High Precision Results for $\alpha_s(m_Z)$

from Jets*, Taus, Lattice, DIS, and Precision Electroweak

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MIT & Harvard

UCSD seminar, San Diego
February 2011

WORKSHOP ON PRECISION MEASUREMENTS OF α_s

Max-Planck-Institute for Physics
Munich, Germany
February 9-11, 2011

EVENT SHAPES AND JET PRODUCTION • LATTICE SIMULATIONS • ELECTROWEAK PRECISION OBSERVABLES
TAU DECAYS • DEEP INELASTIC SCATTERING • FUTURE PERSPECTIVES

Confirmed Speakers and Convenors:

S. Aoki
M. Beneke
J. Blümlein
N. Brambilla
S. Brodsky
M. Davier
Y. Dokshitzer
J. Erler
S. Forte
T. Gehrmann
C. Glasman
M. Golterman
A. Höcker
W. Hollik
M. Jamin
A. Kronfeld
J. Kühn
P. Lepage
V. Mateu
S. Menke
K. Mönig
Y. Nomura
F. Petriello
T. Pich
K. Rabbertz
V. Radescu
G. Salam
H. Schulz
E. Shintani
R. Sommer
M. Steinhauser
J. Stirling
B. Webber
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Organizers:

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J. Schieck
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Program and registration
http://www.mpp.mpg.de/alphas

Excellence Cluster Universe
Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)
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Outline

- $\alpha_s(m_Z)$ Motivation, World Averages
- Precision & Controversy
  - Electroweak Global fits
  - Tau Decays
  - Lattice QCD
  - DIS
- Jets in e^+e^-, with event shapes at $\mathrm{N^3LL} + \mathcal{O}(\alpha_s^3)$
  - perturbation theory
  - power corrections
  - sum large logs
  - pert./power overlaps (renormalons)
  - Global Fit (thrust, heavy jet mass)

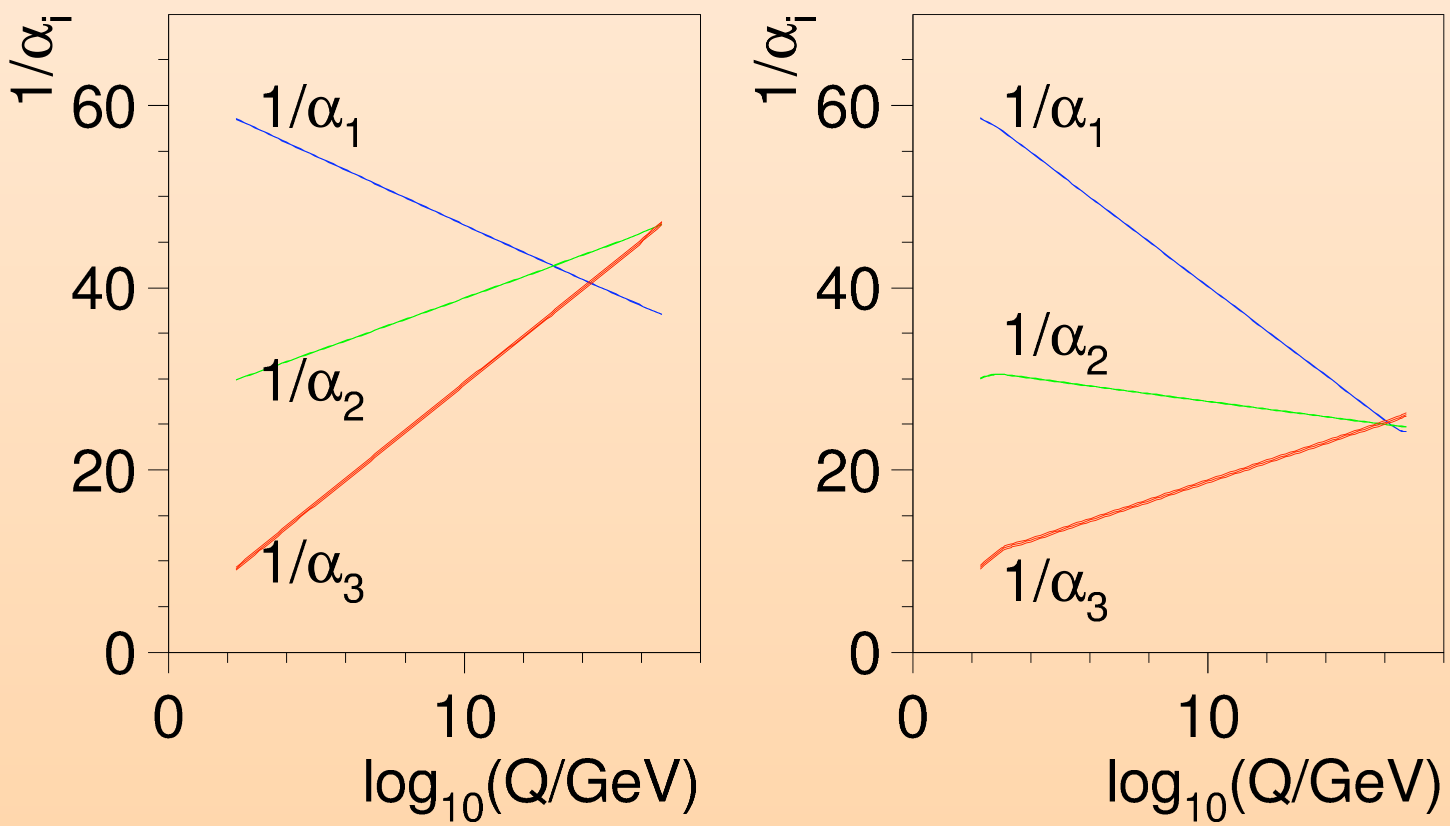
Abbate, Fickinger, Hoang, Mateu, I.S.

arXiv:1006.3080

Motivation

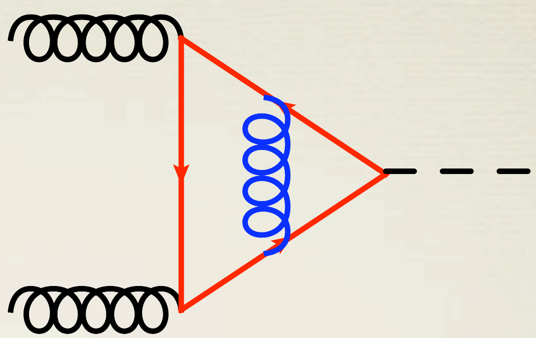
- $\alpha_s(m_Z)$ is a key parameter in the standard model, and enters the analysis of all collider data (LHC, Tevatron, Jlab, RHIC, DESY, B-factories, ILC, ...)

- It also plays a role in searches for new physics
  - indirectly in precision electroweak analyses, $B \to X_s \gamma$
  - directly through the unification of couplings:



eg. Higgs Inclusive Cross Section

(F. Petriello, α_s-workshop)



Sensitivity of gg→H to gluon PDF, α_S:

$$\text{Tevatron:}\ \sigma \sim \alpha_S^3 \times [f_g(0.075)]^2$$

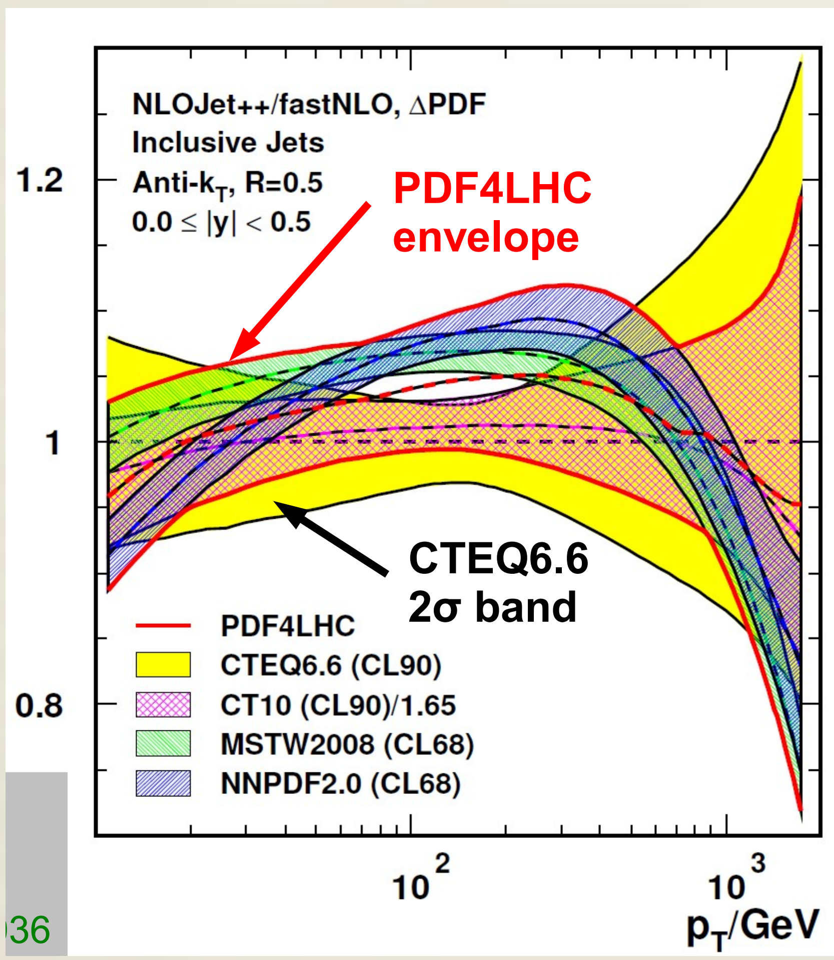
$$\text{LHC:}\ \sigma \sim \alpha_S^{2.5} \times [f_g(0.02)]^2$$

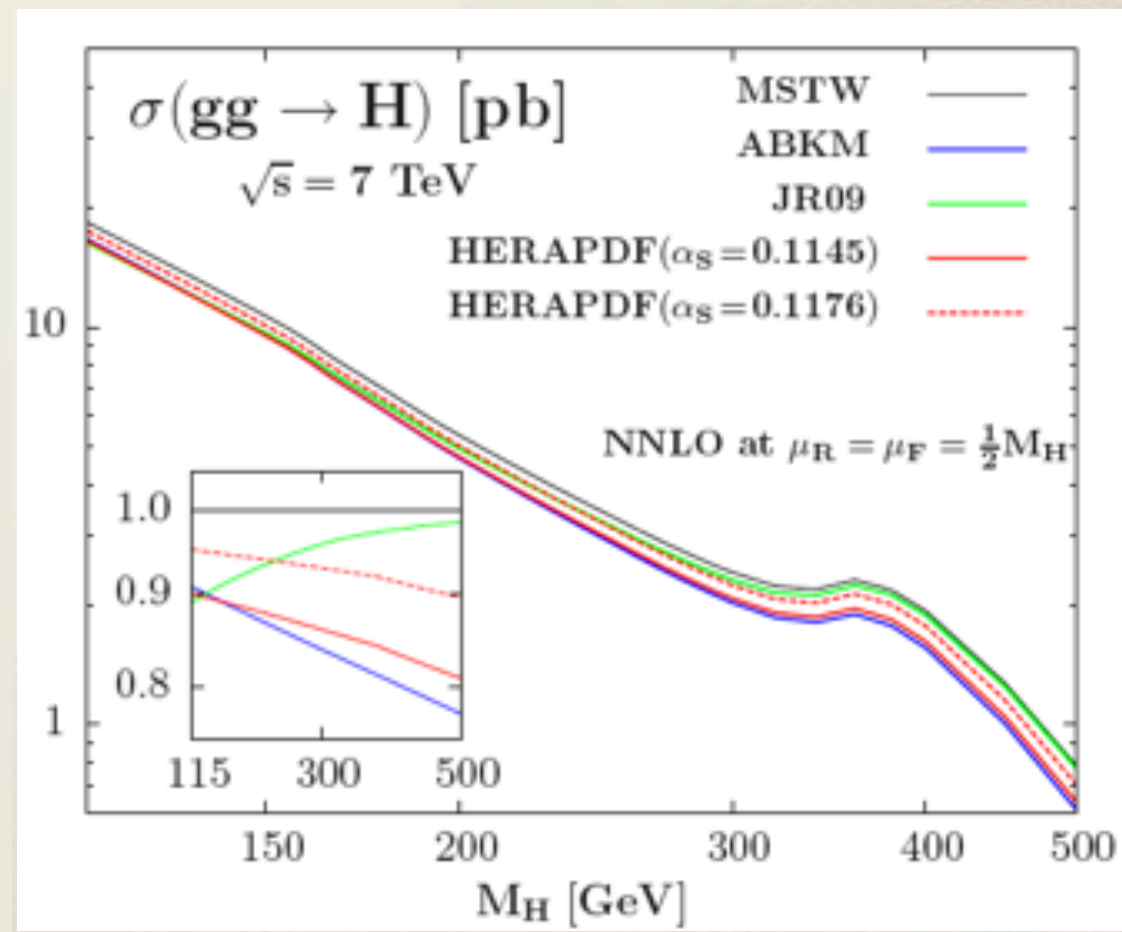
→ because of large higher-order corrections

10-15% (LHC) or 20-40% (TeV) differences from sets with lower α_S

PDF4LHC default

~ 7–8% on $\sigma_{gg\rightarrow H}$ from PDF $+\alpha_s$





from Baglio, Djouadi et al. 2009-2011

eg. Grand Unification

(Y. Nomura, α_s-workshop)

Hard to even quantify the errors

… threshold corrections from the weak **and** unified scales

e.g. Minimal SUSY SU(5)

$$\alpha_3^{-1}(m_Z) = \frac{12}{7}\alpha_2^{-1}(m_Z) - \frac{5}{7}\alpha_1^{-1}(m_Z) - \frac{1}{4\pi}\left\{\frac{18}{7}\ln\frac{M_{H_C}}{(M_V^2 M_\Sigma)^{1/3}} - \frac{19}{7}\ln\frac{m_{\rm SUSY}}{m_Z}\right\}$$

Unknown masses of GUT-scale particles

cf. Hisano, Murayama, Yanagida ('92)

GUT-scale threshold corrections become even larger in extended models …

For "exact unification"

$$\alpha_s(m_Z) = \underline{0.130} + 0.009\left(\frac{m_{t,\rm pole}^2 - (173.1\ \mathrm{GeV})^2}{(173.1\ \mathrm{GeV})^2}\right) - \frac{19\alpha_s^2}{28\pi}\ln\frac{m_{\rm SUSY}}{m_Z}$$

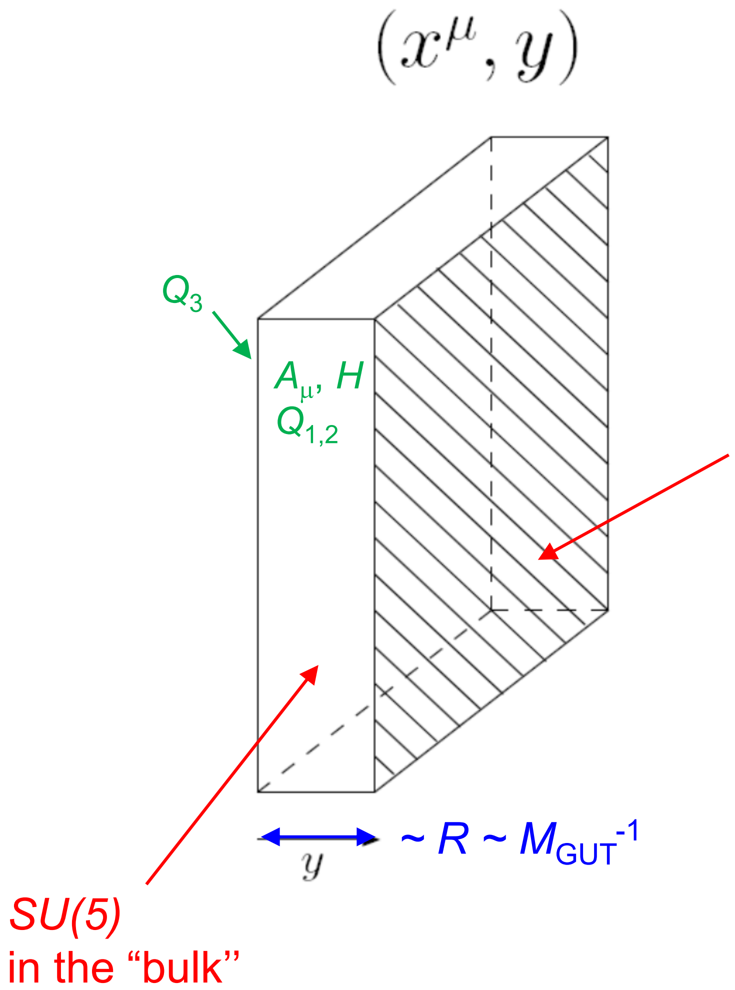
Somewhat large

cf. Langacker, Polonsky ('95)

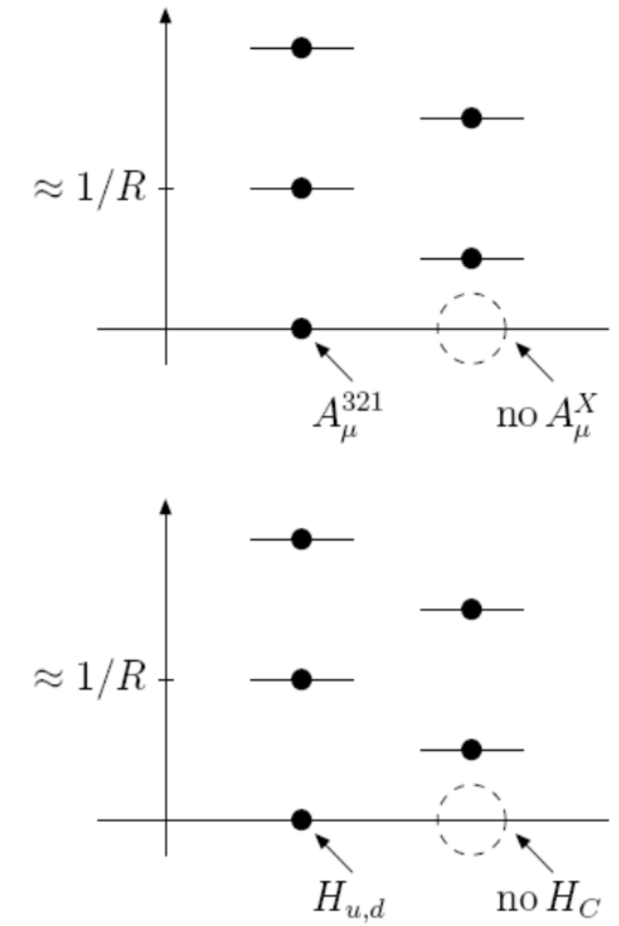
$m_{\rm SUSY}$: "effective" superpartner scale

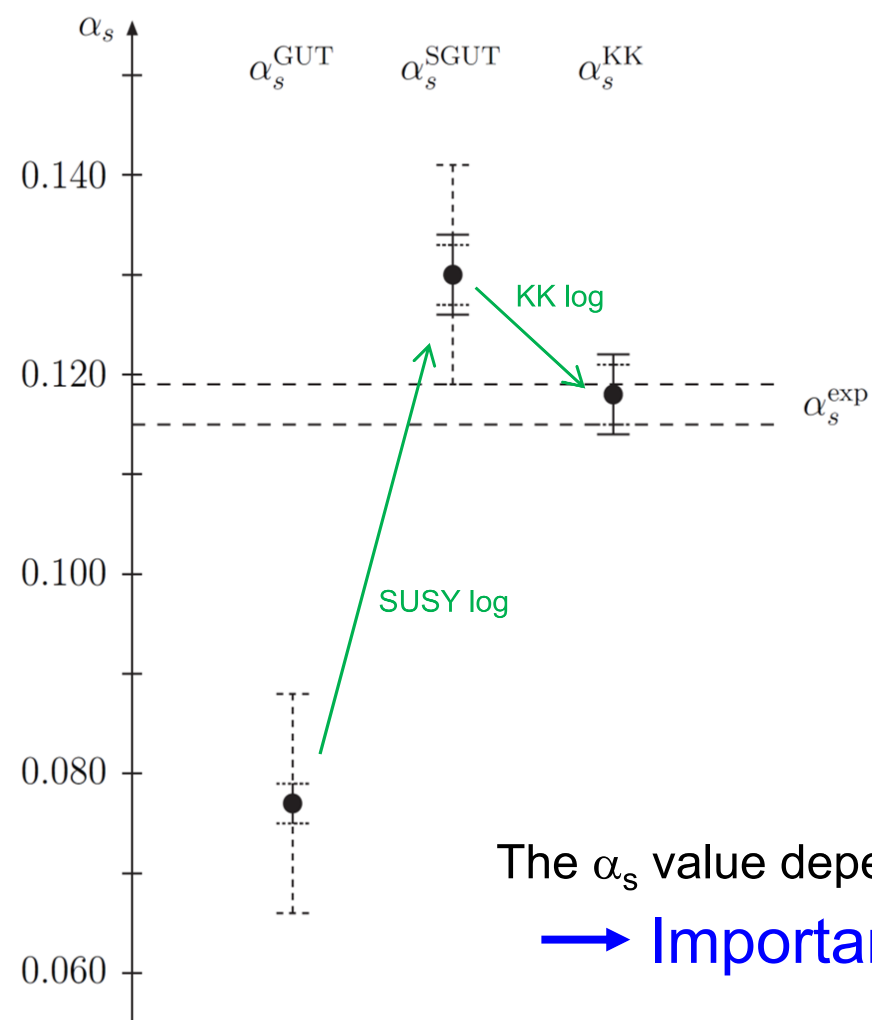
Grand unification in higher dimensions

Hall, Y.N.; Kawamura ('00 - '02)



GUT
scale
masses
fixed





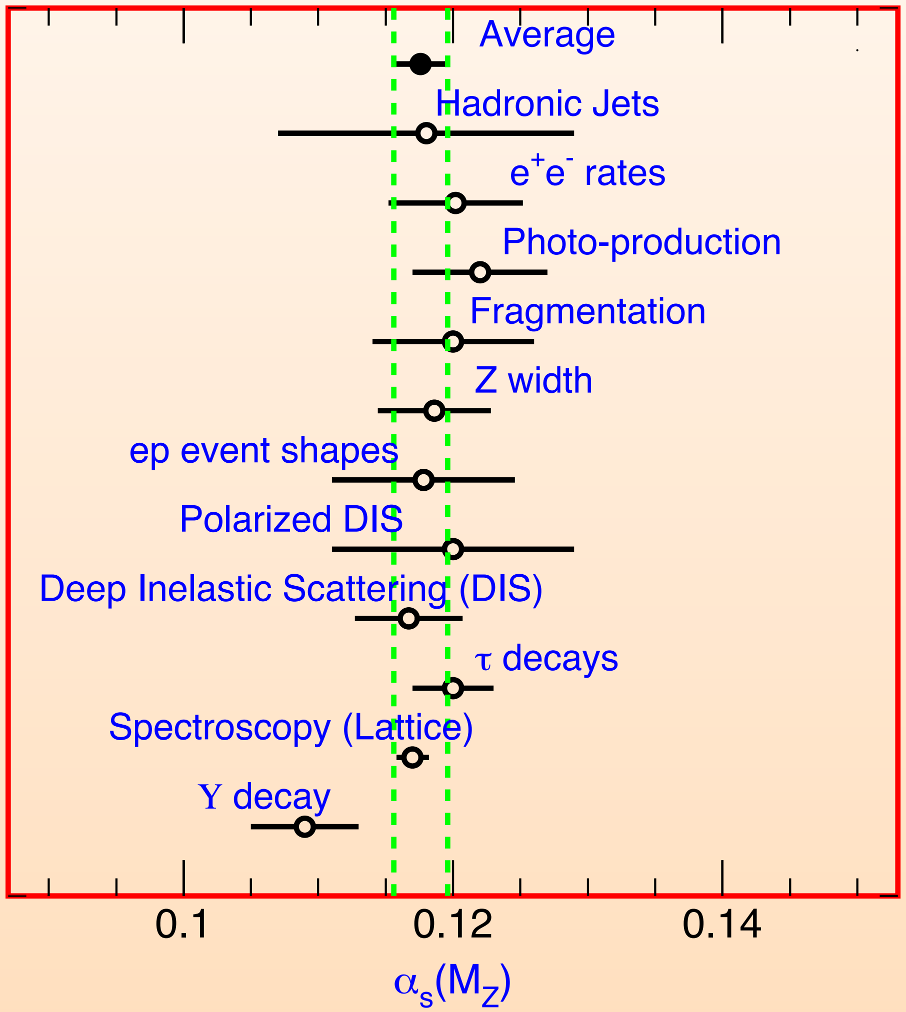
The α_s value depends on gauge group, # of extra dim, …

→ Important window into high energy physics

World Averages

(PDG Average '05)

Hinchliffe

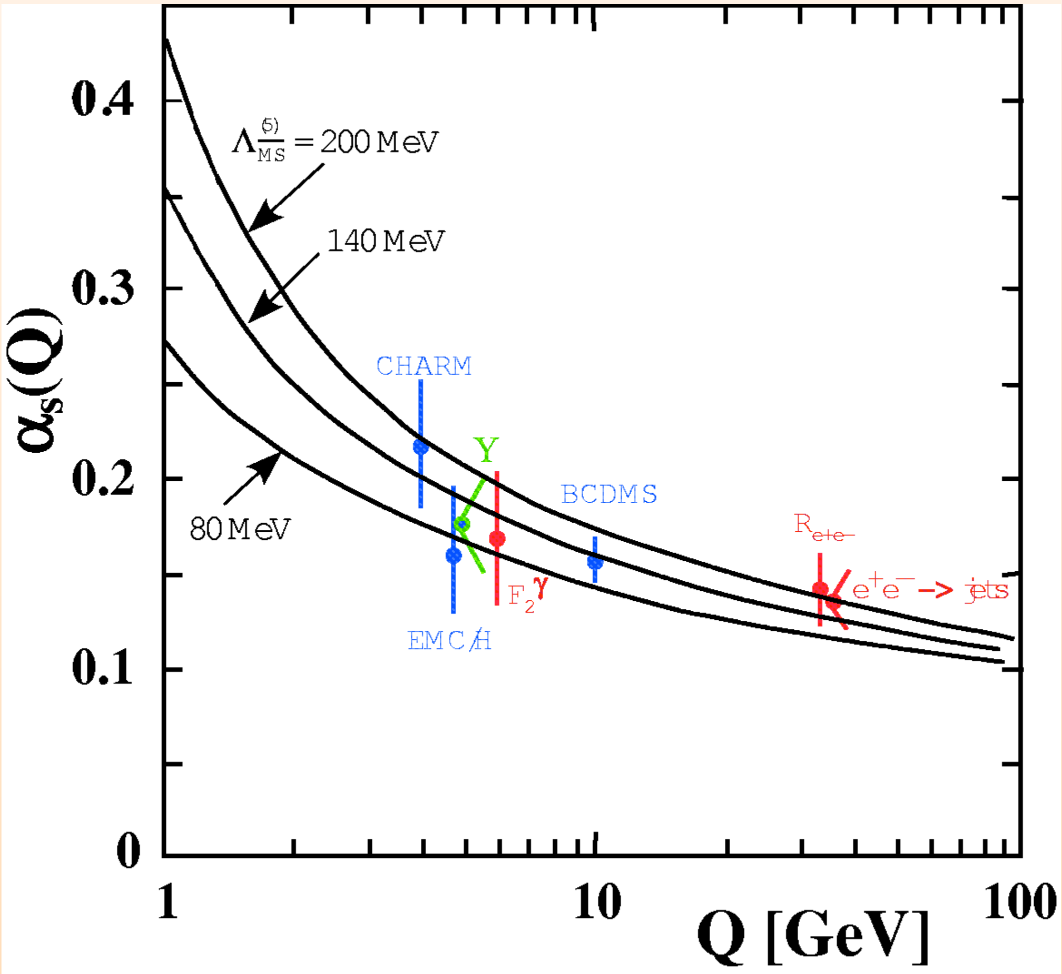


$\overline{\rm MS}$ scheme

$$\alpha_s(m_Z) = 0.1170 \pm 0.0012$$

(S. Bethke)

1989:

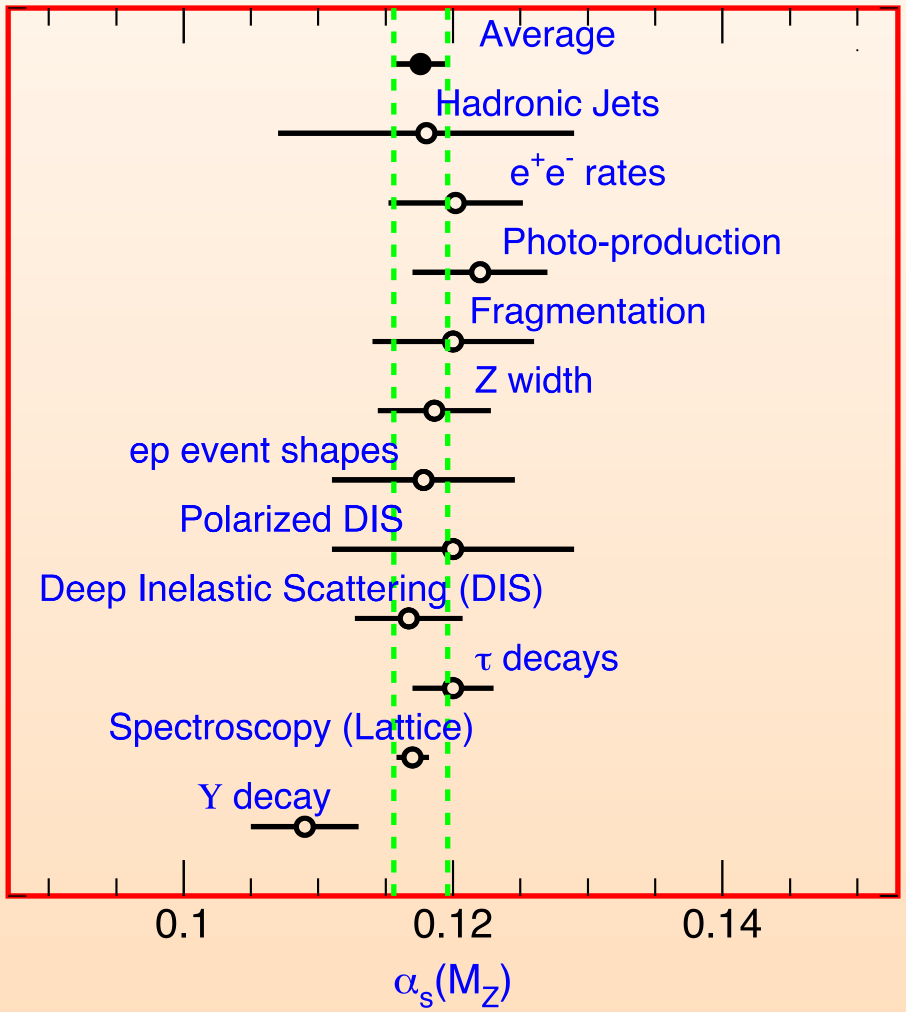


$$\alpha_s(m_Z) = 0.11 \pm 0.01$$

World Averages

(PDG Average '05)

Hinchliffe

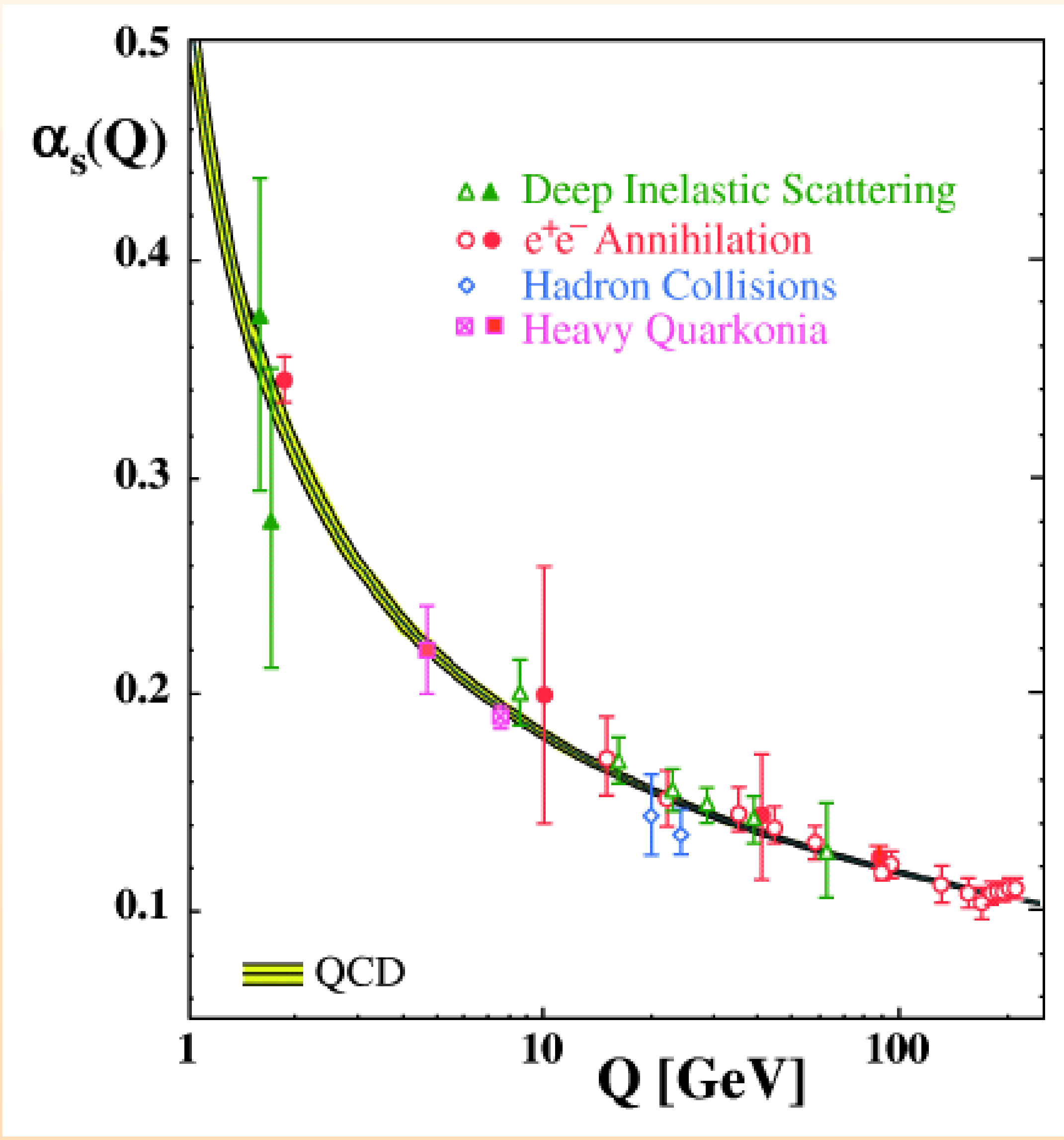


$\overline{\text{MS}}$ scheme

$$\alpha_s(m_Z) = 0.1170 \pm 0.0012$$

(S. Bethke)

2006:

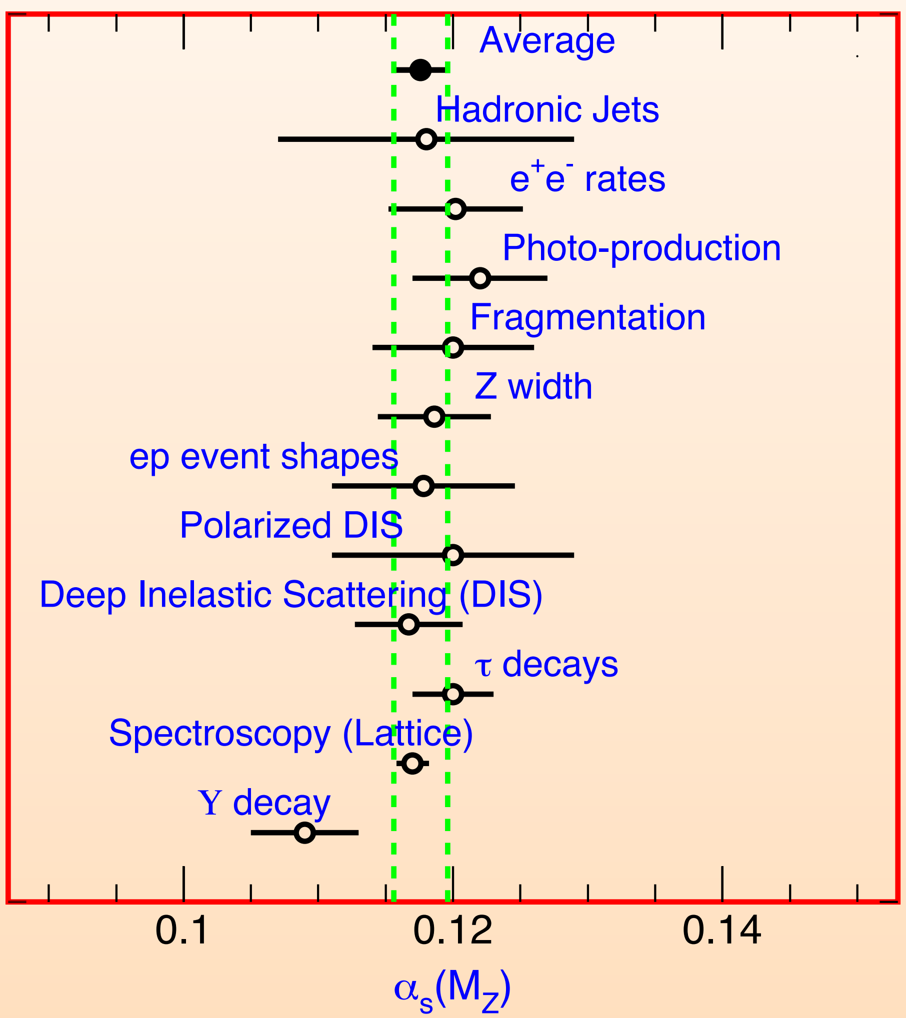


$$\alpha_s(m_Z) = 0.1189 \pm 0.0010$$

World Averages

(PDG Average '05)

Hinchliffe

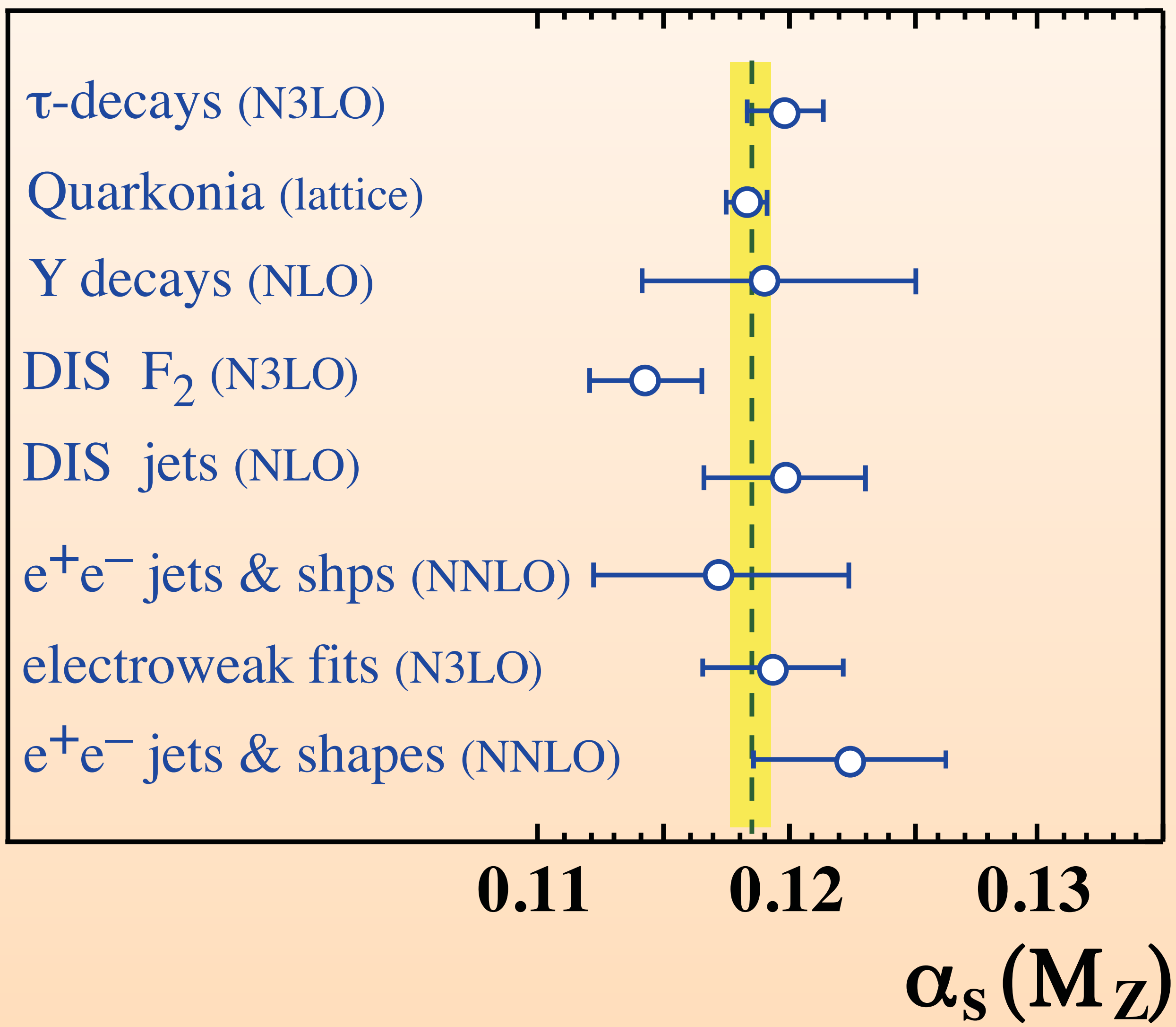


$\overline{\text{MS}}$ scheme

$$\alpha_s(m_Z) = 0.1170 \pm 0.0012$$

(S. Bethke) (arXiv:0908.1135)

2009:



$$\alpha_s(m_Z) = 0.1184 \pm 0.0007$$

Electroweak Fits

α_s from Z decays

α_s from the global EW fit

(J.Erler, K.Monig, α_s-workshop)

Γ_Z, σ_{had}, R_l, $\longrightarrow$ $\alpha_s(m_Z)$

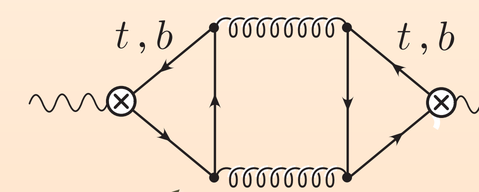
$\sigma_0^{\text{had}} = \frac{12\pi}{m_Z} \frac{\Gamma_e \Gamma_{\text{had}}}{\Gamma_Z^2}$ $R_\ell^0 = \frac{\Gamma_{\text{had}}}{\Gamma_\ell}$

(P. A. Baikov, K. G. Chetyrkin, J. H. Kühn, arXiv:0801.1821)

$\delta\alpha_s \simeq 0.0004$

$$\Gamma_{\text{had}} = \Gamma_{\text{had}}^{\text{no QCD}} \left[1 + \frac{\alpha_s}{\pi} + 1.4 \left(\frac{\alpha_s}{\pi}\right)^2 - 12.7 \left(\frac{\alpha_s}{\pi}\right)^3 - 80.0 \left(\frac{\alpha_s}{\pi}\right)^4 \right]$$

$\Delta_{theo}\alpha_s = \pm 0.00009$ tiny **expt. errors dominate**



- sensitivity to M_H: $M_H \rightarrow 2\times M_H \Rightarrow \Delta\alpha_s = +0.0004$

- axial-vector singlet: $\mathcal{O}(\alpha_s^2)$ *Kniehl, Kühn 1990* $\Delta\alpha_s = +0.0027$
  $\mathcal{O}(\alpha_s^3)$ *Larin, v. Ritbergen, Vermaseren 1995* $\Delta\alpha_s = +0.00043$
  $\mathcal{O}(\alpha_s^4) \sim \mathcal{O}(\alpha_s^3)^2 / \mathcal{O}(\alpha_s^2) \Rightarrow \Delta\alpha_s = \pm 0.00007$ (*dominant*)

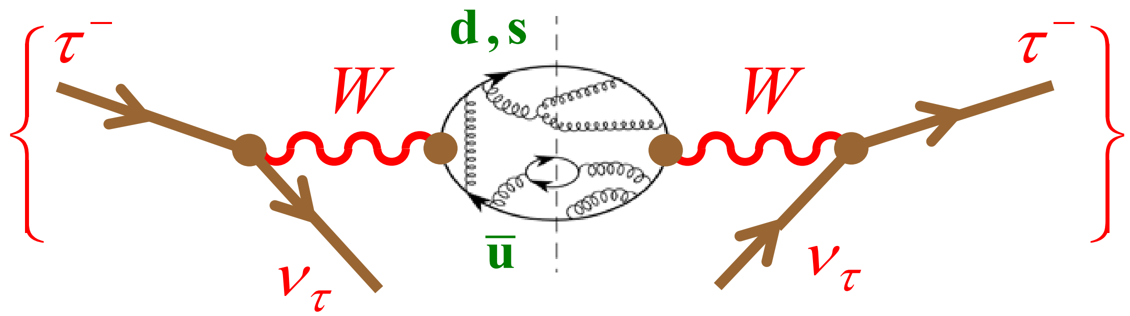
GAPP

$$\alpha_s(m_Z) = 0.1196 \pm 0.0027$$

Zfitter/Gfitter

$$\alpha_s(m_Z) = 0.1192 \pm 0.0028$$

Tau Decays

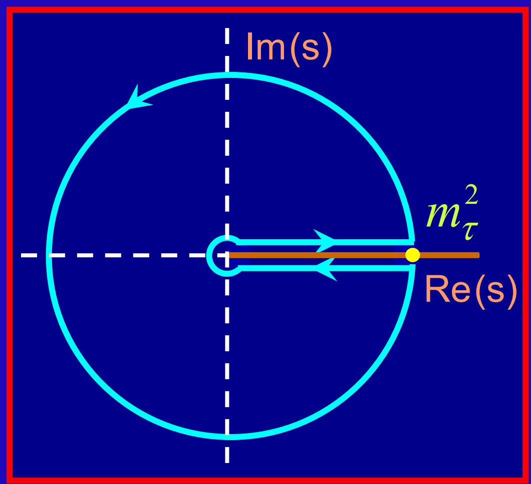


$\Gamma_{\tau\to\nu_\tau+\text{had}} \sim \text{Im}\{\ldots\}$

(T. Pich, α_s-workshop)

$m_\tau = 1.777\,\text{GeV}$!

$$R_\tau \equiv \frac{\Gamma(\tau^-\to\nu_\tau+\text{had})}{\Gamma(\tau^-\to\nu_\tau e^-\bar{\nu}_e)} = 12\pi\int_0^1 dx\,(1-x)^2\left[(1+2x)\,\text{Im}\,\Pi^{(1)}(xm_\tau^2)+\text{Im}\,\Pi^{(0)}(xm_\tau^2)\right]$$



$$R_\tau = 6\pi i\oint_{|x|=1} dx\,(1-x)^2\left[(1+2x)\,\Pi^{(0+1)}(xm_\tau^2)-2x\,\Pi^{(0)}(xm_\tau^2)\right]$$

$$R_\tau = N_C\,S_{\text{EW}}\left(1+\delta_{\text{P}}+\delta_{\text{NP}}\right)$$

$$\delta_{\text{NP}} \approx \frac{-1}{2\pi i}\oint_{|x|=1} dx\,\left(1-3x^2+2x^3\right)\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-xm_\tau^2)^n} = -3\,\frac{C_6\langle O_6\rangle}{m_\tau^6}-2\,\frac{C_8\langle O_8\rangle}{m_\tau^8}$$

$$\delta_{\text{NP}} = -0.0059\pm 0.0014$$

Fitted from data (Davier et al)

$\delta_P \sim 20\%$ A perturbative “issue” dominates the error

FOPT $$\delta^{(0)}_{\rm FO} = \sum_{n=1}^{\infty} a(M_\tau^2)^n \sum_{k=1}^{n} k\, c_{n,k}\, J_{k-1} \qquad J_l \equiv \frac{1}{2\pi i} \oint\limits_{|x|=1} \frac{dx}{x}\, (1-x)^3\, (1+x)\, \ln^l(-x)$$

CIPT $$\delta^{(0)}_{\rm CI} = \sum_{n=1}^{\infty} c_{n,1}\, J_n^a(M_\tau^2) \qquad J_n^a(M_\tau^2) \equiv \frac{1}{2\pi i} \oint\limits_{|x|=1} \frac{dx}{x}\, (1-x)^3\, (1+x)\, a^n(-M_\tau^2 x)$$

Series expansions for $\alpha_s(M_\tau^2) = 0.34$:

	α_s^1		α_s^2		α_s^3		α_s^4		α_s^5		
$\delta^{(0)}_{\rm FO}$ =	0.1082	+	0.0609	+	0.0334	+	0.0174	(+	0.0088)	=	0.2200 (0.2288)
$\delta^{(0)}_{\rm CI}$ =	0.1479	+	0.0297	+	0.0122	+	0.0086	(+	0.0038)	=	0.1984 (0.2021)

Both methods appear to converge, but to different values.

FOPT $$\delta^{(0)}_{\rm FO} = \sum_{n=1}^{\infty} a(M_\tau^2)^n \sum_{k=1}^{n} k\, c_{n,k}\, J_{k-1} \qquad J_l \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1-x)^3\,(1+x)\ln^l(-x)$$

CIPT $$\delta^{(0)}_{\rm CI} = \sum_{n=1}^{\infty} c_{n,1}\, J_n^a(M_\tau^2) \qquad J_n^a(M_\tau^2) \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1-x)^3\,(1+x)\, a^n(-M_\tau^2 x)$$

Pro CIPT:

- Better convergence, smaller scale dependence

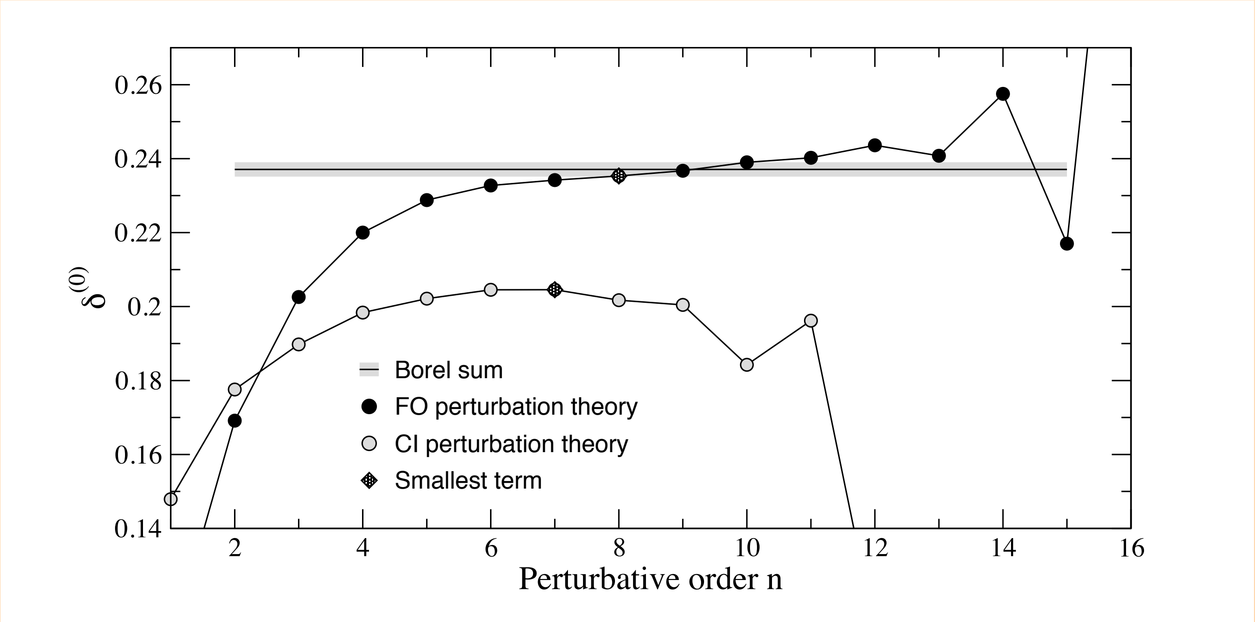
Scale error on $\alpha_s(M_\tau^2)$ from variation of μ in [1,2.5] GeV is ${}^{+0.010}_{-0.005}$ for FO and ${}^{+0.005}_{-0.002}$ for CI.

- Expansion of the running coupling on the circle as used in FO has only a finite radius of convergence [Le Diberder, Pich; 1992]

$$\alpha_s(M_\tau^2 e^{i\pi}) = \frac{\alpha_s(M_\tau^2)}{1 + \frac{\beta_0}{4\pi} i\pi\alpha_s(M_\tau^2)}$$

Pro FOPT:

series are asymptotic. Models for higher terms with u=2 renormalon:



Duality violations in hadronic tau decays

(Golterman, α_s-workshop)

- High precision determination of α_s from tau decays requires understanding of Duality Violations; pinched weights do not suppress DVs sufficiently

$$\int_0^{s_0} ds\, w(s)\, \rho_{V,A}(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\, \Pi^{OPE}_{V,A}(s) \; - \int_{s_0}^{\infty} ds\, w(s)\, \rho^{DV}_{V,A}(s)$$

model:

$$\rho^{DV}_{V,A}(s) = \theta(s - s_{min}) \left[\kappa_{V,A}\, e^{-\gamma_{V,A} s} \sin(\alpha_{V,A} + \beta_{V,A} s)\right]$$

fit params to data

- Assuming our *ansatz* for DVs, we obtain, from vector channel with $w = 1$ preliminary values

$$\alpha_s(M_\tau) = 0.322(25) \Rightarrow \alpha_s(M_Z) = 0.1188(29) \quad \text{(CIPT)}$$
$$\alpha_s(M_\tau) = 0.307(18) \Rightarrow \alpha_s(M_Z) = 0.1169(24) \quad \text{(FOPT)}$$

Recent $\alpha_s(m_\tau)$ Analyses

Reference	Method	δ_P	$\alpha_s(m_\tau)$	$\alpha_s(m_Z)$
Baikov et al	CIPT, FOPT	0.1998 (43)	**0.332 (16)**	**0.1202 (19)**
Davier et al	CIPT	0.2066 (70)	**0.344 (09)**	**0.1212 (11)**
Beneke-Jamin	BSR + FOPT	0.2042 (50)	**0.316 (06)**	**0.1180 (08)**
Maltman-Yavin	PWM + CIPT	–	**0.321 (13)**	**0.1187 (16)**
Menke	CIPT, FOPT	0.2042 (50)	**0.342 (11)**	**0.1213 (12)**
Narison	CIPT, FOPT	–	**0.324 (08)**	**0.1192 (10)**
Caprini-Fischer	BSR + CIPT	0.2042 (50)	**0.321 (10)**	–
Cvetič et al	β_{exp} + CIPT	0.2040 (40)	**0.341 (08)**	**0.1211 (10)**
Pich	**CIPT**	**0.1997 (35)**	**0.338 (12)**	**0.1209 (14)**

CIPT: Contour-improved perturbation theory
FOPT: Fixed-order perturbation theory
BSR: Borel summation of renormalon series
CIPTm: Modified CIPT (conformal mapping)
βexp: Expansion in derivatives of the coupling (β function)
PWM: Pinched-weight moments

Lattice QCD

(A. Kronfeld, α_s-workshop)

Fix parameters. Calibrate.

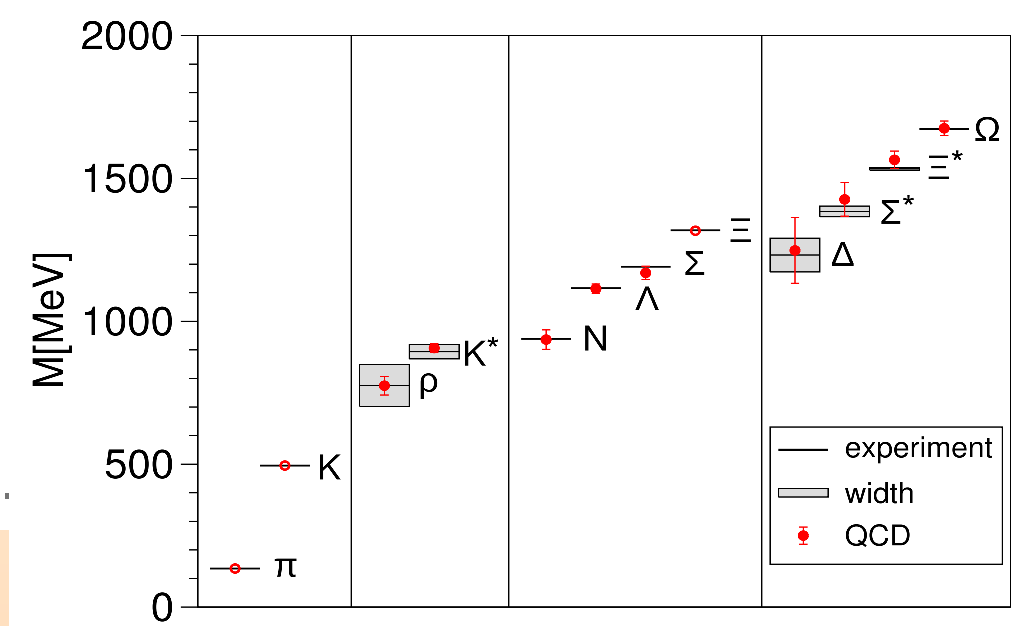
$$\mathcal{L}_{QCD} = \frac{1}{g_0^2}\,\mathrm{tr}[F_{\mu\nu}F^{\mu\nu}] - \sum_f \bar{\psi}_f(\not{D}+m_f)\psi_f$$

m_Ω or $\Upsilon(2S\text{-}1S)$ or f_π or r_1 or ....

m_π, m_K, m_{Ds} or $m_{J/\psi}$, m_{Bs} or m_Υ, ...

- The spectrum results suggest that the calibration step is understood:
  - Continuum limit under control: 3–5 different lattice spacings—up to ×3;
  - Chiral extrapolation under control;
  - Finite-volume effects small (as expected for masses of stable particles);
  - Several groups (MILC, PACS-CS, BMW) with 2+1 spectrum and few % errors.

BMW Collaboration



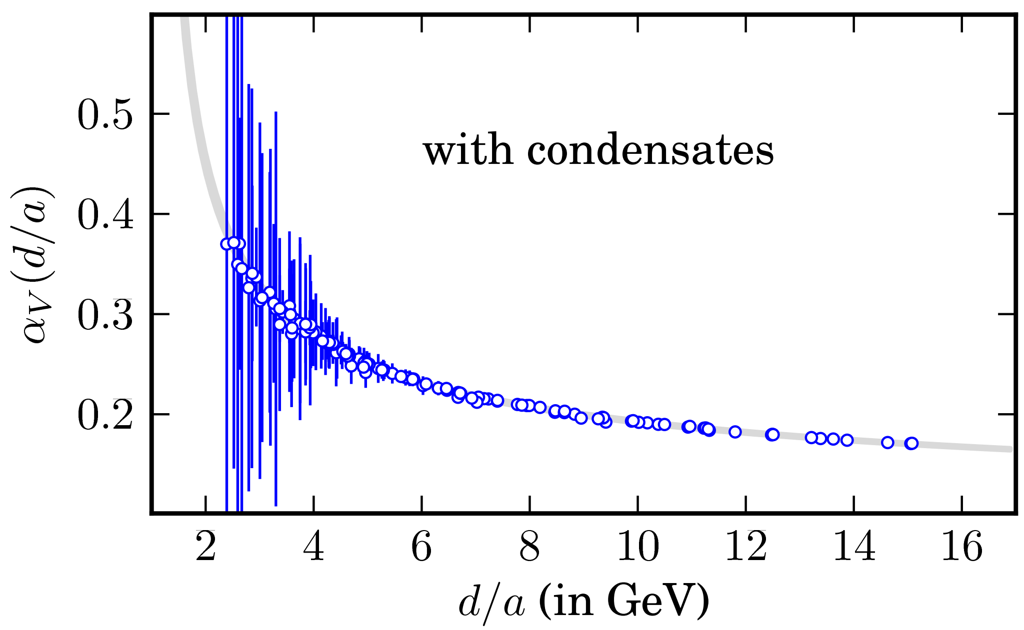
Extract $\alpha_s(m_Z)$

- small Wilson loops
- Current correlators
- Schrodinger functional

Small Wilson loops $Y = \sum_{n=1}^{3} c_n \, \alpha_V^n(d/a)$ (P. Lepage, α_s-workshop)

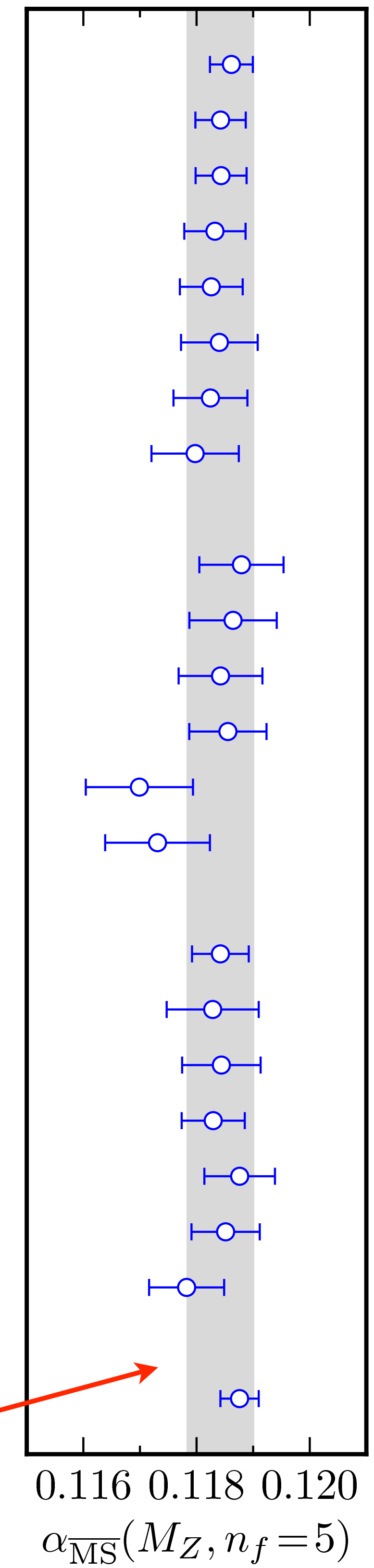
staggered quarks

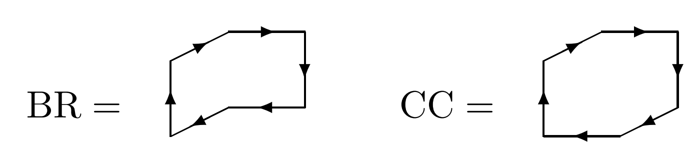
Davies *et al* (HPQCD), Phys. Rev. D**78**, 114507 (2008) [arXiv:0807.1687]
McNeile *et al* (HPQCD) Phys. Rev. D**82**, 034512 (2010) [arXiv:1004.4285]



- 264 different α_Vs.
- Know 3 terms in pert'n theory; allow for 10 in all (only 4 needed).
- Use BLM/LM scale $q^*=d/a$ with α_V.
- n_f=3. Convert to MS-bar and evolve to M_Z n_f=5 using continuum pert'n theory.
- Nonperturbative corrections: chiral (measured) and gluon condensates (sensitivity varies by 100s).

$$\alpha_{\overline{\rm MS}}(M_Z, n_f = 5) \;=\; 0.1184\,(6)$$

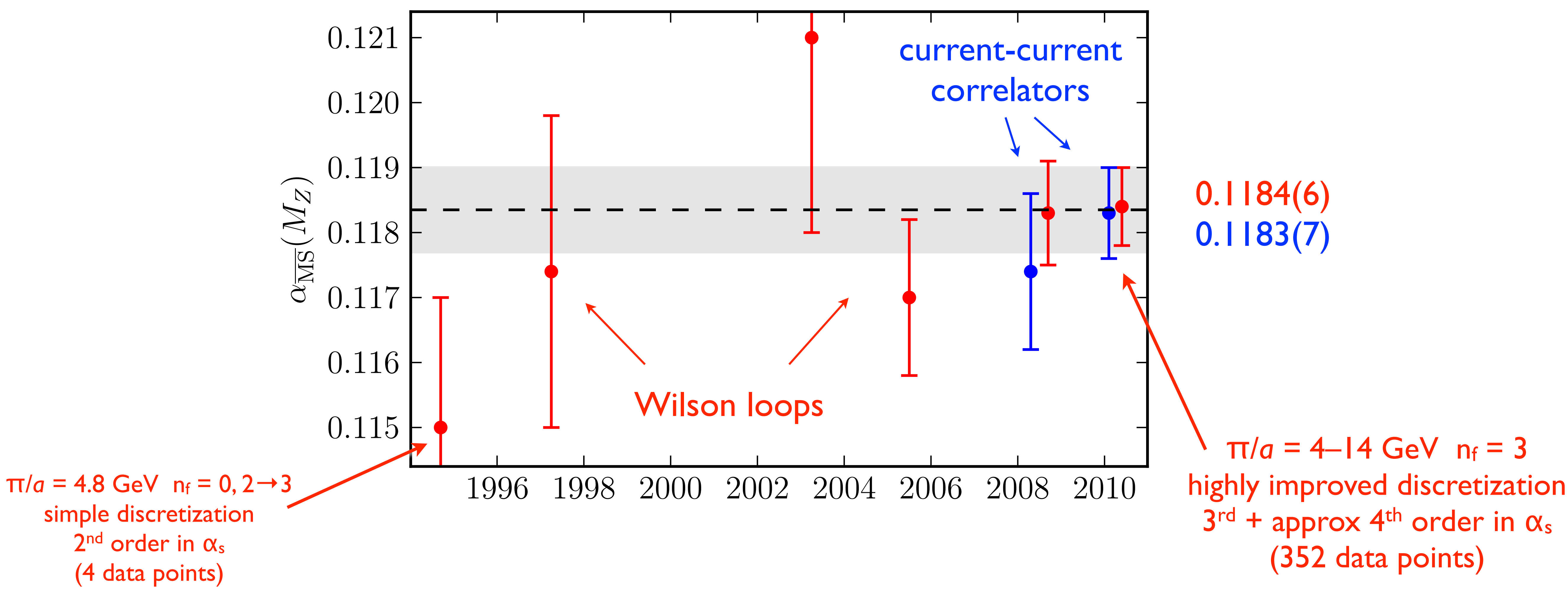




Error Budget

	$\log W_{11}$	$\log W_{12}$	$\log W_{22}$	$\log W_{11}W_{22}/W_{12}^2$	$\log W_{12}/u_0^6$	$\log W_{22}/u_0^8$	$\alpha_{\mathrm{lat}}/W_{11}$
$c_1 \ldots c_3$	0.1%	0.1%	0.1%	0.3%	0.1%	0.1%	0.1%
c_n for $n \geq 4$	0.1	0.1	0.2	0.3	0.2	0.3	0.1
am_q, $r_1 m_q$ extrapolation	0.0	0.0	0.1	0.1	0.1	0.1	0.0
$(a/r_1)^2$ extrapolation	0.0	0.0	0.2	0.3	0.1	0.2	0.0
$(r_1/a)_i$ errors	0.2	0.2	0.2	0.2	0.2	0.2	0.2
r_1 errors	0.1	0.1	0.1	0.1	0.1	0.1	0.1
gluon condensate	0.1	0.2	0.2	0.2	0.2	0.1	0.1
statistical errors	0.0	0.0	0.0	0.1	0.1	0.1	0.0
$V \to \overline{\mathrm{MS}} \to M_Z$	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Total	0.3%	0.4%	0.5%	0.6%	0.4%	0.5%	0.3%

QCD Coupling — HPQCD History



Current Correlators

(S.Hashimoto, α_s-workshop)

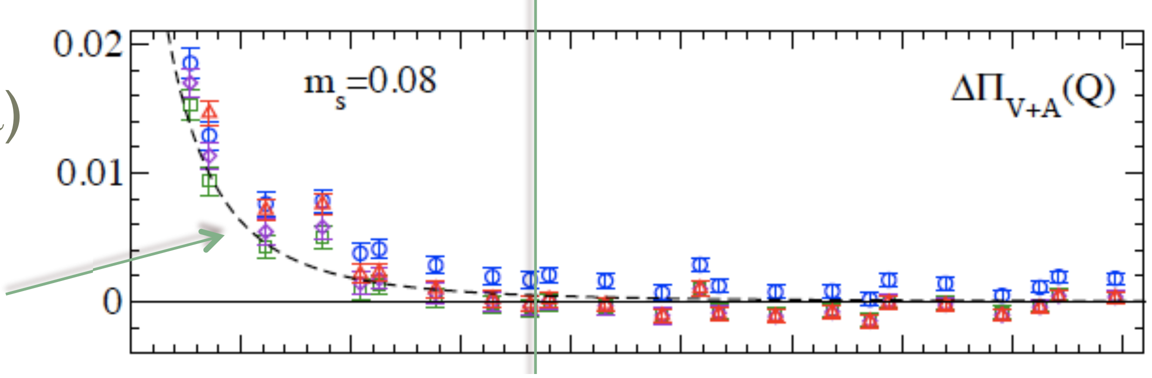
dynamical overlap fermion

E. Shintani et al. [JLQCD collaboration], arXiv:1002.0371 [hep-lat], Phys. Rev. D80, 074505 (2010).

$$\int d^4x e^{iQx} \langle 0 | J^a_\mu(x) J^{b\dagger}_\nu(0) | 0 \rangle = \delta^{ab} \left[(\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu) \Pi^{(1)}_J(Q) - Q_\mu Q_\nu \Pi^{(0)}_J(Q) \right]$$

- Need to be careful about
  - Discretization effects? : more important at high Q^2. how are they estimated?
  - Window? : can we find the region where the pert formula safely applies while disc error is small enough?
  - Enough sensitivity? : can we get enough precision for $\alpha_s(\mu)$

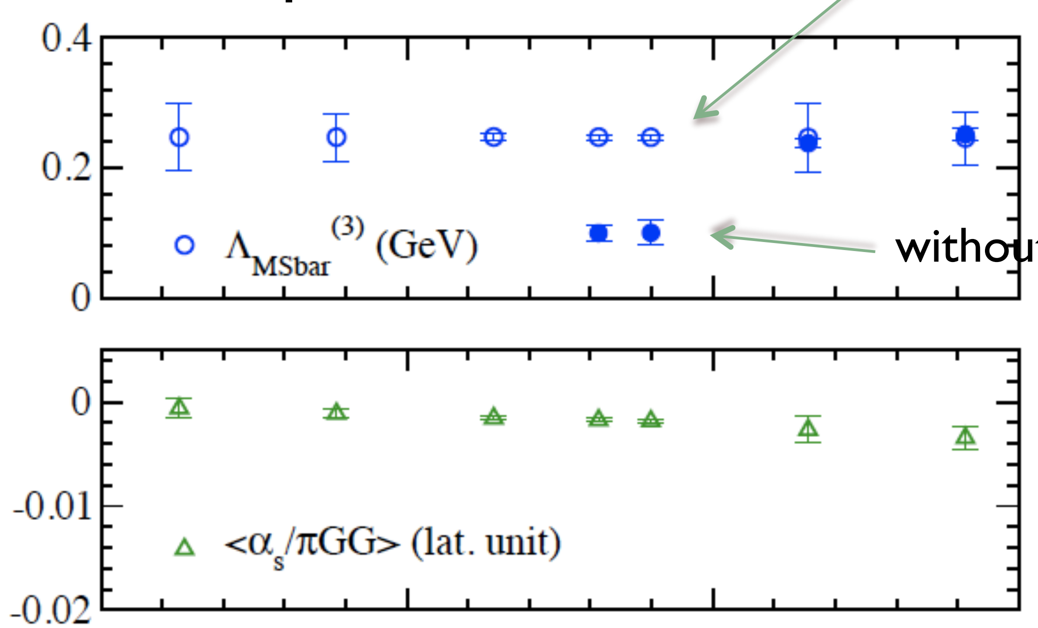
used in the fit with $1/Q^4$



$1/Q^6$ curve significant below $(1\ \mathrm{GeV})^2$

For 3 fit parameters

with $1/Q^4$



without $1/Q^4$

$$\alpha_s^{(5)}(M_Z) = 0.1181(3)(^{+14}_{-12})$$

Systematic errors

- Error to $\alpha_s^{(5)}(M_Z)$

Sources	Estimated error in $\alpha_s^{(5)}(M_z)$
Uncorrelated fit	± 0.0003
Lattice artifact ($\mathcal{O}(a^2)$ effect)	$+0.0003$
$\Delta_{\mu\nu}^{V+A}$	± 0.0002
Quark condensate	± 0.0001
Z_m	± 0.0001
Perturbative expansion	± 0.0003
$1/Q^2$ expansion	< 0.0001
$m_{c,b}$	$^{+0.0001}_{-0.0003}$
Lattice spacing	$^{+0.0013}_{-0.0010}$
Total (in quadrature)	$^{+0.0014}_{-0.0012}$

Dominant error:
$1/a$=
1.83(1) GeV r_0=0.49 fm
1.97(4) GeV f_π
1.76(8) GeV m_Ω

Schroedinger Functional

Wilson fermions

(S. Aoki, α_s-workshop)
PACS-CS

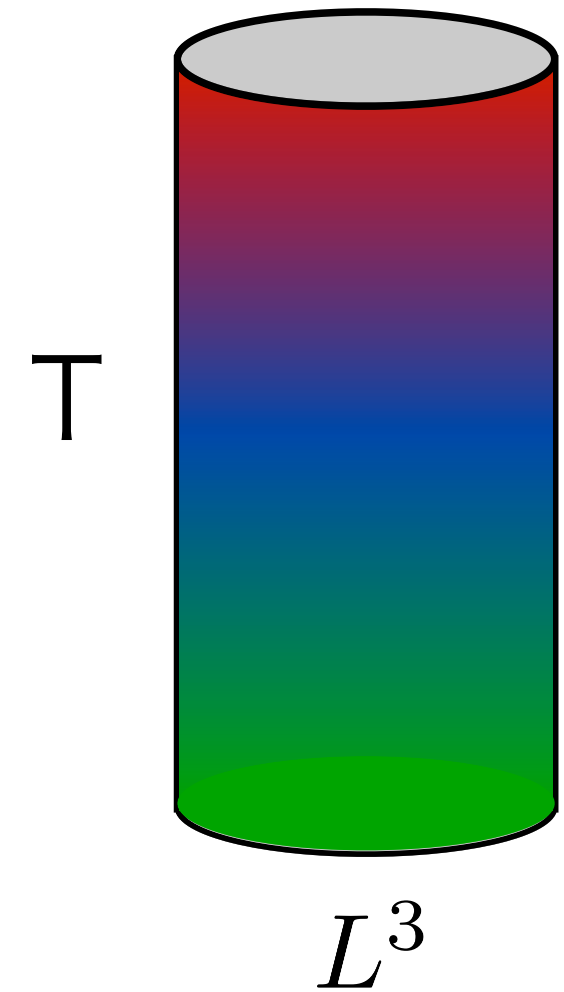
- Parton-hadron duality says energy in can, for $Q \gg \Lambda_{\rm QCD}$, can be computed with partons, *i.e.*, with perturbation theory.

Advantages

Non-perturbative.

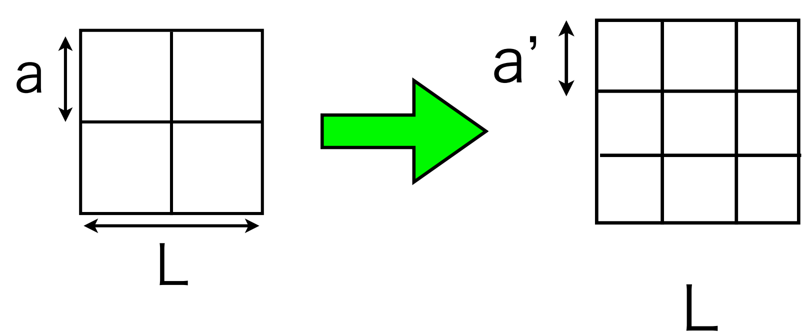
A box size L gives the scale. No other scale is needed.

- Vary $Q = L^{-1}$ over potentially *enormous* range: $\times 10^3$ [*Alpha*].



(1) Step Scaling Function (SSF)

$$\Sigma\left(u, \frac{a}{L}\right) = \bar{g}^2(2L)\Big|_{u=\bar{g}^2(L)}$$



continuum limit

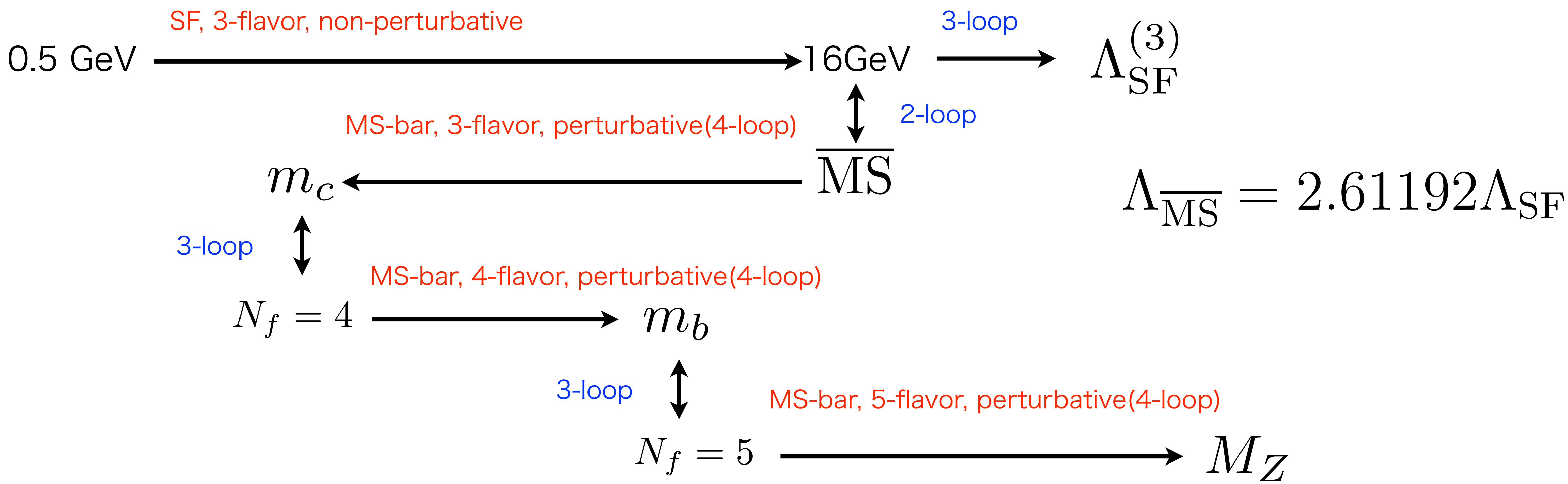
$$\sigma(u) = \lim_{a/L \to 0} \Sigma\left(u, \frac{a}{L}\right)$$

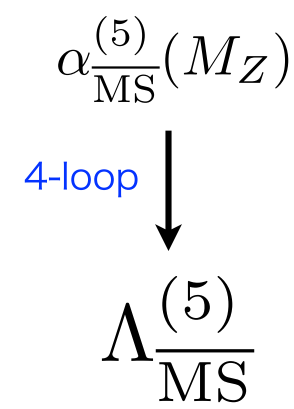
(2) define a reference scale $L_{\rm max}$ through a fixed value of $\bar{g}^2(L_{\rm max})$

non-perturbative $1/L_{\rm max} \sim 0.5$ GeV —SSF(n times)→ perturbative $1/L = 2^n/L_{\rm max} \sim 16$ GeV

(3) $$\Lambda_{\rm SF} = \frac{1}{L}\left(b_0\bar{g}(L)\right)^{-\frac{b_1}{2b_0^2}} \exp\left(-\frac{1}{2b_0\bar{g}(L)}\right) \exp\left(-\int_0^{\bar{g}(L)} dg \left(\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right)\right)$$

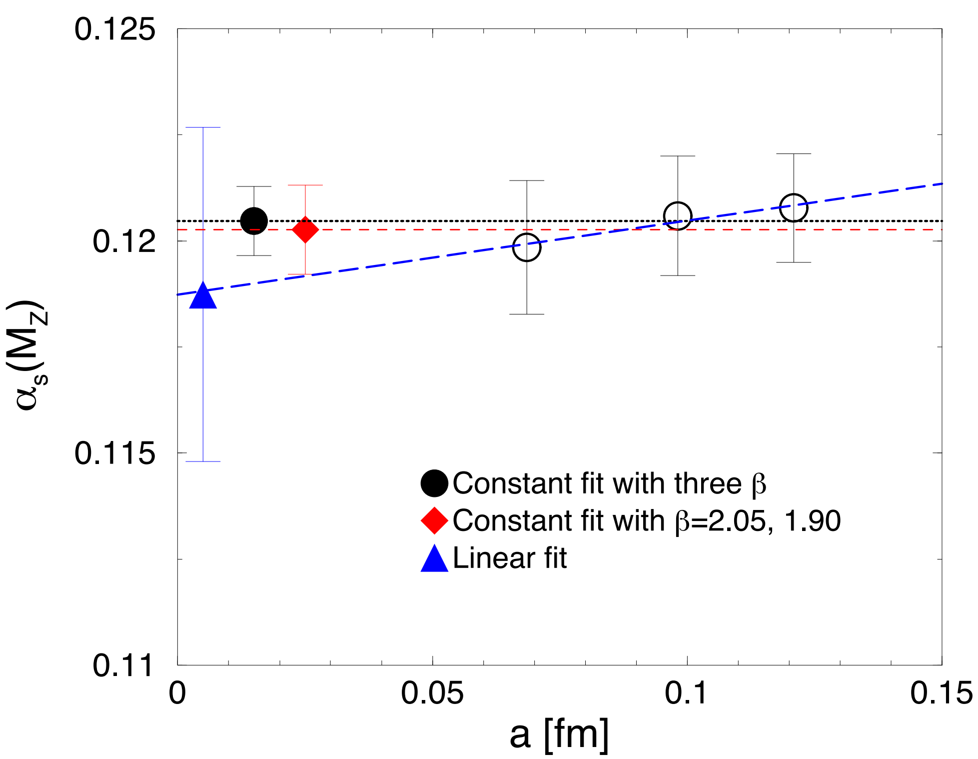
(4) $L_{\rm max}$ in physical unit from hadron mass





Simulations

3-flavor massless QCD
non-perturbatively O(a) improved Wilson quark action
RG improved gauge action
non-perturbative c_A
tree-level boundary terms



matching

$$\alpha_s(M_Z) = 0.12047(81)(48)(^{+0}_{-173})$$

a->0

Selected $\alpha_s(M_Z)$ Results from Lattice QCD

$\alpha^{(5)}_{\overline{\text{MS}}}(M_Z)$	R	Q range	$\mathcal{R}$	sea	collab	when
0.1170(12)	Wilson loops	3	NNLO	2+1 √stag	HPQCD	2005
0.1183(8)	Wilson loops	7	NNLO	2+1 √stag	HPQCD	2008
0.1192(11)	Wilson loops	7	NNLO	2+1 √stag	Maltman ...	2008
0.1174(12)	$\bar{Q}Q$ correlator	1–2	NNLO	2+1 √stag	HPQCD + KIT	2008
0.1183(7)	$\bar{Q}Q$ correlator	3–6	NNLO	2+1 √stag	HPQCD + KIT	2010
0.1181(3)(+14/–12)	Adler	5	NNLO	2+1 overlap	JLQCD	2010
0.1205(8)(5)(+0/–17)	Schrödinger	80	asymptote	2+1 Wilson	PACS-CS	2009
$\Lambda^{(2)}_{\overline{\text{MS}}} = 245(23)\,\frac{0.5\text{ fm}}{r_0}$ MeV	Schrödinger	270	asymptote	2 Wilson	*Alpha*	2004
0.1xxx(y)	Schrödinger	1000	asymptote	2+1+1 Wilson	*Alpha*	2012

- Superseded; re-analysis.

DIS &
inclusive Jets

(J.Blumlein, CP Yuan, S. Forte, K.Monig, A.Martin α_s-workshop)

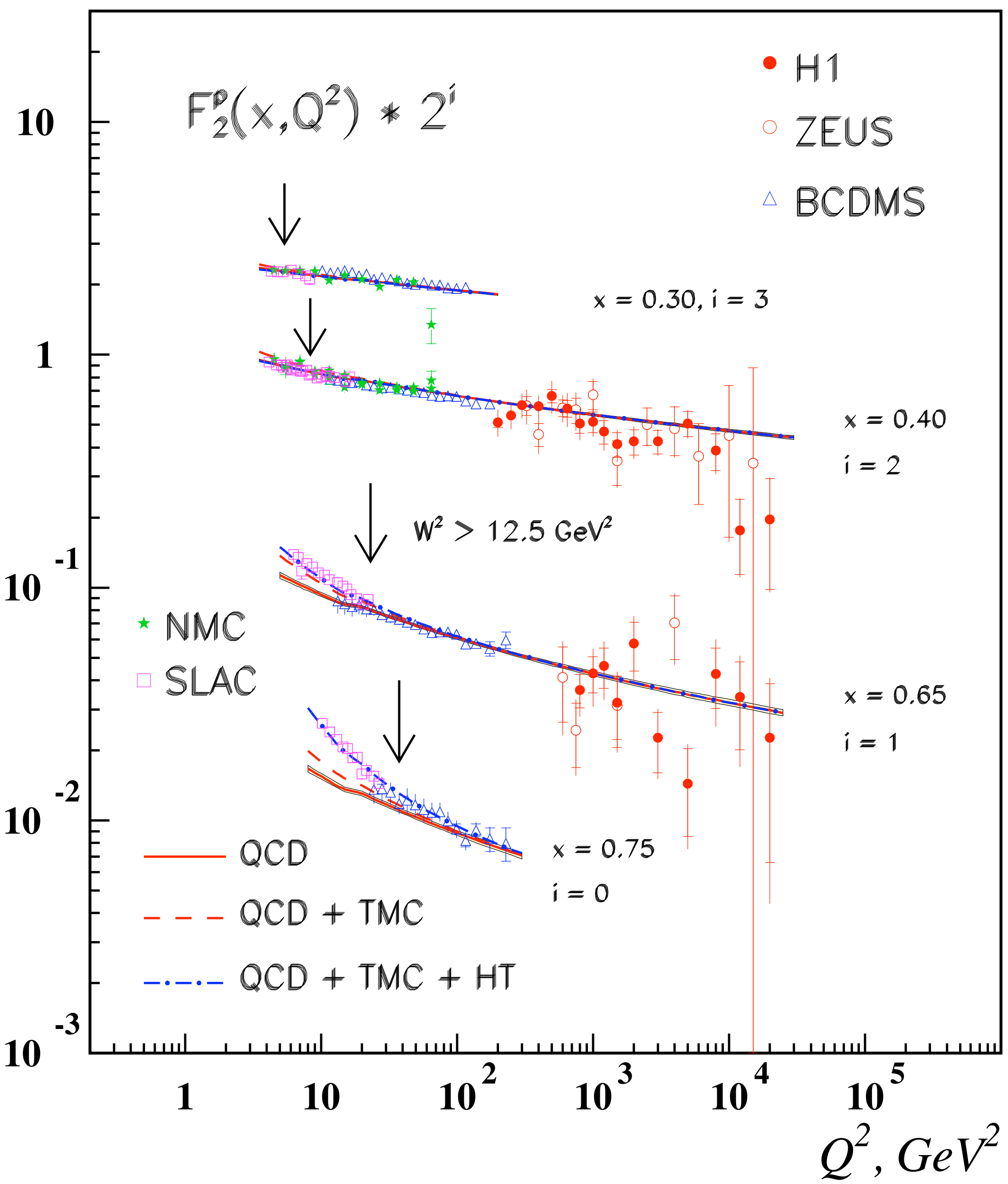
DIS $\alpha_s(m_Z)$ from scaling violation

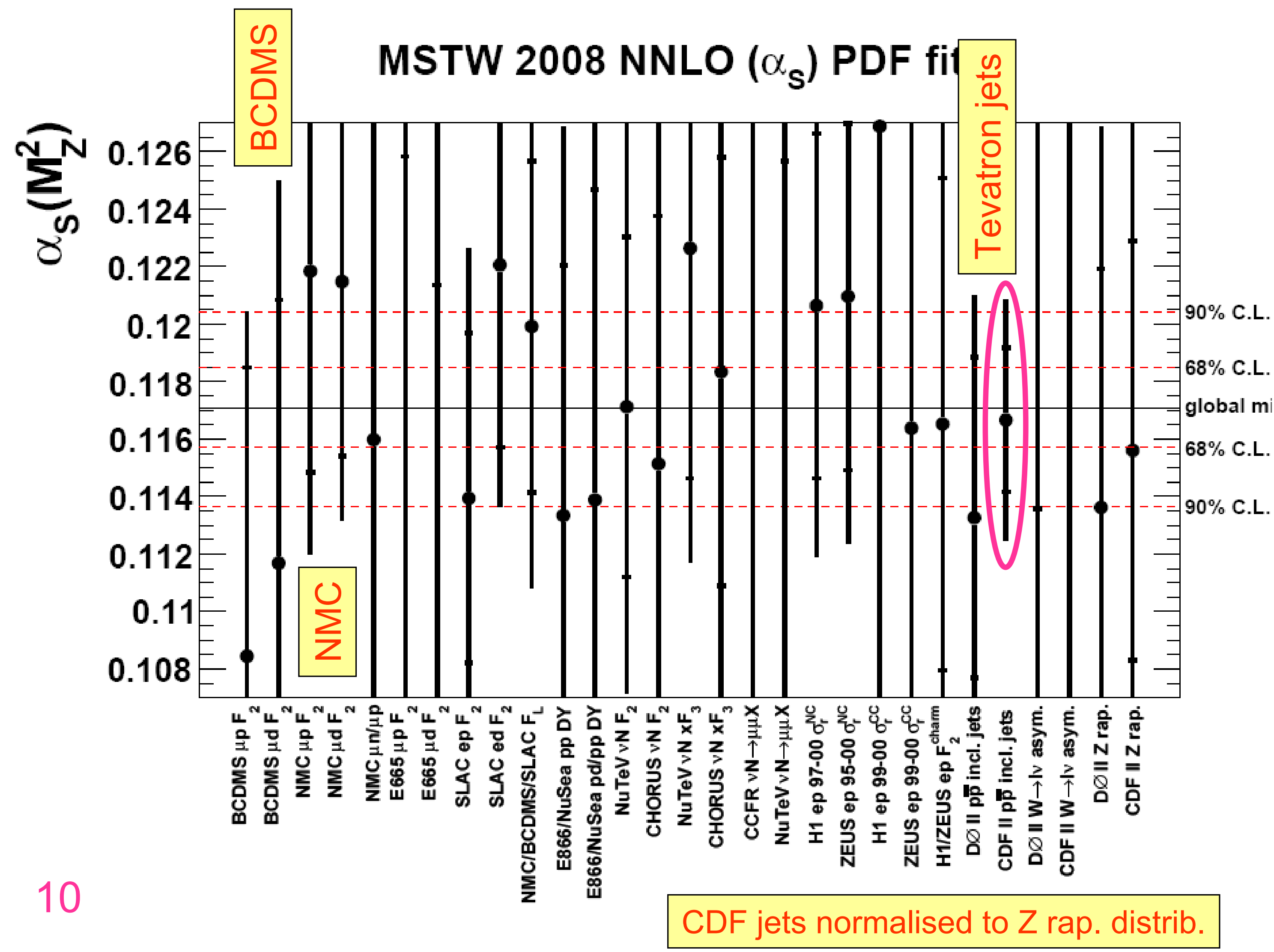
$$F_k(N,Q^2) = f_k(N,Q^2)\sum_{n=0}^{3} C_n(N)\alpha_s^n(Q^2)$$

non-singlet N^{3}LO: 3-loop RGE & Coefficients (Vogt et al.) Pade for 4-loop RGE

Global fits:

- MSTW
- CTEQ
- NNPDF





10

DIS

NLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
CTEQ6	0.1165	±0.0065		[1]
MRST03	0.1165	±0.0020	±0.0030	[2]
A02	0.1171	±0.0015	±0.0033	[3]
ZEUS	0.1166	±0.0049		[4]
H1	0.1150	±0.0017	±0.0050	[5]
BCDMS	0.110	±0.006		[6]
GRS	0.112			[10]
BBG	0.1148	±0.0019		[9]
BB (pol)	0.113	±0.004	$^{+0.009}_{-0.006}$	[7]

NLO at least: scale errors of ±0.0050

NNLO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
MRST03	0.1153	±0.0020	±0.0030	[2]
A02	0.1143	±0.0014	±0.0009	[3]
SY01(ep)	0.1166	±0.0013		[8]
SY01(νN)	0.1153	±0.0063		[8]
GRS	0.111			[10]
A06	0.1128	±0.0015		[11]
BBG	0.1134	$+0.0019/-0.0021$		[9]
N^{3}LO	$\alpha_s(M_Z^2)$	expt	theory	Ref.
BBG	0.1141	$+0.0020/-0.0022$		[9]

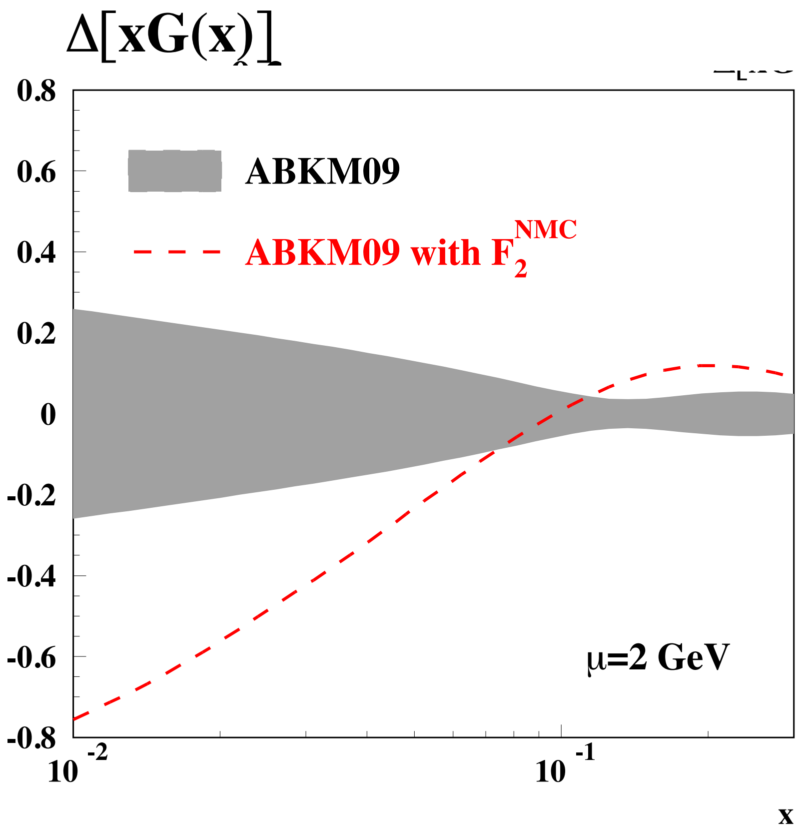
NNLO systematic shifts down

N^{3}LO slight upward shift

- Wrong treatment of $F_L(x,Q^2)$ in NMC F_2 extraction.

BBG (2006)

$\alpha_s(M_Z^2)$	with $\sigma_{\rm NMC}$	with $F_2^{\rm NMC}$	difference
NLO	0.1179(16)	0.1195(17)	$+0.0026 \simeq 1\sigma$
NNLO	0.1135(14)	0.1170(15)	$+0.0035 \simeq 2.3\sigma$
NNLO +F$_L$O(α_s^3)	0.1122(14)	0.1171(14)	$+0.0050 \simeq 3.6\sigma$



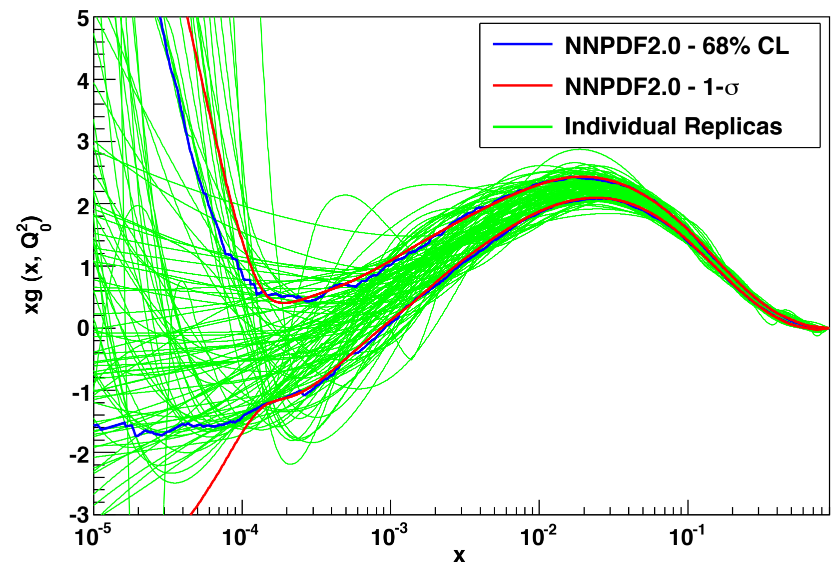
	$\alpha_s(M_Z^2)$	
BBG (2006)	$0.1134 \; ^{+0.0019}_{-0.0021}$	valence analysis, NNLO
ABKM	0.1135 ± 0.0014	HQ: FFS $N_f = 3$
ABKM	0.1129 ± 0.0014	HQ: BSMN-approach
JR (2008)	0.1124 ± 0.0020	dynamical approach
MSTW (2008)	0.1171 ± 0.0014	
HERAPDF (2010)	0.1145	(combined H1/ZEUS data, prelimiary)
ABM (2010)	0.1147 ± 0.0012	(FFN, combined H1/ZEUS data in)
BBG (2006)	$0.1141 \; ^{+0.0020}_{-0.0022}$	valence analysis, N^{3}LO

UNBIASED PDF DETERMINATION: THE NNPDF APPROACH

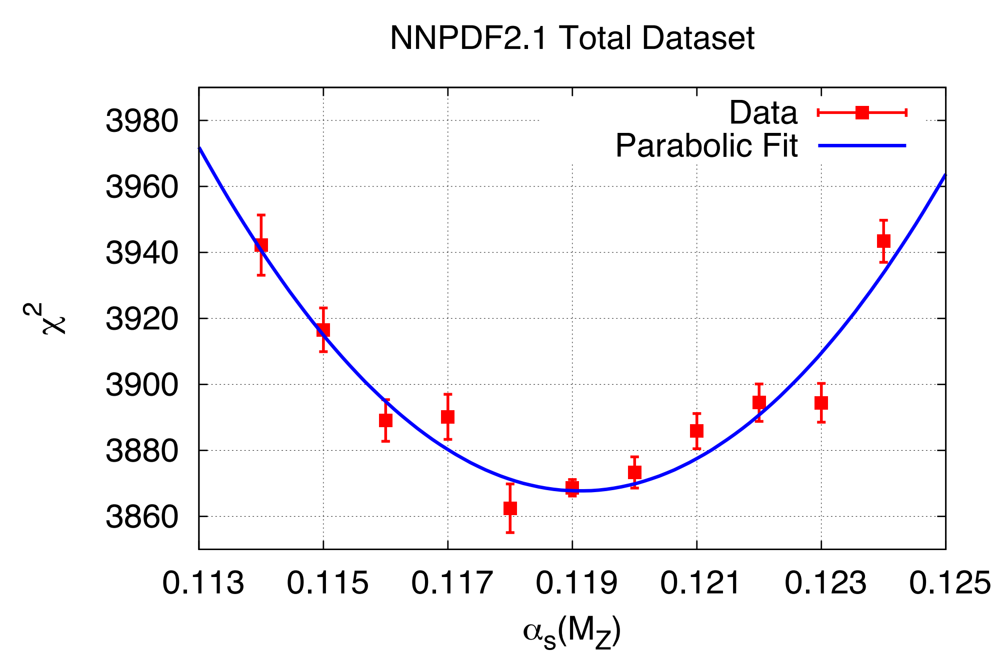
BASIC IDEA: MONTE CARLO SAMPLING
OF THE PROBABILITY MEASURE IN THE (FUNCTION) SPACE OF PDFS

- START FROM MONTE CARLO SAMPLING OF DATA SPACE
- EACH PDF$\leftrightarrow$ NEURAL NETWORK PARAMETRIZED BY 37 PARAMETERS (NNPDF: $37 \otimes 7 = 259$ PARMS)
  "INFINITE" NUMBER OF PARAMETERS$\Rightarrow$ CAN REPRESENT ANY FUNCTION
- FIT STOPS WHEN QUALITY OF FIT TO RANDOMLY SELECTED "VALIDATION" DATA (NOT FITTED) STOPS IMPROVING

CAN DETERMINE BOTH 68C.L.& 1-σ



χ^2 PROFILE VS. α_s



CAVEATS

- χ^2 IS A RANDOM VARIABLE $\Rightarrow$ FLUCTUATES FOR FINITE SAMPLE SIZE $\Rightarrow$ ADDITIONAL UNCERTAINTY DUE TO FINITE-SIZE FLUCTUATIONS

$$\Delta\chi^2 \sim \frac{\sqrt{N_{\rm dat}}}{N_{\rm rep}}$$

Global fits:

- **CTEQ 2010 (CT10.AS)** $\alpha_s(M_Z) = 0.1197 \pm 0.0061$

- **MSTW have highest α_s from DIS fits**

$\alpha_s \longleftrightarrow$ gluon correlation

$\alpha_S(M_Z^2) = 0.1171 \; {}^{+0.0014}_{-0.0014} \; (68\% \text{ C.L.})$ (less than ± 0.002 theory error)

1. More flexible low x parametrization of gluon
   needed by data ($\Delta\chi^2 \sim 80$) --- shape confirmed by NNPDF
   without flexibility } α_s(NLO) 0.1202 → 0.1175
   α_s(NNLO) 0.1171 → 0.1157

theory uncertainty dominant

2. Inclusion of Tevatron jet data

Jet data themselves prefer α_s slightly lower than global α_s
However jets demand more high x gluon (less low x gluon)
which turn a low α_s into a better constrained high α_s

- **NLO NNPDF2.1 global determination** (only stat. error known)

$$\alpha_s(M_z) = 0.1191 \pm 0.0006(\text{stat.})$$

$\chi^2/\text{d.o.f.} = 1.4$ for the parabolic fit

HEAVY QUARKS	$\alpha_s(M_z) = 0.1169 \pm 0.0009(\text{stat.})$	NLO NNPDF2.0 global
deep-inelastic data	$\alpha_s(M_z) = 0.1178 \pm 0.0009(\text{stat.})$	
HERA data only	$\alpha_s(M_z) = 0.1103 \pm 0.0033(\text{stat.})$	

theory uncertainty dominant

Event shapes

$$e^+e^- \to \text{jets}$$

R. Abbate, M. Fickinger, A. Hoang, VM & I. Stewart – arXiv: 1006.3080 [hep-ph]

R. Abbate, A. Hoang, VM, M. Schwartz & I. Stewart – work in progress for HJM

Builds on work by Gehrmann et al & Weinzierl $O(\alpha_s^3)$ and Becher & Schwartz at N^3LL

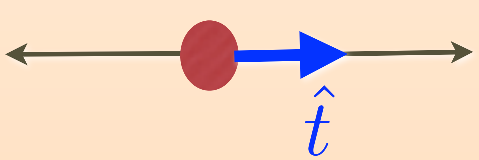
Also builds on work done in SCET community.

Thrust is a classic example of an "event-shape"

$$T = \max_{\hat{t}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \qquad \tau = 1 - T$$

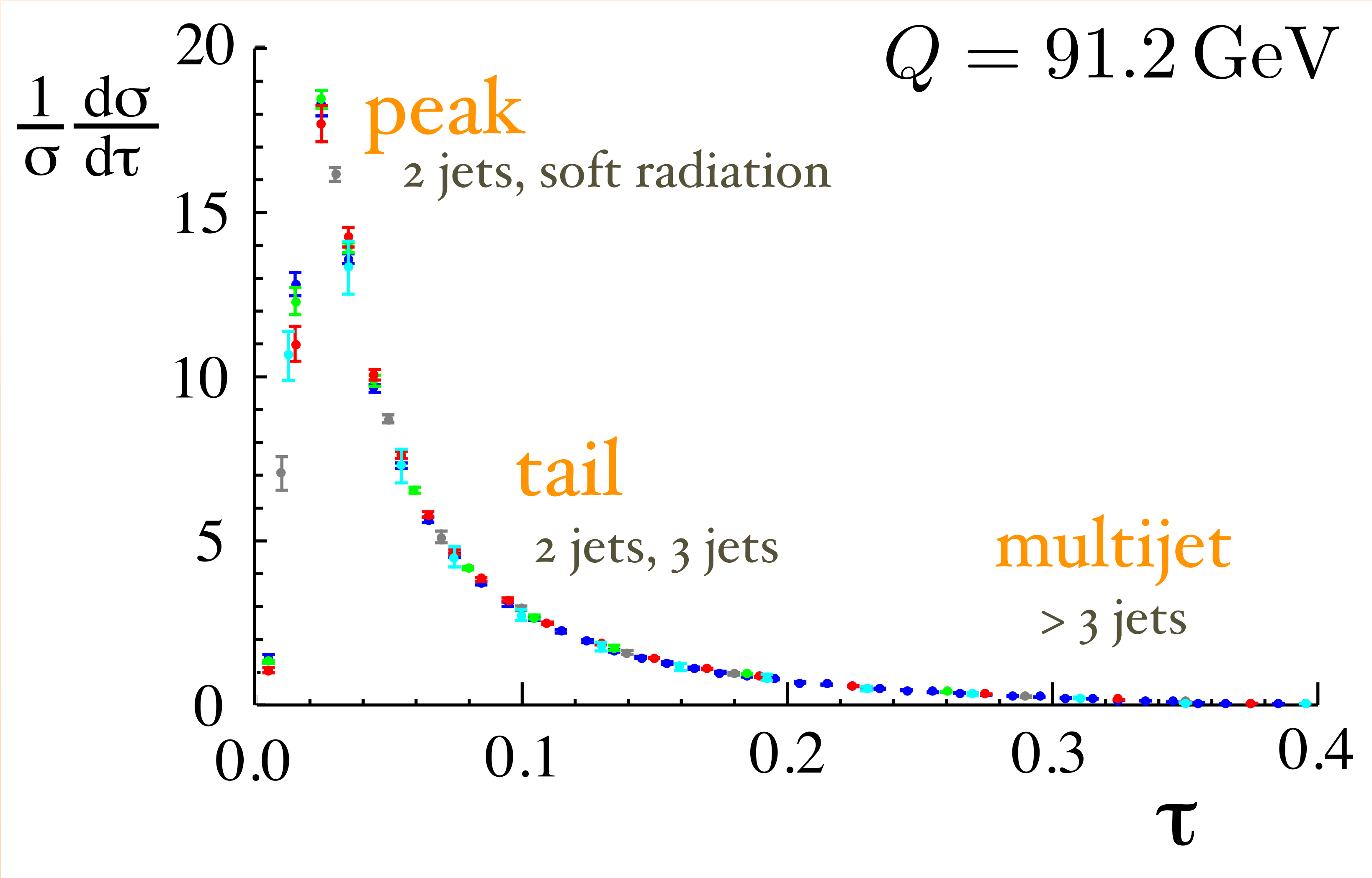
ALEPH, DELPHI, L3, OPAL, SLD

2 jets

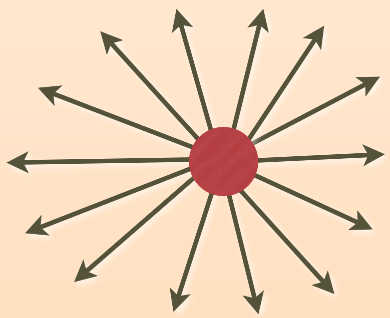


$T = 1$

$\tau = 0$



spherical event



$T = 1/2$

$\tau = 1/2$

Factorization theorem

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right)\left(\tau - \frac{k}{Q} \right) S_\tau^{mod}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right)$$

$$\frac{\Delta \alpha_s}{\alpha_s} \sim 0.5\%$$

$$\frac{d\hat{\sigma}_s}{d\tau} = \sum_n \alpha_s^n \delta(\tau) + \sum_{n,l} \alpha_s^n \left[\frac{\ln^l \tau}{\tau} \right]_+$$

$$= H(\mu_H) \times J(\mu_J) \otimes S(\mu_S)$$

Singular partonic for massless quarks QCD+QED final states

b mass corrections

nonperturbative soft function $,\left(\frac{\Omega_1}{Q\tau} \right)$

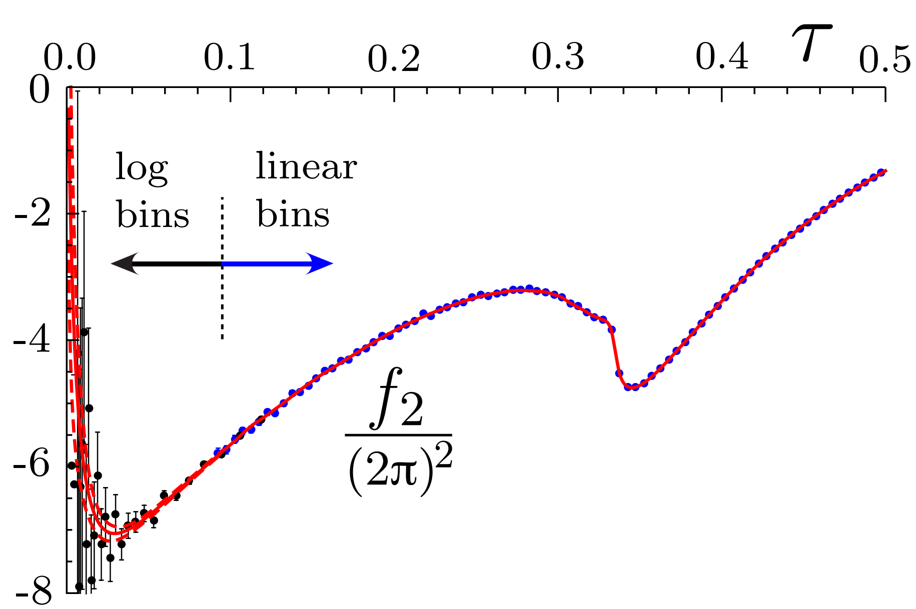
$$\frac{d\hat{\sigma}_{ns}}{d\tau} = \sum_{n,l} \alpha_s^n \ln^l \tau + \sum_n \alpha_s^n f_n(\tau)$$

Nonsingular partonic

(Fixed order) - (Singular) = Nonsingular

$\mathcal{O}(\alpha_s^2)$

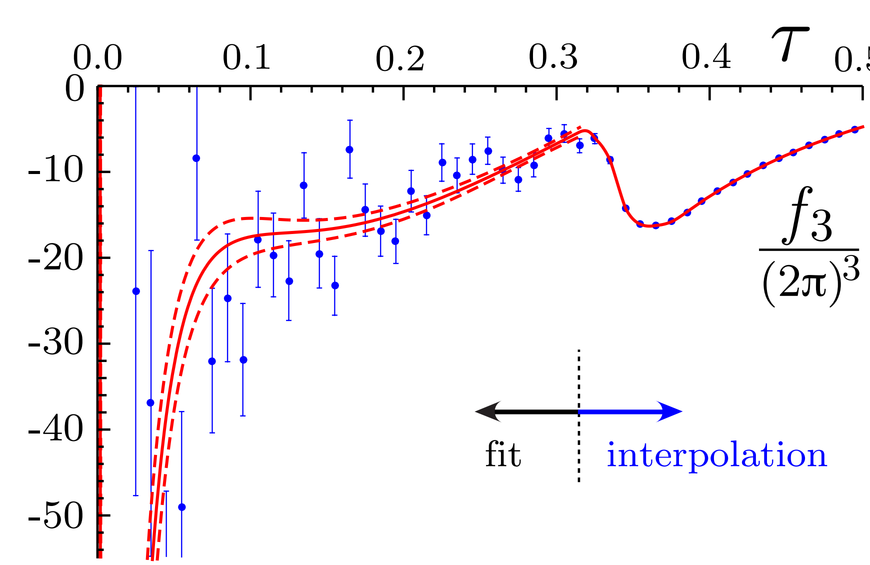
EVENT2



$\mathcal{O}(\alpha_s^3)$

EERAD3

Gehrmann et al.



Factorization theorem

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}_s}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right)\left(\tau - \frac{k}{Q} \right) S_\tau^{\text{mod}}(k - 2\bar{\Delta}) + O\left(\sigma_0 \frac{\alpha_s \Lambda_{QCD}}{Q} \right)$$

$$\frac{\Delta\alpha_s}{\alpha_s} \sim 0.5\%$$

b mass corrections

nonperturbative soft function $, \left(\frac{\Omega_1}{Q\tau}\right)$

$$\frac{d\hat{\sigma}_s}{d\tau} = \sum_n \alpha_s^n \delta(\tau) + \sum_{n,l} \alpha_s^n \left[\frac{\ln^l \tau}{\tau} \right]_+$$
$$= H(\mu_H) \times J(\mu_J) \otimes S(\mu_S)$$

Singular partonic for massless quarks QCD+QED final states

$$\frac{d\hat{\sigma}_{ns}}{d\tau} = \sum_{n,l} \alpha_s^n \ln^l \tau + \sum_n \alpha_s^n f_n(\tau)$$

Nonsingular partonic

Resummation for singular partonic

$$\ln \frac{d\sigma}{dy} = (\alpha_s \ln)^k \ln + (\alpha_s \ln)^k + \alpha_s(\alpha_s \ln)^k + \alpha_s^2(\alpha_s \ln)^k + \ldots$$

$y = \mathrm{FT}(\tau)$ LL NLL NNLL $\mathrm{N^3LL}$

Sum Large Logarithms

Thrust Factorization Theorem:

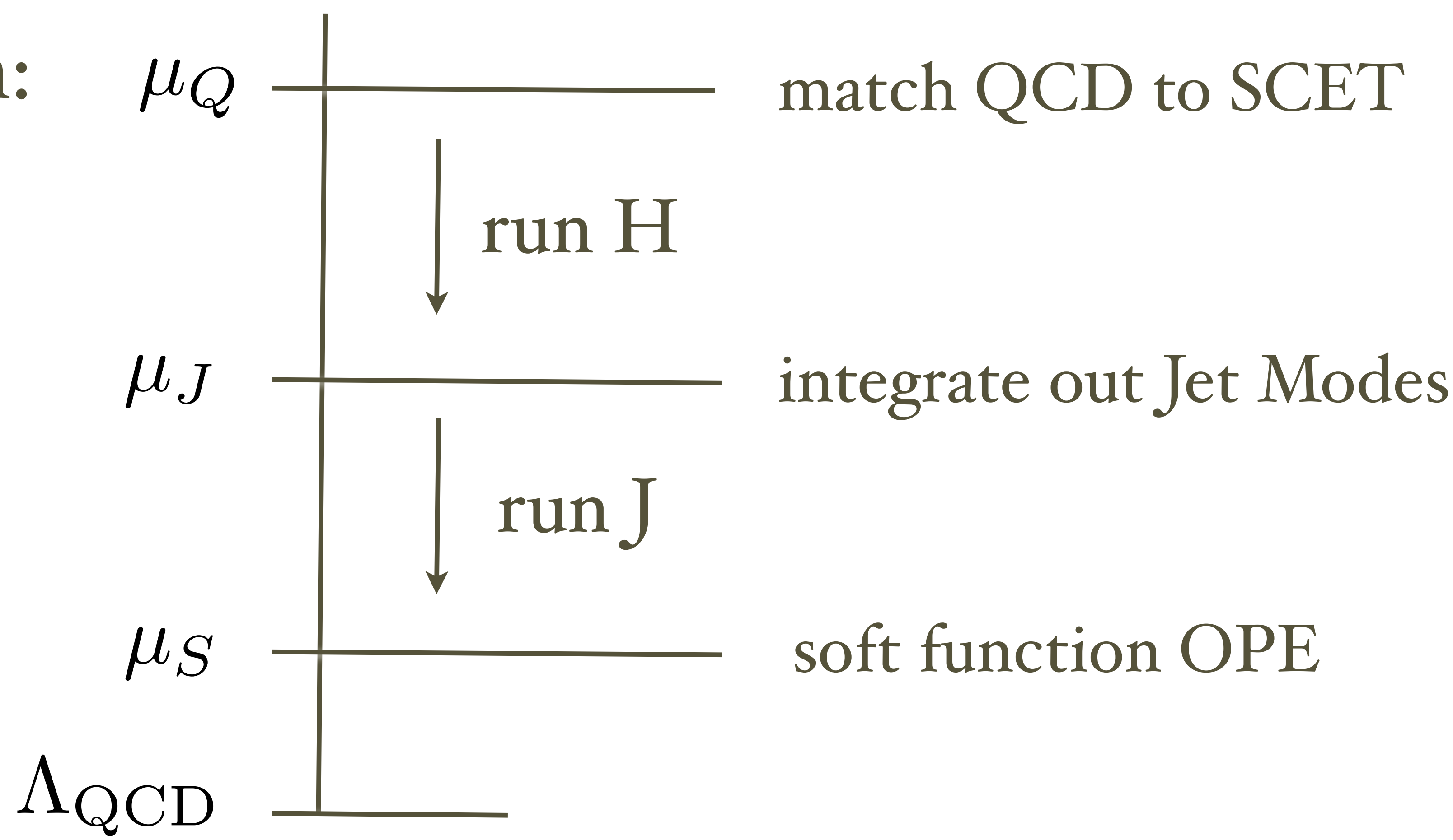
$$\frac{d\sigma}{d\tau} = \sigma_0 H(Q,\mu)\, Q \int d\ell\, J_T\Big(Q^2\tau - Q\ell, \mu\Big) S_T(\ell,\mu)$$

$$p^2 \sim Q^2 \qquad\qquad p^2 \sim Q^2\tau \qquad\qquad p^2 \sim Q^2\tau^2$$

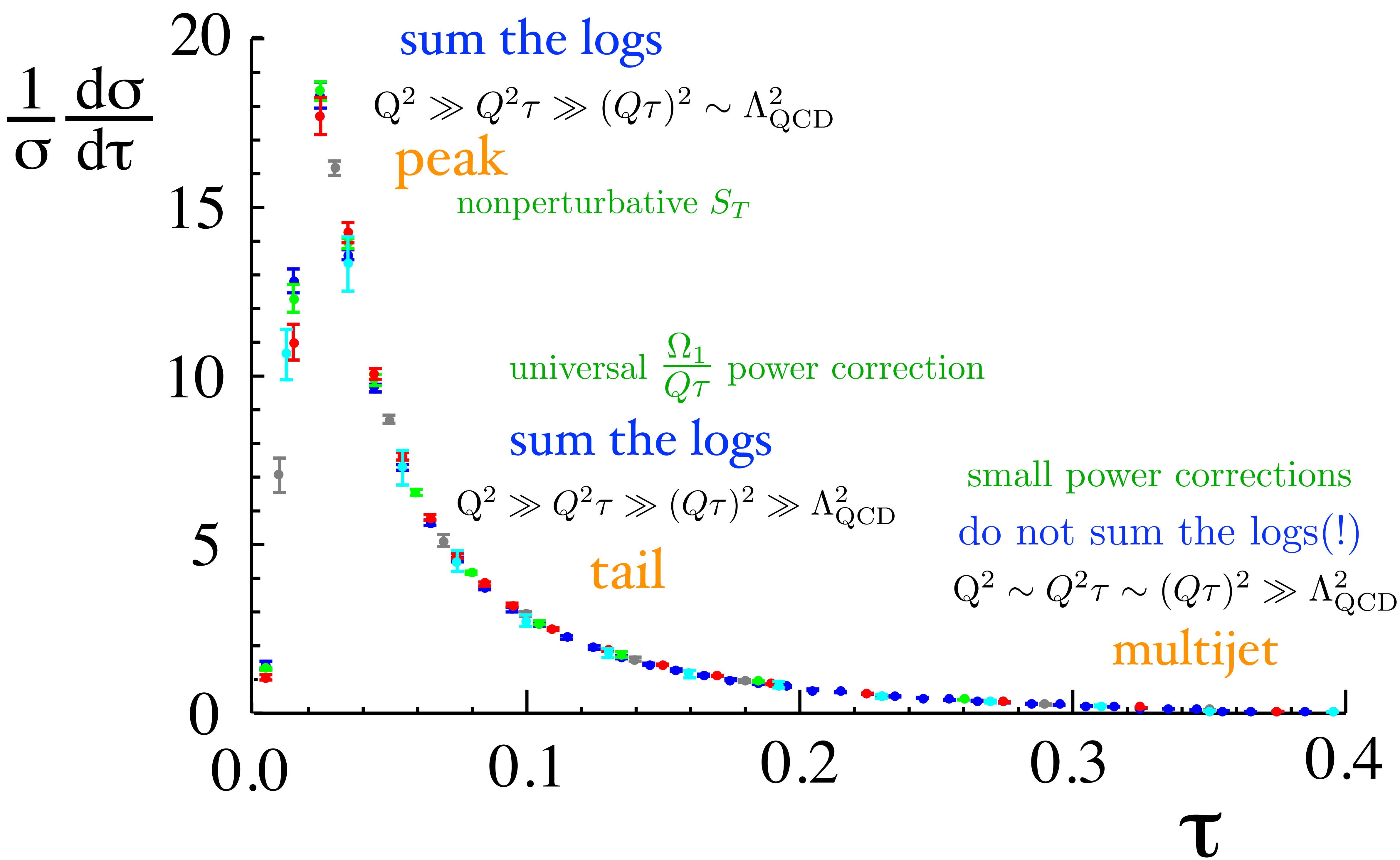
$$\sim \mu_Q^2 \qquad\qquad \sim \mu_J^2 \qquad\qquad \sim \mu_S^2$$

To minimize large logs we want to evaluate these functions at different scales

Match & Run:



Our Three Regions:



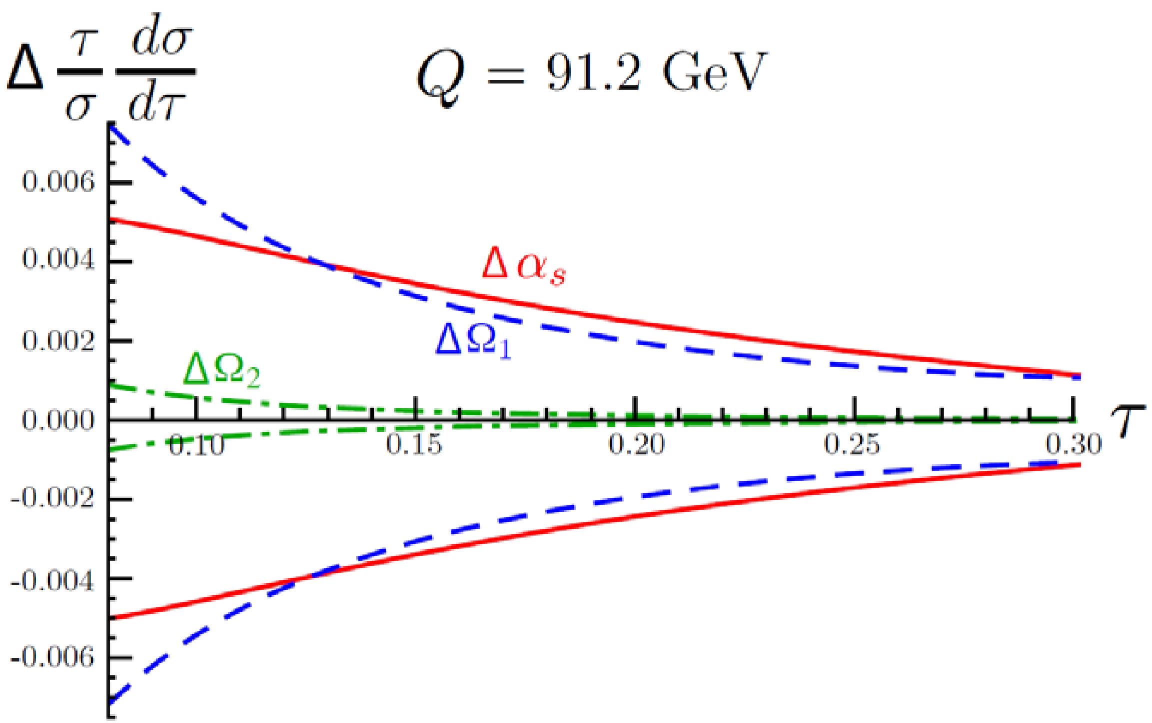
Factorization theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}=\int \mathrm{d}k\left(\frac{\mathrm{d}\hat{\sigma}_{\mathrm{s}}}{\mathrm{d}\tau}+\frac{\mathrm{d}\hat{\sigma}_{\mathrm{ns}}}{\mathrm{d}\tau}+\frac{\mathrm{d}\hat{\sigma}_{b}}{\mathrm{d}\tau}\right)\left(\tau-\frac{k}{Q}\right)S_{\tau}^{\mathrm{mod}}(k-2\bar{\Delta})+O\left(\sigma_0\frac{\alpha_s\Lambda_{\mathrm{QCD}}}{Q}\right)$$

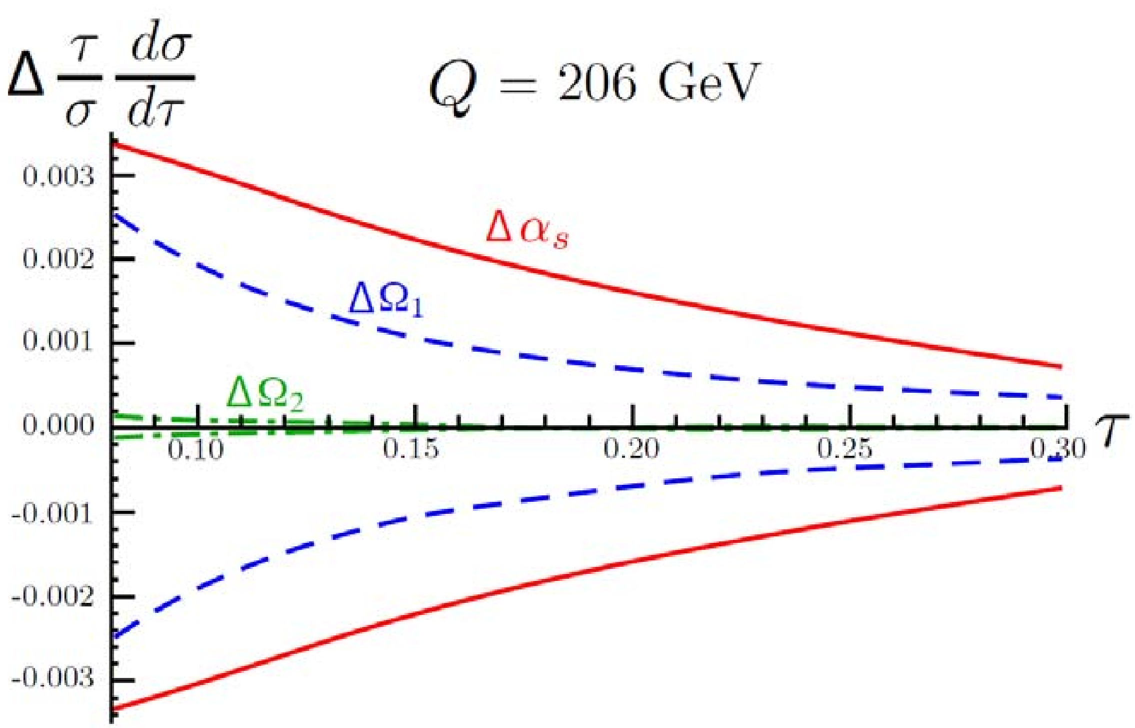
- $O(\alpha_s^3)$ fixed order (nonsingular). Event2 $O(\alpha_s^2)$ and EERAD3 $O(\alpha_s^3)$.
- $O(\alpha_s^3)$ matrix elements. Axial singlet anomaly. Full hard function at 3 loops.
- Resummation at N^3LL. Effective field theory (SCET).
- Correct theory in peak, tail and multijet (profile functions).
- Field theory matrix elements for power corrections.
- Removal of u=1/2 renormalon in leading power correction/soft function.
- QED effects in Sudadok & FSR @ NNLL $O(\alpha_s^2)$ with $\alpha\sim\alpha_s^2$.
- bottom mass corrections with factorization theorem.
- Computation of bin cumulants in a meaningful way.

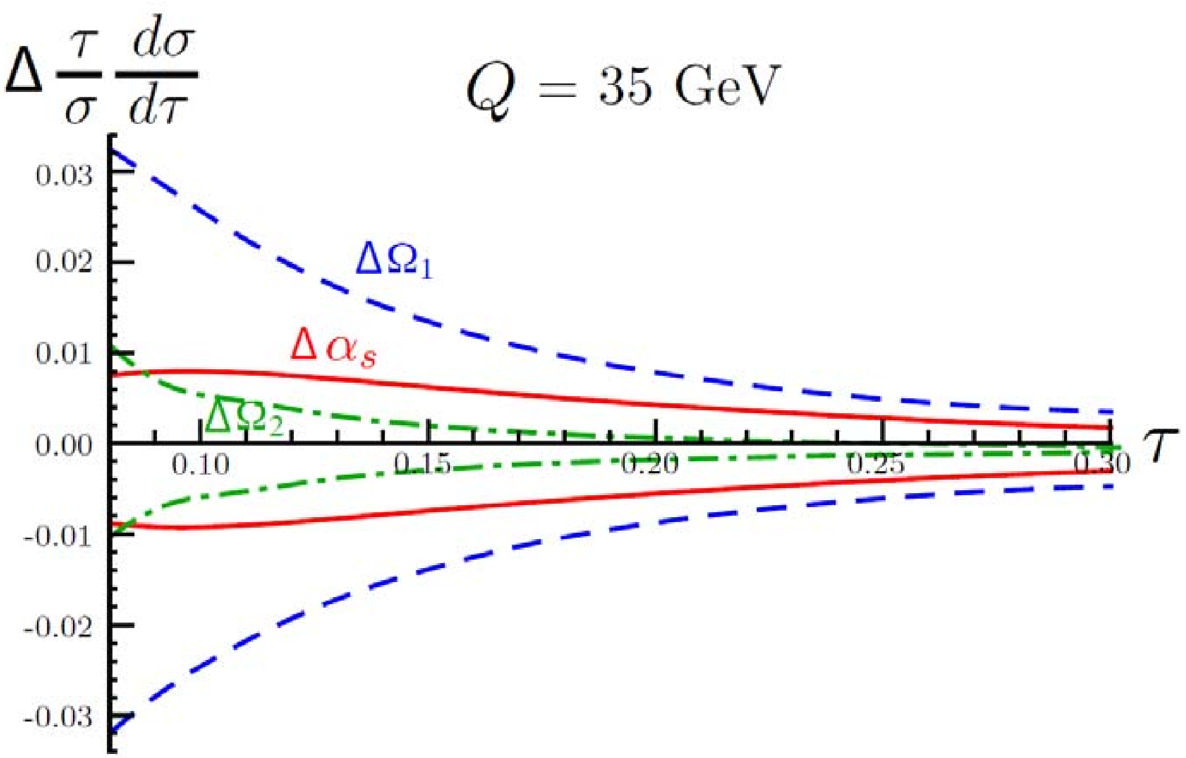
Why a global fit (many Q's)

We fit for Ω_1 & $\alpha_s(m_Z)$ simultaneously. Strong degeneracy lifted by many Q's.

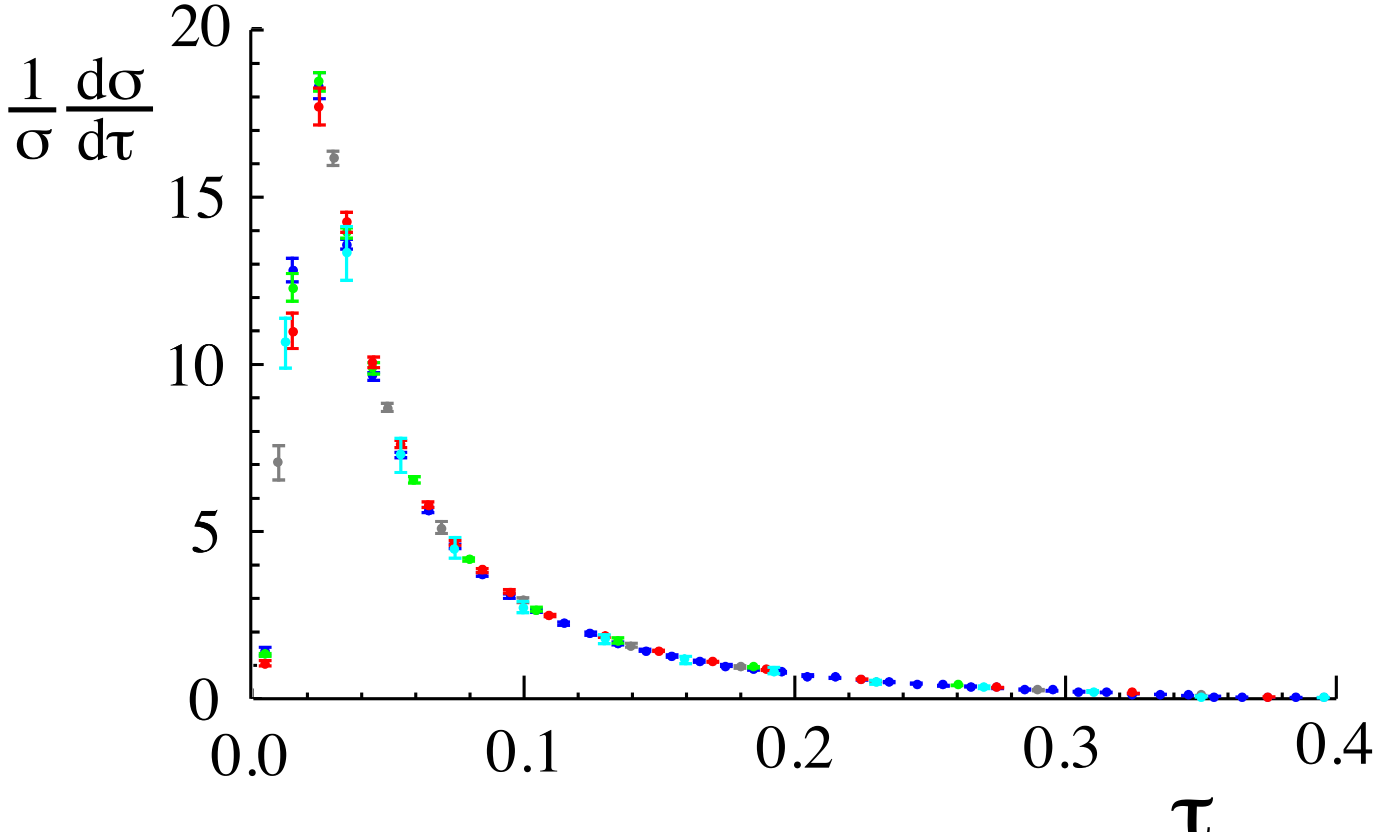


Power correction needed with 20% accuracy to get α_s at the 1% level





	Experiment data	Values of Q $e^+e^- \xrightarrow{Q}$ jets
LEP	ALEPH	{91.2, 133.0, 161.0, 172.0, 183.0, 189.0, 200.0, 206.0}
LEP	DELPHI	{45.0, 66.0, 76.0, 89.5, 91.2, 93.0, 133.0, 161.0, 172.0, 183.0, 189.0, 192.0, 196.0, 200.0, 202.0, 205.0, 207.0}
LEP	OPAL	{91.0, 133.0, 177.0, 197.0}
LEP	L3	{41.4, 55.3, 65.4, 75.7, 82.3, 85.1, 91.2, 130.1, 136.1, 161.3, 172.3, 182.8, 188.6, 194.4, 200.0, 206.2}
SLAC	SLD	{91.2}
DESY	TASSO	{14.0, 22.0, 35.0, 44.0}
DESY	JADE	{35.0, 44.0}
KEK	AMY	{55.2}

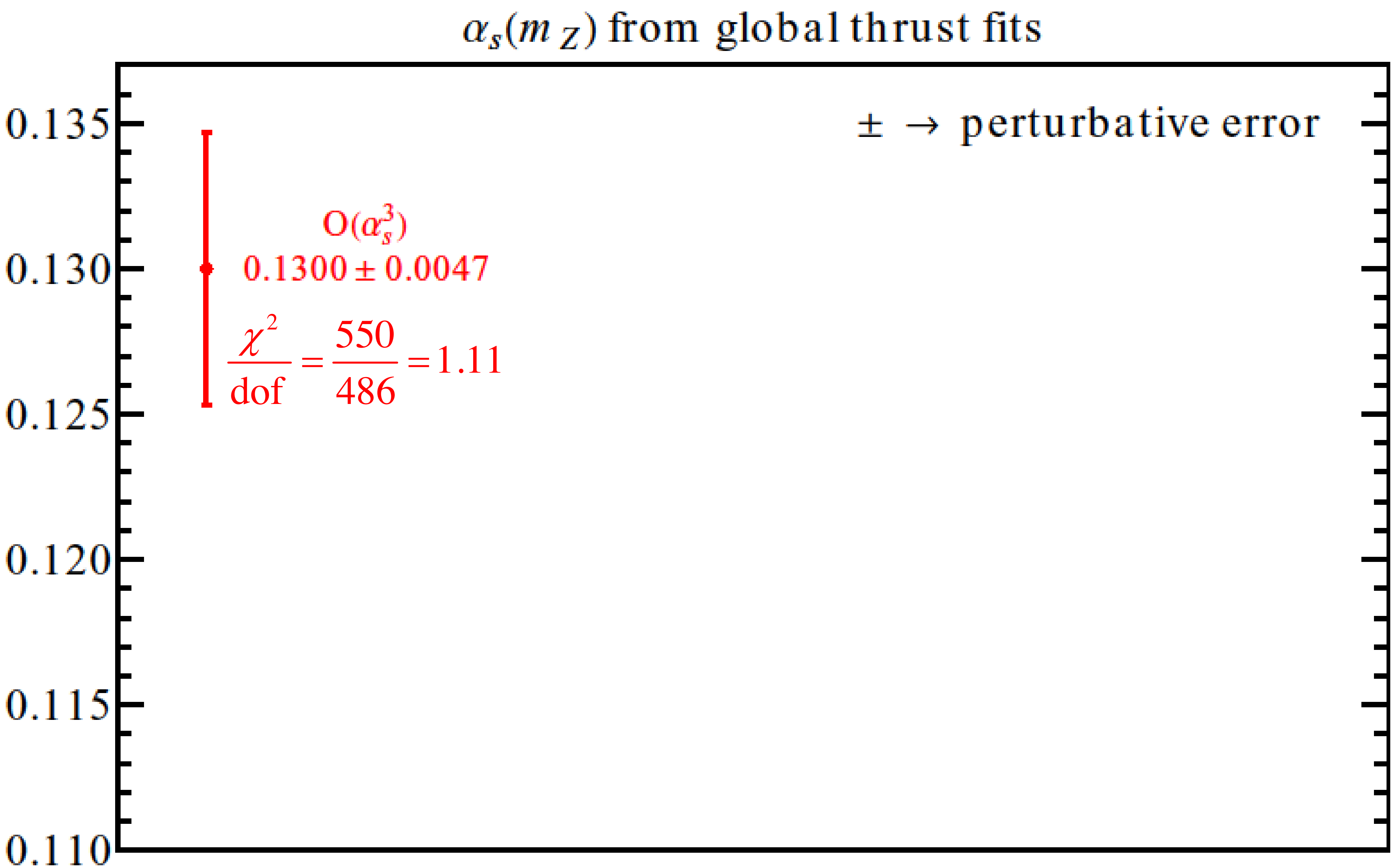


"standard" data set:

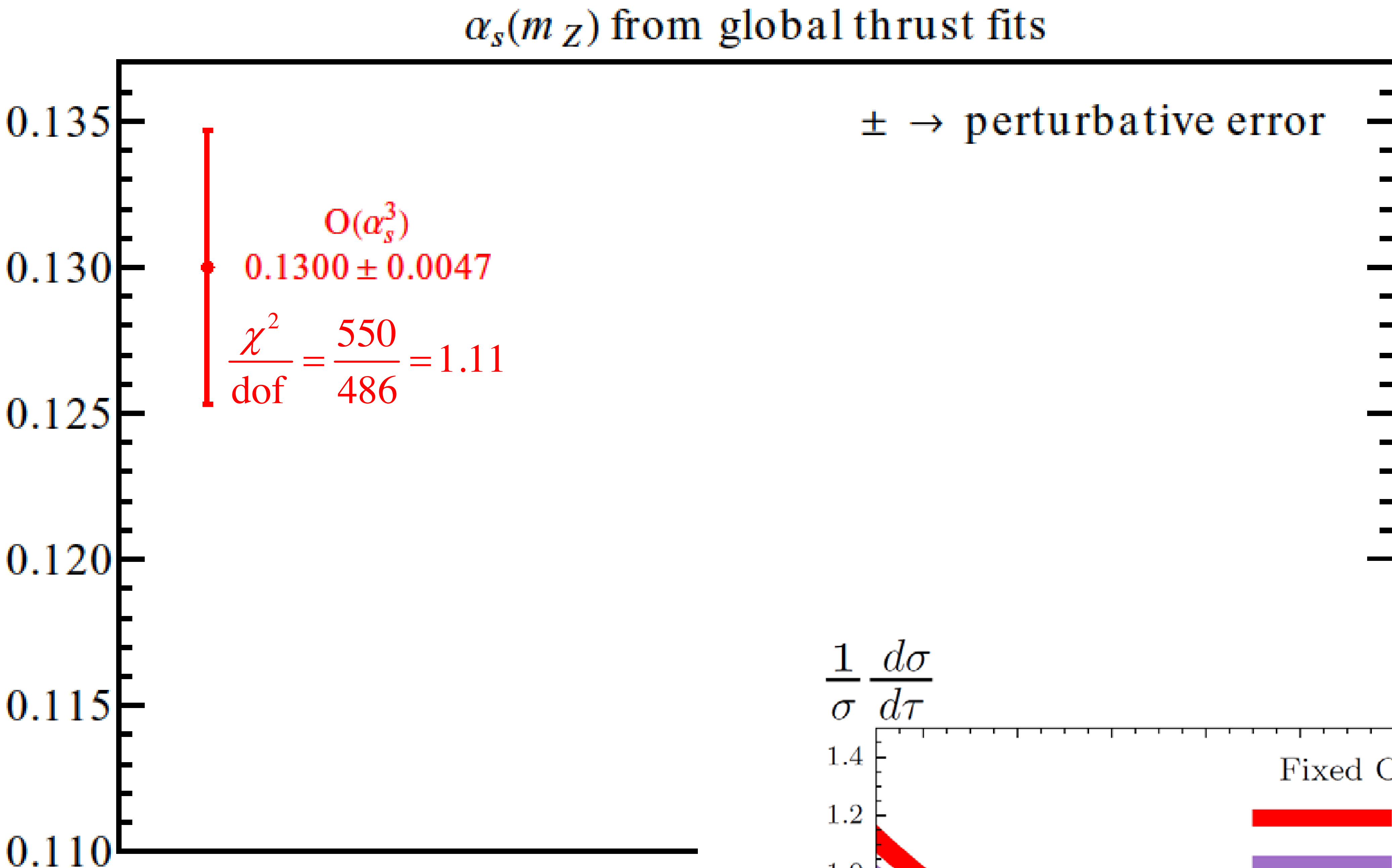
$Q \geq 35\,\mathrm{GeV}$

$\frac{6\,\mathrm{GeV}}{Q} \leq \tau \leq 0.33$

487 bins



- Pure Fixed order



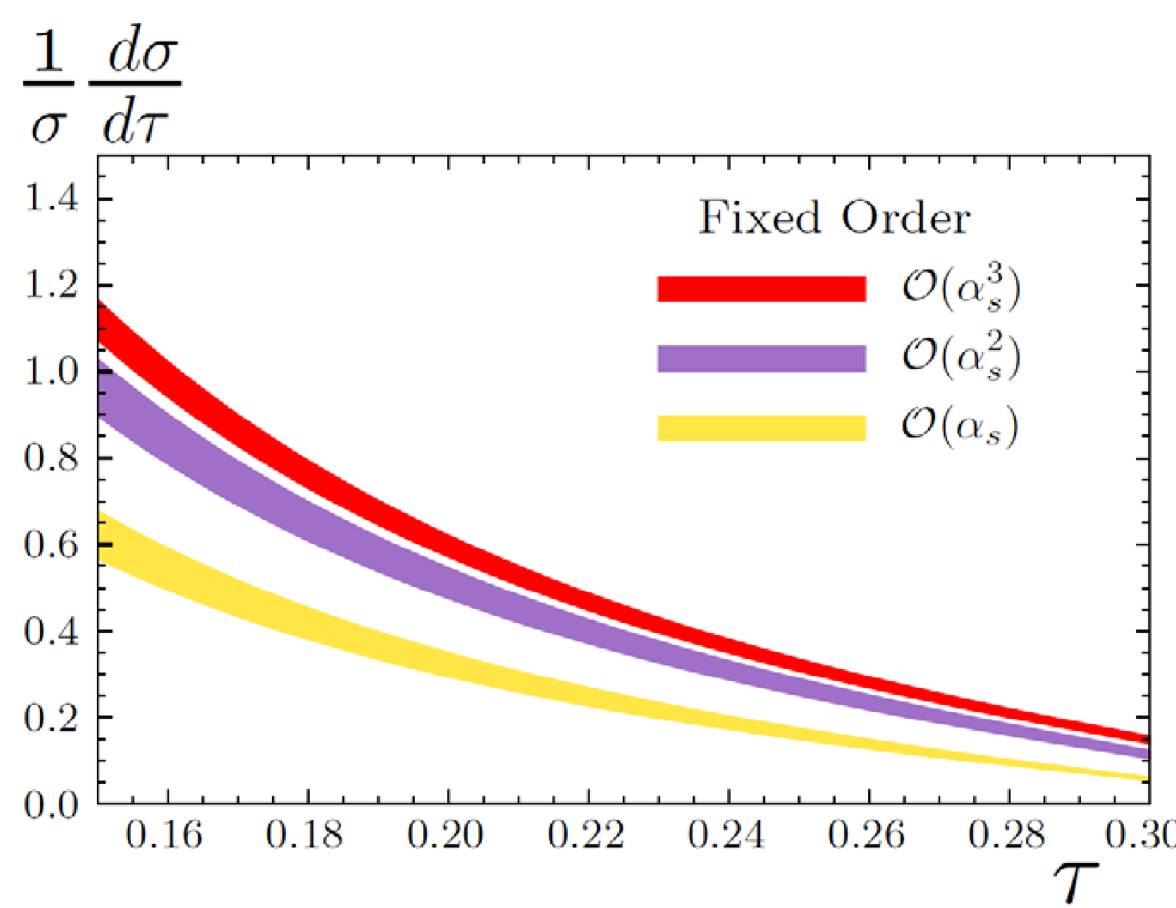
- Pure Fixed order

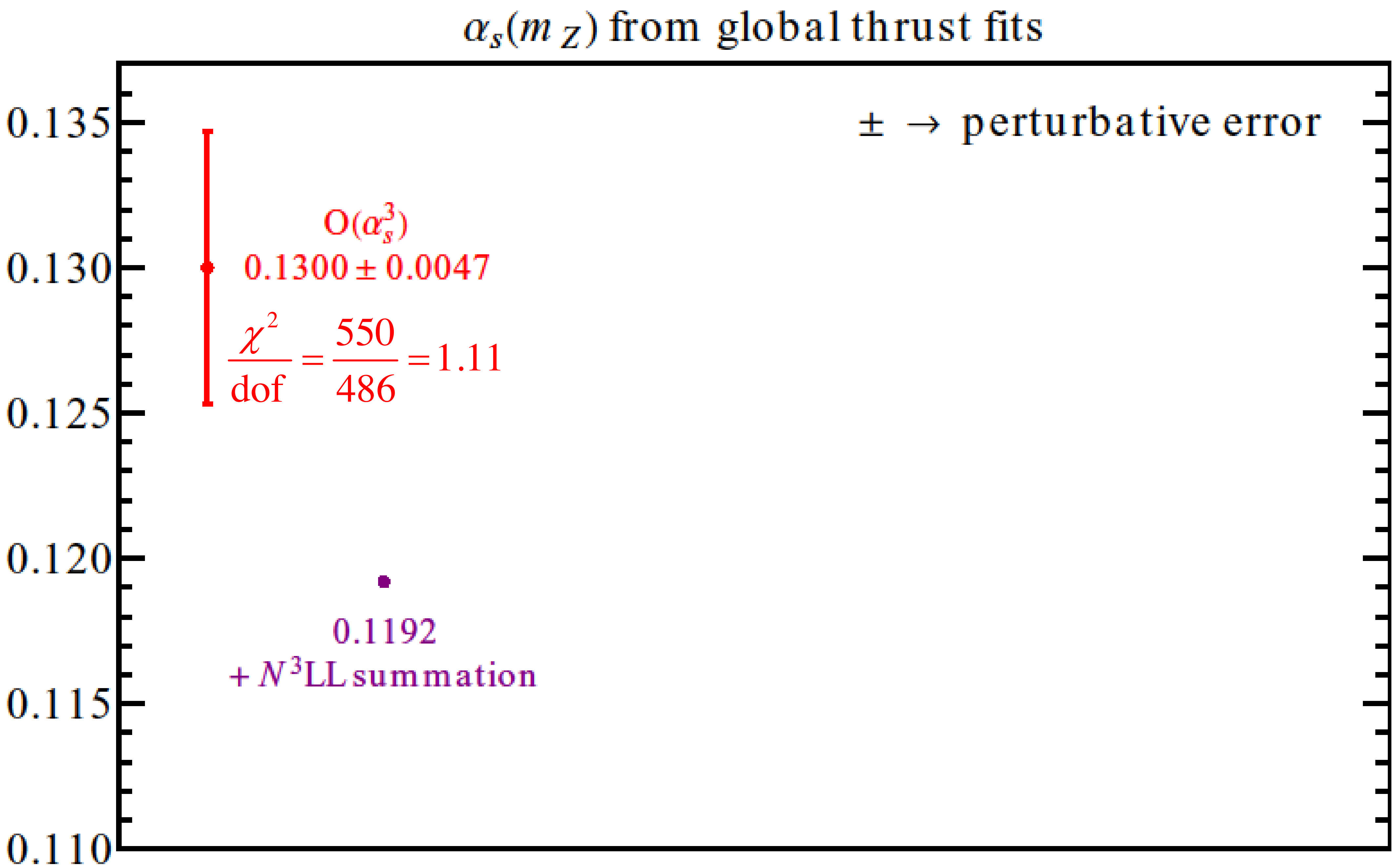
ALEPH Q=m_Z thrust

$0.1274 \pm (0.0042)_{\text{pert}}$

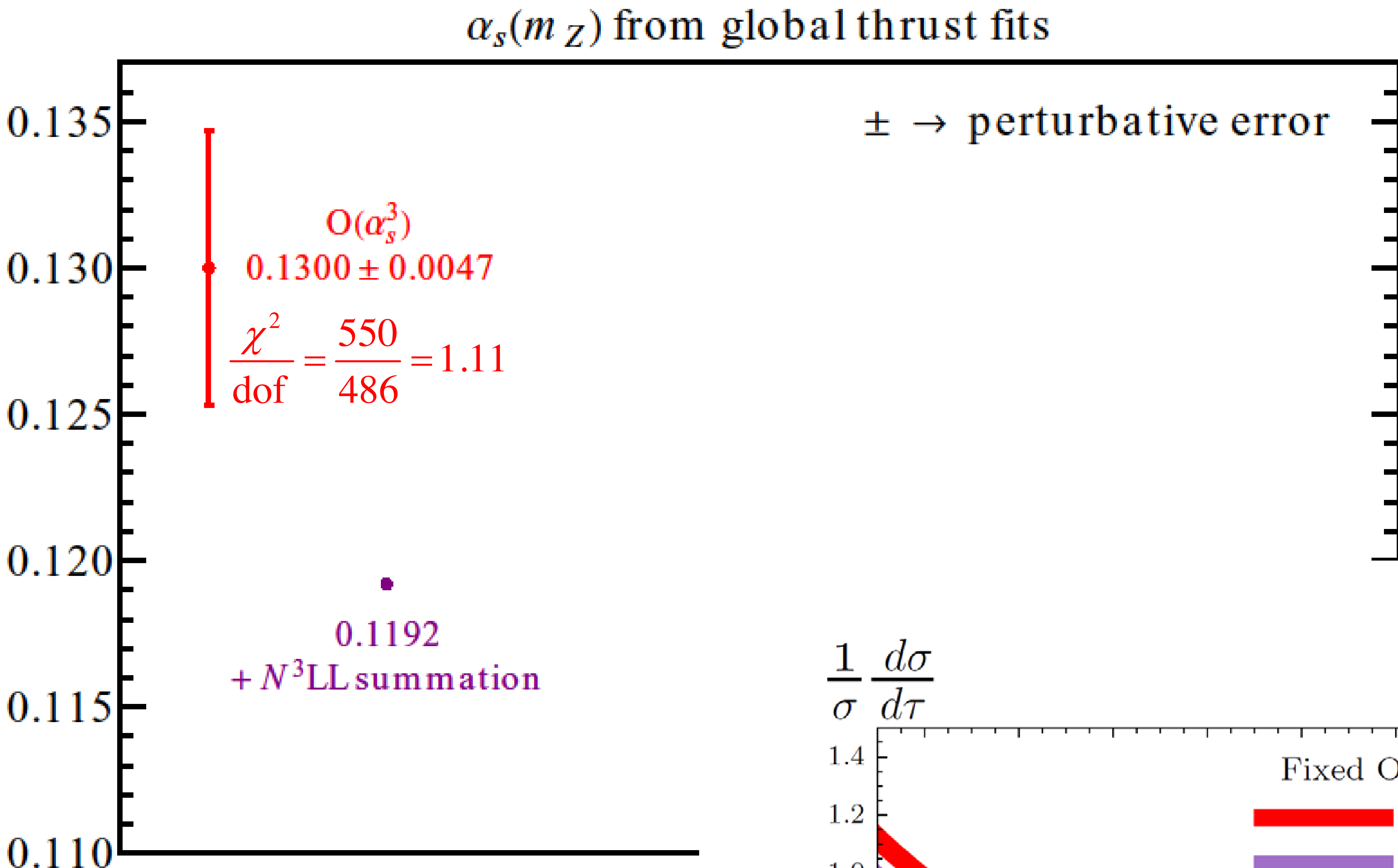
Dissertory et al '07

ALEPH Q's all event shapes $0.1240 \pm (0.0029)_{\text{pert}}$

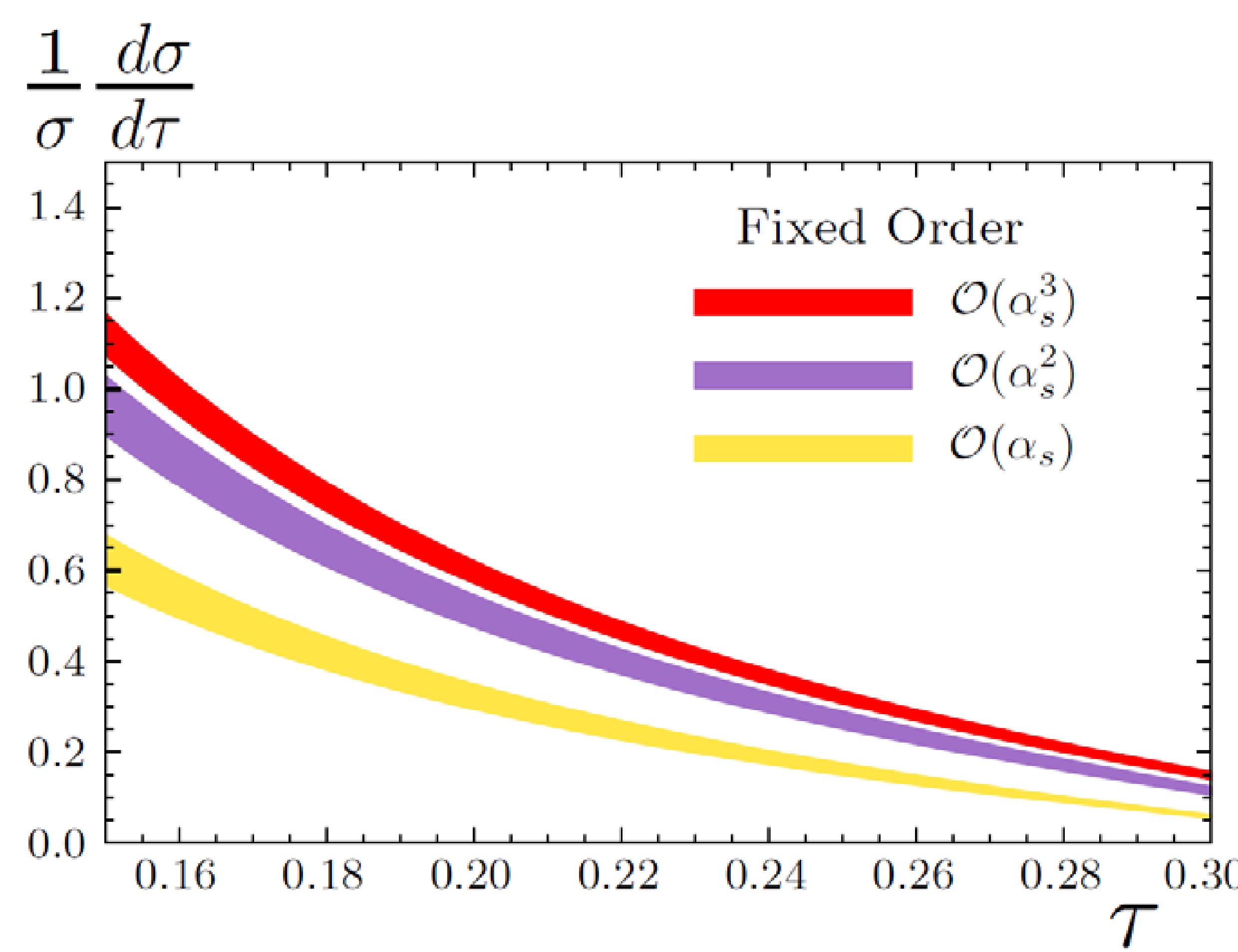


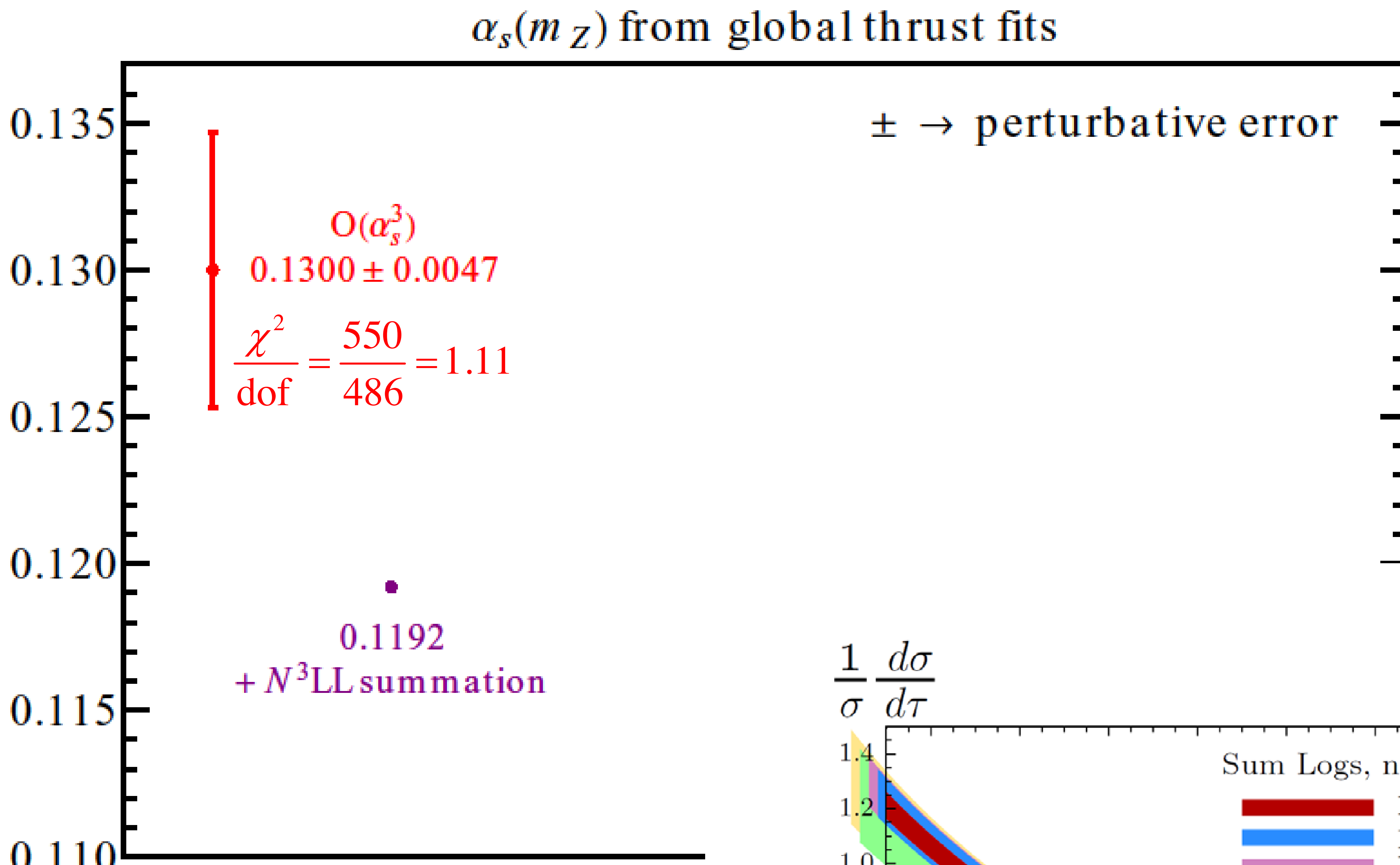


- Resummation at N^3LL



• Resummation at N^{3}LL



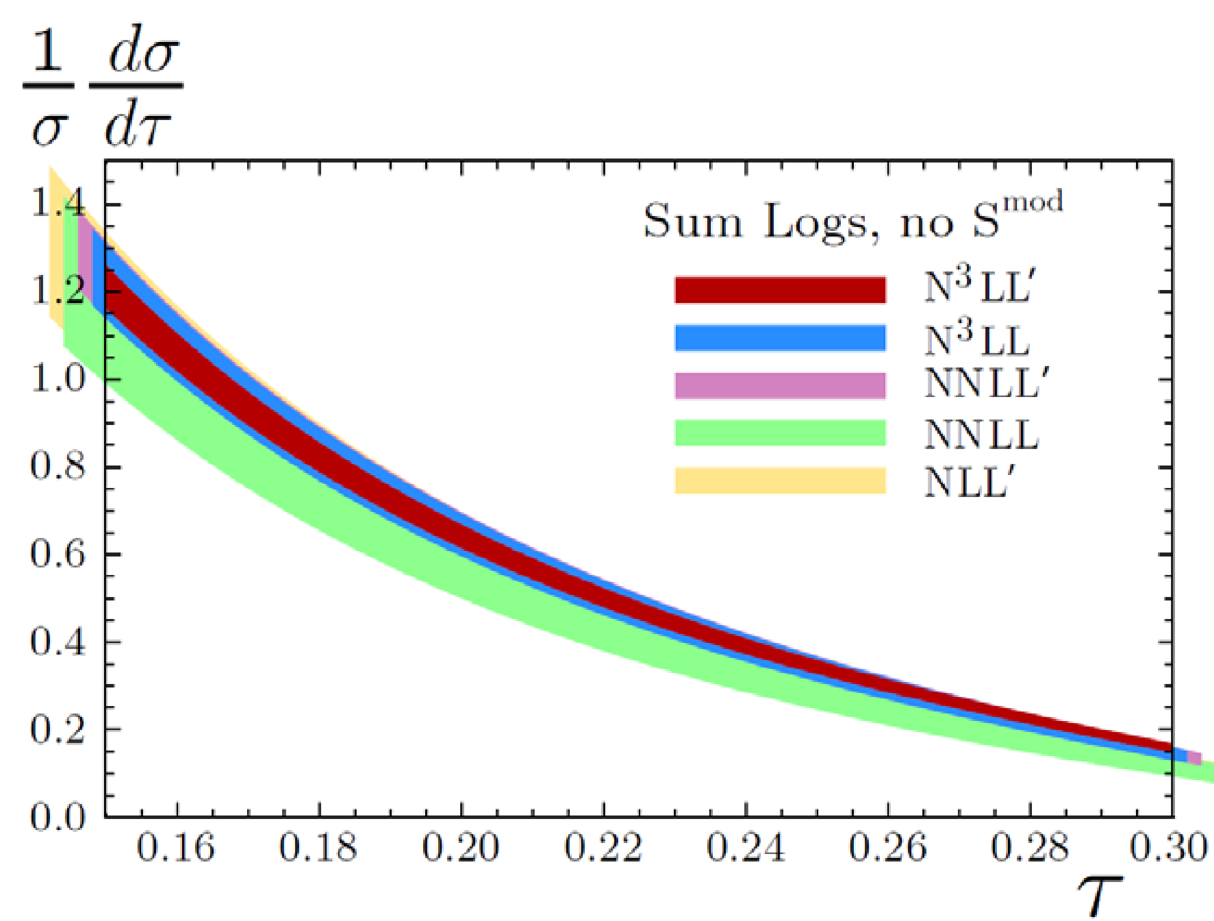


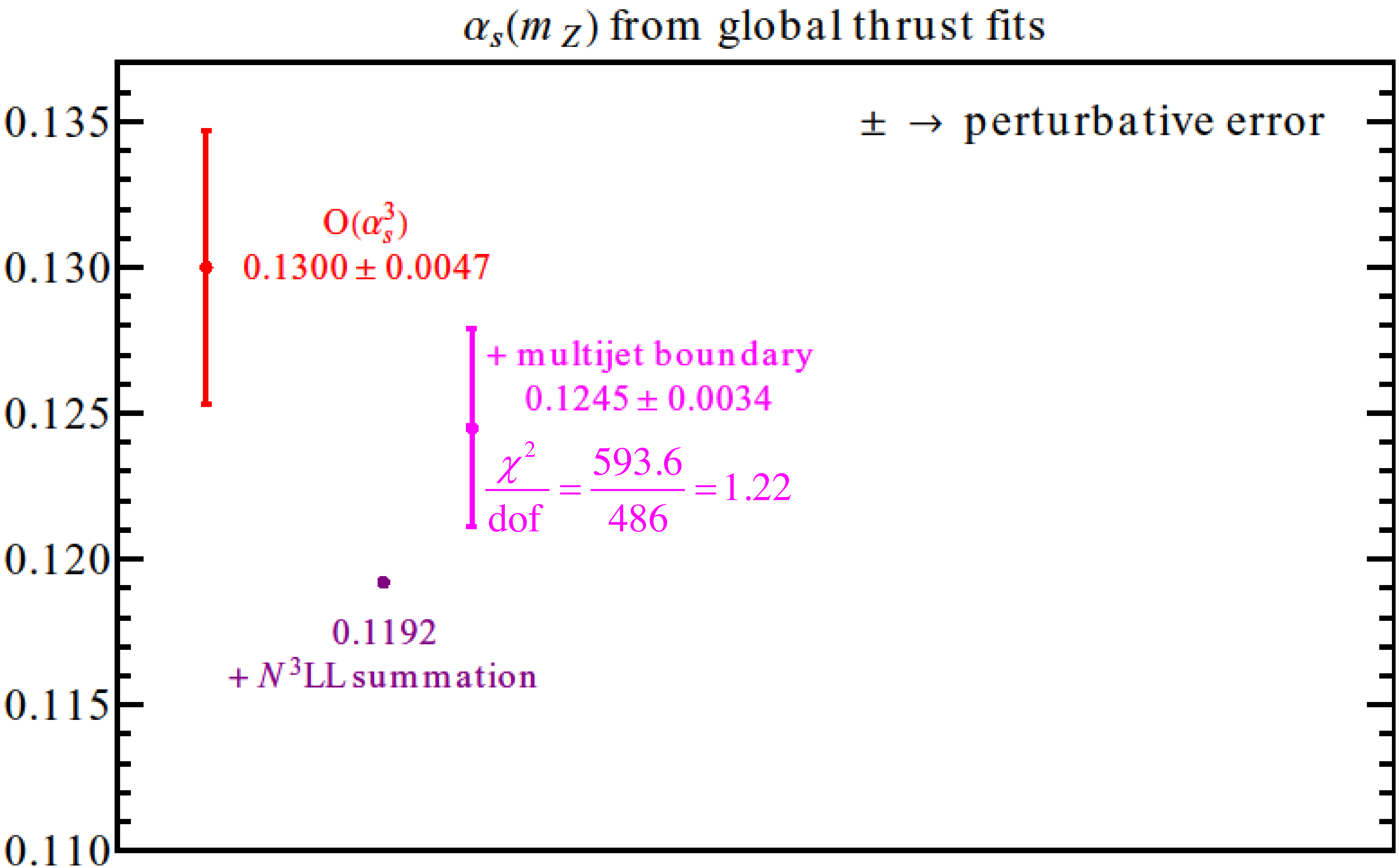
- Resummation at N^3LL

Fit to ALEPH and OPAL

$0.1172 \pm (0.0012)_{\text{pert}}$

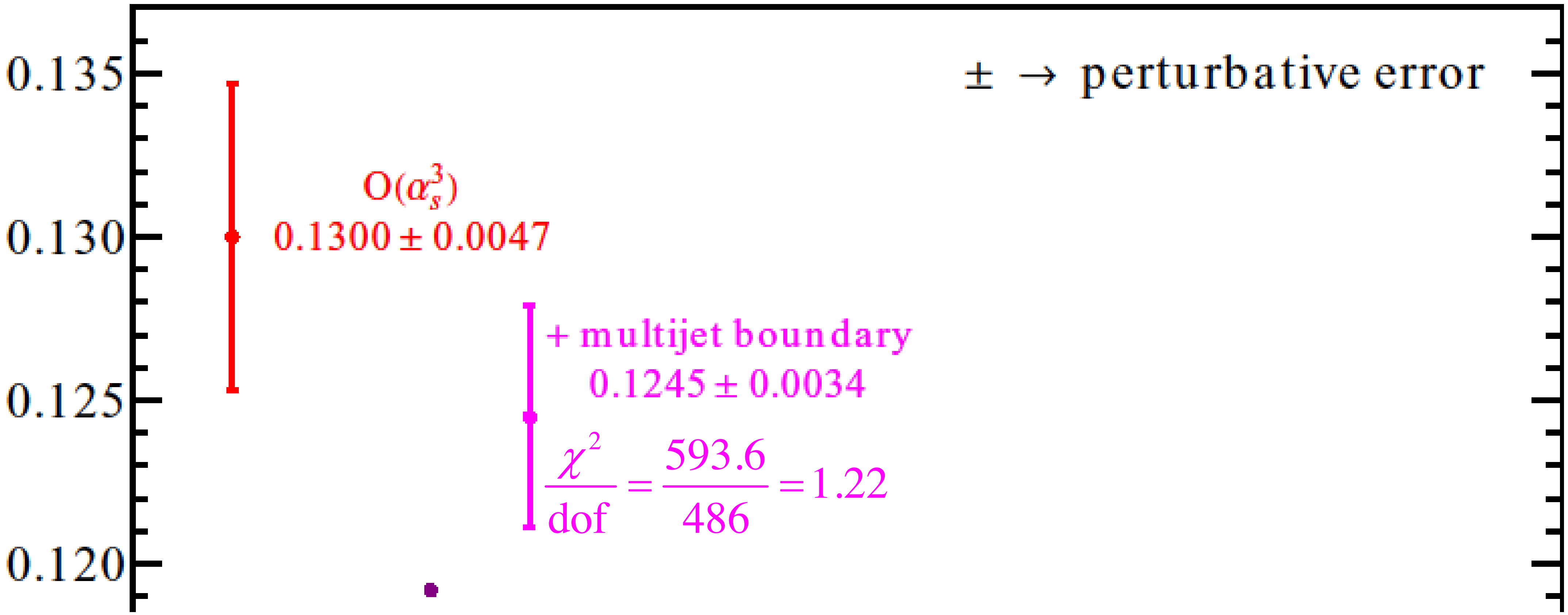
Becher & Schwartz '08



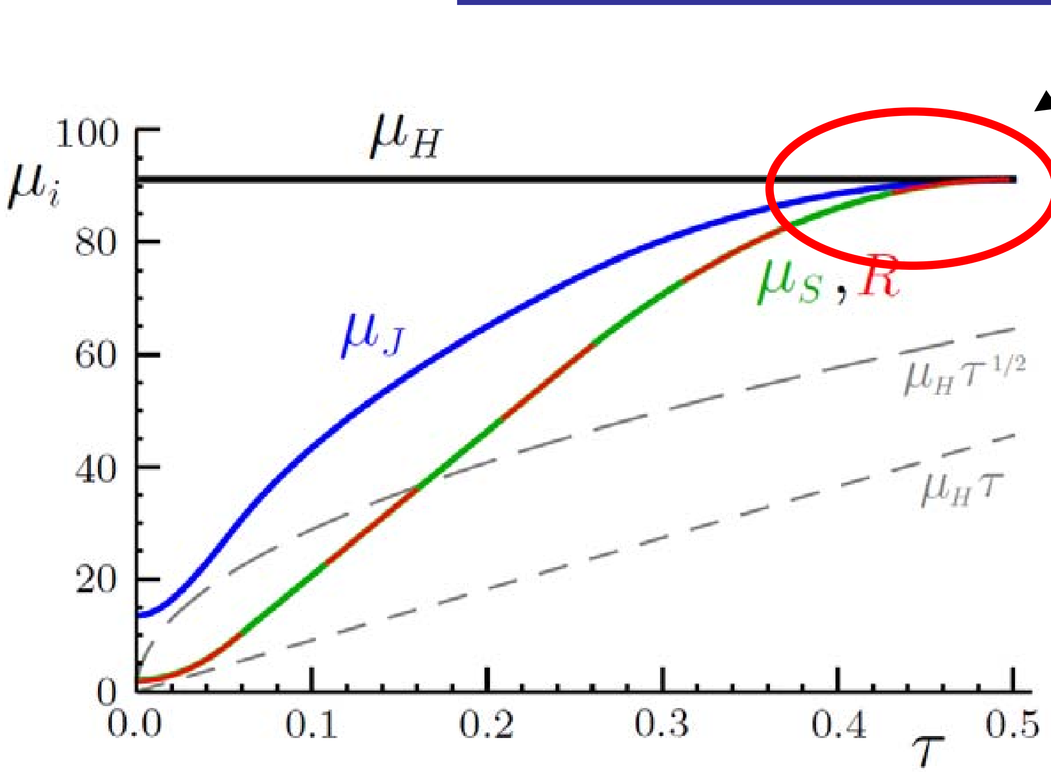


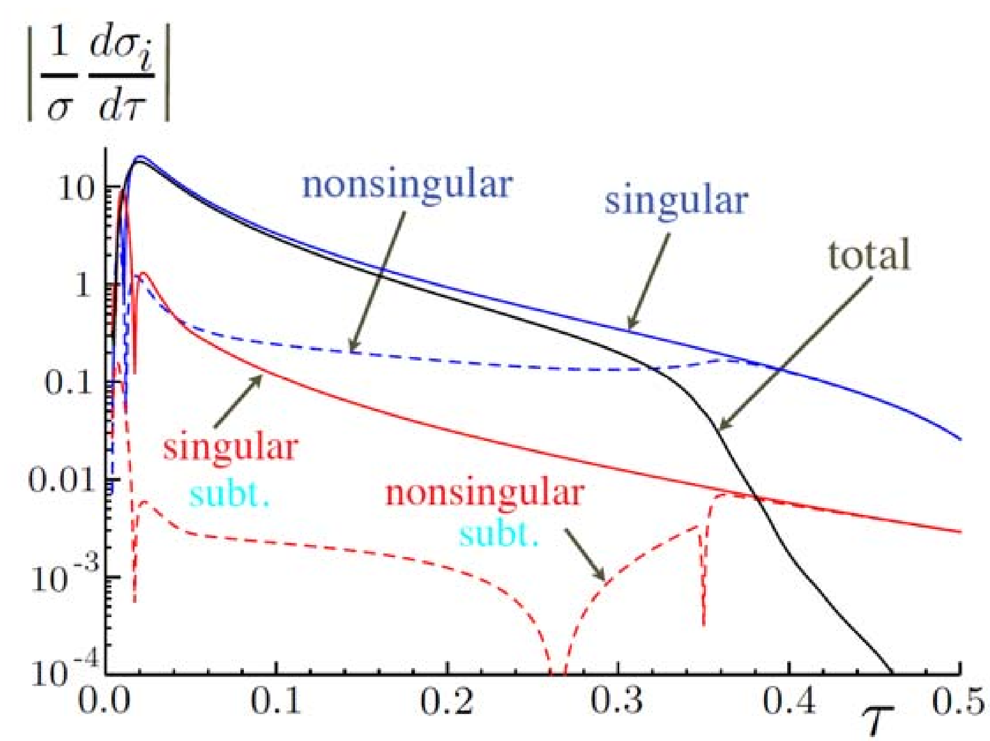
- Resummation at N^3LL
- Multijet boundary condition
- No power corrections
- No renormalon subtraction

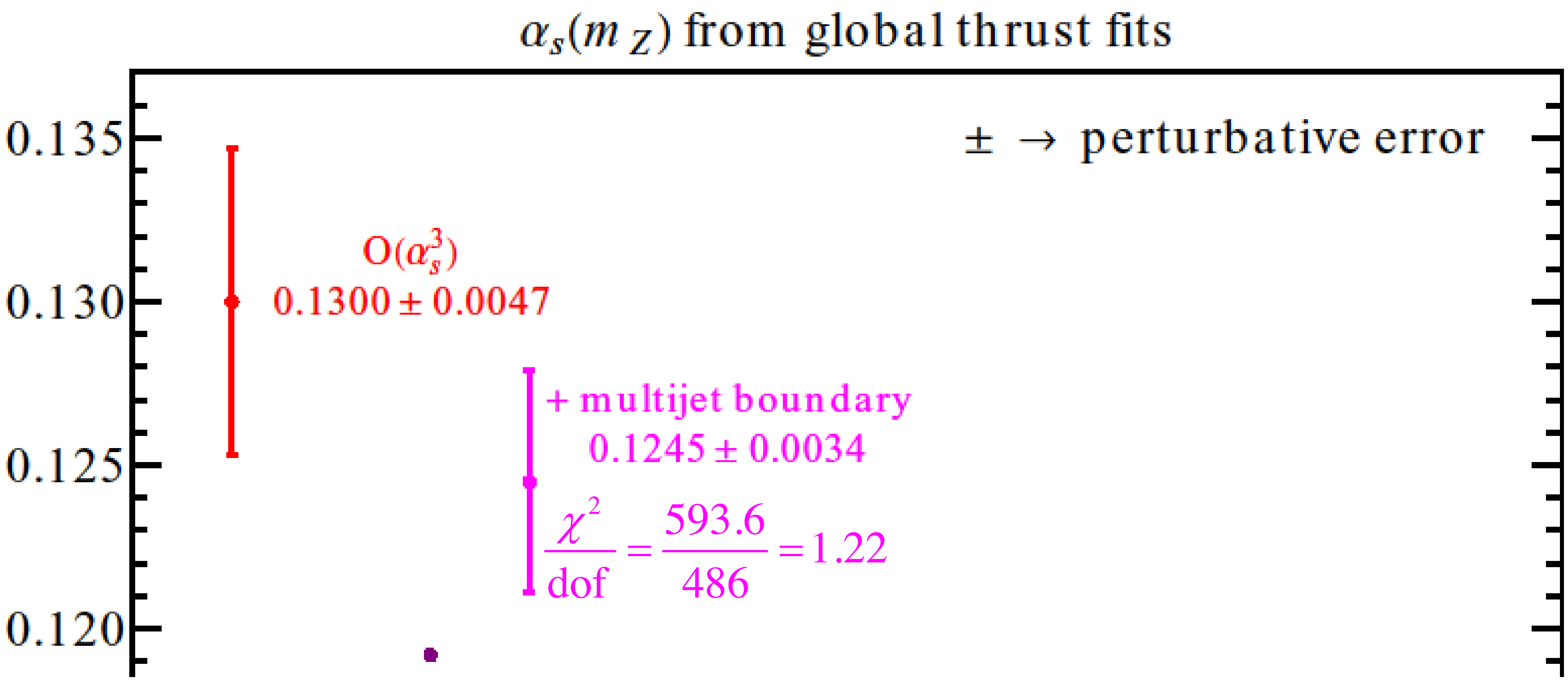
$\alpha_s(m_Z)$ from global thrust fits



We must turn off the resummation in the multijet region







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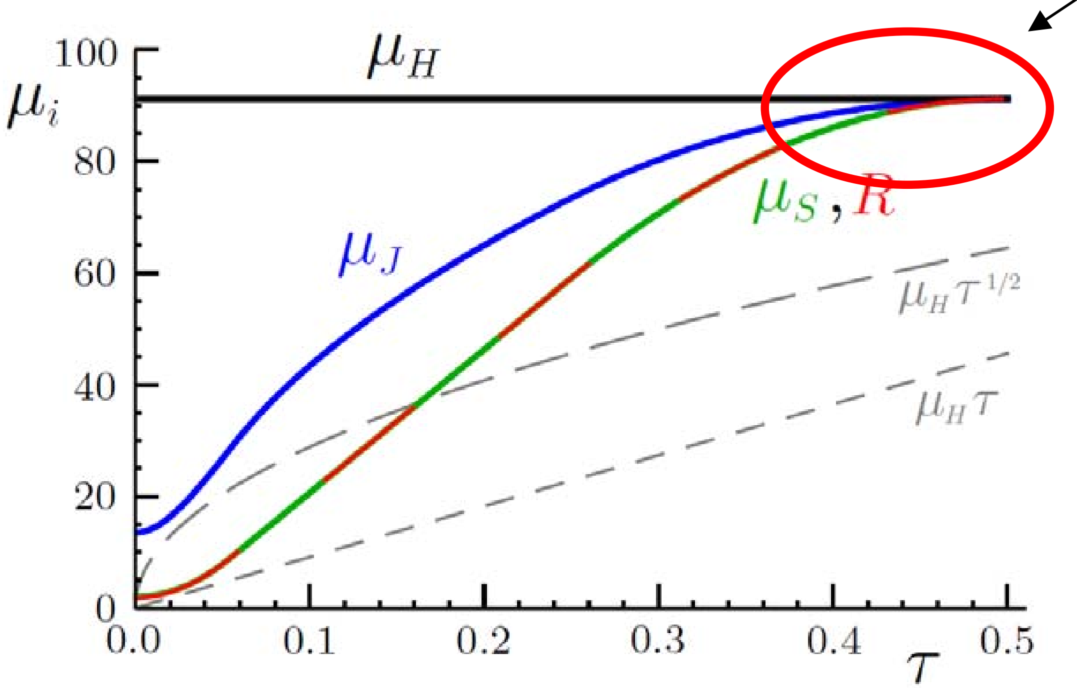
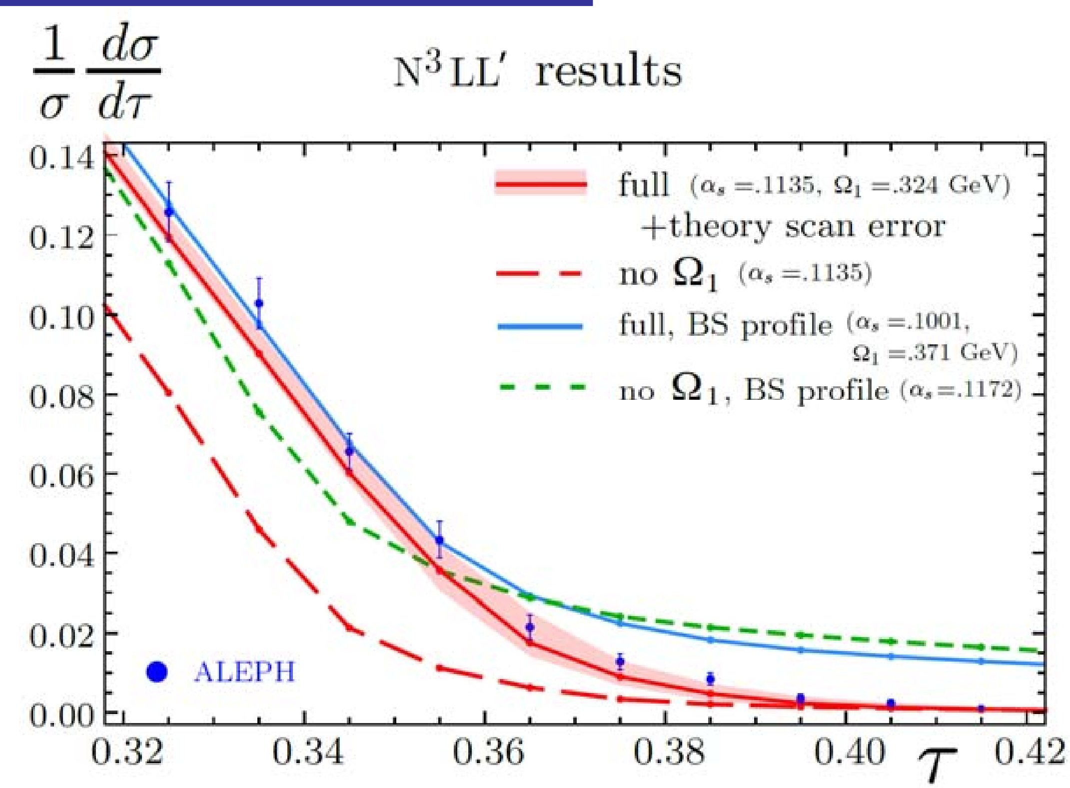
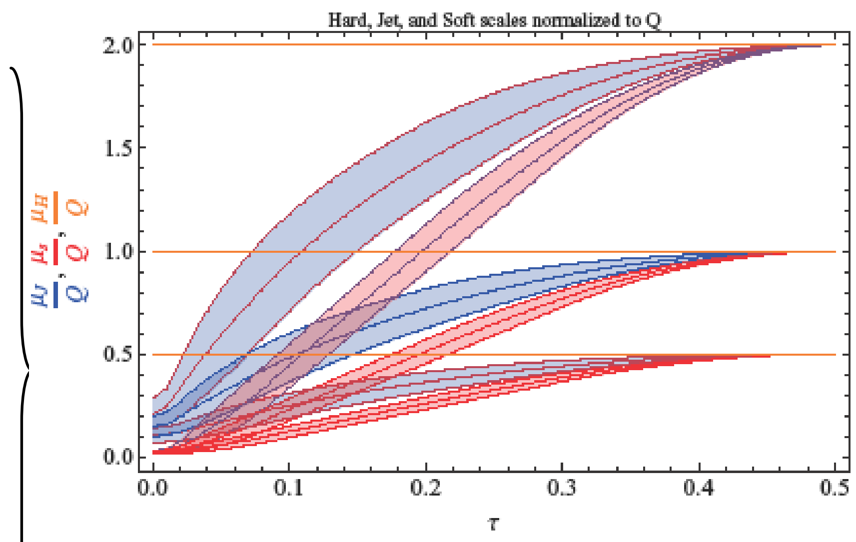




Estimate of perturbative uncertainties

parameter	default value	range of values
μ_0	$2\,\mathrm{GeV}$	1.5 to 2.5 GeV
n_1	5	2 to 8
t_2	0.25	0.20 to 0.30
e_J	0	-1,0,1
e_H	1	0.5 to 2.0
n_s	0	-1,0,1
s_2	-39.1	-36.6 to -41.6
Γ_3^{cusp}	1553.06	-1553.06 to $+4569.18$
j_3	0	-3000 to $+3000$
s_3	0	-500 to $+500$
ϵ_2	0	-1,0,1
ϵ_3	0	-1,0,1

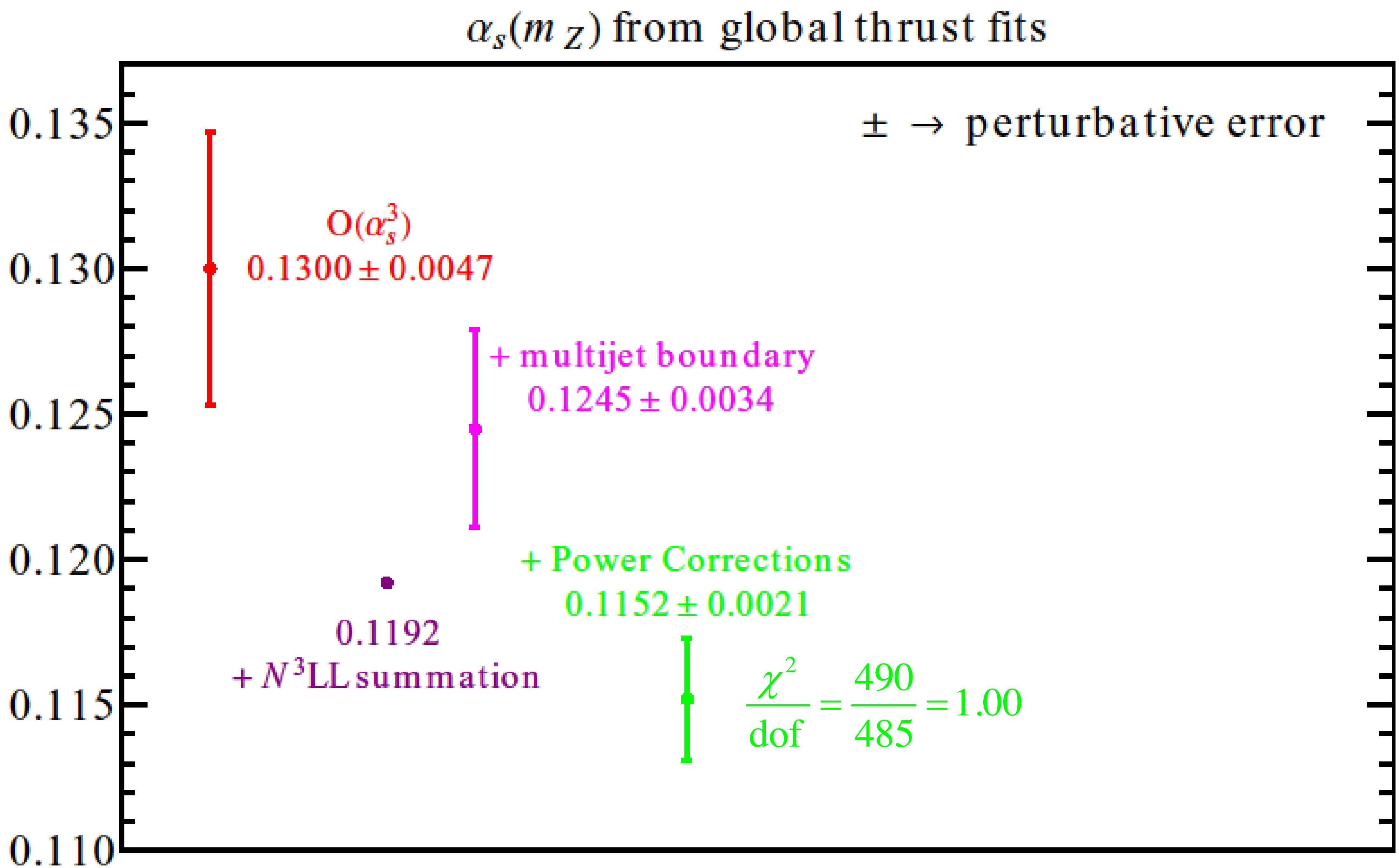


Profile functions (μ_0, n_1, t_2, e_J, e_H, n_s)

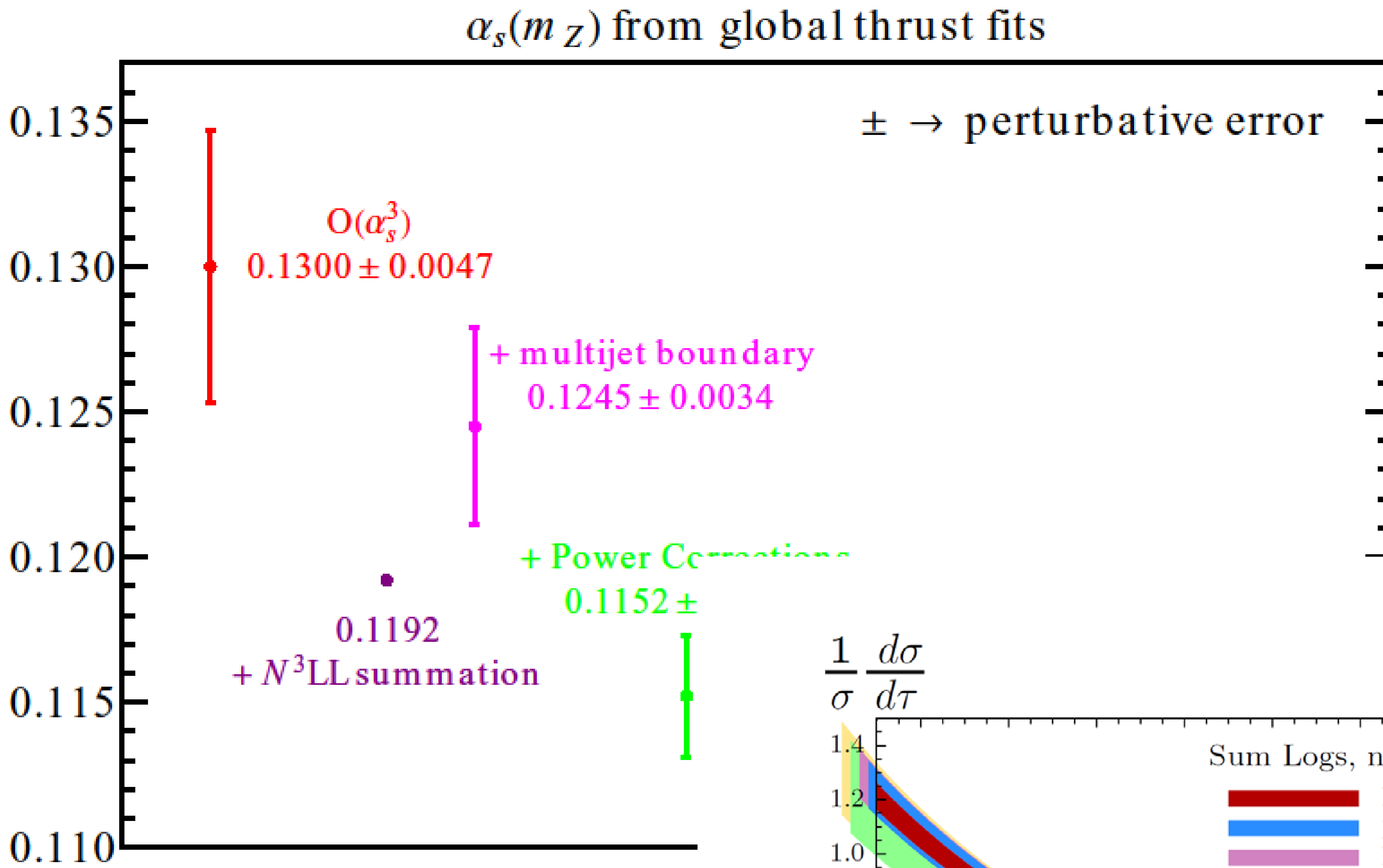
$h_3 = 8998.05$
Baikov et al

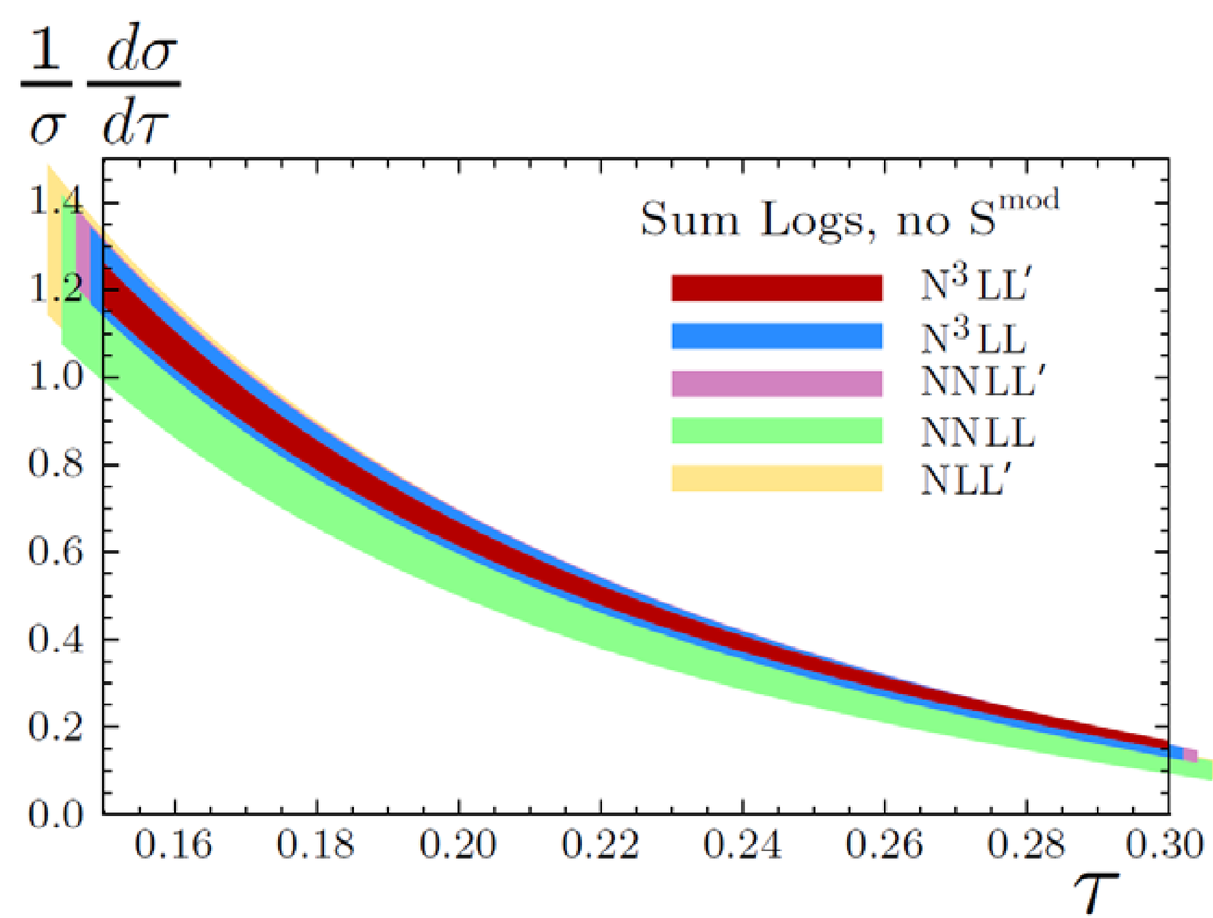
Padè approximants for range (s_2, Γ_3^{cusp}, j_3, s_3)

Nonsingular statistical error (ϵ_2, ϵ_3)



- Resummation at N^3LL
- Multijet boundary condition
- Power corrections give -7.5% shift





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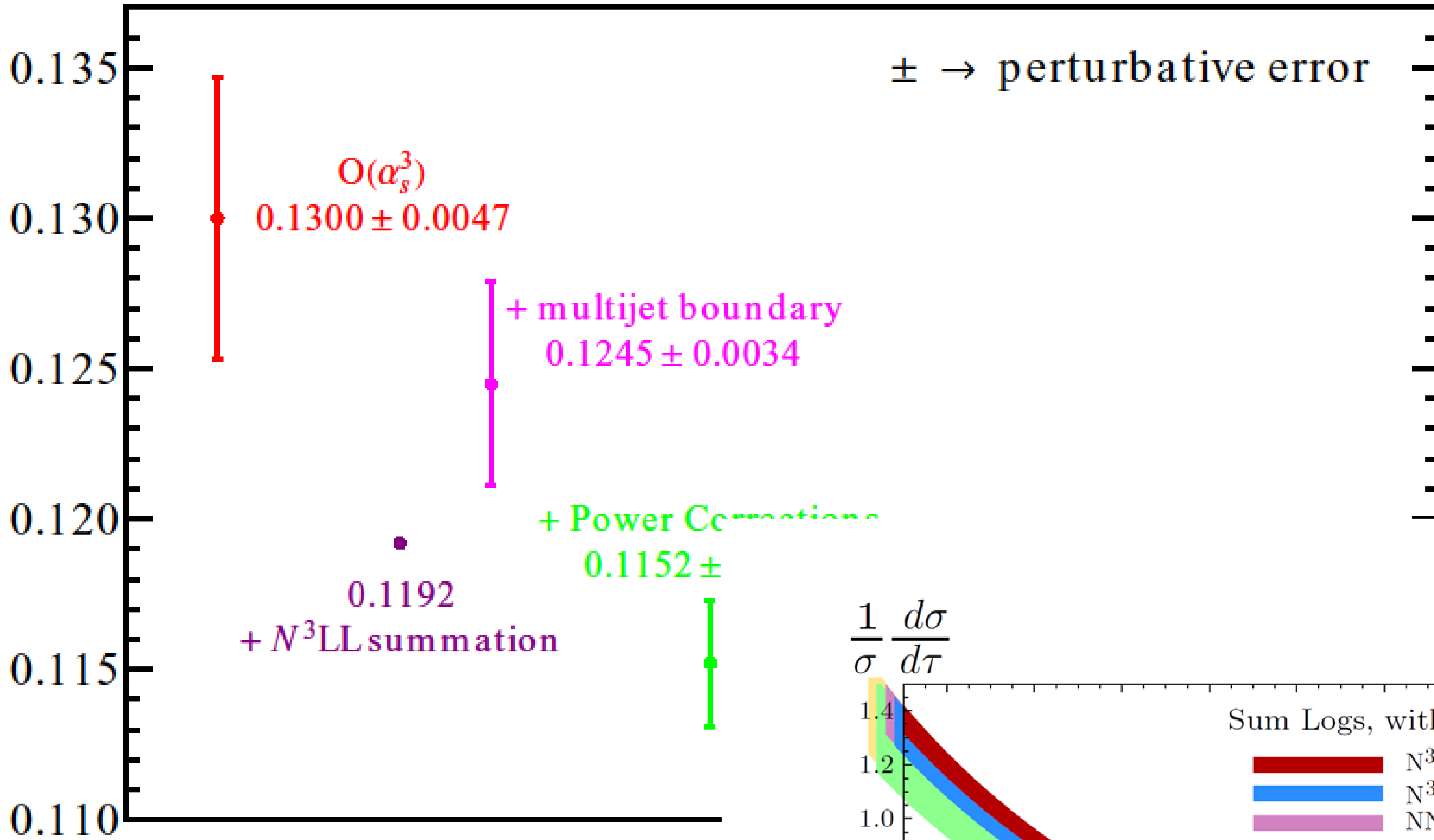
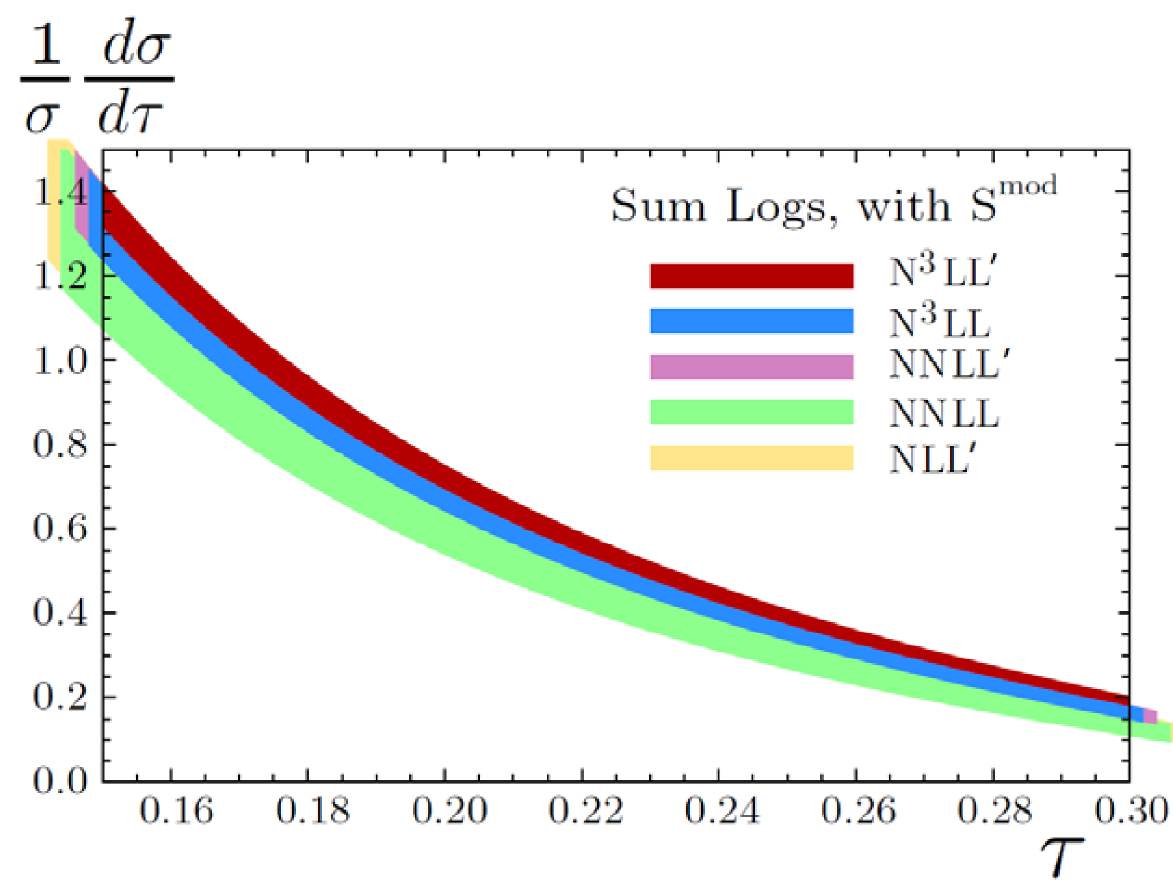
$\alpha_s(m_Z)$ from global thrust fits





- Resummation at N^3LL
- Multijet boundary condition
- Power corrections give -7.5% shift

$$\frac{d\sigma}{d\tau} = \int dk \left(\frac{d\hat{\sigma}}{d\tau} + \frac{d\hat{\sigma}_{ns}}{d\tau} + \frac{d\hat{\sigma}_b}{d\tau} \right) \left(\tau - \frac{k}{Q} \right) S_\tau^{\rm mod}(k - 2\bar{\Delta}) + \mathcal{O}\left(\sigma_0 \frac{\alpha_s \Lambda_{\rm QCD}}{Q} \right)$$

In the tail region $\ell_{\rm soft} \sim Q\,\tau \gg \Lambda_{QCD}$

and we can expand the soft function

Shifts distributions to the right !

$$S(\tau) = S_{\rm pert}(\tau) - S'_{\rm pert}(\tau)\frac{2\Omega_1}{Q} \approx S_{\rm pert}\left(\tau - \frac{2\Omega_1}{Q}\right) \qquad \Omega_1 \sim \Lambda_{QCD}$$

Is a nonperturbative parameter

Ω_1 is defined in field theory

$$\bar{\Omega}_1 \equiv \frac{1}{2N_C} \langle 0 | {\rm tr} \bar{Y}_{\bar{n}}(0) Y_n(0)\, i\partial_\tau Y_n^\dagger(0) \bar{Y}_{\bar{n}}^\dagger(0) | 0 \rangle \qquad \overline{\rm MS}$$

$$i\partial_\tau \equiv \theta(i\bar{n}\cdot\partial - i n\cdot\partial)\, i n\cdot\partial + \theta(i\bar{n}\cdot\partial - i n\cdot\partial)\, i\bar{n}\cdot\partial$$

Consistency check

$$\frac{1}{\sigma}\frac{d\sigma}{d\tau} = h\left(\tau - \frac{2\Lambda}{Q}\right)$$

Perturbative expression

Power correction

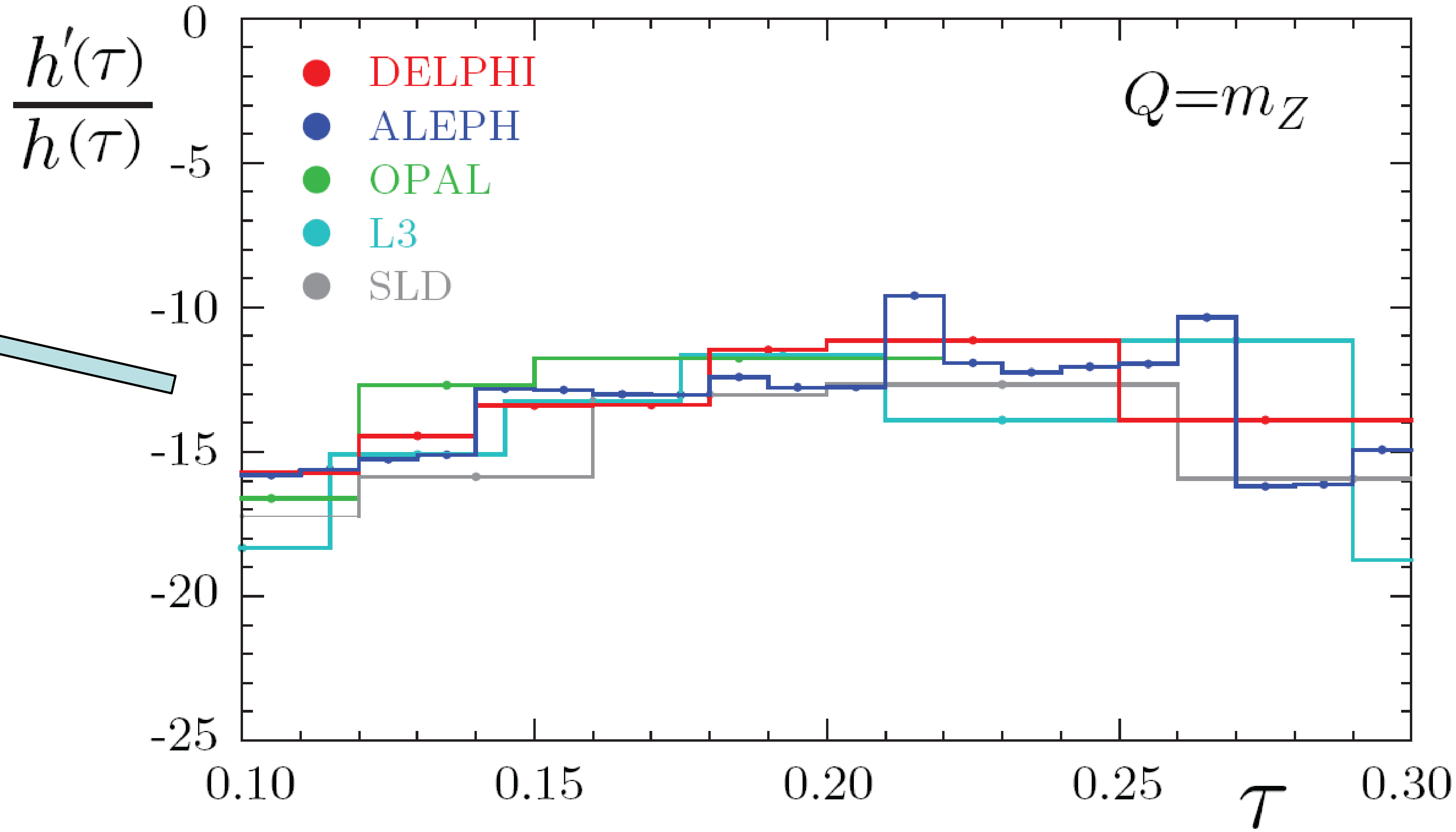
assuming that $h \sim \alpha_s$

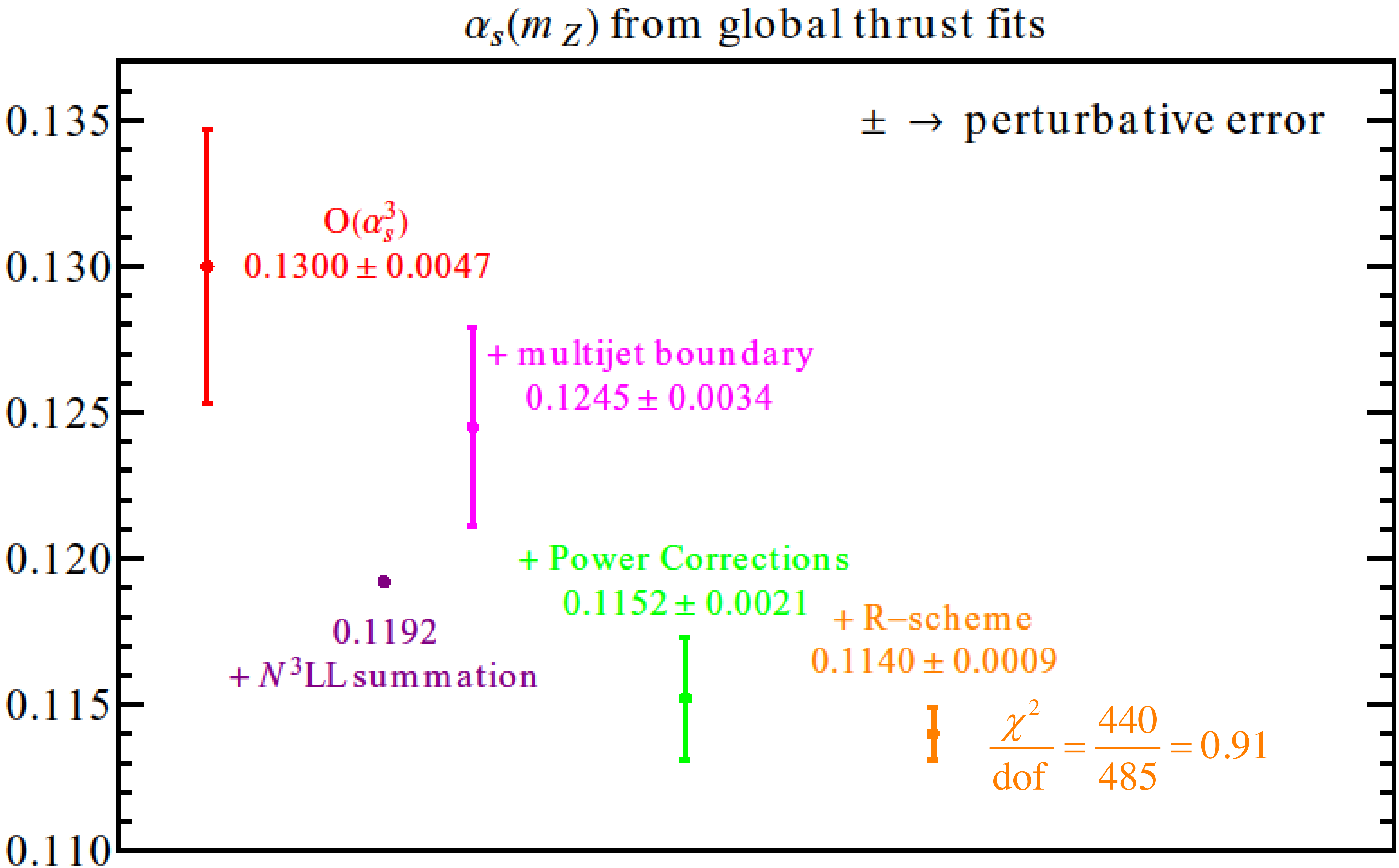
$$\frac{\delta\alpha_s}{\alpha_s} \approx \frac{2\Lambda}{Q}\frac{h'(\tau)}{h(\tau)}$$

$$\frac{h'(\tau)}{h(\tau)} \approx -14 \pm 4$$

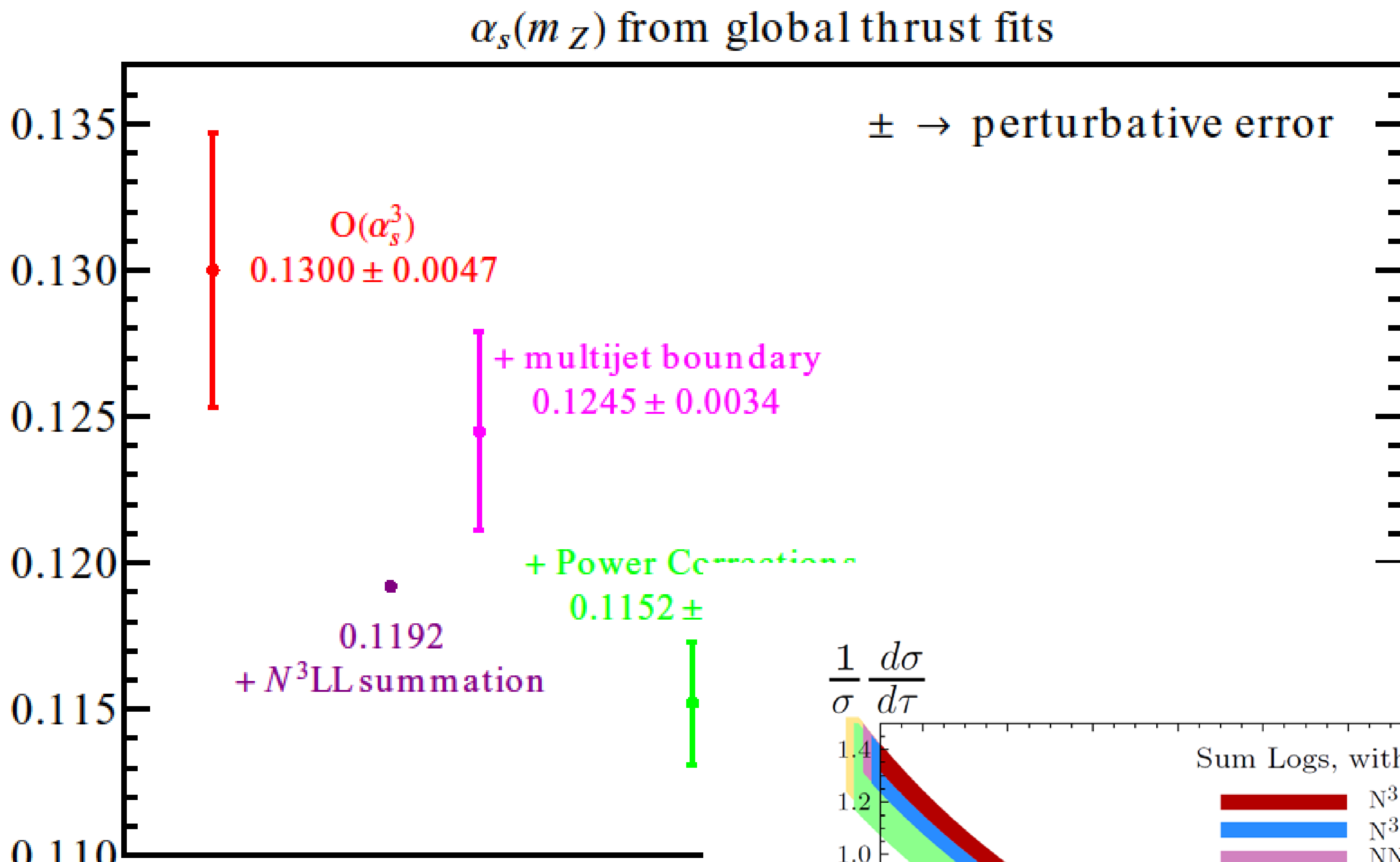
assuming $\Lambda \sim 0.3\,\mathrm{GeV}$

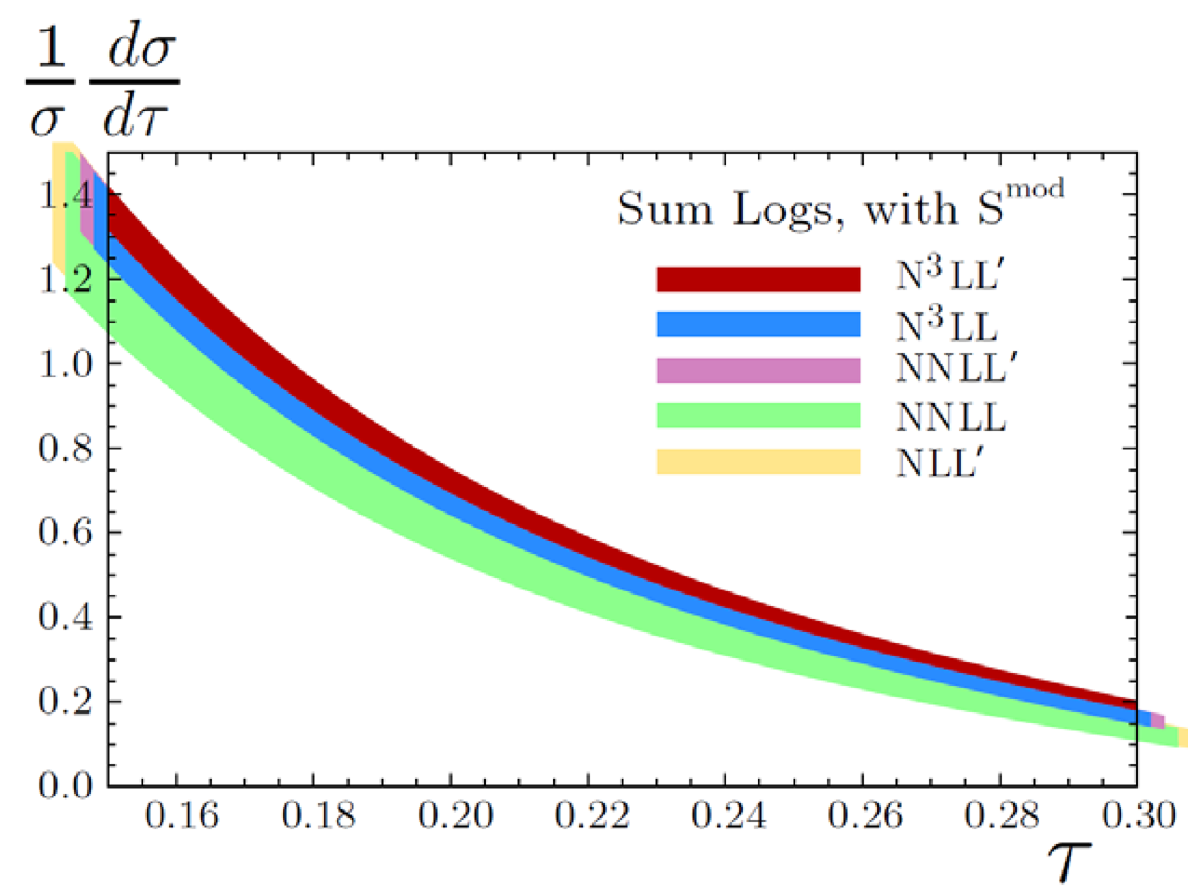
$$\frac{\delta\alpha_s}{\alpha_s} \approx -(9 \pm 3)\ \%$$



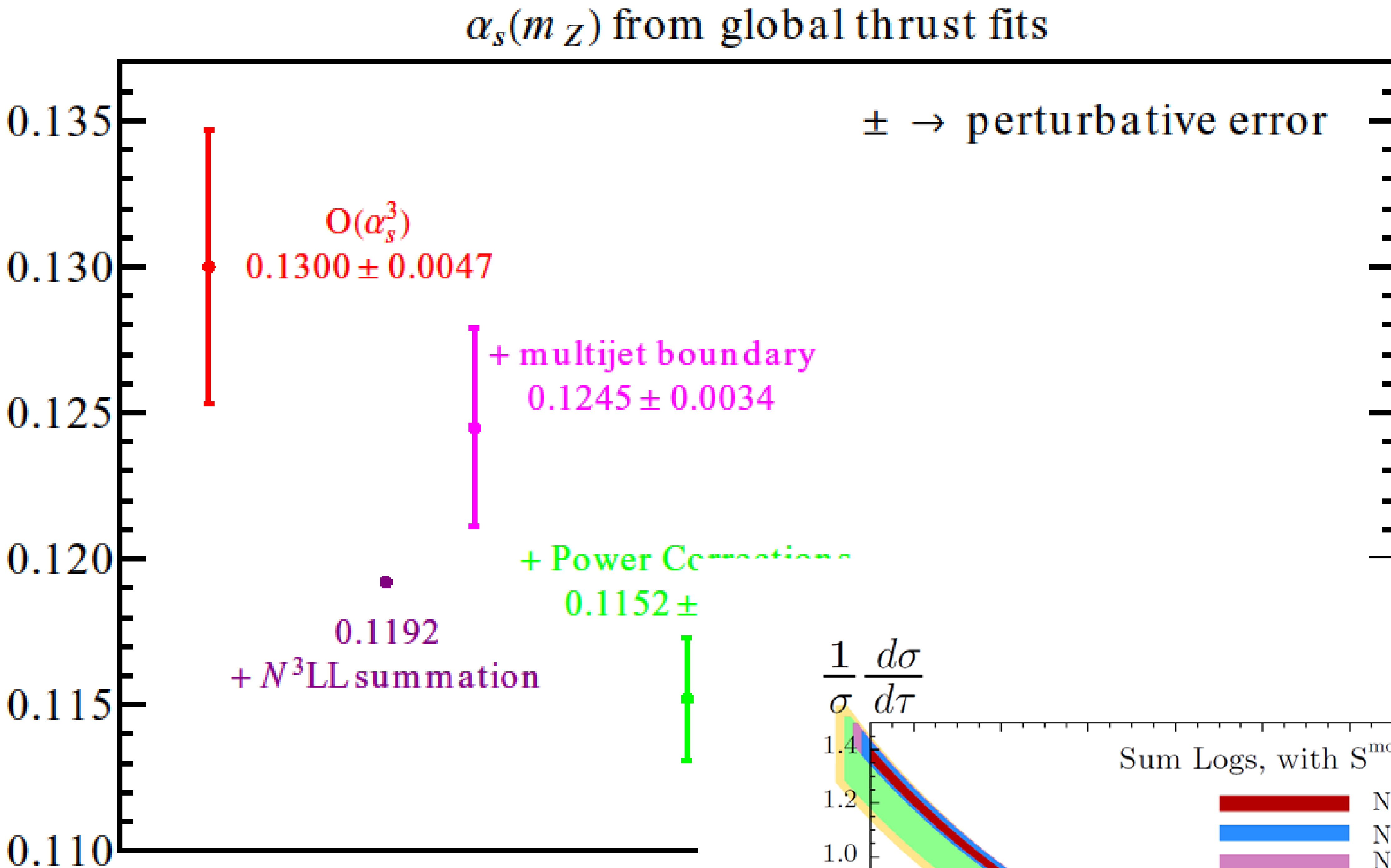


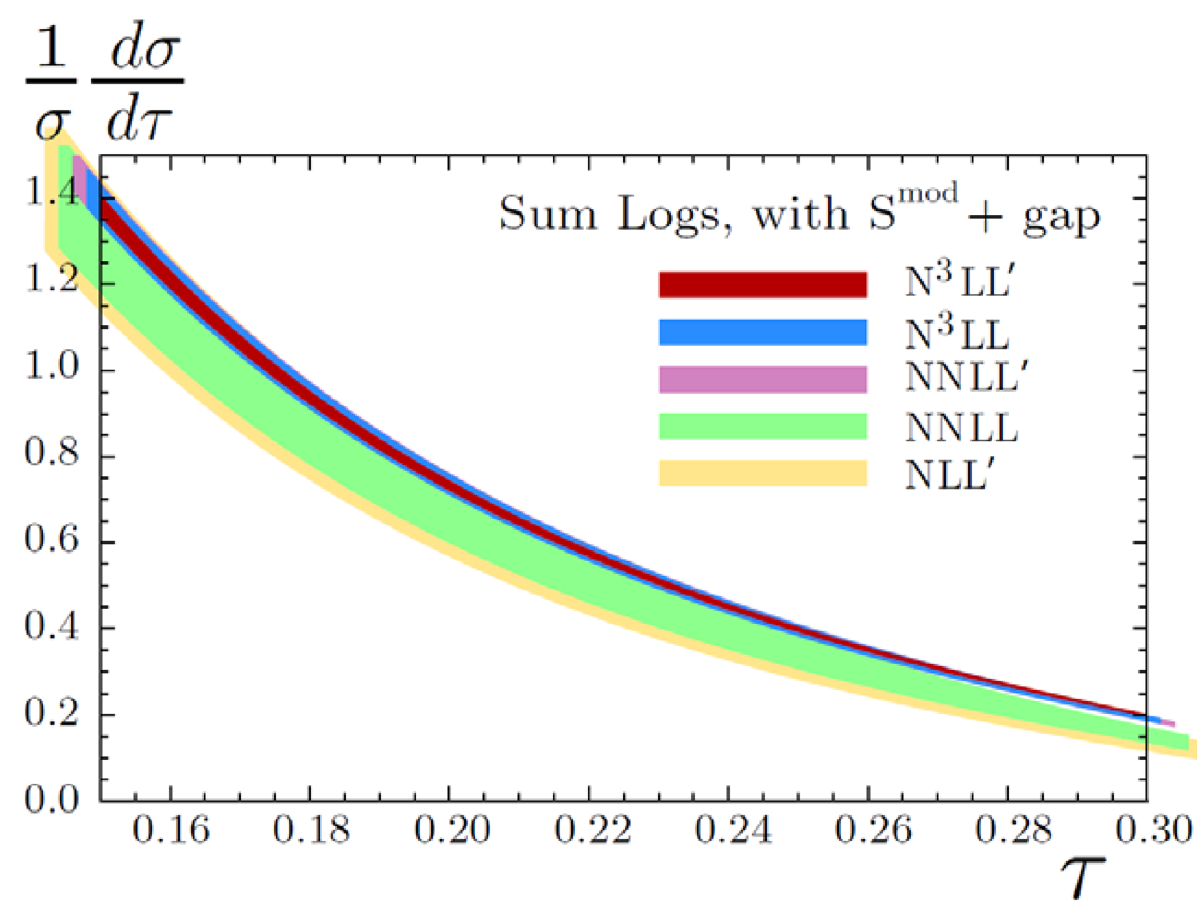
- Resummation at N^3LL
- Multijet boundary condition
- Power correction, in a scheme free of the $O(\Lambda_{QCD})$ renormalon





- Resummation at N^{3}LL
- Multijet boundary condition
- Power correction, in a scheme free of the O(Λ_{QCD}) renormalon





- Resummation at N^{3}LL
- Multijet boundary condition
- Power correction, in a scheme free of the $O(\Lambda_{QCD})$ renormalon

R-scheme

- $$\boxed{\Omega_1(R,\mu_s)} = \bar{\Omega}_1 - \delta(R,\mu_s) \qquad \delta(R,\mu) = \sum_n \left(\frac{\alpha}{4\pi}\right)^n \delta_n(R,\mu)$$

  Renormalon free

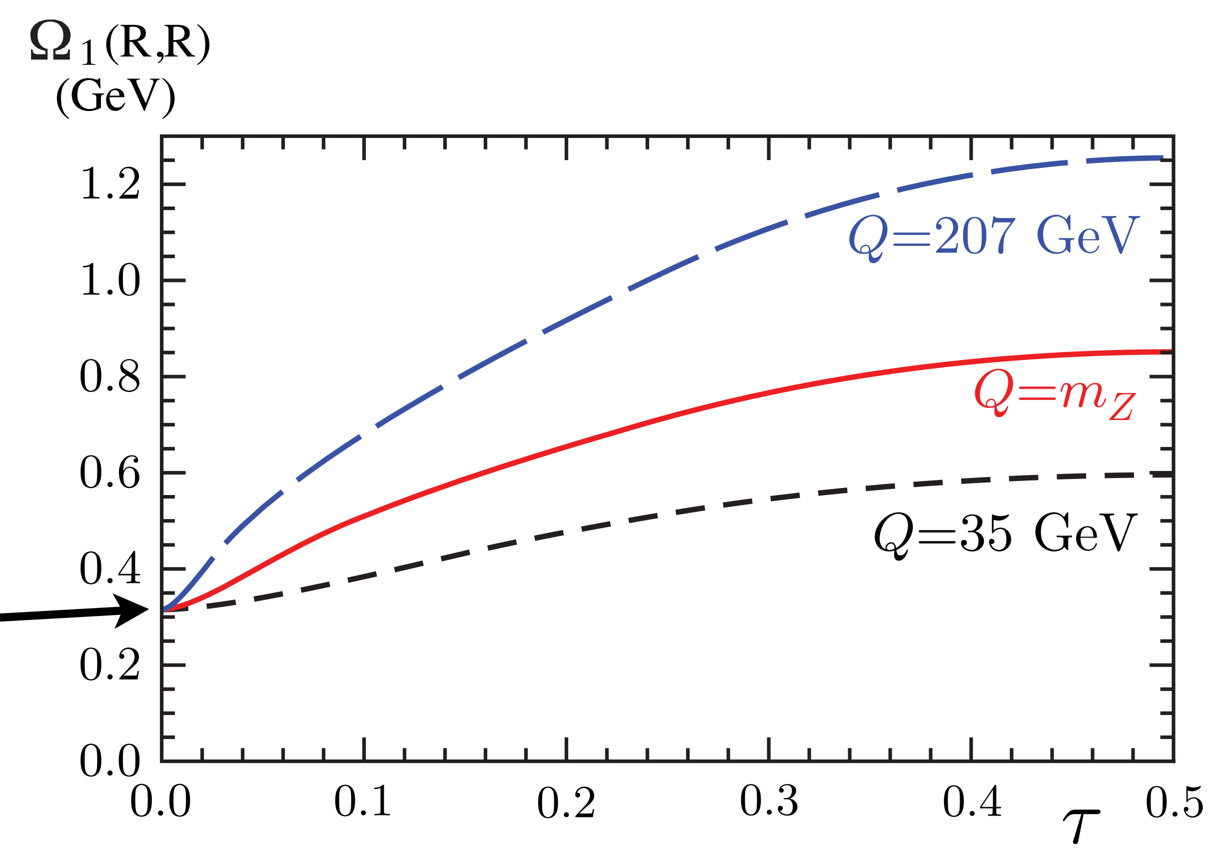
$$S(l,\mu) = \int \mathrm{d}l' \overbrace{\left[e^{-2\delta(R,\mu)\partial/\partial l} S_{\text{part}}(l,\mu)\right]}^{\text{perturbative}} \overbrace{S_\tau^{\text{mod}}(l' - 2\bar{\Delta}(R,\mu), R)}^{\text{non perturbative}}$$

- $$\delta_n(R,\mu) = \mathrm{R}\, e^{\gamma_R} \sum_m \delta_n^m \log^m\left(\frac{R}{\mu}\right) \longrightarrow$$ Can become large / Keep them O(1) $\longrightarrow R \sim \mu$

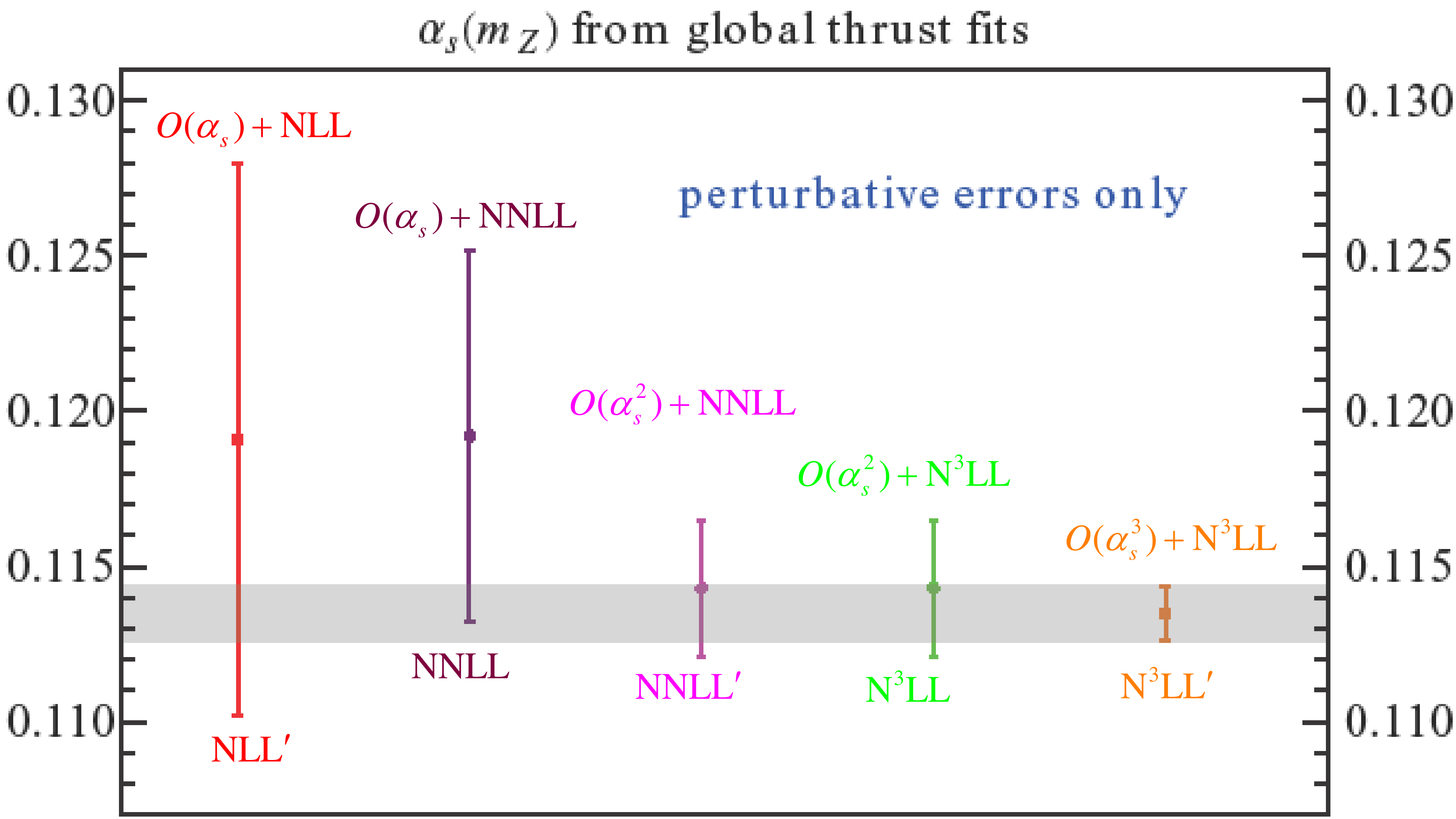
Running equations

- R-running
- μ-running

boundary value is the unknown parameter: $\Omega_1(R_0, R_0)$

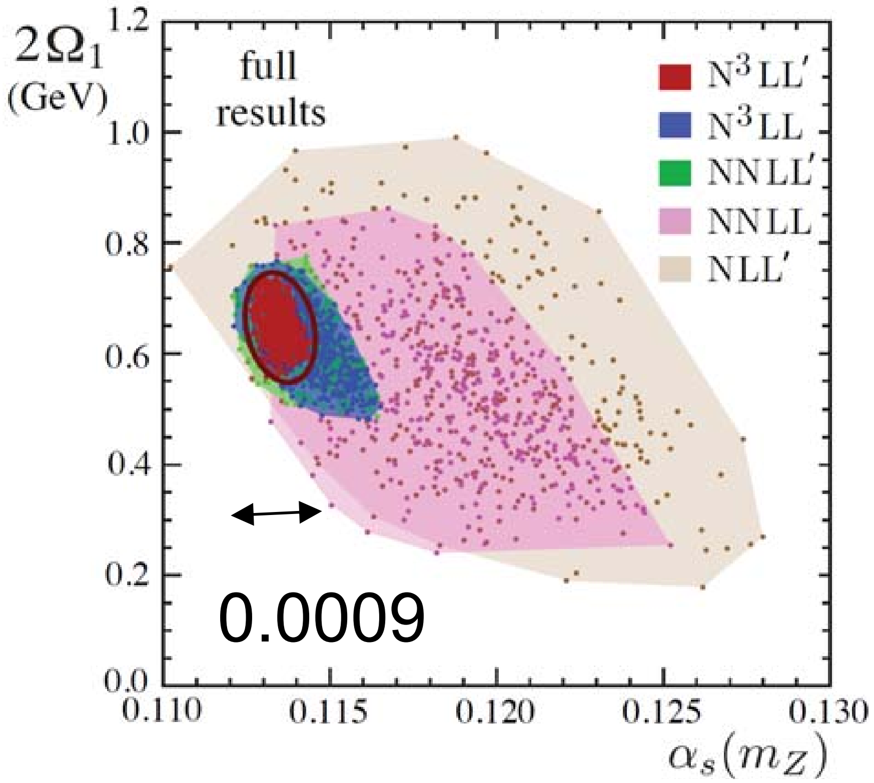


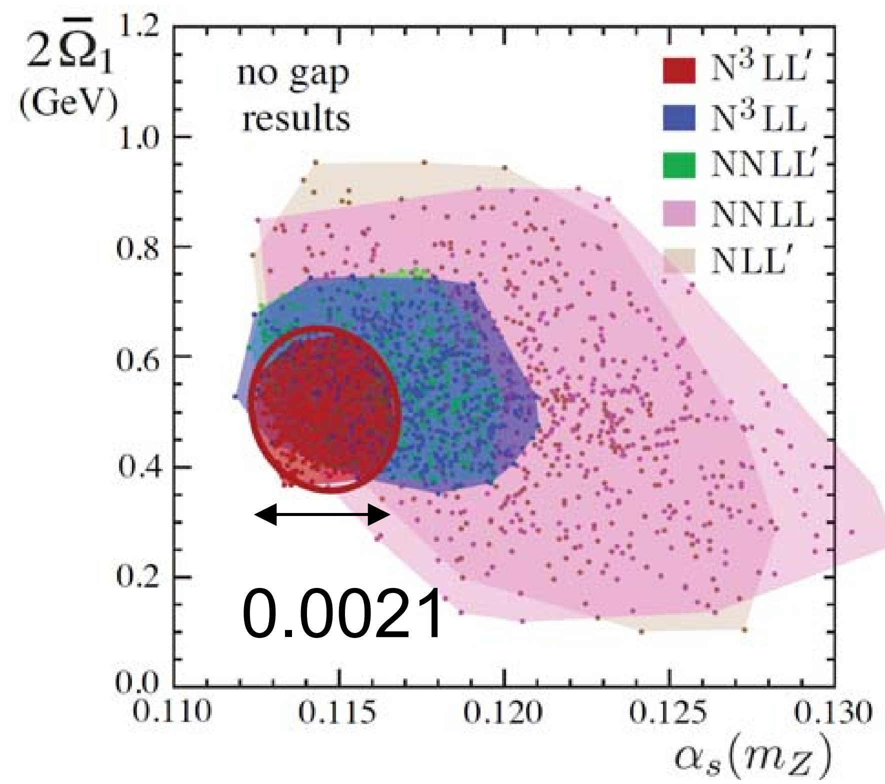
Convergence of results

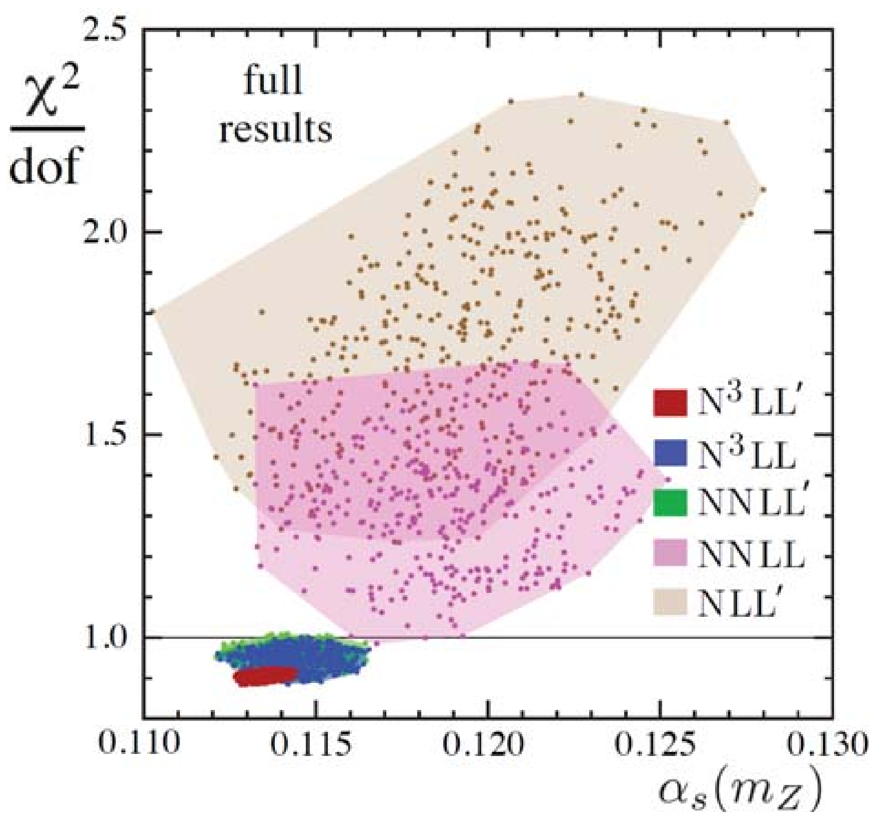


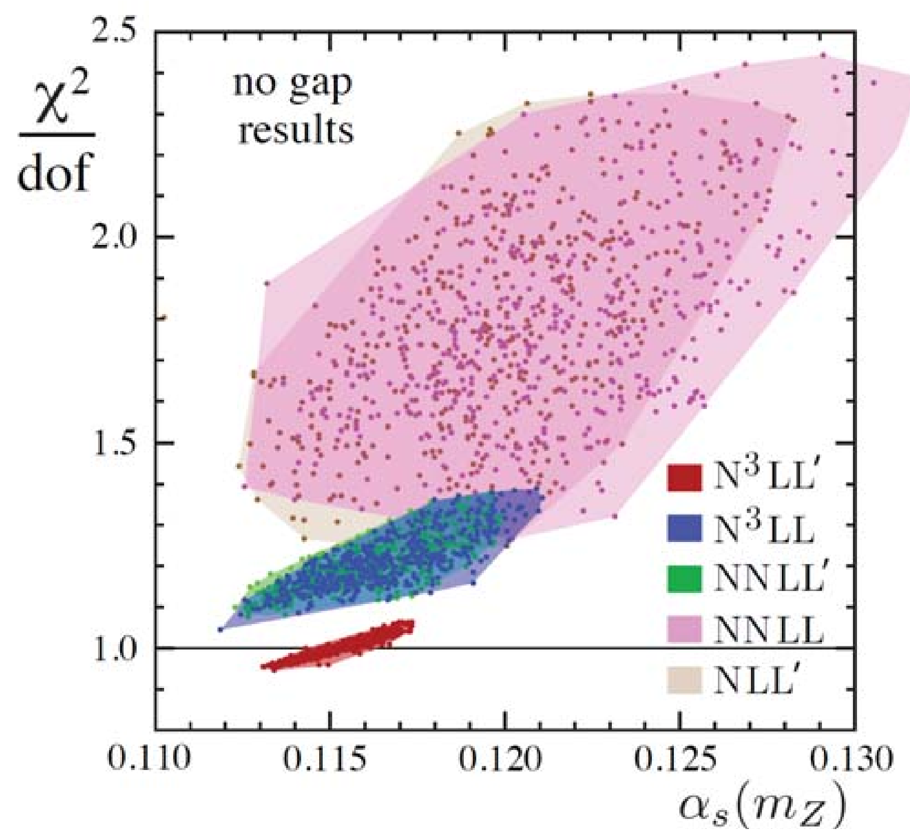
Theory uncertainty is from a flat scan

Renormalon free	With renormalon









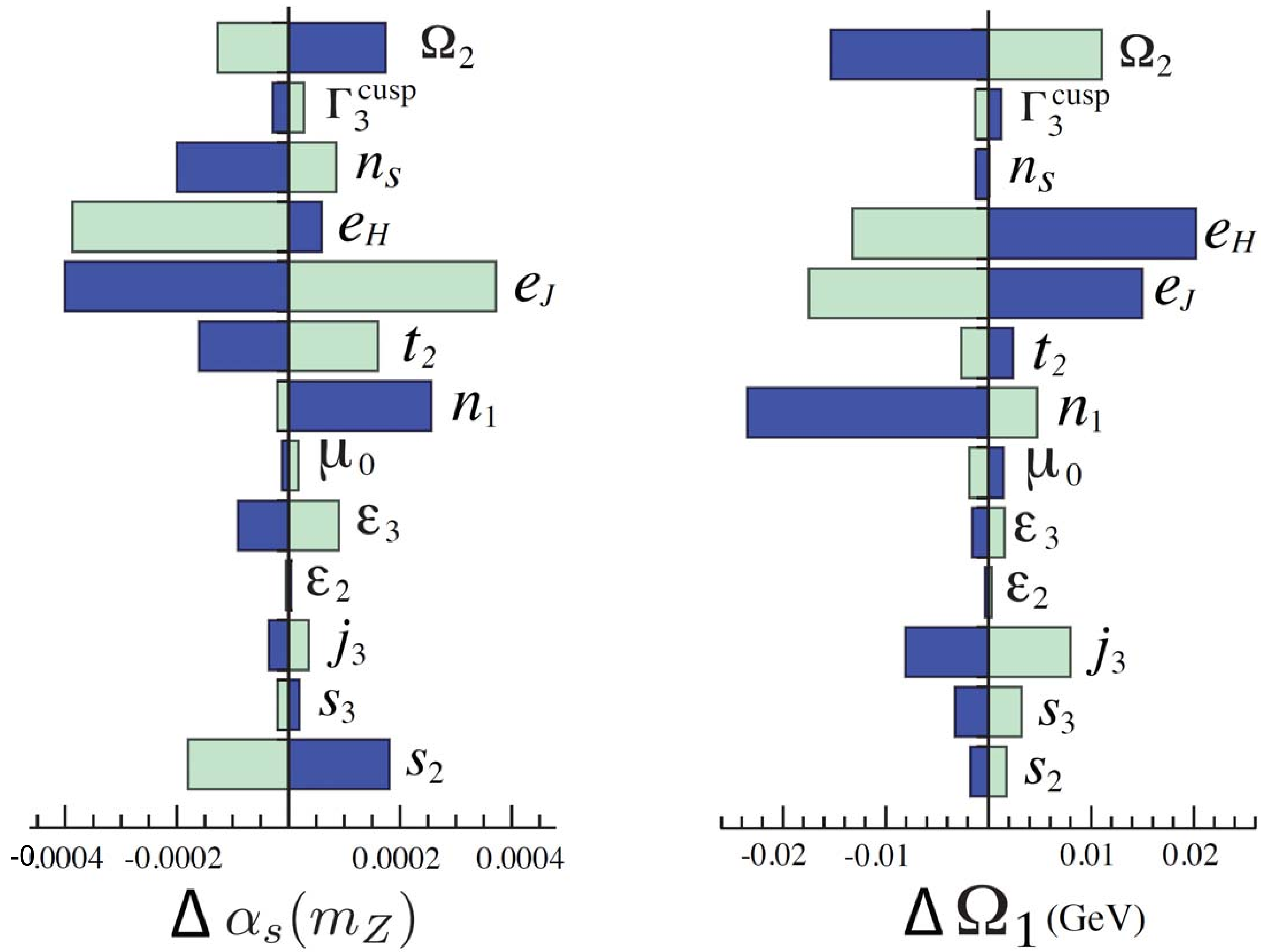
Renormalon-free results have smaller theory errors and better fits

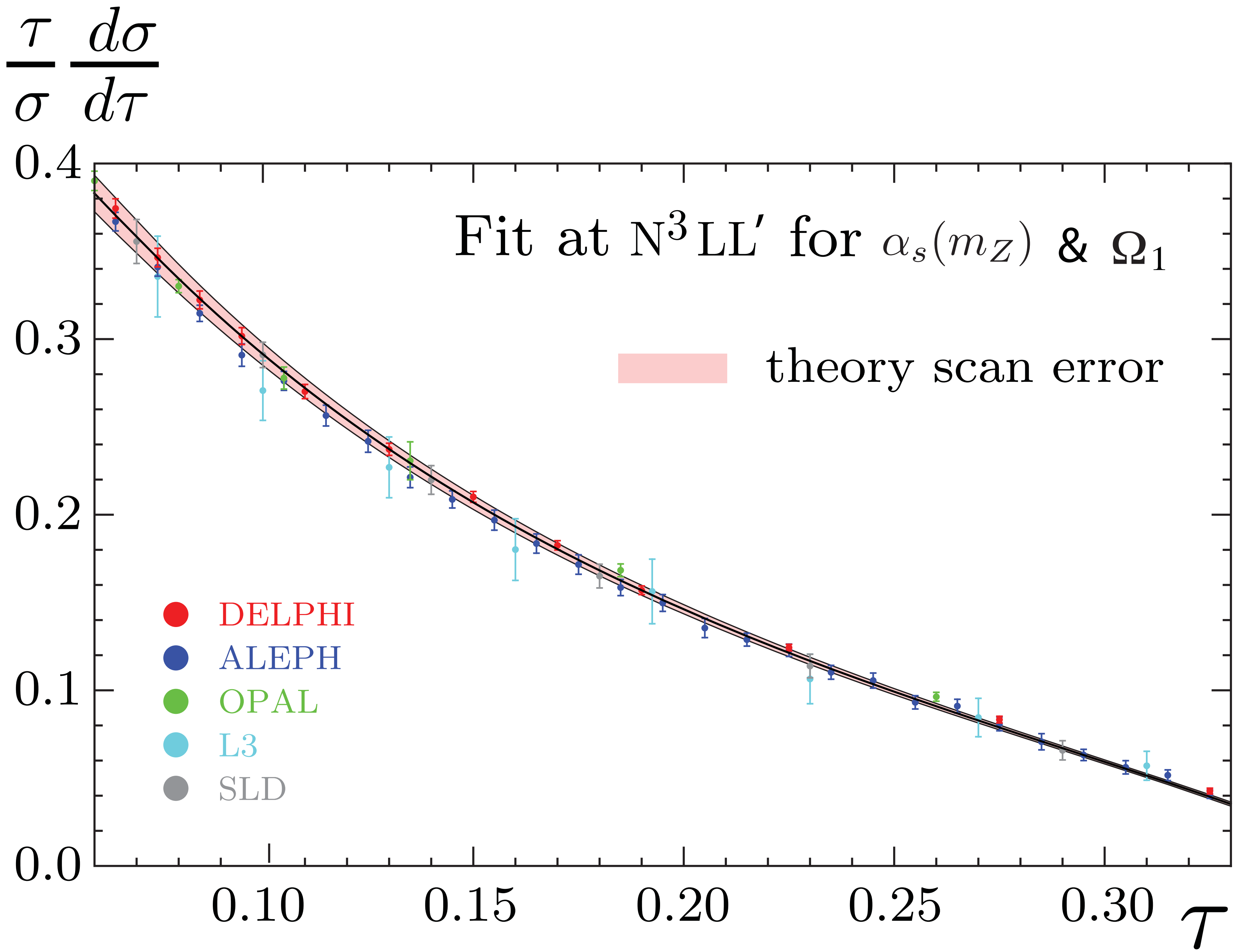
Ω_1 determined to 16% accuracy

500 points random scan per order

Adding individual errors in quadrature gives similar (but smaller) error

Effect of the various scan parameters

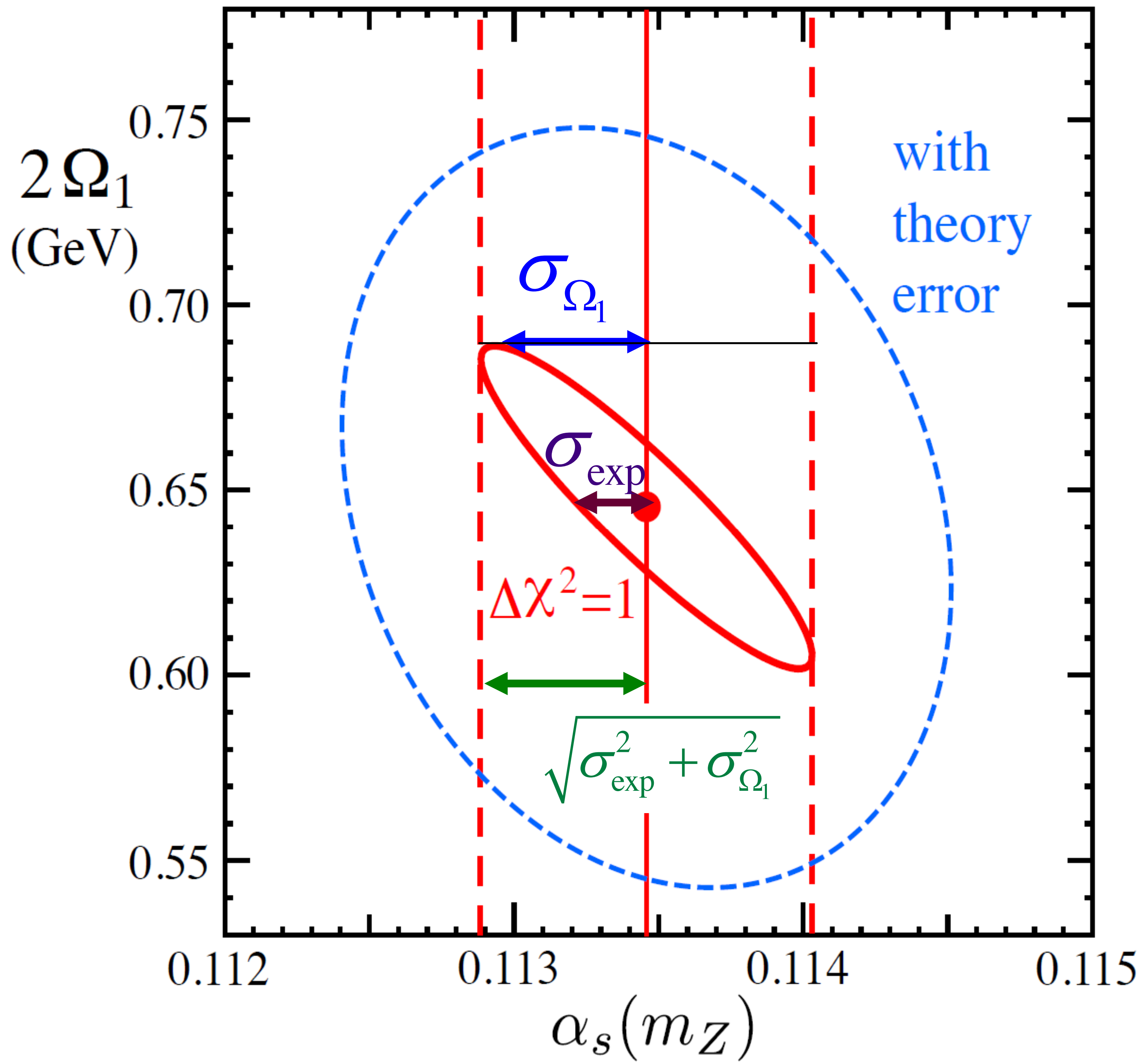



$\frac{\tau}{\sigma}\frac{d\sigma}{d\tau}$
Fit at N^{3}LL′ for $\alpha_s(m_Z)$ & Ω_1
theory scan error
DELPHI
ALEPH
OPAL
L3
SLD
0.0
0.1
0.2
0.3
0.4
0.10
0.15
0.20
0.25
0.30
τ

Experimental error

$$\frac{\chi^2}{\text{dof}} = \frac{440}{485} = 0.91$$

1σ (39% confidence level)



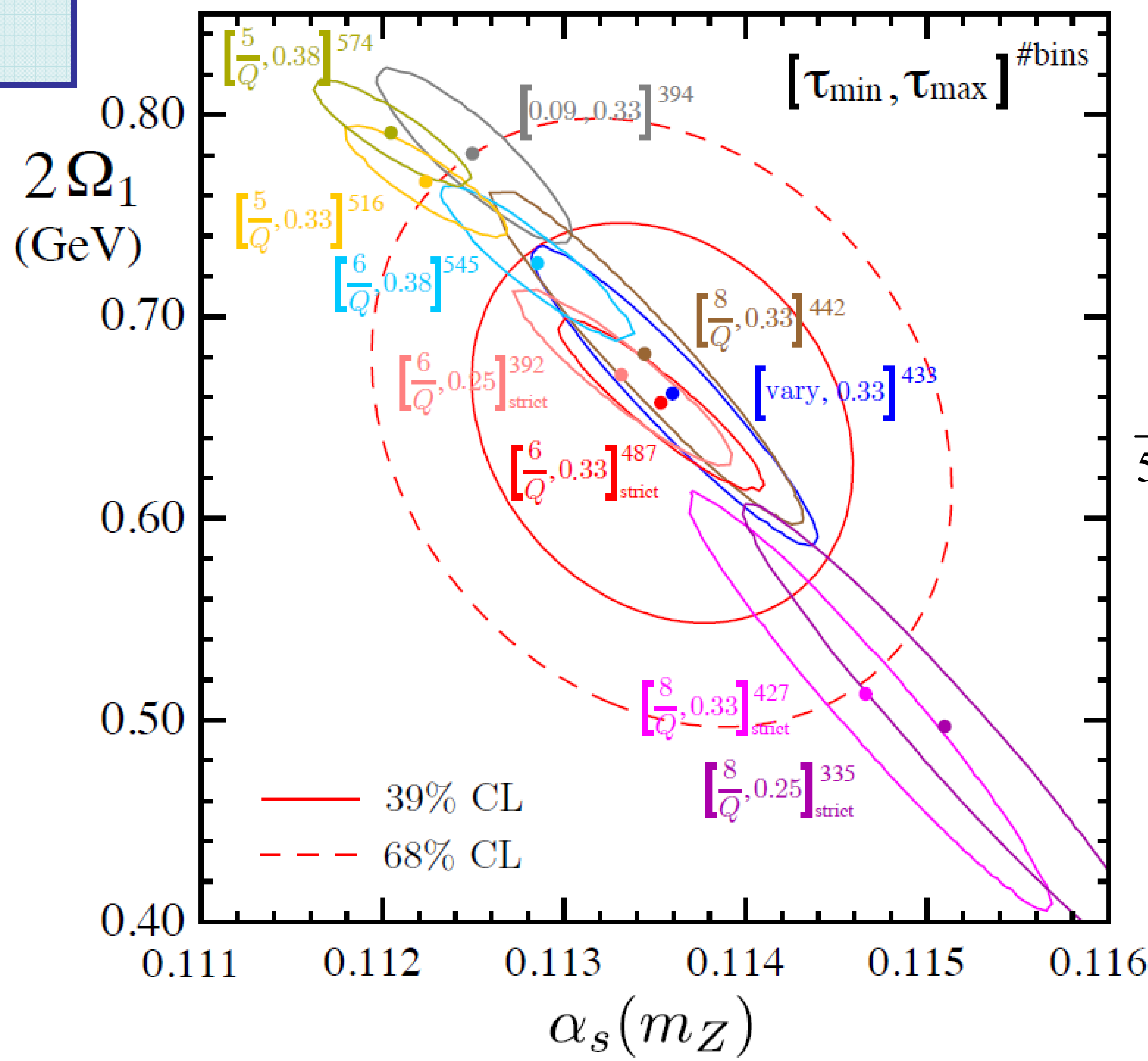
$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$$

mostly Ω_1, and includes Ω_2 (had) — scan (pert)

Fit for bins: different data sets

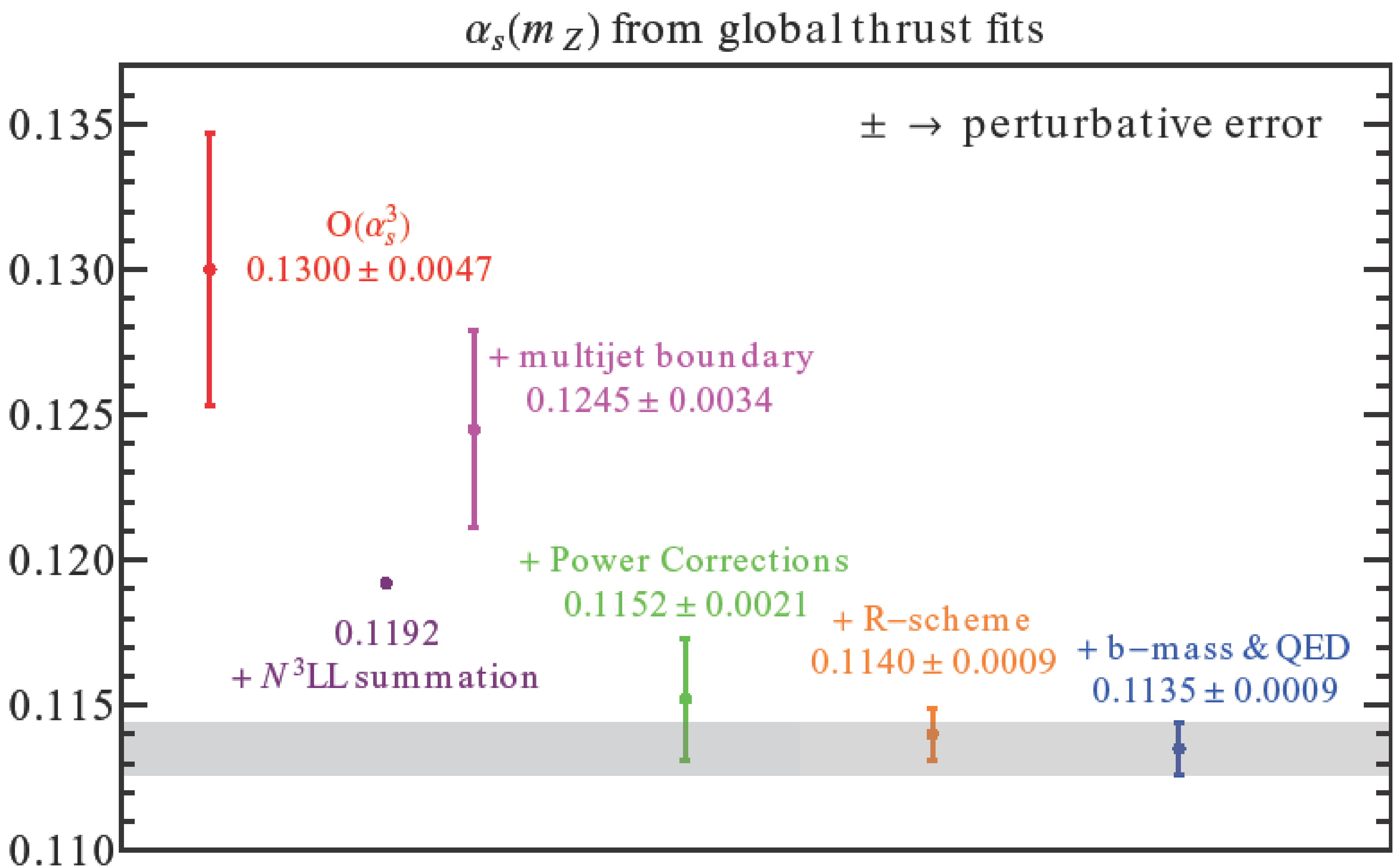
Ω_2 effects increase

Renormalon free

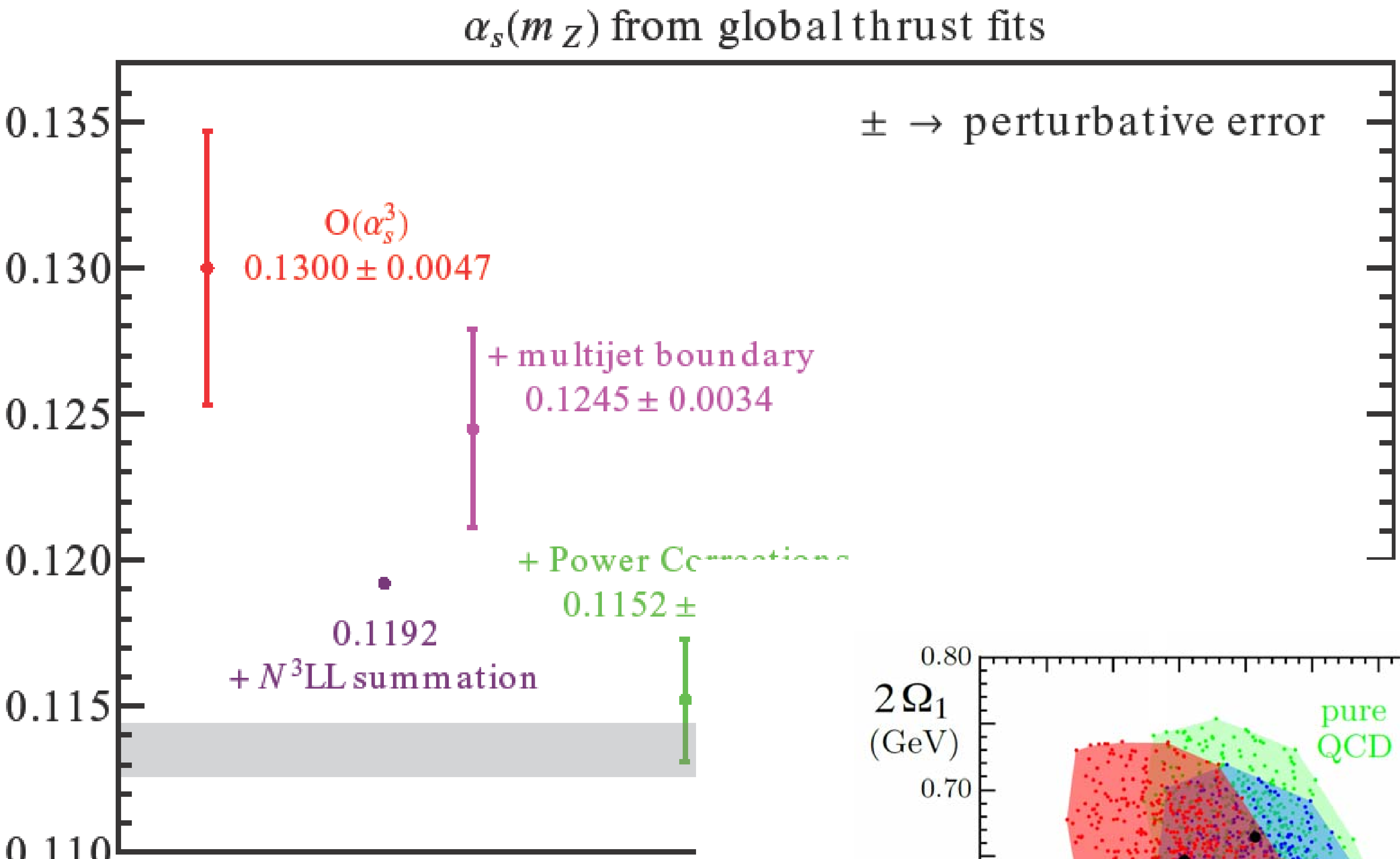


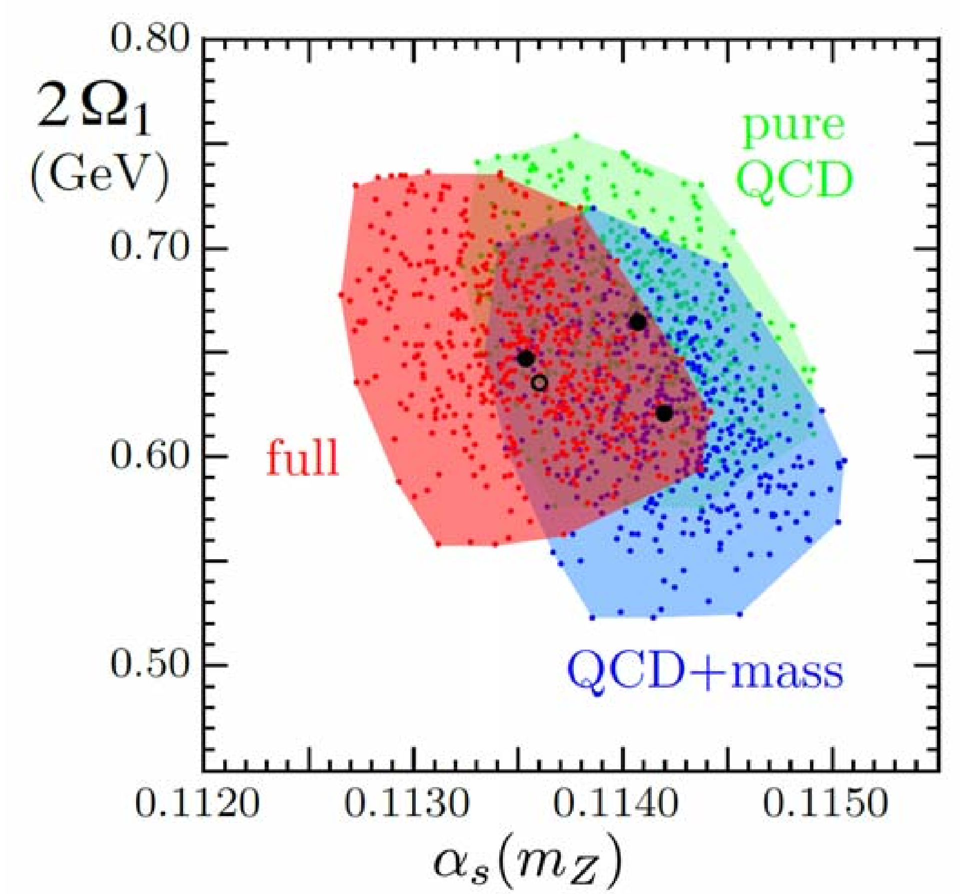
$$\frac{\Omega_1}{50.2\text{GeV}} = 0.1200 - \alpha_s(m_Z)$$

statistical errors decrease



- Resummation at N^3LL
- Multijet boundary condition
- Power correction, in a scheme free of the $O(\Lambda_{QCD})$ renormalon
- QED & bottom mass corrections





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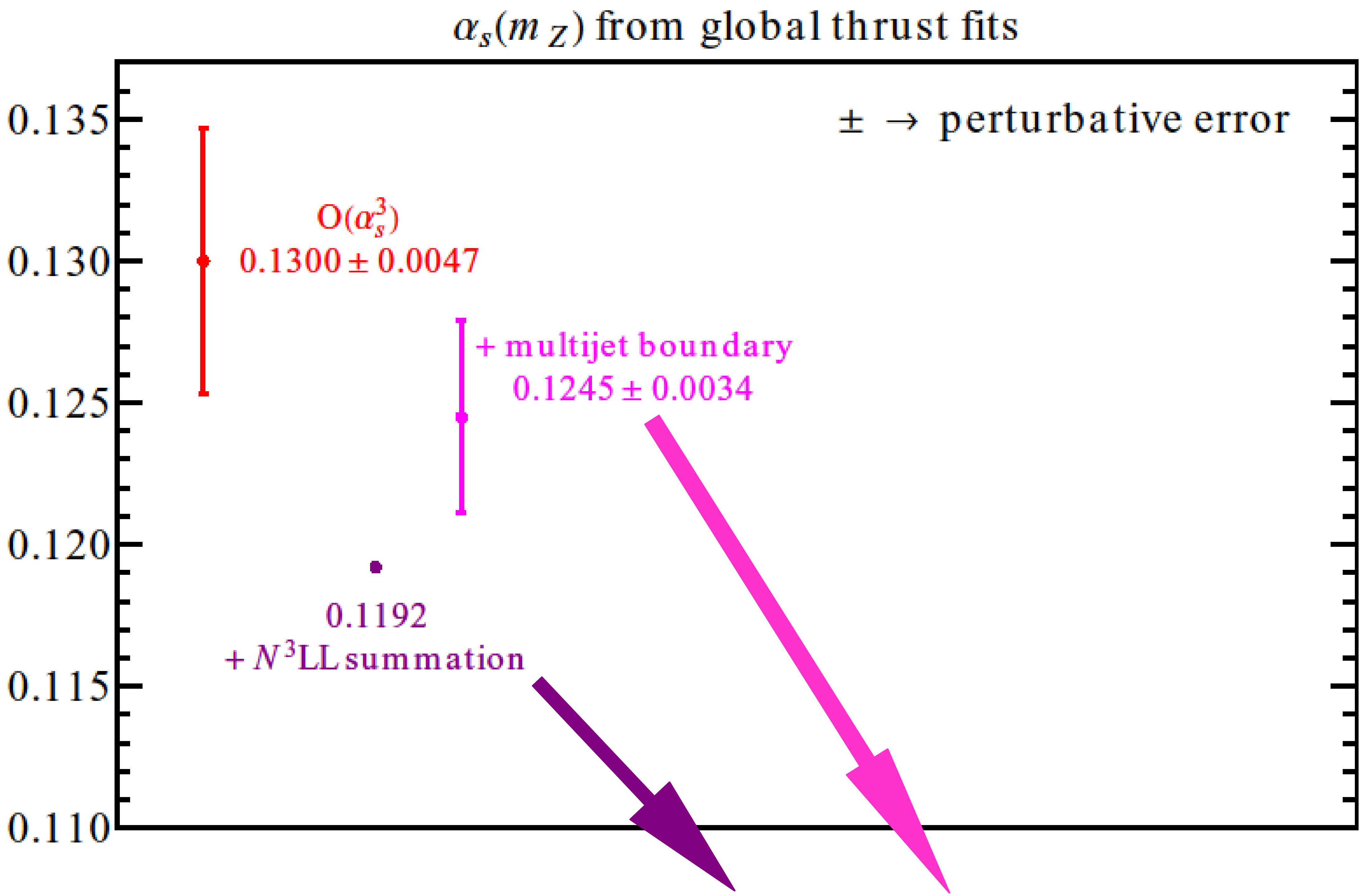
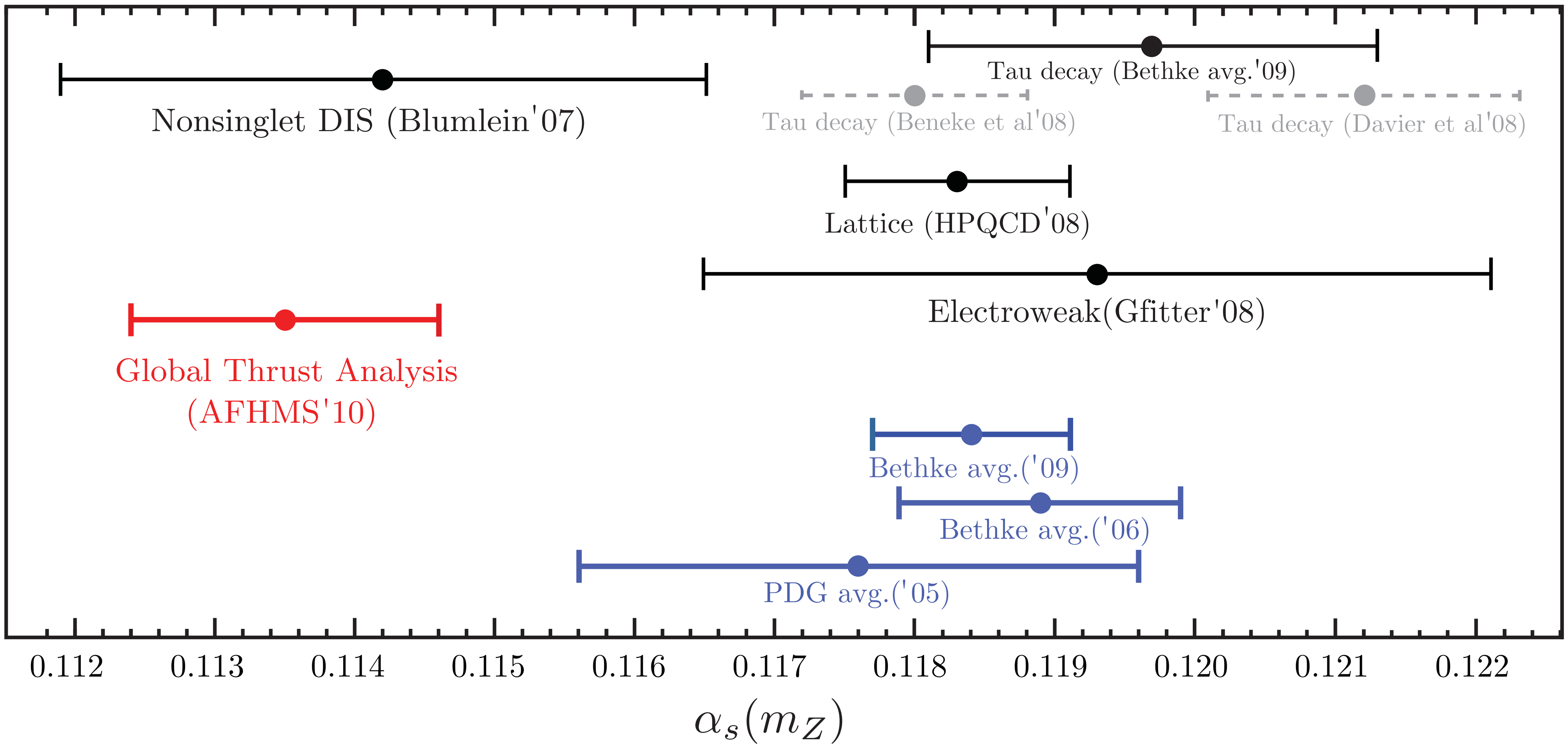


Preliminary Fits to ALEPH data:			
	thrust	0.1169	0.1223
	heavy jet mass	0.1175	0.1220

Perturbative N^3LL agrees*
with Chien & Schwartz '10

Final thrust result

$$\alpha_s(m_Z) = 0.1135 \pm 0.0002_{\text{exp}} \pm 0.0005_{\text{had}} \pm 0.0009_{\text{pert}}$$



Result from jets differs by 3.5σ from the HPQCD lattice result

Summary & Outlook

$\alpha_s(m_Z)$

- Tau Decays (FOPT vs. CIPT; Duality violation)
- Lattice QCD (multiple actions; trustworthy errors)
- DIS & Global (NMC data; gluon pdf parameterization; theory error analysis)
- R ratio & Precision EW (Giga Z? Super B?)

Thrust & Event Shapes

- The Soft-Collinear Effective Theory provides a powerful formalism for deriving factorization theorems and analyzing processes with Jets
- Important to account for nonperturbative effects (not with MC)
- Consistency checks with other event shapes at perturbative level, consistency check for full analysis on the near horizon
- Results are systematically smaller than (some) other extractions

The End