

# Exercises for Neutrino Group Members to Study for the Part III

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## 1 Introduction

The Part III exam tests fluency in the concepts of our field. A problem with the Part III exam is that there are not a lot of good sources of appropriate exercises for you to use to study. Most textbooks books offer extended problems, much like you did in 8.701, 711 and 811, rather than conceptual questions of the type that are asked in this exam. The purpose of this document is to provide a set of conceptual questions for use in your Part III preparation. Like the exam, these cover a broad range of topics. I don't claim the list of topics is complete. The Part III usually has questions on the latest topics of interest in the field, and I have tried to include examples here, even though this means some of the problems will become outdated in the near future. There is extra emphasis on flavor physics in this set of questions since, as neutrino physicists, you especially need to know this topic well. These questions are presented using very similar phrasing to the questions you will hear in the Part III so that you can get used to the conversational style. Hope this is useful.

## 2 Basics (Kinematics, Quantum Numbers, Masses, Fundamental concepts, Etc.)

1. What is the relationship between resonance width and particle lifetime? What is meant by the partial width?
2. Inverse muon decay is  $\nu_\mu + e^- \rightarrow \mu^- + \nu_e$ . Calculate the threshold for this process to occur in the lab frame.
3. If I gave you the energies of 2 muons and the angle between them, what equation would you use to find the invariant mass? Could you identify if it is a  $J/\psi$  or an  $\Upsilon$ ? That is to say, do you know the invariant masses of these two particles (approximately)?
4. A  $K_s^0$  is 60 GeV in the lab frame. What is the largest opening angle in the lab frame of the two pions from the decay?
5. Consider a hydrogen target in a proton beam. What is the minimum beam energy to produce an antiproton?
6. Can you list the masses (more or less) of all of the charged leptons?
7. What are the constituent masses of the quarks? Why do I say “constituent” masses? What other kinds of quark masses are there?
8. What is the mass of each of the bosons (W, Z, H)?
9. What’s the mass of the  $\pi^+$ ,  $\pi^0$  and, roughly, the kaon? What’s the mass of the neutron, proton and delta resonance?
10. What’s the  $c\tau$  for a  $\pi^+$  and a  $K^+$ ? (In case it helps: I only remember one and then remember the other is 1/2 for the first.) Using this, can you work out roughly the fraction of 3 GeV pions and kaons that decay in the 50 m MiniBooNE beamline?
11. If a particle with average lifetime  $\tau$  has not decayed after a time  $\Delta t$ , what is the probability that it decays in the subsequent interval  $\Delta t$ ?
12. Can you use isospin invariance to explain why  $\sigma(pp \rightarrow \pi^+ + d)/\sigma(np \rightarrow \pi^0 d) = 2$ ? (d means deuteron).
13. The delta resonance is wide, the  $\pi^0$  resonance is narrow and a  $\pi^+$  resonance is never even discussed. Explain.
14. Why don’t we discuss top mesons? Every other quark forms a meson... Explain.
15. Discuss what it means for a theory to respect parity. Describe C.S. Wu’s parity violation experiment.
16. Pions have  $-1$  parity. But the  $\pi^0$  decays to 2 photons...? How do you explain this?
17. Does the  $\rho$  decay to two pions? Why or why not?
18. What is the OZI rule and how is it applied? How does the OZI rule explain the narrow  $J/\psi$  resonance?

19. What is a Yukawa potential? How does it enter into our discussions of particle and nuclear physics?
20. What is Fermi's Golden Rule? Can you describe the meaning of the different components of the equation?
21. What do we mean when we say production of a heavy particle is "limited by phase space"?
22. Do you know your units of cross section? Can you convert from barns to  $\text{cm}^2$ ? And for a  $\sim 10$  GeV beam hitting a fixed target, can you estimate, very roughly, the magnitude of the cross section of a  $pp$  interaction and a  $\nu p$  interaction?
23. Do you know the value of  $\hbar c$ ? The value of the fine-structure constant?  $\alpha_s(Q^2 = 1 \text{ GeV}^2)$ ? These values are useful to know.

### 3 W and Z Physics

1. Draw the Feynman diagrams for neutral current and charged current neutrino scattering. Charged current scattering was discovered much earlier than neutral current scattering. Why were neutral current events so difficult to demonstrate?
2. Direct production of the  $W$  and  $Z$  bosons were first observed at what collider? This collider made use of a new concept, which led to a Nobel prize, called stochastic cooling. What is that?
3. What accelerator has allowed the most precise measurements of  $Z$  decays to date?
4. Describe leptonic  $Z$  decays. How do you determine the  $Z$  mass? Now describe leptonic  $W$  decays. How do you measure the  $W$  mass?
5. In the Standard Model the  $W$  coupling allows for flavor change (e.g.  $d \rightarrow u$ ), but the  $Z$  coupling does not. This is consistent with what we see, but is there anything fundamental about this? In another universe, could the  $Z$  coupling also allow for flavor change? (where electric charge is conserved, so  $d \rightarrow s$  for example), without any other major changes to the Standard Model?
6. How do you measure the invisible width of the  $Z$ ? How is this measurement used to show there are 3 active neutrinos?

### 4 Weak Interactions, Neutrinos and Flavor

1. What is the GIM mechanism? What decay (or lack of decay) was it introduced to explain? How does it explain the effect?
2. Why does the  $\pi^+$  preferentially decay to  $\mu^+$  and  $\nu_\mu$ ? Roughly, how suppressed is the decay to  $e^+$  and  $\nu_e$ ? How about  $\tau^+$  and  $\nu_\tau$ ?
3. Here are 3 types of  $K^+$  decays:
  - $K^+ \rightarrow \nu_e e^+$
  - $K^+ \rightarrow \nu_\mu \mu^+$

- $K^+ \rightarrow \pi^0 e^+ \nu_e$

Which do you expect to have the largest branching ratio? Which will have the smallest branching ratio? Explain your ordering.

4. For a 2 flavor mixing model, explain the derivation of the neutrino oscillation probability.
5. Do solar neutrinos oscillate as they travel from the sun to the earth? Why or why not? Without derivations, only with words and sketches, discuss the MSW effect.
6. When we discuss “matter effects” in neutrino oscillations, what are we referring to? How can this help us learn the mass hierarchy? No derivations – explain in words and sketches.
7.  $\bar{\nu}_e + p \rightarrow n + e^+$  is called inverse beta decay. The connection to neutron decay, which has a well-measured lifetime, means that the cross section is extremely well known. Explain the connection. Along with the well-known cross section, a second nice feature is that the energy of the antineutrino can be fully reconstructed in these events. Work out the formula that relates the visible energy in the detector to the antineutrino energy.
8. What is the difference between a Gamow-Teller transition and a Fermi transition in nuclear beta decay? What is an allowed versus a forbidden transition?
9. Kaon decay-at-rest produces a monoenergetic beam of neutrinos at about 235 MeV. How do you make a “KDAR” decay-at-rest neutrino beam? Why is this a pure neutrino beam? What happened to the  $K^-$ ?
10. How would you design a neutrino experiment using  $K_L$ 's where the initial flavor of the neutrino is tagged? Will you see oscillations in a this flavor-tagged neutrino experiment?
11. Why does the  $K_L$  have such a long lifetime compared to the  $K_S$ ? How do you generate some  $K_S$  in an initially pure  $K_L$  beam? Physically what is happening in this process?
12.  $CP$  violation in mesons can occur through mixing or direct  $CP$  violation. Pick your favorite neutral meson and explain the difference between the two types of  $CP$  violation. Draw Feynman diagrams. What is a penguin diagram?
13. What is  $B_s$  mixing? By analogy to neutrino oscillations, can you write down the formula for the probability for  $B_s \rightarrow \bar{B}_s$ ? How do you search for  $B_s$  mixing?
14. The LHCb detector has a very different design than ATLAS or CMS. Can you describe it? How is this different? We say that it “looks like” a fixed target experiment. Why do we say that? Why is its design so different from ATLAS and CMS?
15. Consider  $B^0 \rightarrow \mu^+ \mu^-$ . This is an important rare decay channel result from LHCb. This happens through a weak interaction triangle diagram, Can you draw it?. One can make an invariant mass peak from the 2 muons. What dominates the width of that peak that you measure? Let's break up the question. Considering only first principles interaction decay physics (no detector effects or any other external physical effects at all) what would the invariant mass peak look like? What do physical effects like multiple scattering and momentum resolution do to the peak? Let's discuss momentum resolution... what can you vary about the detector components and layout to affect/improve momentum resolution?

## 5 The Higgs

1. Before the Higgs was found at LHC, many experiments had limited the mass range where a Standard Model Higgs could be found. Can you describe two or three example searches?
2. Many Higgs searches involve jets. These are found by jet algorithms. What's a jet algorithm? Two "classics" are the "cone" and the  $k_t$  algorithms. How do they work?
3. The favored way to search for the Higgs is  $H \rightarrow \gamma\gamma$ . But the Higgs couples to mass! So isn't this a dumb plan? Explain by drawing Feynman diagrams how this decay proceeds. Now consider the problem of detection – there are so many photons produced, won't this signal be overwhelmed? What makes this signal identifiable?
4.  $H \rightarrow \mu\tau$  would be a very exciting signature. Why?
5. What is the ratio between  $Br(H \rightarrow b\bar{b})$  and  $Br(H \rightarrow \tau\tau)$ , for a Higgs mass of about 120 GeV?
6. Now that we have observed the Higgs, name some detailed questions are we asking about the characteristics of this particle and why.

## 6 Beyond Standard Model Searches

1. Missing  $E_t$  is a signature for new physics at colliders. What is missing  $E_t$ ? There are 3 "not-exotic" issues in detectors that can lead to apparent missing  $E_t$ . What are they?
2. What is a graviton? What is its spin and why? Gravitons are massless – why have we not seen them yet? How might we see them?
3. SUSY is a very popular beyond-Standard Model. What is the symmetry associated with supersymmetry? Name the SUSY partners.
4. It was postulated that the SUSY masses were close to the quark and lepton masses in order to solve a specific issue in the Standard Model called the hierarchy problem. What is that problem and would SUSY partners have solved it?
5. A very popular BSM theory in the early 2000's was large extra dimensions. In a handwavey way, what is the theory of large extra dimensions? How do you search for evidence of effects of large extra dimensions in particle physics? (Give a couple of examples)
6. What is a Majorana neutrino? How do we search for them? In very general terms, explain why we find the idea of the existence Majorana neutrinos compelling. For 100 points extra credit plus a gold star, answer this: the condensed matter community also likes to talk about Majorana fermions. Compare and contrast their "Majorana fermions" to ours.
7. Why do we think there is dark matter? Can you name 2 different ways that the existence of dark matter has been indicated?
8. What is a WIMP? Here are 3 ways to look for WIMPs: 1) production 2) direct detection and 3) indirect detection. For each of these, name an example experiment and describe the experimental signature.

9. In direct dark matter searches, people are now worried about the “neutrino floor.” What is the neutrino floor?
10. What is an axion? Without getting into the details, how do you search for axions?
11. Write down a few standard model diagrams that contribute to muon  $g-2$ . Write down a BSM diagram that contributes? How is the muon  $g-2$  measured? Is there an anomaly presently associated with  $g-2$ ?
12. The decay  $\mu \rightarrow e\gamma$  violates lepton flavor. Isn't this just fine, given that neutrinos oscillate? Why are people so excited about this decay for BSM searches? Draw at least one diagram for such decays (either SM or BSM). Describe what are the irreducible backgrounds.
13. What is the physics motivation of DarkLight? What is the physics motivation of the experiment? What kind of beam is involved? How is the experiment laid out?
14. AMS searches for high energy electrons and positrons. How high is the energy? What's the BSM physics motivation for this search? And what experimental anomaly are they investigating?
15. How do neutrons end up with electric dipole moments? Is it very large? Why try to measure the electric dipole moment of the neutron? What new physics can affect this?

## 7 Standard Model Studies

1. What is coherent neutrino scattering? Why might measurements of this be interesting? How might you measure it – what's the signal? What do you think is likely to be the most pernicious background?
2. Describe the Olympus Experiment. Where is it running? What beam and target does it use? What is its primary physics motivation?
3. What is an exotic meson? Have any ever been found? Describe the Glue-X experiment at JLAB.
4. QWeak at JLAB measures the proton's weak charge. What is the proton's weak charge? How is it related to  $\sin^2 \theta_W$ ?
5. There are multiple ways to extract a measure of the radius of the proton. Can you describe one or two? What is the proton radius puzzle?
6. The measurement of  $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  provided confirmation of the basic tenets of QCD. Describe what this measurement determined. Sketch  $R$  as a function of center of mass energy.
7. There are certain types of scattering that you should recognize by name. For each of these, what is the Feynman diagram and how are they used (or what is the primary interest today)?
  - Bhabha scattering
  - Moller scattering
  - Mott scattering

- the Drell-Yan process
8. Show that, in deep inelastic lepton-proton scattering, the fraction of momentum carried by the struck quark in the “infinite momentum frame” (what did I mean by that expression?) is  $x_{bj} = Q^2/2M\nu$ , where  $Q^2$  is the NEGATIVE 4 momentum transfer,  $M$ =mass of the proton, and  $\nu$  is the energy transferred to the parton.
  9. What are parton distributions? What kinematic variables do they depend on? Describe some experiments constructed to measure parton distributions. Do the kinematic variables of these experiments overlap LHC?
  10. In studying ratios of structure functions measured from various nuclei, the plots often go to  $x_{bj} > 1$ . How can this happen? What can cause  $x_{bj} > 1$ ?
  11. What are “higher twist effects”?
  12. What was the “spin crisis”? Is it still a crisis?
  13. What is a fragmentation function? How are they used?
  14. What’s the basic idea behind lattice QCD calculations? Can you name an arena in which Lattice QCD calculations are especially successful or promising?
  15. The coupling constants vary with  $Q^2$ . Can you describe how  $\alpha$  and  $\alpha_s$  vary? Make a sketch. Can you explain why the qualitative behavior is different?

## 8 Connections to Astrophysics/Cosmology

1. In the early universe, what was the QCD phase transition? What types of experiments search for evidence of the QCD phase transition? Has this effect been observed?
2. What do we mean when we talk about “BBN”? Which isotopes were created in BBN? More or less, how good are our BBN predictions? Do they agree with experiments? What type of experiments?
3. What is the GZK cutoff? Can you calculate the approximate energy of the proton at which the GZK cutoff begins? Name some experiments that look for particles beyond the GZK bound.
4. All of the elements from iron up to uranium are made in supernovae. What is different about the elements past iron, compared to those below iron, that results in the fact that they cannot be made as stars burn? The  $R$ -process models do a very good job of describing the elements in a supernova up to uranium. Very briefly, what’s the  $R$ -process?
5. What’s the primary process by which the sun produces energy? Roughly estimate long does it take for energy produced as a gamma ray in the center of the sun to reach the earth. Compare this to how fast neutrinos produced in the sun reach the earth.
6. Can you think of some fundamental questions that we can potentially address by studying supernova neutrinos? How many supernovae do we expect per century in our galaxy (more or less)? When was the most recent supernova neutrino from our galaxy seen?
7. Describe the FRIB accelerator. The goals for experiments at FRIB are closely connected to astrophysical questions. Can you describe an example?

## 9 Nuclear Physics

1. What is the liquid drop model? How does the phenomenology of the liquid drop model map onto the terms in the semi-empirical mass formula?
2. Explain the shell model. Who first developed it? What's a magic number? For lots of extra credit, can you name the first 5 magic numbers?
3. Sketch the binding energy per nucleon versus  $A$ . Explain the basic features.
4. What does the nuclear potential for protons look like? What does the nuclear potential for neutrons look like?
5. Explain the quantum mechanical origin of Fermi Momentum. How is Fermi energy related to Fermi momentum? What is a typical value of either Fermi momentum or Fermi energy? How do we know that Fermi momentum is "real"? i.e. describe an experimental measurement that this will affect.
6. What is the relativistic Fermi Gas Model?
7. A major discovery at RHIC was the production of a "perfect fluid" with sheer viscosity approaching zero. Explain what they were seeing and why this is very exciting.
8. In heavy ion interactions, certain ideas and terms come up. First of all, can you make a sketch of what two colliding heavy ions looks like? What do we mean by centrality? What are participants as opposed to spectator particles, and what kind of particles are they? HI collisions are often discussed in terms of rapidity. What is rapidity?
9. How does a reactor work, more or less? (We are looking for a very general answer! We are not expecting you to compare and contrast different types of reactors!)

## 10 Accelerators

1. What is luminosity? Write down the formula and explain the terms.
2. Compute the average number of interactions per bunch crossing for a luminosity  $L = 2.5 \times 10^{31} \text{ cm}^{-2}\text{s}^{-1}$ , if the total cross section is  $\sigma = 20 \text{ mb}$  and bunch crossings occur every  $T=4 \mu\text{s}$ . Roughly, what is the probability of having zero interactions in a bunch crossing ?
3. Pile-up is now a major problem at LHC. What is the cause of pile-up?
4. The Tevatron was a  $\bar{p}p$  collider. LHC is a  $pp$  collider. What are the pros and cons of  $\bar{p}p$  vs  $pp$ ? Plan to sketch some diagrams to make your case.
5. What is a Free Electron Laser?
6. Explain the present proposed raison d'être for the linear collider. Why is this collider linear?
7. An alternative to the linear collider is a muon collider. What are the pros and cons of a muon collider?
8. Another major "energy frontier" project under discussion is the 100 TeV proton collider. Discuss the case, sketch the layout and consider pros and cons compared to the ILC and muon collider ideas on the table.



9. The  $J/\psi$  was found by two different experimental groups at almost the same time – one at BNL, the other at SLAC. The two groups were running at different kinds of machines. What were the two kinds of machines? These types of machines are still used for discovery today. What are the pros and cons of the two types of machines for discovery today?
10. In a synchrotron accelerator, what is the ratio of energy loss for  $p$  versus  $e$ ? (you can express this in terms of constants, then make an order of magnitude estimate.)

## 11 Detectors

1. Order these detectors in terms of time-response: silicon, plastic scintillator, drift chamber.
2. How does a time-of-flight (TOF) detector work? How good of a measurement of TOF can you extract with today's technology? Let's say that I have pions and kaons of momentum 2 GeV/c and I measure the TOF across a distance of 2 m. If I can resolve 0.2 ns differences, can I separate pions from kaons?
3. What is meant by a "MIP"?
4. Consider a muon. Sketch  $dE/dx$  versus  $\gamma\beta$ ? Describe the features (what causes a rise or fall? where is the MIP region?, etc.). The medium range part of this plot is being described by the Bethe-Block equation ( $0.1\beta\gamma > 1000$ ) Can you write down the form of the formula (up to some constants)?
5. Does a pion ever behave like a MIP particle? Can it ever be described by the Bethe-Block equation?
6. There is something fundamentally different about electrons that leads to a change in the Bethe-Block equation in order to describe their  $dE/dx$  distribution. What is it? (Hints: 1) it is not the mass. 2) It is deeply fundamental, though, and it has to do with scattering). Does the electron have a "MIP" region?
7. How does a drift chamber work? Aside from statistics of the ionized electrons, what factors affect the resolution?
8. A minimum ionizing particle traverses a 2.5 cm thick drift chamber filled with argon-isobutane. Roughly how many ionized electrons result? What is a typical drift velocity for electrons in a drift chamber? What processes happen when the electron gets close to the wire? What is the purpose of the argon as versus the isobutane? What gets added to make this a "magic gas"? Name another example or two of magic gas mixtures.
9. How does a silicon detector work? If a MIP particle traverses the silicon detector, roughly how many electrons will be liberated? What is the advantage of a silicon detector over a drift chamber? What is a big drawback? What experiment has more silicon in it than any other particle physics detector on earth?
10. Explain how a PMT works. What is a "stage"? What's a typical number of stages? What's a typical gain?
11. What sets the Cherenkov threshold? What sets the opening angle of a Cherenkov ring? Roughly, how does the Cherenkov flux vary with wavelength? What wavelength range do we usually use to detect Cherenkov light?

12. What is the source of light produced by electrons traversing lead glass in a lead glass calorimeter?
13. What is transition radiation? How are transition radiation detectors designed? Can you briefly describe the AMS transition radiation detector?
14. In roughly what energy interval does Compton scattering dominates among the processes of interactions between photons and matter? What dominates at lower and higher energies? How does this depend on the material?
15. Let's talk about EM showers. Make a sketch (sideview) of a lead and scintillator calorimeter on the board. Sketch how a shower evolves within this picture. What is the meaning of radiation length? More or less, what is a reasonable thickness for the lead sheets?
16. For an electromagnetic calorimeter, what is the meaning of critical energy? What do we mean by the Moliere radius and how is it related to the radiation length and critical energy? Is the critical energy the same for electrons and muons?
17. Let's talk about hadron calorimeters. Let's say you make your calorimeter out of some material of your own choice interleaved with scintillator. What material would you like to use? How thick will you make each slab? Sketch how a hadronic shower evolves.
18. What is a compensating calorimeter?
19. How does  $dp/p$  depend on the momentum of charged particles tracked in a magnetic field in air? And in iron ?

## 12 Opinion (FYI — Never answer: “I have no opinion!”)

1. Consider your area of physics. List what you think are the top 3 most important questions (you don't have to rank them) and discuss why you think these questions are important.
2. Every other year, NuPaX develops a strategic plan where we identify prime areas for growth. In what area(s) do you recommend we grow? Explain.
3. Tell us about an arxiv paper that you read recently that especially interested you.
4. Consider the LNS colloquia and the lunch time seminars you have heard recently. Was there one that especially interested you? Explain.

### 13 A Quick Exercise: Which of these are possible and why?

5. Fill in the following table:

Reaction	Is this possible?	If so, what mediating force is (forces are) involved?	if not, what conservation law is violated
a) $p\bar{p} \rightarrow \pi^+\pi^0$			
b) $\eta \rightarrow \gamma\gamma$			
c) $\Sigma^0 \rightarrow \Delta\pi^0$			
d) $\Sigma^- \rightarrow n\pi^-$			
e) $e^+e^- \rightarrow \mu^+\mu^-$			
f) $\mu^- \rightarrow e\bar{\nu}_e$			
g) $\Delta^+ \rightarrow p\pi^0$			
h) $\bar{\nu}_e p \rightarrow n e^+$			
i) $e^- p \rightarrow \nu_e n$			
j) $pp \rightarrow \Sigma^+ n K^0 \pi^+ \pi^0$			
k) $p \rightarrow e^+ \pi^0$			
l) $pp \rightarrow ppp\bar{p}$			
m) $n\bar{n} \rightarrow \pi^+\pi^-\pi^0$			
n) $\pi^+ n \rightarrow \pi^- p$			
o) $K^- \rightarrow \pi^-\pi^0$			
p) $\Sigma^+ n \rightarrow \Sigma^- p$			
q) $\mu^- \rightarrow e^- \gamma$			
r) $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$			
s) $\Xi^0 \rightarrow p\pi^-$			
t) $\pi^- p \rightarrow \Lambda K^0$			
u) $\pi^0 \rightarrow \gamma\gamma$			