The Oomph factor in eA: A-dependence of Q_{s}

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Introduction

"Oomph" = increase in nonlinear effects / black body scattering

$$$ Expensive oomph: high energy, small x
¢¢¢ Cheap oomph: large A

Questions:

- How to extend saturation model fits from protons to nuclei?
- What is the A dependence of Q_{s}?

This talk:

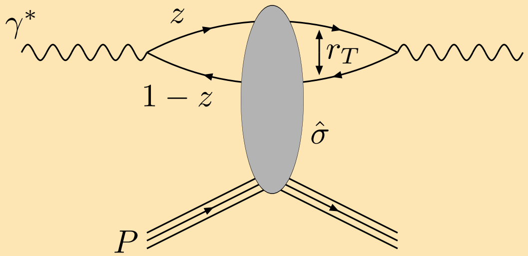
- Simple GBW-like fits to existing eA data
- A more detailed picture of nuclear geometry ▶ the "lumpy" nucleus

For concreteness I will use the KT "impact parameter saturation model" [1], although I believe this simple discussion applies more generally.

[1] H. Kowalski and D. Teaney, *Phys. Rev.* **D68** (2003) 114005 [hep-ph/0304189].

Basics: small-x and dipole cross section

$$\sigma_{\text{dip}}(x, \mathbf{r}_T) = \int \mathrm{d}^2\mathbf{b}_T \frac{\mathrm{d}\sigma_{\text{dip}}(x,\mathbf{r}_T,\mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T}$$
$$\sigma_{\text{dip}}(x, \mathbf{r}_T, \mathbf{\Delta}) = \int \mathrm{d}^2\mathbf{b}_T \frac{\mathrm{d}\sigma_{\text{dip}}(x,\mathbf{r}_T,\mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T} e^{i\mathbf{b}_T\cdot\mathbf{\Delta}},$$
$$\mathbf{\Delta}^2 = -t$$



Total $\gamma^* p$: $\sigma^{\gamma^* p}_{L,T} = \int \mathrm{d}^2\mathbf{r}_T \int \mathrm{d}z \left|\Psi^\gamma_{L,T}(Q^2, \mathbf{r}_T, z)\right|^2 \sigma_{\text{dip}}(x, \mathbf{r}_T)$

Total diff.: $\frac{\sigma^D_{L,T}}{\mathrm{d}t} = \frac{1}{16\pi} \int \mathrm{d}^2\mathbf{r}_T \int \mathrm{d}z \left|\Psi^\gamma_{L,T}(Q^2, \mathbf{r}_T, z)\right|^2 \sigma^2_{\text{dip}}(x, \mathbf{r}_T, \mathbf{\Delta})$

X-cl. diff.: $\frac{\sigma^D_{L,T}}{\mathrm{d}t} = \frac{1}{16\pi} \left| \int \mathrm{d}^2\mathbf{r}_T \int \mathrm{d}z \left(\Psi^\gamma \Psi^{*V}\right)_{L,T} \sigma_{\text{dip}}(x, \mathbf{r}_T, \mathbf{\Delta})\right|^2$

Assumptions:

- S-matrix real
- optical theorem

- $\Psi^\gamma(Q^2, \mathbf{r}_T, z) \sim K_{0,1}(\sqrt{z(1-z)}Q|\mathbf{r}_T|)$ ▶ momentum scale $Q^2 \sim 1/\mathbf{r}_T^2$
- **Same dipole cross section in both inclusive and diffractive**.

Definition of Q_s

Generic behavior of the dipole cross section:

$$\frac{\mathrm{d}\sigma_\mathrm{dip}(x,\mathbf{r}_T,\mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T} \begin{array}{ll} \sim & \mathbf{r}_T^2, \quad \mathbf{r}_T \to 0,\ \text{pQCD} \\ \lesssim & 2, \quad \mathbf{r}_T \to \infty,\ \text{unitarity} \end{array}$$

$\mathbf{r}_T^2 = 1/Q_\mathrm{s}^2$ or $Q^2 \sim Q_\mathrm{s}^2$ characterizes the transition between these regimes.

Model independent (implicit) definition of $Q_\mathrm{s}(x,\mathbf{b}_T)$:

$$\frac{\mathrm{d}\sigma_\mathrm{dip}\left(x,\mathbf{r}_T^2 = \frac{1}{Q_\mathrm{s}^2(x,\mathbf{b}_T)},\mathbf{b}_T\right)}{\mathrm{d}^2\mathbf{b}_T} = 2(1-e^{-1/4}) \approx 0.44$$

The rhs. constant is just (my) convention, chosen to match GBW and others.

Simple impact parameter dependence: GBW

Golec-Biernat & Wüsthoff [2,3] and many others

For total cross section assume factorized impact parameter dependence:

$$\sigma_{\mathrm{dip}}(x, \mathbf{r}_T) = 2 \int \mathrm{d}^2\mathbf{b}_T \left(1 - e^{-\mathbf{r}_T^2 Q_{\mathrm{s}}^2(x)/4}\right) T_p(\mathbf{b}_T), \quad Q_{\mathrm{s}}^2 = Q_0^2 (x/x_0)^{-\lambda}$$

Fit 3 parameters to inclusive cross section

- $\lambda = 0.28$
- $x_0 = 0.4 \times 10^{-4}$
- $\sigma_0 = 2 \int \mathrm{d}^2\mathbf{b}_T T_p(\mathbf{b}_T) = 29\mathrm{mb}$

For diffractive data assume constant diffractive slope

$$\sigma_{\mathrm{dip}}(x, \mathbf{r}_T, \mathbf{\Delta}) = e^{-\frac{1}{2} B_D \mathbf{\Delta}^2} \sigma_{\mathrm{dip}}(x, \mathbf{r}_T, \mathbf{\Delta} = 0)$$

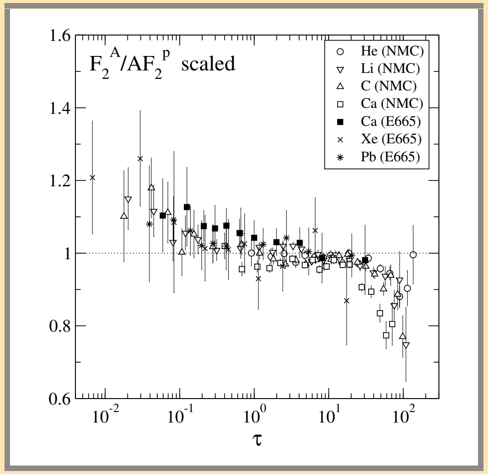
with $B_D = 6/\mathrm{GeV}^2 \Longleftrightarrow$ **Gaussian** proton with rms. radius $R_p \approx 0.7\mathrm{fm}.$

[2] K. Golec-Biernat and M. Wusthoff, *Phys. Rev.* **D59** (1999) 014017 [hep-ph/9807513].
[3] K. Golec-Biernat and M. Wusthoff, *Phys. Rev.* **D60** (1999) 114023 [hep-ph/9903358].

Fitting existing nuclear data à la GBW 1

Freund, Rummukainen, Weigert[4], geometric scaling fit to E665[5,6] and NMC[7] data.

$$F_2^A(x,Q^2) = \overbrace{\left(\frac{x}{x_0}\right)^{-\lambda} A^{1/3}}^{F_2 \sim Q^2 \sigma} \overbrace{A^{2/3+\gamma}}^{\text{area}} F_2^p\left(x_0, \left(\frac{x}{x_0}\right)^{\lambda} \frac{Q^2}{A^{\delta}}\right)$$



- Expectation: $\gamma = 0$ ($\iff \pi R_A^2 \sim A^{2/3}$) and $\delta = 1/3$ ($\iff Q_s^2 \sim A^{1/3}$)
- Fit result: $\gamma = 0.09$ and $\delta = 1/4$

Slower growth of $Q_s^2 \sim A^{1/4}$ compensated by growth of πR_A^2.

[4] A. Freund, K. Rummukainen, H. Weigert and A. Schafer, *Phys. Rev. Lett.* **90** (2003) 222002 [hep-ph/0210139].

[5] **E665** Collaboration, M. R. Adams *et. al.*, *Z. Phys.* **C65** (1995) 225.

[6] **E665** Collaboration, M. R. Adams *et. al.*, *Z. Phys.* **C67** (1995) 403 [hep-ex/9505006].

[7] **New Muon** Collaboration, P. Amaudruz *et. al.*, *Nucl. Phys.* **B441** (1995) 3 [hep-ph/9503291].

Fitting existing nuclear data à la GBW 2

Armesto, Salgado, Wiedemann [8]

$$Q_s^{A\,2} = \left(\frac{A\pi R_p^2}{\pi R_A^2}\right)^\alpha (Q_s^p)^2$$

with $R_A = (1.12A^{1/3} - 0.86A^{-1/3})\text{fm}$.
Fit parameters: R_p, α, i.e. essentially

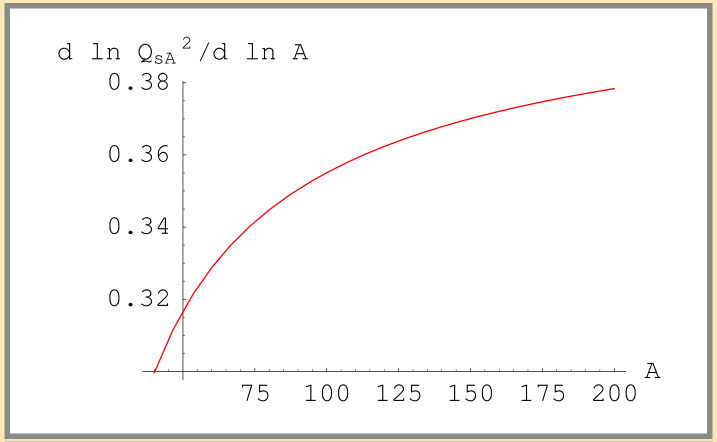
$$Q_s^{A\,2} = C\left(\frac{A}{(A^{1/3} - 0.77A^{-1/3})^2}\right)^\alpha (Q_s^p)^2$$

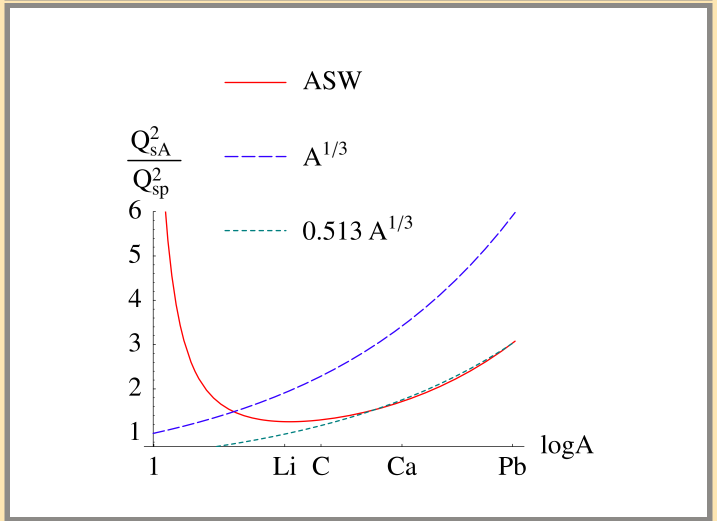
with C and α determined by the fit.
Fit result: $R_p = 0.7\text{fm}$, $\alpha = 1.25$, i.e. $C \approx 0.3$.

Statement: "Favors $Q_s^{A\,2} \sim A^{4/9}$"

Also consistent with $Q_s^{A\,2} \sim A^{1/3}\ln A$

Also close to: $Q_s^{A\,2} \sim 0.5A^{1/3}Q_s^{p\,2}$





$4/9 \approx 0.44$

[8] N. Armesto, C. A. Salgado and U. A. Wiedemann, *Phys. Rev. Lett.* **94** (2005) 022002 [hep-ph/0407018].

KT impact parameter saturation model

See also [9,10]

$$\sigma_{\mathrm{dip}}(x, \mathbf{r}_T) = 2 \int \mathrm{d}^2\mathbf{b}_T \left(1 - \exp\left\{ -\frac{\pi^2}{2N_\mathrm{c}} \alpha_\mathrm{s}(\mu^2) x g(x, \mu^2) T_p(\mathbf{b}_T) \mathbf{r}_T^2 \right\} \right),$$

with DGLAP-evolved $xg(x, \mu^2 = \mu_0^2 + 4/\mathbf{r}_T^2)$.

For nuclei:

$$T_p(\mathbf{b}_T) \Longrightarrow \sum_{i=1}^{A} T_p(\mathbf{b}_T - \mathbf{b}_{Ti}) \text{ with } \mathbf{b}_{Ti} \text{ from } T_A(\mathbf{b}_{Ti})$$

Leads to

$$\frac{\mathrm{d}\sigma^A_{\mathrm{dip}}(x, \mathbf{r}_T, \mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T} \approx 2\left[1 - e^{-\frac{A T_A(\mathbf{b}_T)}{2} \sigma^p_{\mathrm{dip}}} \right] \tag{1}$$

[9] E. Levin and M. Lublinsky, *Nucl. Phys.* **A696** (2001) 833 [hep-ph/0104108].
[10] J. Bartels, K. Golec-Biernat and H. Kowalski, *Phys. Rev.* **D66** (2002) 014001 [hep-ph/0203258].

Why $A^{1/3}$?

$$\frac{\mathrm{d}\sigma^A_{\mathrm{dip}}(x, \mathbf{r}_T, \mathbf{b}_T)}{\mathrm{d}^2\mathbf{b}_T} \approx 2\left[1 - e^{-\frac{A T_A(\mathbf{b}_T)}{2}\sigma^p_{\mathrm{dip}}}\right] \qquad (2)$$

Simplify (2) by approximating

$$\sigma^p_{\mathrm{dip}}(\mathbf{r}_T) = 2\pi R_p^2 \left(1 - e^{-\mathbf{r}_T^2 (Q_\mathrm{s}^p)^2/4}\right)$$

$$T_A(\mathbf{b}_T) = \frac{\theta(R_A - |\mathbf{b}_T|)}{\pi R_A^2} \text{ and } A \gg 1$$

▶ $Q_\mathrm{s}^{A\,2} \approx \frac{A R_p^2}{R_A^2} Q_\mathrm{s}^{p\,2} \approx 0.3 A^{1/3} Q_\mathrm{s}^{p\,2}$

with $R_p = 0.6\mathrm{fm}$ and $R_A = 1.1 A^{1/3}\mathrm{fm}$

Diluteness of nucleus: $R_p A^{1/3} < R_A$ ▶

Yes, $A^{1/3}$, but with a constant ~ 0.3 in front.

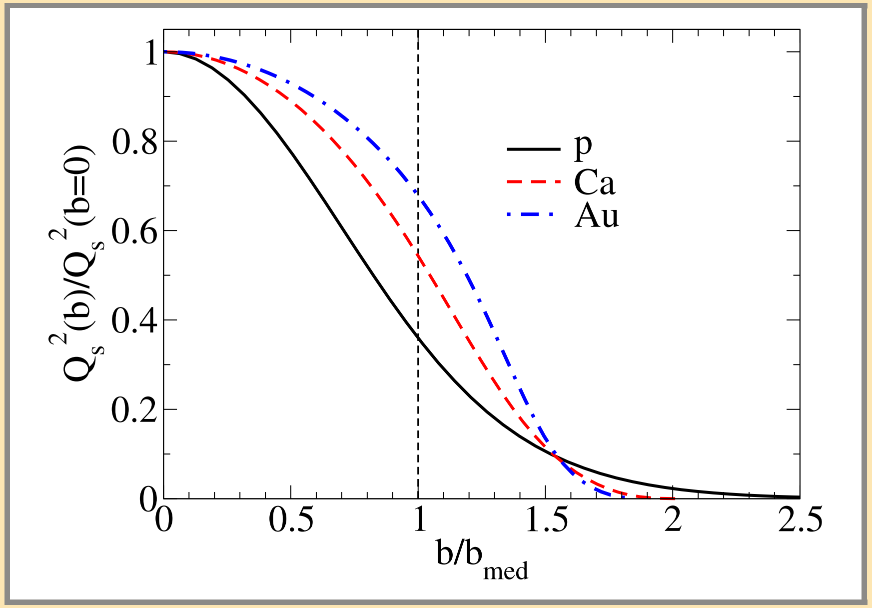
Impact parameter dependence

Nucleus is a large object: dilute edge is less important than in proton.

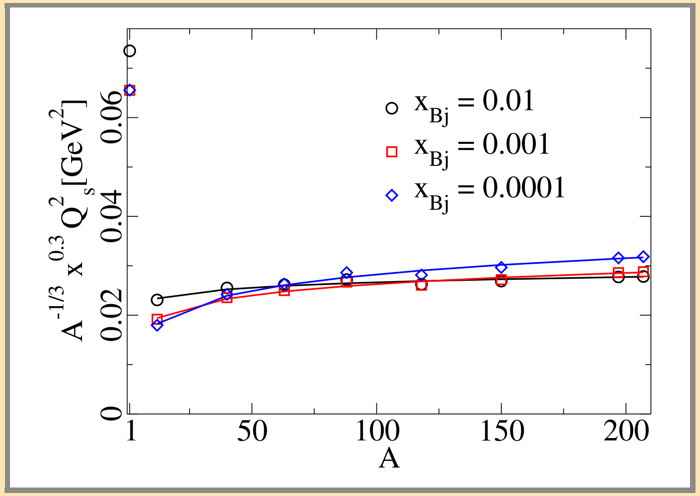
Compare Q_s at median impact parameter b_{med}

▼

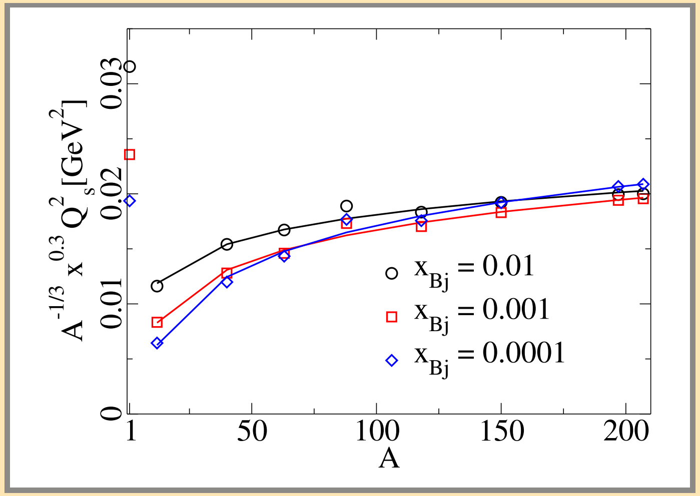
Even at same $Q_s(b=0)$ larger saturation effects in nuclei



Center



median b



Effect of DGLAP

Previous slides: neglected increase of $xg(x, C/\mathbf{r}_T^2 + \mu_0^2)$ with $1/\mathbf{r}_T^2$.

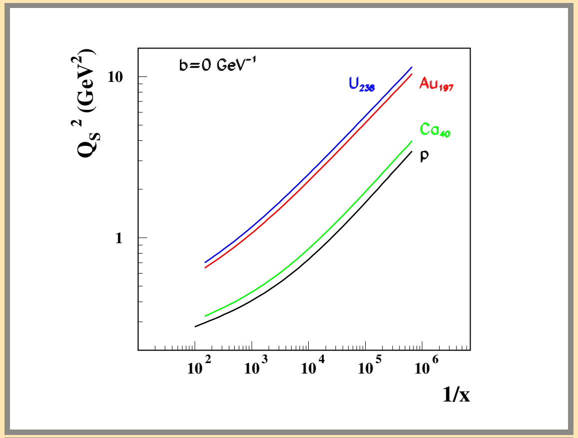
$$\frac{\overbrace{AT_A(\mathbf{b}_T)}^{\sim A^{1/3}} xg(x, Q_{\mathrm{s}}^{A\,2})}{(Q_{\mathrm{s}}^A)^2} = \frac{\overbrace{BT_B(\mathbf{b}_T)}^{\sim B^{1/3}} xg(x, Q_{\mathrm{s}}^{B\,2})}{(Q_{\mathrm{s}}^B)^2}$$

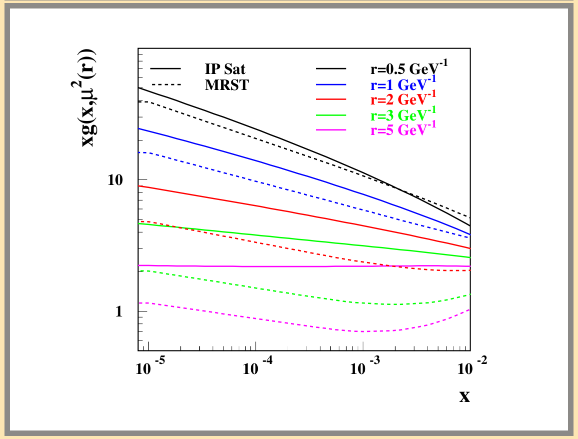
$$\frac{Q_{\mathrm{s}}^{A\,2}}{Q_{\mathrm{s}}^{B\,2}} \sim \left(\frac{A}{B}\right)^{1/3} \times \frac{xg(x, Q_{\mathrm{s}}^{A\,2})}{xg(x, Q_{\mathrm{s}}^{B\,2})}$$

Gold/Calcium: (@ $x = 10^{-4}$):
$2.6 \approx (197/40)^{1/3} \times 1.5$

Bigger effect for smaller x.

BK or JIMWLK evolution (with running coupling) could go in opposite direction [11].





[11] A. H. Mueller, *Nucl. Phys.* **A724** (2003) 223 [hep-ph/0301109].

Conclusions: what is the nuclear “oomph” factor?

Baseline: $Q_s^{A\,2}(b=0) \sim 0.3 A^{1/3} Q_s^{p\,2}(b=0)$

– Pay a price for diluteness of nucleus: $R_p < A^{-1/3} R_A$ ▶ not as much to gain from small nuclei.
+ In nuclei edge contributes less: more oomph at b_{med}.
+ More oomph from DGLAP
– Less oomph from $\ln 1/x$ evolution ?