Electric dipole

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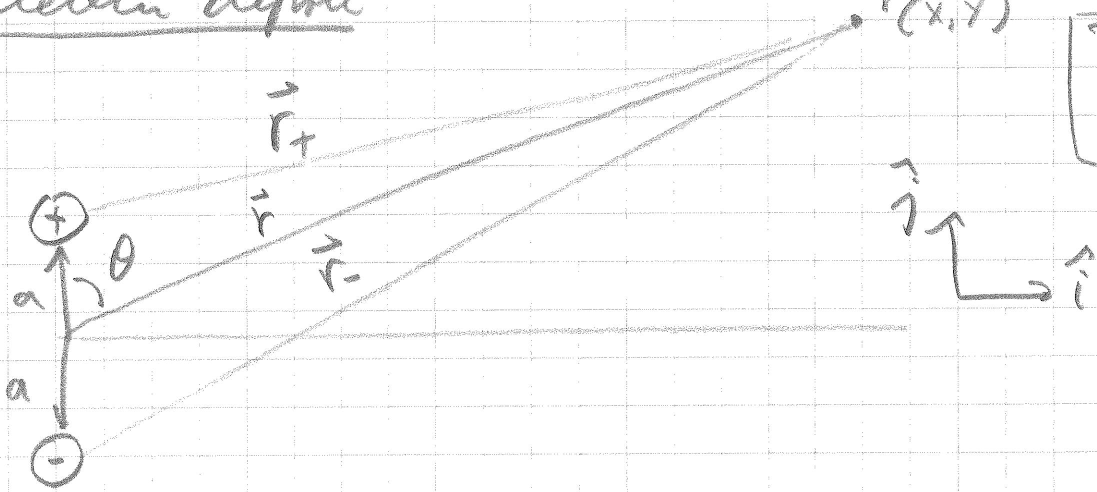

$$\vec{r}_+ = x\hat{i} + (y-a)\hat{j} \qquad \vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{r}_- = x\hat{i} + (y+a)\hat{j}$$

$$\boxed{\begin{aligned} E_x &= kq\left\{\frac{x}{(x^2+(y-a)^2)^{3/2}} - \frac{x}{(x^2+(y+a)^2)^{3/2}}\right\} \\ E_y &= kq\left\{\frac{y-a}{(x^2+(y-a)^2)^{3/2}} - \frac{y+a}{(x^2+(y+a)^2)^{3/2}}\right\} \end{aligned}}$$

Far from the dipole, $x \gg a$, $y \gg a$

Then $x^2+(y-a)^2 = x^2+y^2-2ay+a^2$

$$\simeq x^2+y^2-2ya$$

$$\frac{1}{(x^2+(y-a)^2)^{3/2}} \sim \frac{1}{(x^2+y^2-2ya)^{3/2}} = \frac{1}{(r^2-2ya)^{3/2}}$$

$$= \frac{1}{r^3}\frac{1}{(1-2ya/r^2)^{3/2}}$$

Taylor expansion

$$\frac{1}{\left(1-\frac{2ya}{r^2}\right)^{3/2}} = \frac{1}{(1+\epsilon)^{3/2}} \sim 1 - \frac{3\epsilon}{2}$$

Take $\epsilon = -\frac{2ya}{r^2}$, then $\frac{1}{\left(1-\frac{2ya}{r^2}\right)^{3/2}} \sim 1 + \frac{3ya}{r^2}$

Similarly $\frac{1}{\left(1+\frac{2ya}{r^2}\right)^{3/2}} \sim 1 - \frac{3ya}{r^2}$

For $E_x = kqx \left\{ \frac{1}{(x^2+(y-a)^2)^{3/2}} - \frac{1}{(x^2+(y+a)^2)^{3/2}} \right\}$ $\quad \frac{Nm^2}{C^2} \frac{C\,m}{m^2}$

$$= \frac{kqx}{r^3} \left\{ 1 + \frac{3ya}{r^2} - \left(1 - \frac{3ya}{r^2}\right) \right\}$$

$$= \frac{6qykxa}{r^5} \qquad \cancel{C\,m\,\frac{Nm^2}{C^2}\,\frac{m\,m}{m^4}}$$

$x = r\cos\theta$
$y = r\sin\theta$
$p = 2qa$

$$\boxed{E_x = \frac{3pk\cos\theta\sin\theta}{r^3}}$$

For E_y:

$$E_y = \frac{kq}{r^3} \left\{ (y-a)\left(1+\frac{3ya}{r^2}\right) - (y+a)\left(1-\frac{3ya}{r^2}\right) \right\}$$

$ya^2 \ll y^2a$

$$= \frac{kq}{r^3} \left\{ \left(y - a + \frac{3y^2a}{r^2} - \cancel{\frac{3ya^2}{r^2}}\right) - \left(y + a - \frac{3y^2a}{r^2} - \cancel{\frac{3ya^2}{r^2}}\right) \right\}$$

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$$E_y = \frac{kq}{r^3}\left\{\frac{6y^2a}{r^2} - 2a\right\}$$

$$\boxed{E_y = \frac{kp}{r^3}\left\{3\cos^2\theta - 1\right\}}$$