Comparison of Inclusive Jet Cross Sections with Single Particle Inclusive Production

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Remembrance of Things Past

Compare $\pi^0$s at FNAL with Jets at LHC

FNAL E63 @ C0 1974

LHC ATLAS @ Point 1

7/25/2017

F. E. Taylor MIT
Single Particle Inclusive & Jet Inclusive Production

Particles: $h$

Jets: $X$

LO Dimension

$$E \frac{d^3 \sigma}{dp^3} \sim \frac{d^2 \sigma}{dp_T^2 dy} \sim \frac{d\hat{\sigma}_{ab}(\alpha, \mu_R^2, s/\mu_F^2, s/\mu_R^2)}{dt}$$

$$\sim \frac{cm^2}{GeV^2} \sim \frac{1}{GeV^4}$$

2 → 2 scattering

$$\sim 1/GeV^6$$ for 2 → 3 scattering

Particles: $h$

$$Ed\sigma/d^3p(s, t, u; A + B \rightarrow h + X) = \int_{x_a}^{1} dx_a \int_{x_b}^{1} dx_b G_{A-a}(x_a)G_{B-b}(x_b)D^b_c(z_c) \frac{1}{z_c} \frac{1}{\pi} \frac{d\hat{\sigma}}{dt}(\hat{s}, \hat{t}; q_a + q_b \rightarrow q'_a + q'_b)$$

Frag Had

Jets: $X$

$$E \frac{d^3 \sigma}{dp^3} = \sum_{a,b} \int dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) d\hat{\sigma}_{ab}(\alpha, \mu_R^2, s/\mu_F^2, s/\mu_R^2) \frac{d\hat{\sigma}_{ab}}{dt} \otimes \text{Frag} \otimes \text{Had}$$

QCD Factorization Theorem

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$P_T$ & “Other” Variable: Rapidity & Radial $X_R$

Rapidity and pseudo rapidity:

$$y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) \approx \eta = -\ln \left( \tan \left( \frac{\theta}{2} \right) \right)$$

Radial scaling $X_R$:

$$X_R = \frac{E}{E_{\text{max}}} = \frac{2}{\sqrt{s - m_{QN}^2}} \sqrt{ \frac{p_T^2 \cosh^2(y)(1 + (m_j^2/p_T^2) \tanh^2(y)) + m_j^2}{\sqrt{s - m_{QN}^2}}}$$

$$\approx \frac{2p_T \cosh(y)}{\sqrt{s}} \sqrt{ \left( 1 + \frac{m_j^2}{p_T^2} \tanh^2(y) \right)}$$

$$\approx \frac{2p_T \cosh(\eta)}{\sqrt{s}}$$

$m_{QN} = \text{mass to satisfy QN conservation}$

$x_R$ is a “final state” scaling variable that controls kinematic boundary suppression that is not respected by $x_{||}$ and $x_T$.

E and $E_{\text{max}}$ are energy of jet (particle) and maximum energy, respectively in the COM. $m_j$ is mass of jet (particle).
Radial Scaling in Inclusive p-p $\pi^0$ Production

\[ E \frac{d^3\sigma}{dp^3} = F(s, p_T, x_R) \approx F(p_T, x_R) \sim A(p_T)f(x_R) \]

\[ \sim (1-x_R)^{nx_R} \]

\[ \sim 1/p_T^{npT} \]

D. C. Carey, ... FET Phys. Rev. Lett. 33, No. 5, 327 (29 July 1974)
A(p_T) \sim p_T^{-6}

Find an approximate Radial Scaling for Inclusive Jet Production – similar to that observed in single particle inclusive production.

Works for CDF but with A(p_T) \sim p_T^{-7}
A more refined analysis is to determine $A(p_T)$ by power-law fits in $(1-x_R)$ for constant $p_T$:

$$\frac{d^2\sigma}{p_Tdp_Td\eta} \sim A(p_T)(1-x_R)^{n_xR}$$

Now study the behavior of $A(p_T)$ and $n_{xR}$ as function of $p_T$, $\sqrt{s}$ and process.
Fit Parameters 13 TeV ATLAS Inclusive Jets

\[ A(p_T, s) = \frac{\alpha(s)}{p_T^{n_{pT}}} \]

\[ n_{xR}(p_T, s) = n_{xR0} + \frac{D(s)}{p_T} \]

\[ \alpha(s) = (1.13 \pm 0.02) \times 10^{-4} \text{ (pb/GeV}^2) \text{ TeV}^{n_{pT}} \]

\[ n_{pT} = 6.36 \pm 0.01 \]

\[ D(s) = 0.68 \pm 0.03 \text{ (TeV}^{-1}) \]

\[ n_{xR0} = 3.61 \pm 0.07 \]
ATLAS Jets at Other p-p Vs Energies

\[ \alpha(s) \text{ grows with } s \]
\[ n_{\text{npT}} \approx \text{constant} \]
\[ D(s) \text{ grows with } s \]
\[ n_{\text{nxR0}} \approx \text{constant} \]

**Graphs:**
- **ATLAS Inclusive Jets**
  - Happening at 2.76 TeV, 7 TeV, and 13 TeV.
- **ATLAS Inclusive Jets R=0.4**
  - Data points at 2.76 TeV, 7 TeV, and 13 TeV.
ATLAS p-Pb Jets $\sqrt{s_{nn}} = 5.02$ TeV

- Compare p-fragmentation side with Pb-fragmentation side (0%-90%)

<table>
<thead>
<tr>
<th>$n_{pT}$</th>
<th>p-side</th>
<th>Pb-side</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>6.15 ± 0.04</td>
<td>6.43 ± 0.07</td>
</tr>
<tr>
<td>nxR0</td>
<td>p-side 0.07 ± 0.02 3.2 ± 0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pb-side 0.3 ± 0.1 2.9 ± 0.5</td>
<td></td>
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</tbody>
</table>

$A(p_T)$ the same
### Summary of Inclusive Jets CDF, D0, ATLAS, CMS

<table>
<thead>
<tr>
<th>vs (TeV)</th>
<th>$\alpha$ (pb/GeV$^2$) TeV$^{mpT}$</th>
<th>$n_{nR0}$</th>
<th>$\chi^2$/d.f.</th>
<th>d.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.96 p-p CDF</td>
<td>(0.9 ± 0.2) x 10$^{-6}$</td>
<td>7.03 ± 0.08</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>1.96 $\bar{p}$-p D0</td>
<td>(1.3 ± 0.1) x 10$^{-6}$</td>
<td>6.90 ± 0.05</td>
<td>1.2</td>
<td>25</td>
</tr>
<tr>
<td>2.76 p-p ATLAS</td>
<td>(6.0 ± 1.0) x 10$^{-6}$</td>
<td>6.29 ± 0.06</td>
<td>3.4</td>
<td>8</td>
</tr>
<tr>
<td>7 p-p ATLAS</td>
<td>(3.7 ± 0.2) x 10$^{-5}$</td>
<td>6.21 ± 0.03</td>
<td>32</td>
<td>14</td>
</tr>
<tr>
<td>8 p-p CMS</td>
<td>(2.98 ± 0.04) x 10$^{-5}$</td>
<td>6.73 ± 0.01</td>
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<td>33</td>
</tr>
<tr>
<td>13 p-p ATLAS</td>
<td>(1.13 ± 0.02) x 10$^{-4}$</td>
<td>6.36 ± 0.01</td>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>13 p-p CMS</td>
<td>(1.06 ± 0.04) x 10$^{-4}$</td>
<td>6.40 ± 0.03</td>
<td>2</td>
<td>27</td>
</tr>
</tbody>
</table>

\[
\frac{d^2\sigma}{p_T dp_T dy} (s, p_T, y; \alpha, n_{nR0}, D, n_{xR0}) = \frac{\alpha(s)}{p_T^{n_{nR0}}} (1 - x_R) \frac{D(s)}{p_T + n_{xR0}}
\]

\[
x_R = \frac{E}{E_{\text{max}}} = \frac{2 \sqrt{p_T^2 \cosh^2(y) (1 + (m_j^2/p_T^2) \tanh^2(y)) + m_j^2}}{\sqrt{s}}
\]

\[
\approx \frac{2 p_T \cosh(y)}{\sqrt{s}} \sqrt{1 + \frac{m_j^2}{p_T^2} \tanh^2(y)}
\]

where: $m_j/p_T < R/\sqrt{2} = 0.28$ for

\[
R = \sqrt{\left(\Delta \phi^2 + \Delta \eta^2\right)}
\]

Conclusions:

**p_T - behavior**

\[ n p_T \approx \text{constant} = 6.5 \pm 0.3 \]

\( \alpha(s) \) grows linearly with \( s \)

Quality of power-law fits in \( p_T \) is low.

Better fit:

\[ A(p_T, s) = \exp\left( \beta(s) \left( \ln(p_T) \right)^2 \right) \frac{\alpha(s)}{p_T^{n_{p_T}}} \]

**\( 1-x_R \) behavior**

\[ n x R 0 \approx \text{constant} = 3.3 \pm 0.5 \]

\( D(s) \) grows linearly with \( s \)

Same Vs data combined:

\( (D0 \otimes CDF, ATLAS\otimes CMS) \)
Single Particle Inclusive Reactions

\[
\frac{d^2 \sigma}{p_T dp_T dy}(s, p_T, y; \alpha, \Lambda, n_pT, m, D, n_R0)
\]

\[
= \frac{\alpha(s)}{(\Lambda^2 + p_T^2)^2} \left( 1 - \frac{2\sqrt{(p_T^2 \cosh^2(y)(1 + (m^2/p_T^2) \tanh^2(y)) + m^2)}}{\sqrt{s}} \right)^D(s) \frac{D(s) + n_R0}{p_T^2}
\]

\[
P_T' = \sqrt{(\Lambda^2 + p_T^2)}
\]

Parameterization of cross section at low \( p_T \) needs another term \( \Lambda \) (transverse mass \( \otimes \) intrinsic \( k_T \))

\( \Lambda = 3.6 \pm 0.3 \text{ GeV} \)

\( \Lambda = 7.1 \pm 1.2 \text{ GeV} \)
A(p_T): Single Particle Inclusive 0.063 TeV ≤ \(\sqrt{s}\) ≤ 13 TeV

All single particles: \(n_{pT} = 6.1 \pm 0.6\)

Direct Gamma: \(n_{pT} = 5.7 \pm 0.2\)

Jets: \(n_{pT} = 6.5 \pm 0.3\)

Everything (48): \(n_{pT} = 6.2 \pm 0.6\)

\(n_{pT} = 4\) disfavored by 4 \(\sigma\)
Parameter for prompt single particle inclusive is linear with mass of particle:
\[ \Lambda = 1.1 \, M + 0.4 \, (\text{GeV}) \]
\(\Lambda\) for non-prompt tends to be larger than \(\Lambda\) prompt \((J/\psi)\) but better data needed \(\psi(2S)\).
• The $p_T$-dependence of the invariant inclusive cross sections seems to be independent of process and energy over a wide range as a power law: $1/p_T^{(6.2 \pm 0.6)}$ in the limit $x_R \to 0$. “Higher Twists” have been known to dominate for a long time but this analysis demonstrates this widely. All are consistent with the dimension of $2 \to 3$ scattering.

• The $x_R$ dependence is consistent with a power law $(1-x_R)^{nx_R}$, where $n_{xR}$ is qualitatively dependent on the number of spectator quarks.

• At high $\sqrt{s}$ and HI collisions $nx_R = D/p_T + nx_{R0} \to$ Jets at low $p_T$ strongly suppressed.

• Determining $\Lambda$ gives a hint of production mechanism $\sim$ linear with mass direct, larger for indirect production.

• The data are well-represented by pQCD calculations to NLO (PHYTHIA, SHERPA, JetPhox, PeTer). Experiment authors show agreement with simulations but generally do not try to factorize the $p_T$-dependence from the $y$ or $x_R$ dependence. Doing so would show commonalities and highlight differences. What about 100 TeV?
Backup
Inclusive Jets & Inclusive Single Particles

\[ \frac{d^2 \sigma}{p_T dp_T dy}(s, p_T, y; \alpha, \Lambda, n_{pT}, m, D, n_{xR0}) = \frac{\alpha(s)}{(\Lambda^2 + p_T^2)^{n_{pT}}/2} \left( 2 \sqrt{\frac{p_T^2 \cosh^2(y)(1 + (m^2/p_T^2) \tanh^2(y)) + m^2}{\sqrt{s}}} \right) \right) D(s) \frac{\Lambda^2 + n_{xR0}^2}{p_T + n_{xR0}} \]

\[ = \frac{\alpha(s)}{(\Lambda^2 + p_T^2)^{n_{pT}}/2} \left( 1 - x_R \right) \frac{D(s)}{p_T + n_{xR0}} \]

- \( n_{pT} \approx 6 \) independent of process in \( x_R \rightarrow 0 \) limit.
- \( \Lambda \) depends linearly on mass of particle.
- \( \alpha(s) \) and \( D(s) \) are roughly linear in \( s \) for jets.
- \( n_{xR0} \approx \) constant for inclusive jets

Preliminary version of this work:
F. E. Taylor, “Radial Scaling in Inclusive Jet Production at Hadron Colliders”,
Jets are produced by hard parton scattering (qq → qq, gg → gg, gq → gq). The scattered parton hadronizes into a jet of particles.

QCD Factorization Theorem

\[
E \frac{d^3\sigma}{dp^3} = \sum_{a,b} \int dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \frac{d\hat{\sigma}_{ab}(\alpha_s(\mu_r^2), s/\mu_r^2, s/\mu_r^2)}{dt} \otimes \text{Frag} \otimes \text{Had}
\]

These 10s of parameters and factors are put together in simulations of inclusive jet production at the LHC.

Would expect invariant cross section to show \(1/p_T^4\) behavior if LO scattering dominates

Dimensions:

\[
E \frac{d^3\sigma}{dp^3} \sim \frac{d^2\sigma}{dp_T^2 dy} \sim \frac{d\hat{\sigma}_{ab}(\alpha_s(\mu_r^2), s/\mu_r^2, s/\mu_r^2)}{dt} \otimes \text{Frag} \otimes \text{Had}
\]

\[
\sim \frac{cm^2}{GeV^2} \sim \frac{1}{GeV^4}
\]
ATLAS Inclusive Jet Production at 13 TeV

- Jets defined by anti-\( k_t \) algorithm with \( R = (\Delta \phi^2 + \Delta y^2)^{1/2} = 0.4 \)
- Pythia 8.186 with A14 tune, NLOjet++. Involves integrations & summations using Monte Carlo methods
- Data compared to NLO pQCD calculation 2 -> 2 + NLO processes, leading logarithmic \( p_T \)-ordered parton shower, hadronization with the Lund string model.
Line Counting, Higher Twists, Diquarks

- Dimensional Analysis
  \[ M \sim [\text{cm}]^{n_A - 4} \]

  \[ \frac{d^2 \sigma}{p_T dp_T dy} \sim \frac{1}{p_T^{4n_A - 4}} \]

  \[ n_A = 4 \quad 2 \rightarrow 2 \text{ scattering} \]

  \[ \text{HIDDEN } x_R \rightarrow 0 \]

  \[ \frac{d^2 \sigma}{p_T dp_T dy} \sim \frac{1}{p_T^4} \]

  \[ n_A = 5 \quad 2 \rightarrow 3 \text{ scattering} \]

  \[ \text{DOMINATES } x_R \rightarrow 0 \]

After Arleo – Moriond QCD 2010

(Diquarks: Selem, Wilczek, Jaffe, Berger, Gottschalk, Sivers, Brodsky, ...)
Jets in p-p, p-A, A-A have similar behavior

\[
\frac{(1 - x_R)}{(D/p_T + n_{0xR})}
\]

All behave:

13 TeV
7 TeV
2.76 TeV

Only fitting errors shown
If there is a hard $2 \rightarrow 2$ scattering core by naive dimensional analysis then:

$$
\frac{d\sigma(ab \rightarrow x)}{dQ^2} \sim \frac{1}{Q^4} \rightarrow \frac{d^2\sigma(pp \rightarrow Jets)}{p_T dp_T dy} \sim \frac{1}{p_T^4}
$$

Thus would expect:

$$
p_T^4 \left( \frac{d^2\sigma(pp \rightarrow Jets)}{p_T dp_T dy} \sim \frac{1}{p_T^4} \right) \sim F(x_R)
$$

Note: Have approximated $\eta$ by $y$
**Physical Picture: Inclusive Jets & Pions**

Choose 4 points in phase space: \((p_{\text{T}_{\text{L,H}}}, x_{R1}, x_{R2})\)

\[
R(p_T;\{x_{R1}, x_{R2}\}) = \frac{\sigma_1}{\sigma_2} = \frac{(1 - x_{R1})^{\frac{D(p_T)}{p_T} + n_{xR0}}}{(1 - x_{R2})^{\frac{D(p_T)}{p_T} + n_{xR0}}}
\]

"Beam fragmentation region"

"Beam fragmentation region" augmented by increasing \(\sqrt{s}\) and/or by increasing beam A in Heavy Ion Collisions.

Jet quenching in both cases. Same Physics?

Jet strongly attenuated on approach to kinematic boundary because of large "D" term

Jet less strongly attenuated on approach to kinematic boundary because "D" term \(\to 0\)

7/25/2017  
F. E. Taylor MIT
\[ \eta(x_R, s, p_T) = \ln \left( \frac{x_R \sqrt{s}}{2 p_T} + \sqrt{\frac{x_R s}{4 p_T^2}} - 1 \right) \]

\[ \eta_{\text{max}} = \ln \left( \frac{\sqrt{s}}{2 p_T} + \sqrt{\frac{s}{4 p_T^2}} - 1 \right) \]

Analyses in constant \( \eta \) couples \( p_T \) and \( x_R \) so that the hard scattering part of \( \frac{d^2\sigma}{dp_T dp_T d\eta} \) that is characterized by \( p_T \) is entangled with a change in \( x_R \) – the kinematic boundary parameter.
BRAHMS $\pi^+$ from Ag-Ag Collisions 62.4 GeV

$A(p_T)$ vs. $p_T$

$A(p'_T)$ vs. $p'_T = (\Lambda^2 + p_T^2)^{1/2}$

$nxR(1/p_T)$ vs. $1/p_T$

$\Lambda = 0.56 \pm 0.07$ GeV

$npT = 5.7 \pm 0.5$

$nxR = (3.33 \pm 0.18) / p_T$ (GeV) + 1.2 ± 0.3

$A(p_T) = \alpha_0 \frac{\Lambda^{n_{p_T}-4}}{\left(\Lambda^2 + p_T^2\right)^{\frac{n_{p_T}}{2}}}$
Naively, does not indicate hard 2 → 2 scatterings – such as:

- $qq \rightarrow qq$
- $gg \rightarrow gg$
- $gq \rightarrow gq$

are dominating.

Note: plotted errors are statistical and systematic errors added in quadrature.
“Uncertainties in NLO + parton shower matched simulations of inclusive jet and dijet production”; Stefan Hoche, Marek Schonherr
Radial scaling analysis reveals systematic difference in n(1/pT).

Data: $\alpha = (1.608 \pm 0.434) \times 10^{-5}$
$n_{pT} = 6.499 \pm 0.0125$

SHERPA: $\alpha = (1.895 \pm 0.353) \times 10^{-5}$
$n_{pT} = 6.380 \pm 0.089$

SHERPA underestimates D

Data: $D = 0.125 \pm 0.0112$
$n_{0xR} = 3.03 \pm 0.16$

SHERPA: $D = 0.082 \pm 0.015$
$n_{0xR} = 3.19 \pm 0.21$
Power Law in $p_T$ not ‘Perfect’

ATLAS 13 TeV R=0.4 $A(p_T)$ vs. $p_T$

Fit is good over 8 decades but there is a systematic deviation from the power law of ± 20%
$A(p_T, s) \text{ Quadratic Fit in } \ln(p_T)$

\[
\ln(A(p_T, s)) = \beta(s) \ln(p_T)^2 - n_{pT} \ln(p_T) + \rho(s)
\]

\[
A(p_T, s) = \exp\left(\beta(s) \left(\ln(p_T)\right)^2\right) \frac{\alpha(s)}{p_T^{n_{pT}}}
\]

<table>
<thead>
<tr>
<th>vs (TeV)</th>
<th>(\beta)</th>
<th>(\alpha (\text{pb/GeV}^2 \text{ TeV}^{n_{pT}}))</th>
<th>(n_{pT})</th>
<th>(\chi^2/\text{d.f.})</th>
<th>d.f.</th>
</tr>
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<tr>
<td>1.96 (\bar{p}-p) CDF</td>
<td>0.03 ± 0.2</td>
<td>((1.6 \pm 0.8) \times 10^{-6})</td>
<td>6.7 ± 0.6</td>
<td>0.92</td>
<td>38</td>
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<tr>
<td>7 p-p ATLAS</td>
<td>-0.38 ± 0.05</td>
<td>((1.0 \pm 0.1) \times 10^{-5})</td>
<td>7.8 ± 0.2</td>
<td>2.50</td>
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<td>8 p-p CMS</td>
<td>-0.38 ± 0.02</td>
<td>((2.1 \pm 0.1) \times 10^{-5})</td>
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<td>4.3</td>
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<tr>
<td>13 p-p ATLAS</td>
<td>-0.26 ± 0.01</td>
<td>((9.2 \pm 0.1) \times 10^{-5})</td>
<td>6.92 ± 0.02</td>
<td>0.77</td>
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<tr>
<td>13 p-p CMS</td>
<td>-0.32 ± 0.04</td>
<td>((8.7 \pm 0.2) \times 10^{-5})</td>
<td>7.03 ± 0.07</td>
<td>0.48</td>
<td>26</td>
</tr>
</tbody>
</table>

Residuals of Quadratic $\ln(p_T)$ Fit - 13 TeV ATLAS
\[
\frac{d\sigma}{d\eta} = \int_{p_T^{\text{min}}}^{p_T^{\text{max}}} \frac{d^2\sigma}{p_T dp_T d\eta} p_T dp_T = \int_{p_T^{\text{min}}}^{p_T^{\text{max}}} \frac{a}{n p_T} \left(1 - \frac{2 p_T}{\sqrt{s}} \cosh(\eta)\right)^{n_{xR}} p_T dp_T
\]

\[
\frac{d\sigma(p_T^{\text{min}}, p_T^{\text{max}})}{d\eta} = a F \left( p_T^{\text{min}}, p_T^{\text{max}}, \frac{\cosh(\eta)}{\sqrt{s}} \right)
\]

\(p_T^{\text{min}}\) is the minimum transverse momentum cut (\(p_T \geq p_T^{\text{min}}\))

For fixed \(p_T^{\text{min}}\) and parameter \(a\), all \(\eta\) dependence through \(\cosh(\eta)/\sqrt{s}\)
Pseudo-rapidity Plateau in Toy Model

Width of plateau controlled by kinematic limit:

\[ \eta_{\text{max}} = \ln \left( \frac{\sqrt{s}}{2 p_T} + \sqrt{\frac{s}{4 p_T^2} - 1} \right) \]

\[ \frac{dN}{d\eta} \text{ on plateau } \eta = 0 \text{ grows by kinematics – (no QCD required)} \]
\( \frac{dN}{d\eta} \) is a function of \( \cosh(\eta)/\sqrt{s} \)

- **Toy Model**
  - \( n_{pT} = 6 \)
  - \( n_{xR} = 4 \)
  - \( p_{T\text{min}} = 10 \text{ GeV} \)

- First shown in (1979):
  “Interpretation of the Rise in Central Rapidity Density in Terms of Radial Scaling”,

  R. W. Ellsworth,

  16th International Cosmic Ray Conference, Vol. 7. Published by the Institute for Cosmic Ray Research, University of Tokyo

  [http://adsabs.harvard.edu/abs/1979ICRC....7..333E](http://adsabs.harvard.edu/abs/1979ICRC....7..333E)
Check of Rapidity Distribution of Jets

• Fit: $p_T > 0.1$ TeV with numerical integration of fit function un-normalized.

$$\frac{d^2\sigma}{p_T dp_T d\eta} \sim A(p_T)(1 - x_R)^n$$

• Data:

$$\frac{dN}{d\eta} \sim \sum_i \frac{d^2\sigma_i}{p_{T_i} dp_T d\eta} p_{T_i} \Delta p_T$$
Pseudo-rapidity Distribution for Measured Jets

Used the fits of the inclusive jet cross sections: \{\alpha(\sqrt{s}), npT(\sqrt{s}), D(\sqrt{s}), n_{0xR}(\sqrt{s})\} CDF & ATLAS

Rapidity Plateau (Parameters for \( pT > 100 \) GeV)

13 TeV is different because of large ‘D’ term for \( nxR = D/pT + n_{0xR} \)
PHOBOS $dN/d\eta$

B.B. Black, et al.
arXiv:nucl-ex/0509034v1 28 Sep 2005
B-field = 0 (very low $p_{T\text{min}}$)

$dN/d\eta$ Phobos-RHIC Ag-Ag 6% most central collisions

$dN/d\eta$ vs. $\cosh(\eta)/\sqrt{s}$ PHOBOS-RHIC Ag-Ag

Region of scaling is high $\eta$. Note that $\cosh(\eta)/\sqrt{s}$ scaling similar to $\eta'$ scaling – see backup.
What about the $x_R$-Dependence

- Inclusive cross section roughly factorizes: $\sigma \sim A(p_T) \left(1-x_R\right)^{n_{xR}}$

- Would expect that $n_{xR} = n_{xR}(s, p_T, \text{process})$ to characterize the fragmentation and hadronization of primordial quark/gluon.

- Quark line-counting rules suggest $n_{\text{spectator}}$, the number of non-participating quarks in the primary collision, controls the $(1-x_R)$ power:

$$\frac{d^2\sigma}{p_Tdp_Tdy} \sim A(p_T) \left(1-x_R\right)^{2n_{\text{spectator}}^{-1}}$$
Summary of \((1-x_R)^{n_{xR}}\) Power

Notes:

1. Qualitatively \(n_{xR} \approx 2 n_{\text{spectator}} - 1\)

2. In cases where \(n_{xR}\) is roughly independent of \(p_T\) the average values and standard deviations are plotted.

3. In cases where there is a significant \(1/p_T\) dependence the value \(n_{xR0}\) is plotted, where: \(n_{xR}(1/p_T) = D/p_T + n_{xR0}\) and the error of \(n_{xR0}\) is shown.

4. Caveat: \(J/\psi\) data show inconsistencies among experiments. Trend shown is consistent but details not clear. See backup.
Parton-Parton Elastic Scattering – 2 Examples

Functions of the Mandelstam variables $s$, $t$, $u$ and $\alpha_s$. All have dimensions of $(\text{energy})^{-4}$.

\[
\frac{d\hat{\sigma}(\hat{s}, \hat{t}, \hat{u}; ud \rightarrow ud)}{d\hat{t}} = \frac{4\pi\alpha_s^2}{9\hat{t}^2} \left(\hat{s}^2 + \hat{u}^2\right)
\]

\[
\frac{d\hat{\sigma}(\hat{s}, \hat{t}, \hat{u}; gg \rightarrow gg)}{d\hat{s}} = \frac{9\pi\alpha_s^2}{2\hat{s}^2} \left(3 - \frac{\hat{t}\hat{u}}{\hat{s}^2} - \frac{\hat{s}\hat{u}}{\hat{t}^2} - \frac{\hat{s}\hat{t}}{\hat{u}^2}\right)
\]

\[
\hat{s} = (p_a + p_b)^2 = \frac{s}{4}(x_1 + x_2)^2
\]

\[
\cos \theta = \left(1 - \frac{p_T^2}{\hat{s}}\right)^{1/2}
\]

\[
\hat{t} = -\frac{\hat{s}}{2}(1 - \cos \theta)
\]

\[
\hat{u} = -\frac{\hat{s}}{2}(1 + \cos \theta)
\]
PDF and DGLAP Evolution and Splitting Functions

Parton Distribution Functions (mostly from DIS Lepton-Nucleon Scattering):

DGLAP evolution and splitting functions:

\[
\frac{\partial f_a}{\partial \ln \mu^2} \sim \frac{\alpha_s(\mu^2)}{2\pi} \sum_b (P_{ab} \otimes f_b)
\]

\[
P_{qs}(x) = \frac{1}{2} \left[ x^2 + (1-x)^2 \right]...
\]

These 10s of parameters and factors are put together in simulations of inclusive jet production at the LHC.
Integrate over $x_R$ to find $p_T$ Dependence

- J. Thaler suggested:

$$
\frac{1}{p_T^{\text{neff}}} \sim \int_{x_{R\text{min}}}^{1} \frac{d^2 \sigma}{p_T dp_T dy} \left( \begin{array}{c} p_T \\ y \\ p_T \end{array} \right)_{J} dx_R
$$

$$
= \int_{x_{R\text{min}}}^{1} \frac{d^2 \sigma}{p_T dp_T dy} \frac{2}{\sqrt{x_R^2 - x_{R\text{min}}^2}} dx_R
$$

$$
x_{R\text{min}} = \frac{2p_T}{\sqrt{s}}
$$

$p_T$ increases from 6.0 to 6.45.

Tested with toy model.

Interesting suggestion – integration can be extended to determine the moments of the “fragmentation” function $(1-x_R)^{n_R}$. 

$$\text{Tested with toy model.}$$

7/25/2017

F. E. Taylor MIT
p_bar-p Inclusive Jet Production

• Valence q-anti-q scattering/annihilation

D0 1.96 TeV 3 or more points

1.96 TeV CDF A(pT)
An Example of $x_R$-dependence near Kinematic Boundary

- CMS Inclusive Jets 8 TeV / 7 TeV

CMS Preliminary

CMS PAS SMP-14-001

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Studied the approach to $x_T$ scaling, evident for small $|y|$ but misses the main feature. Scaling is in $x_R$ not $x_T$ namely $F(x_T, \theta) = F(x_R)$ and the jet cross sections grow with increasing $s$ through the amplitude term $\alpha(s)$.

Replication of the Analysis – Assume $x_T$ Scaling

\[
\sigma^{\text{inv}} = E \frac{d^3 \sigma}{dp^3} (AB \rightarrow CX) = \frac{F(x_T, \theta)}{p_T^n}
\]

\[
\sigma^{\text{inv}} (AB \rightarrow CX) \propto \frac{(1 - x_T)^{2n_{\text{spectator}} - 1}}{p_T^{2n_{\text{active}} - 4}}
\]

\[
n^{\text{exp}} = \frac{-\ln \left( \sigma^{\text{inv}}(x_T, \sqrt{s_1}) / \sigma^{\text{inv}}(x_T, \sqrt{s_2}) \right)}{\ln \left( \sqrt{s_1} / \sqrt{s_2} \right)}
\]
x_T analysis: power of p_T depends on x_T and process.

h^±/\pi^0 – circles
\gamma – squares
Jets – triangles

The x_R analysis finds power of p_T independent of process within errors:

n_{pT} = 6.5 \pm 0.4

Fig. from Arleo, et al.; arXiv:0911.4604v2, PRL 105,06200 (2010)
n_{eff} without correction term using ATLAS Jet Fits

Use the full parameterization of the inclusive Jet cross section – including the $s$-dependent terms. Plots are for different angles in COM.

Find $n_{eff} \rightarrow 4$ as $x_T \rightarrow 0$.
Must formulate the cross sections with this:

\[ \sigma^{\text{inv}} \equiv E \frac{d^3 \sigma}{dp^3} (AB \rightarrow CX) = \frac{\alpha(\sqrt{s})(1 - x_R)^{n_x R(\sqrt{s}, p_T)}}{p_T^n} \]

\[ n^{\text{exp}} = -\ln \left( \frac{\sigma^{\text{inv}}(x_T, \sqrt{s_1})/\sigma^{\text{inv}}(x_T, \sqrt{s_2})}{\alpha(\sqrt{s_1})/\alpha(\sqrt{s_2})} \right) + \ln \left( \frac{\sqrt{s_1}}{\sqrt{s_2}} \right) \]
\[ n_{\text{eff}} \text{ with the } \alpha(s) \text{ cross section amplitude term} \]

Hence \( n_{\text{eff}} \rightarrow 4 \) as \( x_T \rightarrow 0 \) is a result of neglecting the ‘\( \alpha(s) \)’ term that contains important overall normalization that corrects \( n_{\text{eff}} \approx 4 \) to \( n_{\text{eff}} \approx 6 \).
Diquarks

Flavor Decomposition of the Elastic Nucleon Electromagnetic Form Factors
G. D. Cates, C. W. de Jager, S. Riordan, and B. Wojtsekhowski

Diquark correlations in baryons on the lattice with overlap quarks
Ronald Babich, et al.
arXiv:hep-lat/0701023v2 19 Oct 2007

Strong diquark correlations inside the proton
Jorge Segovia
EPJ Web of Conferences 113, 05025 (2016)

Hadron Systematics and Emergent Diquarks
Alexander Selema and Frank Wilczek

Cates, et al. conclude that d-quark contribution to the proton form-factor appears to be suppressed from no-diquark assumption.