=0}^{n} a_{nk} \left[\frac{\alpha_s(\mu)}{4\pi}\right]^n \ln^k\!\left(\frac{\mu}{R}\right)$$

$$R \sim \Gamma$$

- Recently we studied an RGE for R, which allows us to smoothly connect these schemes to $\overline{\rm MS}$ where $R = \overline{m}(\mu)$

Hoang, Jain, Scimemi, I.S. (arXiv:0803.4214)

[c.f. the kinetic mass of Bigi et.al.]

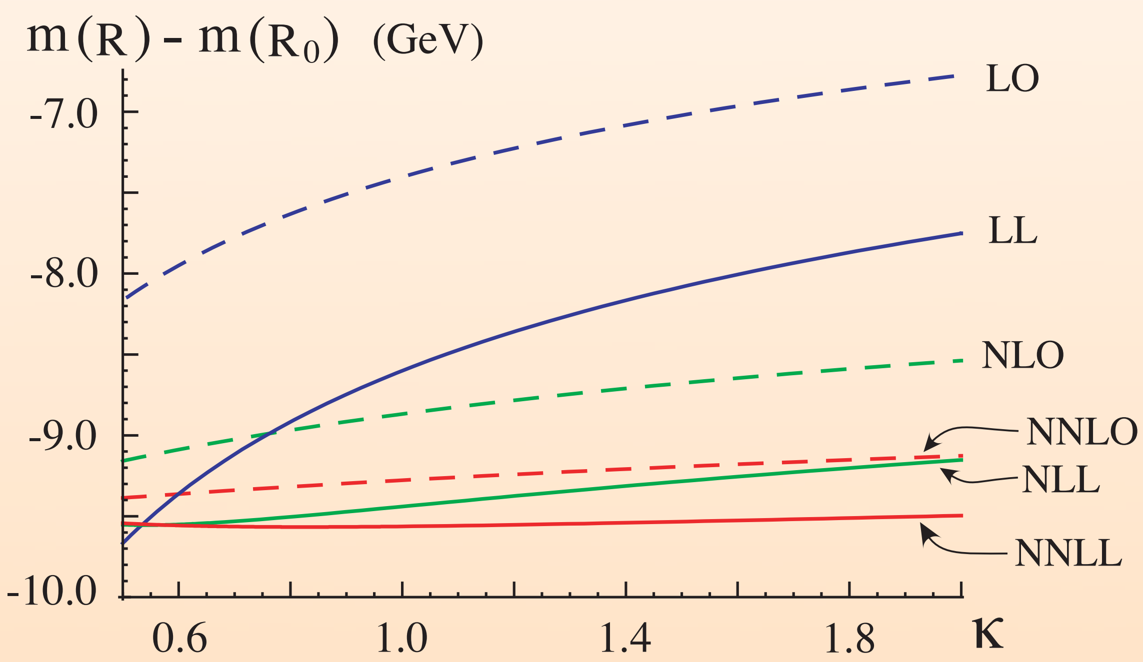
- We can estimate the scheme uncertainty of the Tevatron measurement by varying the initial $R = R_0 = 3^{+6}_{-2}\,{\rm GeV}$ (since any such mass is an equally good short distance scheme)

(See the talk by A. Hoang tomorrow at 9:30am in the Flavor workshop for further details)

$m_t(R_0)$ = 172.6 ± 1.4 GeV

assume Tevatron measures a top-resonance mass

$$\overline{m}_t(\overline{m}_t) = 163.0 \pm 1.3 \, {}^{+0.6}_{-0.3} \pm 0.05 \, \text{GeV}$$



scheme uncertainty

$R_0 = 3^{+6}_{-2}\,\text{GeV}$

conversion uncertainty is small (3 loop with RGE)

Similar issues at the LHC in most methods

The pole mass is what we get for $R_0 = 0$, but is very likely not what the Tevatron measures. If we demand that the measurement corresponds to a pole mass, then an additional uncertainty of

$\sim \Lambda_{QCD} \sim 600\,\text{GeV}$ from the renormalon should be added to those above.

Summary & Outlook

Top Jets

- Discussed a factorization theorem for invariant mass distributions for massive unstable particles: $e^+e^- \to t\bar{t}$ separation of perturbative and non-perturbative effects for ILC

- Systematic relation of peak to a Lagrangian mass parameter:

  What mass is measured? "Jet mass"

- Effective Field Theory: can be extended to higher orders in the power and perturbative expansions

Progress for massless event shapes:

- Reexamine LEP massless jet data with calculations at NNLL, ...
  (Becher & Schwartz; Gehrmann-De Ridder et.al.)

Future:

- Extension to large pT events for LHC, and to Monte-Carlo
- Technique can be used to study other processes with jets and massive underlying particles