Inclusive Jet Production at the Large Hadron Collider meets an Old Friend – the $\pi^0$
Jets at LHC & $\pi^0$s at FNAL

LHC ATLAS

FNAL E63
Prospective

• Some 45 years ago the highest energy in proton-proton collisions was at the Intersecting Storage Ring (ISR) at CERN at energy ~ 60 GeV. FNAL and the SPS at CERN were fixed target machines and could achieve COM energies of ~ 27 GeV.
  • The concepts of Jets, the Gluon and QCD were just being developed in this era.

• Many experiments were performed at that time to measure the inclusive rate of single particle production – such as $p + p \rightarrow \pi^0 + X$, where only the $\pi^0$ was measured. These experiments were hadronic analogs to deep inelastic electron scattering: $e^- + p \rightarrow e^- + X$.

• Is there any similarity between the systematics observed at these low energies with those of experiments now performed at the large hadron collider?

• In the era of highly sophisticated QCD analyses by large analysis teams is there anything that can be learned by “just looking” at the data?
The Paradigm for Single Particle Inclusive Production

\[
E d\sigma/d^3 p(s, t, u; A + B \rightarrow h + X) = \int_{x_{a}^{\text{min}}}^{1} dx_a \int_{x_{b}^{\text{min}}}^{1} dx_b G_{A-a}(x_a)G_{B-b}(x_b)D_c^h(z_c) \frac{1}{z_c} \frac{d\hat{\sigma}}{dt}(\hat{s}, \hat{t}; q_a + q_b - q'_a + q'_b)
\]

Field and Feynman

Quark elastic scattering as a source of high - transverse - momentum mesons, R. D. Field and R. P. Feynman, PRD 15, 2590 (1977)
Jets are produced by hard parton scattering (qq → qq, gg → gg, gq → gq). The scattered parton hadronizes into a jet of particles.

*QCD Factorization Theorem*

\[
E \frac{d^3 \sigma}{dp^3} = \sum_{a,b} \int dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \frac{d\hat{\sigma}_{ab}(\alpha_s(\mu_R^2), s/\mu_R^2, s/\mu_F^2)}{dt} \otimes \text{Frag} \otimes \text{Had}
\]

These *10s of parameters and factors* are put together in simulations of inclusive jet production at the LHC.

Dimensions:

\[
E \frac{d^3 \sigma}{dp^3} \sim \frac{d^2 \sigma}{dp_T^2 dy} \sim \frac{d\hat{\sigma}_{ab}(\alpha_s(\mu_R^2), s/\mu_R^2, s/\mu_F^2)}{dt} \otimes \text{Frag} \otimes \text{Had}
\]

\[
\sim \frac{cm^2}{GeV^2} \sim \frac{1}{GeV^4}
\]
ATLAS Inclusive Jet Production at 13 TeV

- Jets defined by anti-$k_t$ algorithm with $R=(\Delta \phi^2 + \Delta y^2)^{1/2} = 0.4$
- Pythia 8.186 with A14 tune, NLOjet++. Involves integrations & summations using Monte Carlo methods
- Data compared to NLO pQCD calculation including 2 $\rightarrow$ 2 processes, leading logarithmic $p_T$-ordered parton shower, hadronization with the Lund string model.
Broad energy and angle coverage provided an “aerial photography of kinematic landscape”:

\[ \theta = 30 \text{ to } 275 \text{ milli-radians}, \ P_{\text{beam}} = 50 \text{ to } 400 \text{ GeV} \]

Detected single \( \gamma \) and used Sternheimer analysis to determine \( \pi^0 \) kinematics:

\[ \sigma_{\pi}(k) \sim -k \frac{\partial \sigma_{\gamma}(k)}{\partial k} \]

FIG. 10. $\pi^0$ invariant cross sections as a function of transverse momentum for various incident proton beam momenta, at laboratory angles (a) 30 mrad, (b) 65 mrad, (c) 100 mrad, and (d) 200 mrad.
Radial Scaling variable $X_R$

Rapidity and pseudo rapidity:
\[ y = \frac{1}{2} \ln \left( \frac{E + p_T}{E - p_T} \right) \approx \eta = -\ln \left( \tan \left( \frac{\theta}{2} \right) \right) \]

Radial scaling $x_R$:
\[ x_R = \frac{E}{E_{\text{max}}} = \frac{2\sqrt{p_T^2 \cosh^2(y)(1 + m_j^2/p_T^2 \tanh^2(y)) + m_j^2}}{\sqrt{s - m_{QN}^2}} \]
\[ \approx \frac{2p_T \cosh(y)}{\sqrt{s}} \sqrt{1 + \frac{m_j^2}{p_T^2} \tanh^2(y)} \]
\[ \approx \frac{2p_T \cosh(\eta)}{\sqrt{s}} \]

$x_R$ is a “final state” scaling variable that controls kinematic boundary effects that affect $x_{\text{Feynman}}$ and $x_T$.

E and $E_{\text{max}}$ are energy of jet (particle) and maximum energy, respectively in the COM. $m_j$ is mass of jet (particle).

$m_{QN}$=mass to satisfy QN conservation
$\eta$ verses $x_R$

\[ \eta(x_r, s, p_T) = \ln \left( \frac{x_r \sqrt{s}}{2p_T} + \sqrt{\frac{x_r s}{4p_T^2} - 1} \right) \]

\[ \eta_{\text{max}} = \ln \left( \frac{\sqrt{s}}{2p_T} + \sqrt{\frac{s}{4p_T^2} - 1} \right) \]

Analyses in constant $\eta$ couples $p_T$ and $x_R$ so that the hard scattering part of $d^2\sigma/p_T dp_T d\eta$ that is characterized by $p_T$ is entangled with a change in $x_R$ – the kinematic boundary parameter.
Radial Scaling in Inclusive p-p $\pi^0$ Production

$$E \frac{d^3 \sigma}{dp^3} = F(s, p_T, x_R) \approx F(p_T, x_R) \sim A(p_T) f(x_R)$$

FIG. 2. The functions $f(x_R)$ calculated for twelve fixed $p_T$ values for the 80-mrad data are compared with each other and with the functional form $(1-x_R)^4$.

FIG. 3. The functions $g(p_T)$ calculated for four incident momenta at 80 mrad are compared with each other and with the functional form $(p_T^2 + 0.86)^{-4.5}$.

D. C. Carey, ... FET Phys. Rev. Lett. 33, No. 5, 327 (29 July 1974)
If there is a hard $2 \rightarrow 2$ scattering core by naive dimensional analysis then:

$$\frac{d\sigma(ab \rightarrow x)}{dQ^2} \sim \frac{1}{Q^4} \rightarrow \frac{d^2\sigma(pp \rightarrow Jets)}{p_Tdp_Tdy} \sim \frac{1}{p_T^4}$$

thus:

$$p_T^4 \left( \frac{d^2\sigma(pp \rightarrow Jets)}{p_Tdp_Tdy} \sim \frac{1}{p_T^4} \right) \sim F(x_R)$$

Note: Have approximated $\eta$ by $y$
Using $A(p_T) \sim p_T^{-4}$

Naively, does not indicate hard $2 \rightarrow 2$ scatterings – such as:

- $qq \rightarrow qq$
- $gg \rightarrow gg$
- $gq \rightarrow gq$

are dominating.

Note: plotted errors are statistical and systematic errors added in quadrature.
Try $A(p_T) \sim p_T^{-6}$

$A(p_T) = \frac{1}{\left(1 + \frac{p_T^2}{\Lambda^2}\right)^{n_{pT}/2}}$

$\Lambda = 0.01$ TeV, $n_{pT}/2 = 3.0$, $p_T$ in TeV

13 TeV $R=0.4$ ATLAS Inclusive Jets

$X_R = 2 p_T \cosh(y)/\sqrt{s}$
Other Measurements: ATLAS & CDF

1.0E-08
1.0E-07
1.0E-06
1.0E-05
1.0E-04
1.0E-03
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Λ = 0.01 TeV
n = 3.1

1.0E-08
1.0E-07
1.0E-06
1.0E-05
1.0E-04
1.0E-03
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Λ = 0.025 TeV
n = 3.25

1.0E-08
1.0E-07
1.0E-06
1.0E-05
1.0E-04
1.0E-03
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Λ = 0.01 TeV
n = 3.2

1.0E-08
1.0E-07
1.0E-06
1.0E-05
1.0E-04
1.0E-03
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Λ = 0.01 TeV
n = 3.5
Refine the Analysis as in 1976

The behavior of \( A(p_T) \) conveys information about the hard scattering and separates primordial hard scattering from fragmentation. Note that the limit \( x_R \to 0 \) is extrapolating behavior smaller than \( x_{R \text{min}} = 2p_T/vs \) and is effectively letting \( \sqrt{s} \to \infty \) for finite \( p_T \) with \( p_T >> \Lambda \).

Plot: \[
\frac{d^2\sigma}{dp_Tdp_Tdy} \sim A(p_T)(1-x_R)^{n_{xR}}
\]
for constant \( p_T \) as a function of \((1-x_R)\) to determine \( A(p_T) \).
Behavior of $\pi^0$

$$\frac{E d^3\sigma}{dp^3} \sim A(p_T) F(x_R)$$

$$A(p_T) \sim \left(\frac{1}{p_T}\right)^{7.02 \pm 0.23} \text{ for } p_T \geq 1.25 \text{ GeV}$$

$$F(x_R) \sim \left(1-x_R\right)^{4.0 \pm 1.0} \text{ (no } p_T \text{ cut)}$$

Table IV from FET et al. PRD 14, 5, 1217, (1976)
13 TeV ATLAS Jets – Constant $p_T$ vs. $(1-x_R)$

Find the same behavior as seen in the $\pi^0$ study 40 years ago.

$$\frac{d^2\sigma}{p_T dp_T d\eta} \sim A(p_T) (1-x_R)^{n_{xR}}$$

Now study the behavior of $A(p_T)$ and $n_{xR}$ as function of $p_T$, $\sqrt{s}$ and process $x_R < 0.9$
Fit Parameters 13 TeV ATLAS Inclusive Jets

13 TeV ATLAS $A(p_T)$

$$A(p_T) = \frac{(9.97 \pm 0.44) \times 10^{-5}}{p_T^{6.45 \pm 0.04}} \frac{pb}{GeV^2 (TeV)^{6.45}}$$

ATLAS $n_{xR}$ vs. $p_T$ 13 TeV

$$n_{xR}(p_T) = 3.877 \pm 0.077 + \frac{(0.671 \pm 0.021) TeV}{p_T}$$

ATLAS 13 TeV $n_{xR}$ from $\eta$ vs. $1/p_T$

Low $p_T$ Jets suppressed
Radial scaling analysis reveals systematic difference in $n(1/p_T)$.

Data:

- $\alpha = (1.608 \pm 0.434) \times 10^{-5}$
- $n_{pT} = 6.499 \pm 0.0125$

SHERPA:

- $\alpha = (1.895 \pm 0.353) \times 10^{-5}$
- $n_{pT} = 6.380 \pm 0.089$

Data: $D = 0.125 \pm 0.0112$
- $n_{0xR} = 3.03 \pm 0.16$

SHERPA: $D = 0.082 \pm 0.015$
- $n_{0xR} = 3.19 \pm 0.21$

SHERPA underestimates $D$.
Power Law in $p_T$ not ‘Perfect’

ATLAS 13 TeV R=0.4 A($p_T$) vs. $p_T$

Fit is good over 8 decades but there is a systematic deviation from the power law of ± 20%
A(\(p_T\)) for Single Particle Inclusive Production in p-p Collisions

\[ p + p \rightarrow \pi^+ + X, \ldots, \text{from F.E.T. et al. PRD 14, 1217 (1976)} \]

\( p_T \) power law \( p_T > 1.25 \text{ GeV} \)

F. E. Taylor MIT

1/24/2017
$A(p_T)$ Single Particle Inclusive Production in p-$p$

$p + p \rightarrow K^+ + X \ldots$, from F.E.T. et al. PRD 14, 1217 (1976)
Summary of $p_T$ Power Law using Radial Scaling

$n_{pT}$ vs. Process

$n_{pT}$ seems ~ independent of process ($\gamma$?) over a wide range of $\sqrt{s}$ and $\neq 4$. 

Index | Process | $\sqrt{s}$ (TeV) | $n_{pT}$ | Error
---|---|---|---|---
1 | Ref[1] $\pi^+$ 10 GeV to 63 GeV ($p_T>1.25$ GeV) | 0.063 | 6.34 | 0.09
2 | Ref[1] $\pi^0$ 10 GeV to 63 GeV ($p_T>1.25$ GeV) | 0.063 | 7.02 | 0.23
3 | Ref[1] $\pi^-$ 10 GeV to 63 GeV ($p_T>1.25$ GeV) | 0.063 | 6.38 | 0.06
4 | Ref[1] $K^+$ 10 GeV to 63 GeV ($p_T>1.25$ GeV) | 0.063 | 5.83 | 0.29
5 | Ref[1] $K^-$ 10 GeV to 63 GeV ($p_T>1.25$ GeV) | 0.063 | 6.11 | 0.08
6 | Ref[1] $p_{bar}$ 10 GeV to 63 GeV ($p_T>1.25$ GeV) | 0.063 | 6.65 | 0.67
7 | DO: Inclusive Jets $p_{bar}$-p 1.80 TeV | 1.800 | 6.75 | 0.12
8 | DO: Inclusive Jets $p_{bar}$-p 1.96 TeV | 1.960 | 6.84 | 0.04
9 | CDF: InclusiveJets $p_{bar}$-p 1.96 TeV | 1.960 | 7.31 | 0.16
10 | ATLAS: Inclusive Jets p-p 2.76 TeV | 2.760 | 6.46 | 0.12
11 | ATLAS: Inclusive Jets p-Pb Pb-forward 5.02 TeV | 5.020 | 6.78 | 0.21
12 | ATLAS: Inclusive Jets p-Pb p-forward 5.02 TeV | 5.020 | 6.62 | 0.23
13 | ATLAS: Inclusive Jets p-p 7 TeV | 7.000 | 6.50 | 0.12
14 | CMS: Prompt $\gamma$ | 7.000 | 5.24 | 0.03
15 | CMS: InclusiveJets p-p ($p_T<1.95$ TeV) 8 TeV | 8.000 | 6.80 | 0.05
16 | ATLAS: Prompt $\gamma$ | 8.000 | 5.68 | 0.03
17 | ATLAS: InclusiveJets p-p 13 TeV | 13.000 | 6.46 | 0.04
18 | CMS: InclusiveJets p-p ($p_T<1.38$ TeV) | 13.000 | 6.37 | 0.08
19 | MC: InclusiveJets p-p SHERPA 7 TeV | 7.000 | 6.38 | 0.09


$\langle n_{pT} \rangle = 6.5 \pm 0.5$
Line Counting, Higher Twists, Diquarks

• Dimensional Analysis \( M \sim [\text{cm}]^{n_A - 4} \)

\[ \frac{d^2 \sigma}{p_T dp_T dy} \sim \frac{1}{p_T^4} \]

\( n_A = 4 \quad 2 \to 2 \) scattering

HIDDEN \( x_R \to 0 \)

\[ \frac{d^2 \sigma}{p_T dp_T dy} \sim \frac{1}{p_T^6} \]

\( n_A = 5 \quad 2 \to 3 \) scattering

DOMINATES \( x_R \to 0 \)

\[ \frac{M^2}{s^2} \]

\[ \frac{1}{p_T^{2n_A - 4}} \]
s-dependence of ATLAS Inclusive Jets

A(pT) vs. pT (TeV)

ATLAS Jets n_{xR}(1/p_T)

- 2.76 TeV ATLAS
- 7 TeV ATLAS
- 13 TeV ATLAS

A(p_T) (TeV \cdot p_T \cdot pb/GeV^2)

n_{xR}(1/p_T)

0.01 0.10 1.00 10.00

0 10 20 30 40 50

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s-dependence of $p_T$ – dependence of jets

$$A(p_T) = \frac{\alpha}{p_T^{n_{p_T}}}$$

\[
\langle n_{p_T} \rangle = 6.7 \pm 0.3
\]
Rapid growth with $\sqrt{s}$! What will be the $D$ value at $\sqrt{s} = 100$ TeV? Probably related to $N_{\text{Jets}}(s)$ and multiple parton scatterings.

$$ (1 - x_R)^{D/p_T + n_{0xR}} $$
Check of Rapidity Distribution of Jets

- Fit: $p_T > 0.1$ TeV with numerical integration of fit function un-normalized.

$$\frac{d^2\sigma}{p_T dp_T d\eta} \sim A(p_T)(1-x_R)^n$$

- Data:

$$\frac{dN}{d\eta} \sim \sum_i \frac{d^2\sigma_i}{p_{Ti} dp_T d\eta} p_{Ti} \Delta p_T$$

13 TeV ATLAS $dN/d\eta$
\[ \frac{d\sigma}{d\eta} = \int_{p_{T\text{min}}}^{p_{T\text{max}}} \frac{d^2\sigma}{p_Tdp_Td\eta}p_Tdp_T = \int_{p_{T\text{min}}}^{p_{T\text{max}}} \frac{a}{p_T^{n_{pT}}} \left(1 - \frac{2p_T}{\sqrt{s}} \cosh(\eta)\right)^{n_{xR}} p_Tdp_T \]

\[ \frac{d\sigma(p_{T\text{min}}, p_{T\text{max}})}{d\eta} = aF\left(p_{T\text{min}}, p_{T\text{max}}, \frac{\cosh(\eta)}{\sqrt{s}}\right) \]

\( p_{T\text{min}} \) is the minimum transverse momentum cut \( (p_T \geq p_{T\text{min}}) \)

For fixed \( p_{T\text{min}} \) and parameter \( a \), all \( \eta \) dependence through \( \cosh(\eta)/\sqrt{s} \)
Pseudo-rapidity Plateau in Toy Model

Width of plateau controlled by kinematic limit:

\[
\eta_{\text{max}} = \ln \left( \frac{\sqrt{s}}{2p_T} + \sqrt{\frac{s}{4p_T^2} - 1} \right)
\]

\[dN/d\eta\] on plateau \(\eta = 0\) grows by kinematics – (no QCD required)
dN/dη is a function of cosh(η)/√s

Have perfect cosh(η)/√s scaling if all s-dependence in x_R term

First shown in (1979): “Interpretation of the Rise in Central Rapidity Density in Terms of Radial Scaling”,

R. W. Ellsworth,

16th International Cosmic Ray Conference, Vol. 7. Published by the Institute for Cosmic Ray Research, University of Tokyo

http://adsabs.harvard.edu/abs/1979ICRC....7..333E

Toy Model

n_pT = 6
n_xR = 4
p_{Tmin} = 10 GeV
Pseudo-rapidity Distribution for Measured Jets

Used the fits of the inclusive jet cross sections: \{\alpha(\sqrt{s}), npT(\sqrt{s}), D(\sqrt{s}), n_{0xR}(\sqrt{s})\} CDF & ATLAS

Rapidity Plateau (Parameters for pT> 100 GeV)

\[ \frac{d\sigma}{d\eta} \]

1.96 TeV
2.76 TeV
7 TeV
13 TeV

\[ \frac{d\sigma}{d\eta} = \frac{\cosh(\eta)}{\sqrt{s}} \]

Rapidity Plateau (Parameters for pT> 100 GeV)

13 TeV is different because of large ‘D’ term for \( nxR = D/pT+n0xR \)
PHOBOS $dN/d\eta$

B.B. Black, et al.
arXiv:nucl-ex/0509034v1 28 Sep 2005
B-field = 0 (very low $p_{T\text{min}}$)

$dN/d\eta$ Phobos-RHIC Ag-Ag 6% most central collisions

$s$-dependence confined to $\cosh(\eta)/\sqrt{s} < 0.2$

Region of scaling is high $\eta$. Note that $\cosh(\eta)/\sqrt{s}$ scaling similar to $\eta'$ scaling – see backup.
What about the $x_R$-Dependence

• Inclusive cross section roughly factorizes: $\sigma \sim A(p_T) (1-x_R)^{n_{xR}}$

• Would expect that $n_{xR} = n_{xR}(s, p_T, \text{process})$ to characterize the fragmentation and hadronization of primordial quark/gluon.

• Quark line-counting rules suggest $n_{\text{spectator}}$, the number of non-participating quarks in the primary collision, controls the $(1-x_R)$ power:

$$\frac{d^2 \sigma}{p_T dp_T dy} \sim A(p_T)(1-x_R)^{2n_{\text{spectator}}-1}$$
Summary of $(1-x_R)^{n_{xR}}$ Power

Notes:

1. Qualitatively $n_{xR} \approx 2 n_{\text{spectator}} - 1$

2. In cases where $n_{xR}$ is roughly independent of $p_T$ the average values and standard deviations are plotted.

3. In cases where there is a significant $1/p_T$ dependence the value $n_{xR0}$ is plotted, where: $n_{xR}(1/p_T) = D/p_T + n_{xR0}$ and the error of $n_{xR0}$ is shown.

4. Caveat: $J/\psi$ data show inconsistencies among experiments. Trend shown is consistent but details not clear. See backup.
Applications of Radial Scaling

• Heavy Ions – particles and jets
  • Examine the $p_T$, $x_R$ and $y$ dependence – differences with p-p would indicate ‘heavy ion physics’
    • Naively $p_T$ dependence should be the same in p-p, p-HI and HI-HI collisions
    • $n_{xR}$ perhaps different and would be sensitive to a different hadronization and/or jet quenching

• Inclusive Charm Production
  • Several sources of $J/\psi$ – direct production and feed-down from bottom decays
    • Heavy quarkonium production a test of non-relativistic QCD effective field theory
    • $\psi(2S)$ essentially free from feed-down decays of higher mass quarkonium states
    • Should be able to measure the mass of parent in decay production by $\Lambda$ term in $p_T$ spectrum

\[
A(p_T) = \alpha_0 \frac{\Lambda^{n_{pT} \rightarrow 4}}{\left(\Lambda^2 + p_T^2\right)^{n_{pT}/2}}
\]
BRAHMS $\pi^+$ from Ag-Ag Collisions 62.4 GeV

\[ A(p_T) = \alpha_0 \frac{\Lambda_{n_T}^{n_T-4}}{\left(\Lambda^2 + p_T^2\right)^{n_T/2}} \]

$\Lambda = 0.56 \pm 0.05$ GeV

$np_T = 5.66 \pm 0.03$

$y = 6.6926 \times 10^2 e^{-4.4104 \times 10^0 x}$

$R^2 = 9.9669 \times 10^{-1}$

$nxR(1/pT)$ vs. $1/pT$

$nxR = (3.33 \pm 0.18) / pT (GeV) + 1.2 \pm 0.3$

$y = 1.5850 \times 10^1 x - 5.6568 \times 10^0$

$R^2 = 9.9966 \times 10^{-1}$

$y = 3.329 x + 1.204$

$R^2 = 0.953$

F. E. Taylor MIT
A($p_T$) for 5.02 TeV p-Pb Inclusive Jets

ATLAS 5.02 TeV proton side $A(p_T)$ vs. $p_T$

$y = 7.3105E-13x^6.6229E+00$
$R^2 = 9.8370E-01$

ATLAS 5.02 TeV Pb side $A(p_T)$ vs. $p_T$

$y = 8.5602E-13x^{-6.7820E+00}$
$R^2 = 9.9085E-01$

$A(p_T)$ has same shape Pb vs. p
Evidence of Jet Quenching p-Pb Collisions

Interpretation:
Jet co-moving with nuclear remnant undergoes multiple interactions which soften its xR dependence.

Jet co-moving with proton remnant does not experience ‘extra’ interactions – hence xR distribution is the same as p-p scattering.

Using $p_T$ and $x_R$ makes this distinction quite obvious.

Low $p_T$ Jets suppressed like p-p jets would be at $\sqrt{s} = 10$ TeV
p-p, p-A, A-A scattering: Analogous Behavior

All behave: \[
\left(1 - x_R \right) \left( \frac{D}{p_T} + n_{0xR} \right)
\]
Jet strongly attenuated on approach to kinematic boundary because of large “D” term

\[ R = R(p_{T_{Low}}) = R(p_{T_{High}}) = \frac{(1 - x_{R2})}{(1 - x_{R1})} \]

“Beam fragmentation region” augmented by increasing √s and/or by increasing beam A in Heavy Ion Collisions.

Jet quenching in both cases. Same Physics?

Jet less strongly attenuated on approach to kinematic boundary because “D” term -> 0
CHARM Production at LHC

- Can separate ‘prompt’ production $\tau(\mu\mu) \sim 0$ from ‘non-prompt’ production where $\tau(\mu\mu) > 0$.
- Can separately measure $J/\psi$ and $\psi(2S)$.
- Can estimate the mass of the parent particle by shape of $p_T$-spectrum at low $p_T$.
- ATLAS, CMS and LHCb contribute but data seem inconsistent. See backups.

$$A(p_T) = \alpha_0 \frac{\Lambda^{n_{p_T}-4}}{\left(\Lambda^2 + p_T^2\right)^{n_{p_T}/2}}$$
CHARM – Prompt & Non-Prompt p-p Data

ATLAS A(pT) vs. pT 5.02 TeV prompt J/Psi

\[ y = 1.321E+06x - 6.976E+00 \]
\[ R^2 = 9.994E-01 \]

ATLAS 5.02 TeV Non-prompt J/Psi

\[ y = 3.572E+05x - 6.402E+00 \]
\[ R^2 = 9.989E-01 \]

\[ \Lambda = 3.6 \pm 0.3 \text{ GeV} \]

\[ \Lambda = 7.1 \pm 0.9 \text{ GeV} \]

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F. E. Taylor MIT
### Summary of $p_T$ Power Law with Form Factor

\[ A(p_T) = \alpha_0 \frac{\Lambda^{n_{pT}-4}}{\left(\Lambda^2 + p_T^2\right)^{n_{pT}/2}} \]

- **Form factor parameter $\Lambda$**: proportional to the mass of the parent particle for heavy quark production in quadrature with intrinsic $k_T$.

#### Table of Results

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<td>Ref[1] $p+$ 10 GeV to 63 GeV</td>
<td>0.063</td>
<td>0.602</td>
<td>0.012</td>
<td>6.93</td>
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<tr>
<td>4</td>
<td>Ref[1] $p+$ 10 GeV to 63 GeV</td>
<td>0.063</td>
<td>0.613</td>
<td>0.054</td>
<td>6.04</td>
<td>0.12</td>
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<td>5</td>
<td>Ref[1] $K+$ 10 GeV to 63 GeV</td>
<td>0.063</td>
<td>0.776</td>
<td>0.091</td>
<td>6.58</td>
<td>0.09</td>
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<tr>
<td>6</td>
<td>Ref[1] $\bar{p}$ bar 10 GeV to 63 GeV</td>
<td>0.063</td>
<td>0.892</td>
<td>0.071</td>
<td>6.79</td>
<td>0.28</td>
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<td>7</td>
<td>BRAHMS RHIC $p+$ Ag-Ag</td>
<td>0.062</td>
<td>0.56</td>
<td>0.05</td>
<td>5.66</td>
<td>0.03</td>
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<td>0.56</td>
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<tr>
<td>8</td>
<td>ATLAS: prompt $J/\psi$</td>
<td>5.020</td>
<td>3.57</td>
<td>0.25</td>
<td>6.98</td>
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<td>9</td>
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<td>7.000</td>
<td>3.25</td>
<td>1.20</td>
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<td>3.57</td>
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<td>6.68</td>
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<td>11</td>
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<td>3.01</td>
<td>1.22</td>
<td>6.34</td>
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<td>12</td>
<td>LHCb: prompt $J/\psi$</td>
<td>13.000</td>
<td>4.44</td>
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<td>7.02</td>
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<td>ATLAS: prompt $\psi(2S)$</td>
<td>7.000</td>
<td>4.10</td>
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<td>1.10</td>
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<td>5.020</td>
<td>7.10</td>
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<td>16</td>
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<td>LHCb: non-prompt $J/\psi$</td>
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<td>4.62</td>
<td>0.24</td>
<td>5.72</td>
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<td>19</td>
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<td>7.000</td>
<td>4.10</td>
<td>2.00</td>
<td>5.58</td>
<td>0.05</td>
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<td>20</td>
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<td>8.000</td>
<td>11.40</td>
<td>0.10</td>
<td>6.83</td>
<td>0.13</td>
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<td>7.75</td>
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</table>

**Notes:**
- $\psi(2S)$: 3.686 GeV
- $\text{BR}(\psi(2S) \rightarrow J/\psi(1S))$ 60%

**References:**
Observations through the Prism of Radial Scaling

• Inclusive jet production at the LHC is quite similar to light quark single particle inclusive production studied > 40 years ago.

• The $p_T$-dependence of the invariant inclusive cross sections seems to be independent of process and energy over a wide range as a power law: $1/p_T^{(6.5 \pm 0.5)}$ in the limit $x_R \to 0$.

• The $x_R$ dependence is consistent with a power law $(1-x_R)^{n_{xR}}$, where $n_{xR}$ is qualitatively dependent on the number of spectator quarks as well as $p_T$ and $\sqrt{s}$s at high $\sqrt{s}$s. At high $\sqrt{s}$s and HI collisions (Charm ?) $n_{xR} = D/p_T+n_{xR0}$.

• Inclusive Charm in p-p collisions has the same behavior as $\pi^+$ and jets in heavy ion collisions.

• Radial scaling determines the pseudo-rapidity plateau and provides a separation of rise of the central plateau by kinematics from pQCD by means of the scaling variable $\cosh(\eta)/\sqrt{s}$.
The $p_T$-dependence of jets/particles again - 3 views

- pQCD agrees with data – so why care that $1/p_T^6$ dominates rather than $1/p_T^4$:
  - The underlying paradigm of the standard model works.
  - Jets and single particles in p-p collisions are governed by the same physics.
  - But there are 10’s of tuned parameters and a mound of processes contributing. How unique?
  - Is there a minimum set of parameters sufficient? Simulations are tuned to data.

- There is a diquark in the nucleon that is either intrinsic or emergent:
  - Hence the 2→3 scattering dominates to make the $1/p_T^6$ dependence.
  - Lattice QCD and Jlab proton form factor data give evidence of a diquark system inside the proton.
  - But what about single $\gamma$ production where $n_{p_T} \sim 5.6$?
  - How can Charm and anti-proton production also come from (exotic) diquarks?

- The ‘extra’ $p_T$ powers come from $p_T$ dependence in the fragmentation and hadronization:
  - The $p_T$-dependence is really not a power law but something that looks like one and can be fit by a quadratic in $\log(p_T) \sim \log$-normal
  - Single $\gamma$ is different because there is no fragmentation and hadronization.
  - Why does this work so well – why so precocious in $\sqrt{s}$?
A formulation is given of inclusive Jet production in p-p collisions that controls the kinematic boundary so that the underlying dynamics can be studied:

$$\frac{d^2\sigma}{dp_T^2 dy}(s,m) = \alpha_0(s) \Lambda(s)^{n_{pT} - 4} \left( \frac{\Lambda(s)^2 + p_T^2}{2} \right)^{n_{pT}} \left(1 - \frac{2\sqrt{p_T^2 \cosh^2(y)(1 + (m^2/p_T^2) \tanh^2(y)) + m^2}}{\sqrt{s - m_{QN}^2}} \right) \frac{D(s)}{p_T^{n_{xR} R^0}}$$

- Can be applied to jets as well as single particle inclusive production.
- Formulation seems useful in studying heavy ion collisions.
- Surprising that such a simple idea works so well – but controlling known kinematic boundary effects would be the first thing one would do.
“To travel hopefully is a better thing than to arrive” - RLS

• In looking at LHC data I found considerable differences between experiments that claim to measure the same thing:
  • For example ATLAS, CMS and LHCb all have data on $J/\psi$ prompt and non-prompt production. The data are not consistent – perhaps because of different acceptance corrections, etc.
  • I recommend that experiments compare data and plots and work on understanding the differences in the measurements – they may reveal new physics.
  • Small inconsistencies can be leads to better understandings.

• Many studies are of limited kinematic range – for example $Z$ production in either a limited range of $|y|$ or integrated over a wide range in $y$. Neither case is useful for determining the fine-grained systematics of the process and in comparing to other measurements.
  • Measure processes over a wide kinematic range & post cross sections on web.

• Conclusions are frequent stated as such: “Our data agree with simulations of NNLO with parton set XYZ” or “with the model given in Ref[25]”.
  • Where is the physics? Experimentalists should not be shy in interpreting results. That should encourage theorists to get it right and make it understandable.
Backup
The spirit of this study is to see how far a simple idea could be applied to LHC and other data without sophisticated analysis machinery in order to uncover patterns – if they exist

- No ‘raw’ data were used – all information from the public domain
- Excel was used for tabulation and plotting
- Mathematica was used to determine closed-form expressions
- When available tabulated data were used but when not available plots were scanned using ImageJ – freeware distributed by NIH. The accuracy of scanned plots is estimated to be < 1%.
- Numerical integrations were calculated by simple sums
- Parameter errors were underestimated – fits of power laws were performed in linearized expressions using LINEST – an Excel fitting program of the central values without systematic or statistical errors but the resultant error reflects the fluctuations of points with equal weight about the fitted form.
Parton-Parton Elastic Scattering – 2 Examples

Functions of the Mandelstam variables $s$, $t$, $u$ and $\alpha_s$. All have dimensions of $(\text{energy})^{-4}$.

\[
\frac{d\hat{\sigma}(\hat{s},\hat{t},\hat{u};ud \rightarrow ud)}{d\hat{t}} = \frac{4\pi\alpha_s^2}{9\hat{t}^2} \frac{\hat{s}^2 + \hat{u}^2}{\hat{s}_t^2}
\]

\[
\frac{d\hat{\sigma}(\hat{s},\hat{t},\hat{u};gg \rightarrow gg)}{d\hat{s}} = \frac{9\pi\alpha_s^2}{2\hat{s}_t^2} \left(3 - \frac{\hat{t}\hat{u}}{\hat{s}_s} - \frac{\hat{s}\hat{u}}{\hat{t}_s} - \frac{\hat{s}\hat{t}}{\hat{u}_u}\right)
\]

\[
\hat{s} = (p_a + p_b)^2 = \frac{s}{4}(x_1 + x_2)^2
\]

\[
\cos \theta = \left(1 - \frac{p_T^2}{\hat{s}}\right)^{1/2}
\]

\[
\hat{t} = -\frac{\hat{s}}{2}(1 - \cos \theta)
\]

\[
\hat{u} = -\frac{\hat{s}}{2}(1 + \cos \theta)
\]
PDF and DGLAP Evolution and Splitting Functions

Parton Distribution Functions (mostly from DIS Lepton-Nucleon Scattering):

DGLAP evolution and splitting functions:

\[ \frac{\partial f_a}{\partial \ln \mu^2} \sim \frac{\alpha_s(\mu^2)}{2\pi} \sum_b \left( P_{ab} \otimes f_b \right) \]

\[ P_{qs}(x) = \frac{1}{2} \left[ x^2 + (1-x)^2 \right] \ldots \]

These *10s of parameters and factors* are put together in simulations of inclusive jet production at the LHC.
$A(p_T)_{jets}$: Power law $1/p_T^{n p T}$ or Quadratic in $\log(p_T)$?

- Power law:
  \[
  \log(A(p_T)) = b_0 \log(p_T) + c_0
  \]
- Or a quadratic in $\log(p_T)$:
  \[
  \log(A(p_T)) = a \log^2(p_T) + b \log(p_T) + c
  \]

- Note: $-b_0$ and $-b$ seem to converge to $n_{pT} \approx 6.5$ (no evidence of $1/p_T^4$ term)

<table>
<thead>
<tr>
<th>log-log fits</th>
<th>$1/\sqrt{s}$</th>
<th>$\sqrt{s}$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$b_0$</th>
<th>$c_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear (a)</td>
<td>0.510</td>
<td>1.960</td>
<td>-1.326</td>
<td>-9.275</td>
<td>-6.920</td>
<td>-7.310</td>
<td>-6.286</td>
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<td>Linear (b)</td>
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<td>2.760</td>
<td>-0.914</td>
<td>-8.308</td>
<td>-6.269</td>
<td>-6.406</td>
<td>-5.463</td>
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<td>Linear (c)</td>
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<td>-7.730</td>
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<td>Linear (b0)</td>
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<td>-6.908</td>
<td>-4.021</td>
<td>-6.496</td>
<td>-4.002</td>
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</table>

\[y = a \log^2(p_T) + b \log(p_T) + c\]
Integrate over $x_R$ to find $p_T$ Dependence

- J. Thaler suggested:

$$\frac{1}{p_T^{neff}} \sim \int_{x_{R\text{min}}}^{1} \frac{d^2\sigma}{p_Tdp_Tdy} \left( \begin{array}{c} p_T \\ y \\ p_T \\ x_R \end{array} \right)_J dx_R$$

$$= \int_{x_{R\text{min}}}^{1} \frac{d^2\sigma}{p_Tdp_Tdy} \frac{2}{\sqrt{x_R^2 - x_{R\text{min}}^2}} dx_R$$

$$x_{R\text{min}} = \frac{2p_T}{\sqrt{s}}$$

$n_{p_T}$ increases from 6.0 to 6.45. Tested with toy model.

Interesting suggestion – integration can be extended to determine the moments of the “fragmentation” function $(1-x_R)^{nx_R}$. 

$p_T$ - Dependence of Integral over $x_R$
Λ vs. Process

Vs (TeV) and Λ vs. Process

- ATLAS: non-prompt y(2S)
- ATLAS: non-prompt y(2S)
- LHCb: non-prompt J/y
- ATLAS: non-prompt J/y
- ATLAS: non-prompt J/y
- ATLAS: non-prompt J/y
- ATLAS: prompt y(2S)
- ATLAS: prompt y(2S)
- LHCb: prompt J/y
- ATLAS: prompt J/y
- CMS: prompt J/y
- ATLAS: prompt J/y
- ATLAS: prompt J/y
- ATLAS: prompt J/y
- BRAHMS RHIC p+ Ag-Ag
- Ref[1] p_bar 10 GeV to 63 GeV
- Ref[1] K- 10 GeV to 63 GeV
- Ref[1] K+ 10 GeV to 63 GeV
- Ref[1] p- 10 GeV to 63 GeV
- Ref[1] p0 10 GeV to 63 GeV
- Ref[1] p+ 10 GeV to 63 GeV
• Valence q-anti-q scattering/annihilation

D0 1.96 TeV 3 or more points

\[ y = 1.37676 \times 6.84012 \times 10^{-6} \times x \]
\[ R^2 = 9.99373 \times 10^{-1} \]

1.96 TeV CDF A(pT)

\[ y = 5.178 \times 7.310 \times 10^{-7} \times x \]
\[ R^2 = 9.941 \times 10^{-1} \]
Inclusive Jet Production $p(p\_\text{bar})$-$p$ Scattering

$\approx 2.5$ at high $p_T$
s-dependence in Perfect Radial Scaling

• In perfect radial scaling entire s-dependence is in the $x_R$ term:

$$x_R = \frac{E}{E_{\text{max}}} \approx \frac{2p_T \cosh(\eta)}{\sqrt{s}} \approx \frac{2p_T \cosh(y)}{\sqrt{s}} \sqrt{1 + \frac{m_j^2}{p_T^2} \tanh(y)}$$

$$\frac{d^2 \sigma}{p_T dp_T dy} \sim A(p_T)(1 - x_R)^{n_{xR}}$$

• This is roughly true for $\pi^0$ production in E63 ($10 < \sqrt{s} < 27$ GeV) but is broken by QCD evolution.

• Studying cross sections using $x_R$ makes QCD evolution clear since radial scaling controls kinematic boundary.
An Example of $x_R$-dependence near Kinematic Boundary

- CMS Inclusive Jets 8 TeV / 7 TeV

![Graphs showing comparison between CMS Preliminary and Toy Model results for different $y$ and $p_T$ values.](image-url)
Agreement generally good over most of the $y$-region except at high rapidity.
Deviations from $p_T$ Power Law

13 TeV ATLAS Residuals of Power Law

$y = -2.438E-01x^2 - 4.134E-01x - 2.560E-02$

$R^2 = 8.408E-01$

5.02 TeV ATLAS p-Pb p-forward

$y = -2.532E+00x^2 - 3.303E+00x - 3.244E-01$

$R^2 = 6.036E-01$

1.96 TeV CDF

$y = -2.7659x^2 - 3.9795x - 1.1805$

$R^2 = 0.3486$

2.76 TeV ATLAS

$y = -2.1773x^2 - 4.4319x - 1.9037$

$R^2 = 0.5859$

7 TeV ATLAS Residuals vs. log($p_T$)

$y = -3.3871E-01x^2 - 1.0812E+00x - 2.3180E-01$

$R^2 = 8.2603E-01$

1/24/2017

F. E. Taylor MIT
13 TeV CMS Inclusive Jets R=0.4

• Compared to Theory
• LHS: NLOJET++ based on the CT14 PDF (similar to ATLAS)
• RHS: POWHEG(PH) + PYTHIA8 (P8)
• Data set quite similar to the 13 TeV ATLAS inclusive jets

https://hepdata.net/record/ins1459051
13 TeV CMS Inclusive Jets

CMS jet reconstruction seems to have large uncorrected systematic errors – OR power law in \((1-xR)\) not a good model

\[
x_R < 0.9
\]

\[
n_{pT} = 6.366 \pm 0.0076
\]
# s-dependence – CDF, D0, ATLAS, CMS

<table>
<thead>
<tr>
<th>Process</th>
<th>$\sqrt{s}$</th>
<th>$\alpha , (\text{TeV}^{npT} , \text{pb}/\text{GeV}^2)$</th>
<th>error</th>
<th>$n_{pT}$</th>
<th>error</th>
<th>$D , (\text{TeV}^{-1})$</th>
<th>error</th>
<th>$n_xR$</th>
<th>error</th>
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<td>Inclusive Jets p-bar-p CDF</td>
<td>1.960</td>
<td>5.178E-07</td>
<td>1.694E-07</td>
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<tr>
<td>Inclusive Jets p-bar-p D0</td>
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<td>1.377E-06</td>
<td>1.262E-07</td>
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<td>0.044</td>
<td>0.022</td>
<td>0.015</td>
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<td>Inclusive Jets p-p ATLAS R=0.4</td>
<td>2.760</td>
<td>3.447E-06</td>
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<td>3.295</td>
<td>0.288</td>
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<tr>
<td>Inclusive Jets p-p ATLAS R=0.4</td>
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<td>1.608E-05</td>
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<td>0.125</td>
<td>0.125</td>
<td>0.011</td>
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<tr>
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<td>No fit</td>
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<td>0.672</td>
<td>0.021</td>
<td>3.875</td>
<td>0.077</td>
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</table>
Compilation of $A(p_T)$ for Various Jet Studies

$A(p_T)$ 13 TeV ATLAS

$y = 9.9607 \times 10^{-6.456}\ E+00$

$R^2 = 9.9889 \times 0.01$

$A(PT)$ 7 TeV ATLAS

$y = 1.608 \times 10^{-6.499}\ E+00$

$R^2 = 9.949 \times 0.01$
Compilation of $A(p_T)$ for Various Jet Studies -2

ATLAS 2.76 TeV $A(p_T)$

$y = 3.447E-06x^{6.461E+00}$
$R^2 = 9.970E-01$

1.96 TeV CDF $A(p_T)$

$y = 5.178E-07x^{-7.310E+00}$
$R^2 = 9.941E-01$

1/24/2017
F. E. Taylor MIT
A(p_T) for 5.02 TeV p-Pb Inclusive Jets

**Proton Forward**

\[ y = -0.7076x^2 - 9.065x - 29.346 \]

\[ R^2 = 0.9925 \]

\[ y = -6.6229x - 27.944 \]

\[ R^2 = 0.9837 \]

**Pb Forward**

\[ y = -0.794x^2 - 9.3363x - 29.336 \]

\[ R^2 = 0.9977 \]

\[ y = -6.782x - 27.786 \]

\[ R^2 = 0.9909 \]

A(pT) has same shape Pb vs. p
Single photons are separated from background by an isolation cut. In a cone R=0.4 the $E_{T\text{iso}} < 4.8 \text{ GeV} + 4.2 \times 10^{-3} E_{T\gamma}$

Prompt photons are either from direct sources of the primordial scattering or from parton bremsstrahlung.
Prompt $\gamma$ Production ATLAS 8 TeV

ATLAS 8 TeV Direct $\gamma$

$n$ vs. $ET$

$y = 3.9203 \times 10^{1.681}$
$R^2 = 0.1351$

$y = 0.199 \times 10^{0.9601}$
$R^2 = 0.176$

$y = 0.0037 \times 10^{4.8035}$
$R^2 = 0.3389$

$y = 0.0002 \times 10^{4.5694}$
$R^2 = 0.6259$

$y = 1 \times 10^{-0.5}$
$R^2 = 0.8565$

$y = 1 \times 10^{-0.06}$
$R^2 = 0.9068$

$y = 1.1901 \times 10^{1.0}$
$R^2 = 9.9949 \times 10^{-01}$

$n \approx 4$
Isolated Prompt $\gamma$ Production CMS 7 TeV

S. Chatrchyan et al.
PHYSICAL REVIEW D 84, 052011 (2011)
Isolated Prompt $\gamma$ Production CMS 7 TeV

$y = 9.5930 E +05x^{-5.2374 E +00}$

$R^2 = 9.9943 E -01$

Isolation cut $E < 5$ GeV in $R=0.4$ cone

CMS 7 TeV prompt photon

$n_{XR}$ vs. $p_T$

$n \approx 3$
$n_{XR}$: Inclusive Jet Production $p$-$p$ Scattering (1976)

$\pi^+$

$\pi^-$

$\pi^0$

$K^+$

$K^-$

$p_{\bar{p}}$

$n_{\text{spectator}} \approx 2.5$

$n_{\text{spectator}} \approx 2.5$

$n_{\text{spectator}} \approx 2.5$

$n_{\text{spectator}} \approx 2.5$

$n_{\text{spectator}} \approx 4.5$

$n_{\text{spectator}} \approx 5.5$
# Table of $(1-x_R)$ Powers

<table>
<thead>
<tr>
<th>Index</th>
<th>Process</th>
<th>$\sqrt{s}$ (TeV)</th>
<th>nxR</th>
<th>error</th>
<th>$&lt;nxR&gt;$</th>
<th>nxR0</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>$\pi^+$ 10 GeV to 63 GeV</td>
<td>0.063</td>
<td>4.1</td>
<td>1.6</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td>$\pi^0$ 10 GeV to 63 GeV</td>
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<td>4.0</td>
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<tr>
<td>3</td>
<td>$\pi^-$ 10 GeV to 63 GeV</td>
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<td>5.5</td>
<td>1.4</td>
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<tr>
<td>4</td>
<td>$K^+$ 10 GeV to 63 GeV</td>
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<td>3.9</td>
<td>1.8</td>
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<tr>
<td>5</td>
<td>$K^-$ 10 GeV to 63 GeV</td>
<td>0.063</td>
<td>7.4</td>
<td>1.6</td>
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<td>6</td>
<td>$p_\bar{p}$ 10 GeV to 63 GeV</td>
<td>0.063</td>
<td>10.7</td>
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<tr>
<td>7</td>
<td>DO: Inclusive Jets $p_\bar{p}$-p 1.96 TeV</td>
<td>1.960</td>
<td>4.0</td>
<td>0.1</td>
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<td></td>
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<td>8</td>
<td>CDF: Inclusive Jets $p_\bar{p}$-p 1.96 TeV</td>
<td>1.960</td>
<td>3.6</td>
<td>0.2</td>
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<td>9</td>
<td>ATLAS: Inclusive Jets p-p 2.76 TeV</td>
<td>2.760</td>
<td>3.3</td>
<td>0.3</td>
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<td>10</td>
<td>ATLAS: Inclusive Jets p-Pb Pb-forward 5.02 TeV</td>
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<td>0.4</td>
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<td>2.8</td>
<td>0.6</td>
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<tr>
<td>12</td>
<td>ATLAS: Inclusive Jets p-p 7 TeV</td>
<td>7.000</td>
<td>3.0</td>
<td>0.2</td>
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<td>13</td>
<td>CMS: Inclusive Jets p-p (pT&lt;1.95 TeV) 8 TeV</td>
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<td>3.7</td>
<td>0.1</td>
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<tr>
<td>14</td>
<td>ATLAS: Inclusive Jets p-p 13 TeV</td>
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<td>0.1</td>
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<td>MC: Inclusive Jets p-p SHERPA 7 TeV</td>
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<td>0.2</td>
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<td>CMS: Prompt $\gamma$</td>
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<td>0.2</td>
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<tr>
<td>21</td>
<td>ATLAS: non-prompt $J/\psi$</td>
<td>7.000</td>
<td>23.7</td>
<td>1.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
y vs. $\eta$ 13 TeV Jets

- ATLAS 13 TeV jets used $y$-bins. Thus to determine $x_R$ one has to know the jet mass, $m_j$; but $m_j$ has been integrated out in the data analyzed.

$$m_j^2 = (\Sigma p_i)^2$$

$$\frac{1}{\sin(\theta)} = \cosh(y) \left[ 1 + \frac{m_j^2}{p_T^2} \tanh^2(y) \right]^{1/2} = \cosh(\eta)$$

- The jet mass can be bounded by $m_j/p_T < R/\sqrt{2} = 0.28$ (Kolodrubetz, et al. arXiv:1605.08038v1) for $R=0.4$. 

Analyzing 13 TeV Jets with $\gamma$

$$m_{J/p_T < R/\sqrt{2}} = 0.28$$

---

**$\eta$ analysis:** $n_{p_T} = 6.456 \pm 0.040$

**$\gamma$ analysis:** $n_{p_T} = 6.450 \pm 0.037$

---

**$\eta$ analysis:** $D = 0.671 \pm 0.021$

**$\gamma$ analysis:** $D = 0.663 \pm 0.022$

---

**$\eta$ analysis:** $n_{xR} = 3.877 \pm 0.077$

**$\gamma$ analysis:** $n_{xR} = 3.551 \pm 0.081$
Quadratic fit parameters of residual fits

- Fit parameters vs. $\sqrt{s}$

$$\frac{A(p_T) - A_{fit}(p_T)}{A_{fit}(p_T)} = a \log(p_T)^2 + b \log(p_T) + c$$

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (TeV)</th>
<th>$A_{fit}$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.960</td>
<td>-2.766</td>
<td>-3.979</td>
<td>-1.181</td>
</tr>
<tr>
<td>2.760</td>
<td>-2.177</td>
<td>-4.432</td>
<td>-1.904</td>
</tr>
<tr>
<td>5.020</td>
<td>-2.532</td>
<td>-3.303</td>
<td>-0.324</td>
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<tr>
<td>7.000</td>
<td>-0.339</td>
<td>-1.081</td>
<td>-0.232</td>
</tr>
<tr>
<td>13.000</td>
<td>-0.244</td>
<td>-0.413</td>
<td>-0.026</td>
</tr>
</tbody>
</table>

Fit to Residuals (%) of power law vs. $\sqrt{s}$

- $y = 1.4023\ln(x) - 3.8027$  
  $R^2 = 0.737$
- $y = 2.2125\ln(x) - 6.0987$  
  $R^2 = 0.8566$
- $y = 0.8733\ln(x) - 2.0978$  
  $R^2 = 0.6879$
Rescaling $\sqrt{s}$ to $\sqrt{s^*}$ to $\sqrt{s_a}$

- Interpret the strong $1/p_T$ dependence in 13 TeV $n_{x_R}$ as caused by a ‘drain’ in $\sqrt{s}$ available for primary collision. Force $(1-x_R)^4$ behavior to find effective $\sqrt{s^*}$. ISR, FSR or multiple parton interactions would lead to $N_{\text{Jet}}$ increasing. The ‘available’ $\sqrt{s_a}$ is given by:

$\sqrt{s^*} = 2p_T \cosh(\eta) \left[ 1 - \left( 1 - \frac{2p_T \cosh(\eta)}{\sqrt{s}} \right)^4 \right]^{-1}$

$\sqrt{s_a} = \sqrt{s} - \sqrt{s^*}$
Arleo, et al.* – $x_T$ Analysis to Determine $n_{pT}$

Studied the approach to $x_T$ scaling, evident for small $|y|$ but misses the main feature. Scaling is in $x_R$ not $x_T$ namely $F(x_T, \theta) = F(x_R)$

*[Arleo,Brodsky,Hwang and Sickles; arXiv:0911.4604v2, PRL 105,06200 (2010)]
Using $x_T$ to Determine $n_{\text{eff}}$ - replication of analysis

\[ \sigma = E \frac{d^3 \sigma}{dp^3} = \frac{1}{p_T^{n_{\text{eff}}}} F(x_T, \theta) \]

\[ p_T = \frac{\sqrt{s}}{2} x_T \]

\[ \ln \left( \frac{\sigma_1}{\sigma_2} \right) = -n_{\text{eff}} \ln \left( \frac{\sqrt{s_1}}{\sqrt{s_2}} \right) + \ln \left( \frac{F(x_T, \theta_1)}{F(x_T, \theta_2)} \right) \]

\[ n_{\text{eff}} = \frac{-\ln \left( \sigma_1 / \sigma_2 \right) + \ln \left( F(x_T, \theta_1) / F(x_T, \theta_2) \right)}{\ln \left( \sqrt{s_1} / \sqrt{s_2} \right)} = \frac{-\ln \left( \sigma_1 / \sigma_2 \right) + \ln \left( F(x_{R1}) / F(x_{R2}) \right)}{\ln \left( \sqrt{s_1} / \sqrt{s_2} \right)} \]

• Assume

• Neglect the ‘F’ term:
$x_T$ analysis: power of $p_T$ depends on $x_T$ and process.

$h^\pm/\pi^0$ – circles

$\gamma$ – squares

Jets – triangles

The $x_R$ analysis finds power of $p_T$ independent of process within errors:

$n_{p_T} = 6.5 \pm 0.4$

Fig. from Arleo, et al.; arXiv:0911.4604v2, PRL 105,06200 (2010)
$n_{\text{eff}}$ without correction term using ATLAS Jet Fits

Use the full parameterization of the inclusive Jet cross section – including the $s$-dependent terms. Plots are for different angles in COM.

Find $n_{\text{eff}} \rightarrow 4$ as $x_T \rightarrow 0$.
$n_{\text{eff}}$ with the F-correction term

Hence $n_{\text{eff}} \to 4$ as $x_T \to 0$ is a result of neglecting the ‘F’ term that contains important overall normalization $\alpha(s)$ term that corrects $n_{\text{eff}} \approx 4$ to $n_{\text{eff}} \approx 6$. 

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The ‘Drell-Yan’ Limit

Computed for $p_{\text{min}} = 0.01 \text{ TeV}$:

$$M^4 \frac{d\sigma}{dM^2} = M^4 \int \int \left( \frac{d}{dM^2} \right) \frac{d^2\sigma}{dp_T^2 dy} dp_T dy$$

Typical point calculation with $\Lambda = \text{Quad}(\sqrt{s})$:

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (TeV)</th>
<th>M (TeV)</th>
<th>$\tau$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.00</td>
<td>0.9900</td>
<td>2.0002E-02</td>
<td>6.2735E+02</td>
</tr>
<tr>
<td>$\Lambda$ (TeV)</td>
<td>0.0350</td>
<td>$p_{\text{min}}$ (TeV)</td>
<td>0.010</td>
</tr>
</tbody>
</table>

\[d^2\sigma/dM^2 d\eta\]

Drell-Yan $M^4 d\sigma/dM^2$ vs. $\tau = M^2/s$
$\eta' = \eta - \tanh^{-1}(\beta_{Beam})$

$\eta' \propto \ln \left( \frac{2 \cosh(\eta)}{\sqrt{s}} \right)$

$y = 5.339E-01 e^{9.917E-01 x}$

$R^2 = 9.999E-01$
Diquarks

Flavor Decomposition of the Elastic Nucleon Electromagnetic Form Factors
G. D. Cates, C. W. de Jager, S. Riordan, and B. Wojtsekhowski

Diquark correlations in baryons on the lattice with overlap quarks
Ronald Babich, et al.
arXiv:hep-lat/0701023v2 19 Oct 2007

Strong diquark correlations inside the proton
Jorge Segovia
EPJ Web of Conferences 113, 05025 (2016)

Hadron Systematics and Emergent Diquarks
Alexander Selema and Frank Wilczek

Cates, et al. conclude that d-quark contribution to the proton form-factor appears to be suppressed from no-diquark assumption.
ATLAS 5.02 TeV Direct $\Lambda = 0$
ATLAS 7 TeV Direct $\Lambda = 0$
7 TeV CMS Prompt $\Lambda = 0$
J/Psi-comparison of $\left(1-x_R\right)$ Power

ATLAS, CMS LHCb $n(pT)$ vs. $pT$
13 TeV LHCb Direct $\Lambda = 4.4 \pm 0.4$ GeV
ATLAS 5.02 J/Ψ Decay $\Lambda = 7.1 \pm 1.5$ GeV

1/24/2017

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ATLAS 7 TeV Decay $\Lambda = 5.8 \pm 1.6$ GeV
13 TeV LHCb Decay $\Lambda = 4.6 \pm 0.3$ GeV
CHARM $n_{xR}$

CHARM $\psi(2S)$ 7 TeV ATLAS

LHCb 13 TeV ‘Direct’ $J/\psi$

13 TeV LHCb Decay

$y = -0.0516x^2 + 2.5373x + 4.5766$
$R^2 = 0.4177$

$y = -0.1498x^2 + 1.4865x + 25.147$
$R^2 = 0.8161$

$y = -0.0455x^2 - 0.8027x + 44.183$
$R^2 = 0.6703$
CHARM – Directly Produced $\Lambda = 3.6 \pm 0.3$ GeV

ATLAS $A(p_T)$ vs. $p_T$ 5.02 TeV prompt J/Psi

$A(p_T) = \alpha_0 \frac{\Lambda^{n_{p_T} - 4}}{(\Lambda^2 + p_T^2)^{\frac{n_{p_T}}{2}}}$

$y = 1.321E+06x^{-6.976E+00}$

$R^2 = 9.994E-01$

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CHARM from b-decay $\Lambda = 7.1 \pm 0.9$ GeV

\[ A(p_T) = \alpha_0 \frac{\Lambda^{\alpha_1 - 4}}{(\Lambda^2 + p_T^2)^{\alpha_2 / 2}} \]

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