Saturation Physics

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Outline

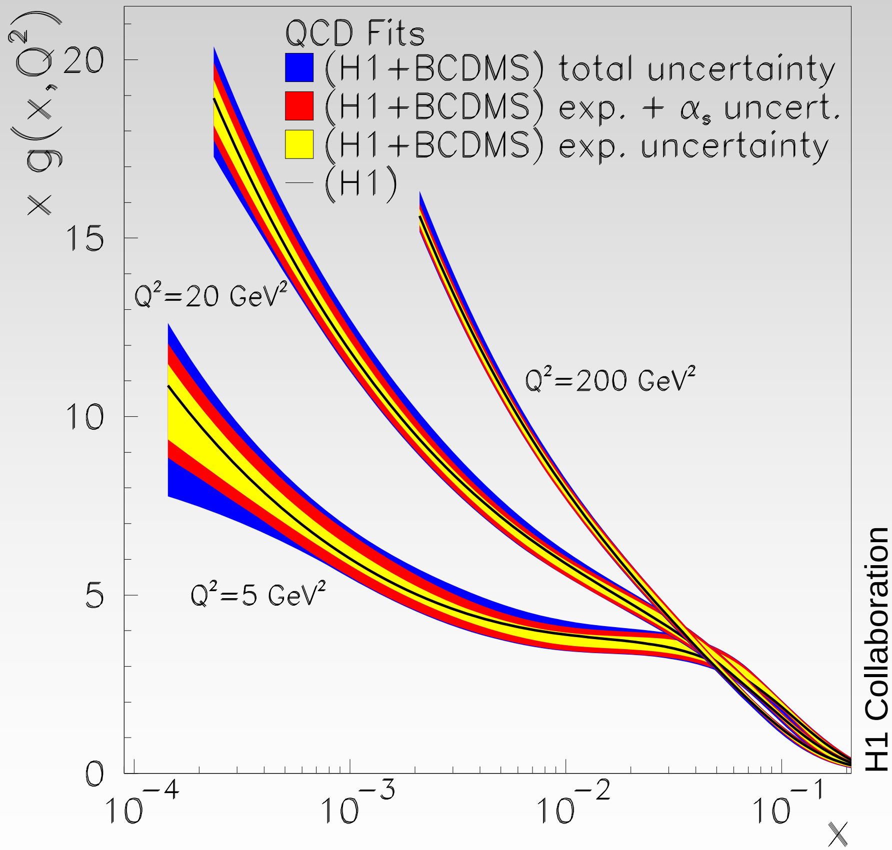
- Breakdown of DGLAP evolution at small-x and low Q^2.
- What we expect to find at small-x in nuclei: strong gluon fields, nonlinear dynamics, near-black cross sections.
- What we have seen so far.
- How EIC can solve many of the existing puzzles.
- Open theory questions.

Standard Approach: DGLAP Equation

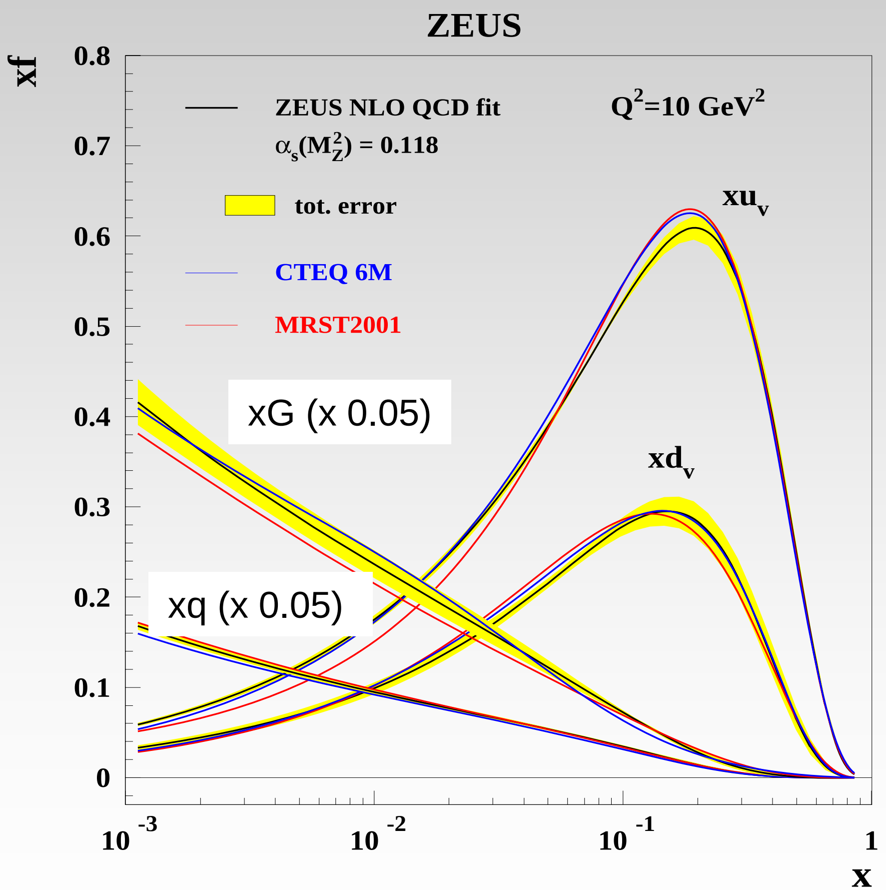
Gluons and Quarks at Low-x

Distribution functions $xq(x,Q^2)$ and $xG(x,Q^2)$ rise steeply at low Bjorken x.

Gluons only



Gluons and Quarks



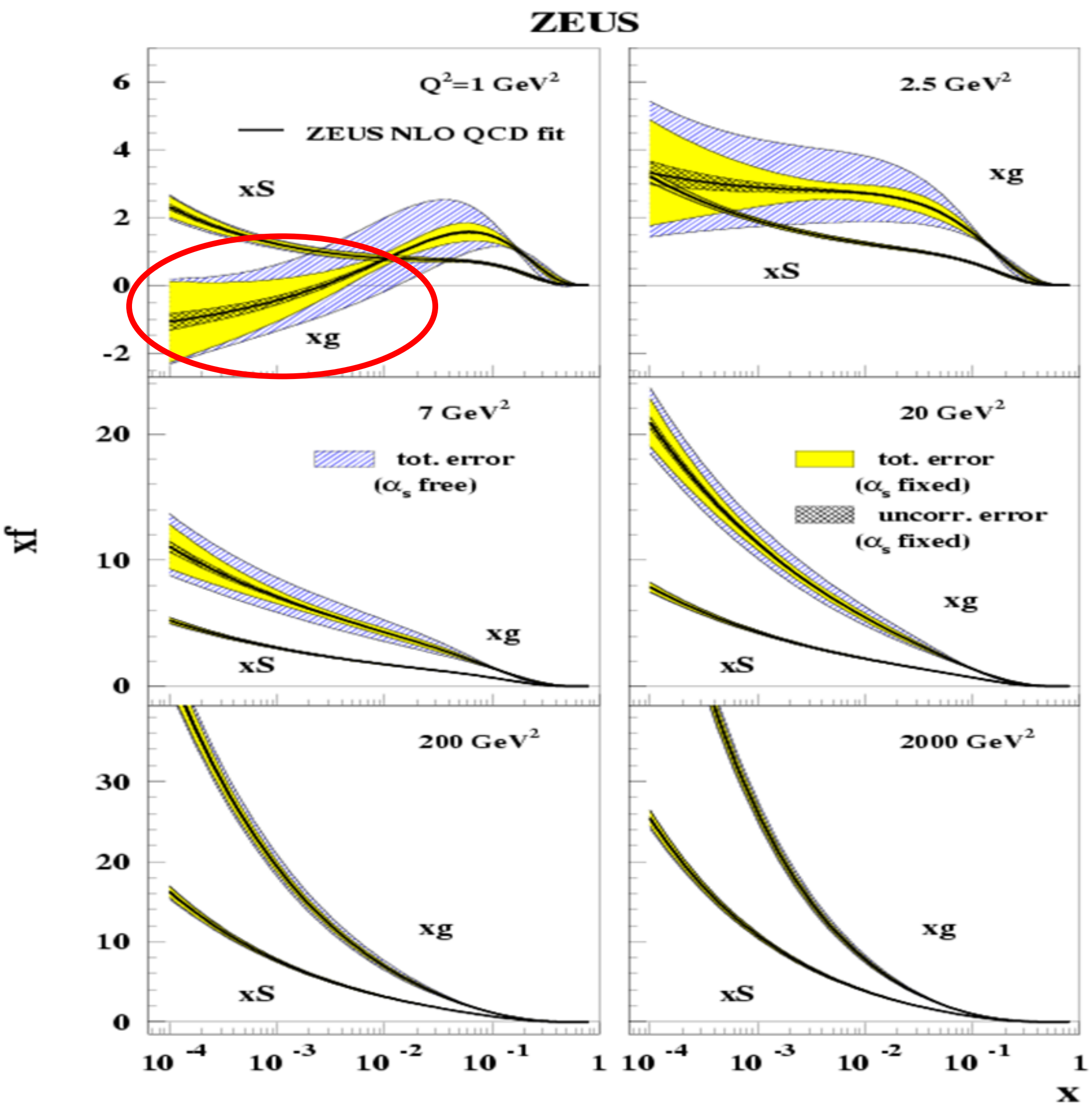
Is all this well-described by the standard DGLAP evolution?

Negative gluon distribution!

- NLO global fitting based on leading twist DGLAP evolution leads to negative gluon distribution
- MRST PDF's have the same features

Does it mean that we have no gluons at $x < 10^{-3}$ and Q=1 GeV?

No!



Why does DGLAP fail?

- Indeed we know that at low Q^2 the higher twist effects scaling as $\sim 1/Q^2$ become important.

- These higher twist corrections are enhanced at small-x:

$$\sim \frac{\Lambda^2}{Q^2} \frac{1}{x^\lambda}$$

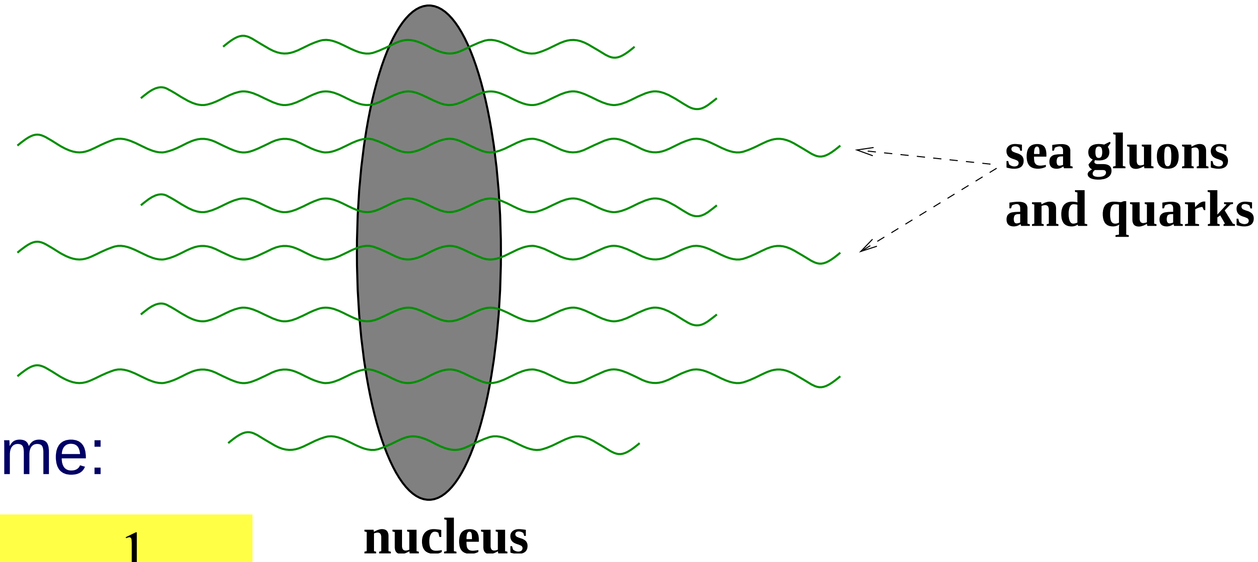
- For large nuclei there is also enhancement by the atomic number A:

$$\sim \frac{\Lambda^2}{Q^2} \frac{A^{1/3}}{x^\lambda}$$

What We Expect to See at Small-x in Nuclei

Nuclear/Hadronic Wave Function

Imagine an UR nucleus or hadron with valence quarks and sea gluons and quarks.

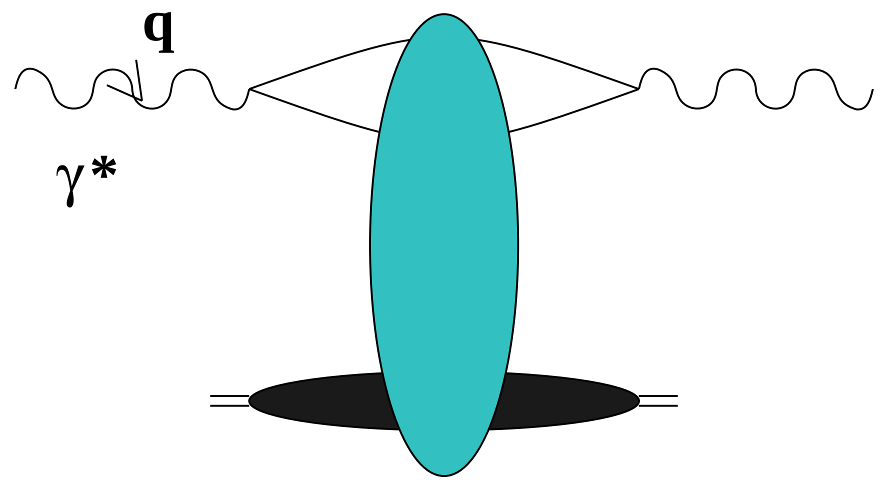


Boost to the rest frame:

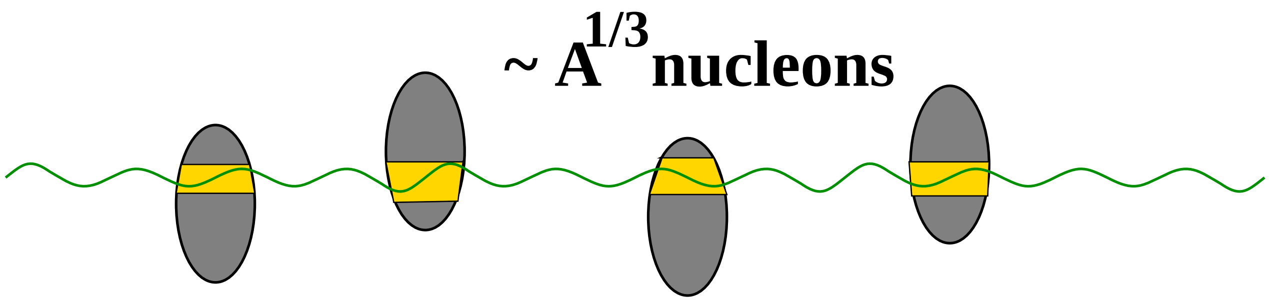
$$l_{coh} \sim \frac{1}{k_+} \sim \frac{1}{x_{Bj}\ p_+} \sim \frac{1}{x_{Bj}\ m_N}$$

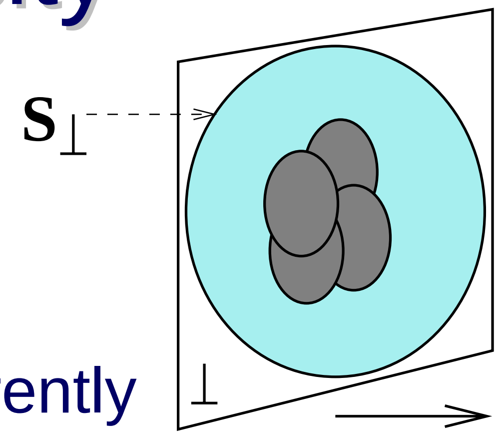
for small enough x_{Bj} we get with R the nuclear radius. (e.g. for $x=10^{-3}$ get l_{coh}=100 fm)

$$l_{coh} \approx \frac{1}{2\,m_N\ x_{Bj}} >> R$$



Color Charge Density





Small-x gluon "sees" the whole nucleus coherently in the longitudinal direction! It "sees" many color charges which form a net effective color charge Q = g (# charges)$^{1/2}$, such that $Q^2 = g^2$ #charges (random walk). Define color charge density

$$\mu^2 = \frac{Q^2}{S_\perp} = \frac{g^2 \text{ \# charges}}{S_\perp} \sim g^2 \frac{A}{S_\perp} \sim A^{1/3}$$

McLerran
Venugopalan
'93-'94

such that for a large nucleus (A>>1)

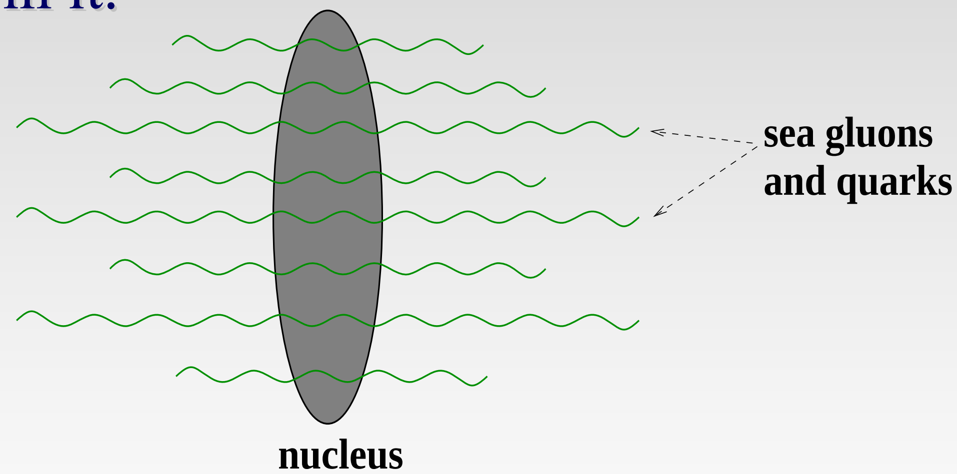
$$\mu^2 \sim \Lambda^2_{QCD} A^{1/3} >> \Lambda^2_{QCD} \quad \Rightarrow \quad \alpha_S(\mu^2) << 1$$

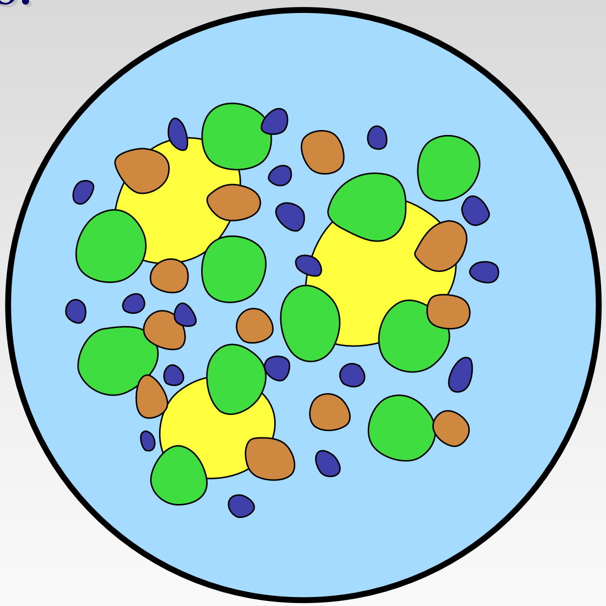
Nuclear small-x wave function is perturbative!!!

McLerran-Venugopalan Model

As we have seen, the wave function of a single nucleus has many small-x quarks and gluons in it.

In the transverse plane the nucleus is densely packed with gluons and quarks.



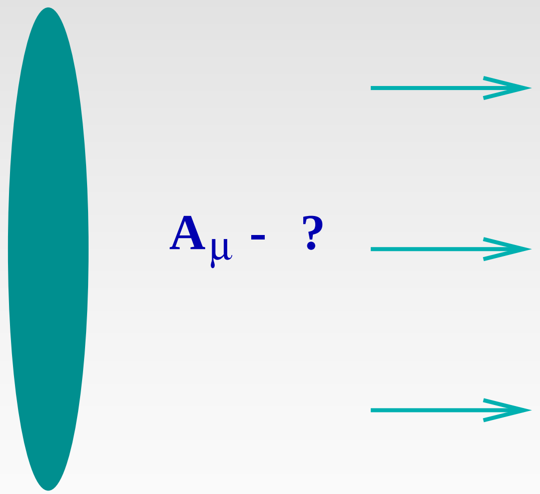


Large occupation number ⇨ Classical Field

McLerran-Venugopalan Model

- Leading gluon field is classical! To find the classical gluon field A_μ of the nucleus one has to solve the non-linear analogue of Maxwell equations – the Yang-Mills equations, with the nucleus as a source of color charge:

$$D_\nu F^{\mu\nu} = J^\mu$$



nucleus is Lorentz contacted into a pancake

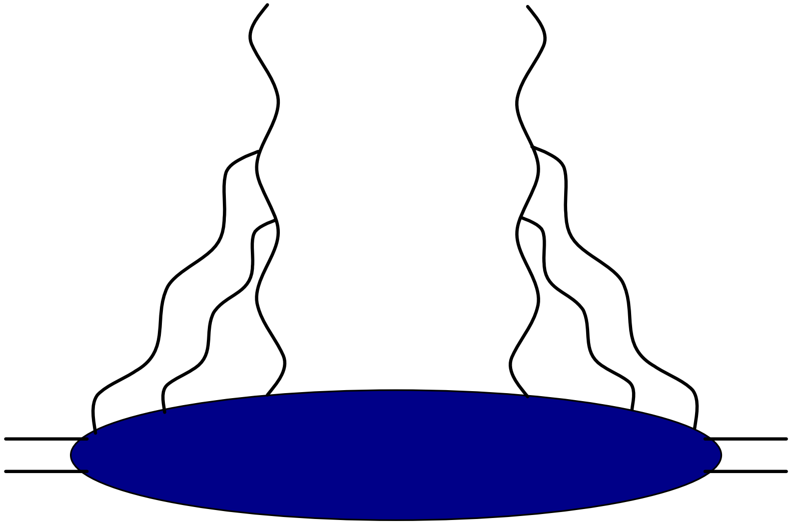
Yu. K. '96

J. Jalilian-Marian et al, '96

Classical Gluon Field of a Nucleus

Using the obtained classical gluon field one can construct corresponding gluon distribution function

$$\phi_A(x,k^2) \sim \left\langle \underline{A}(-k) \cdot \underline{A}(k) \right\rangle$$

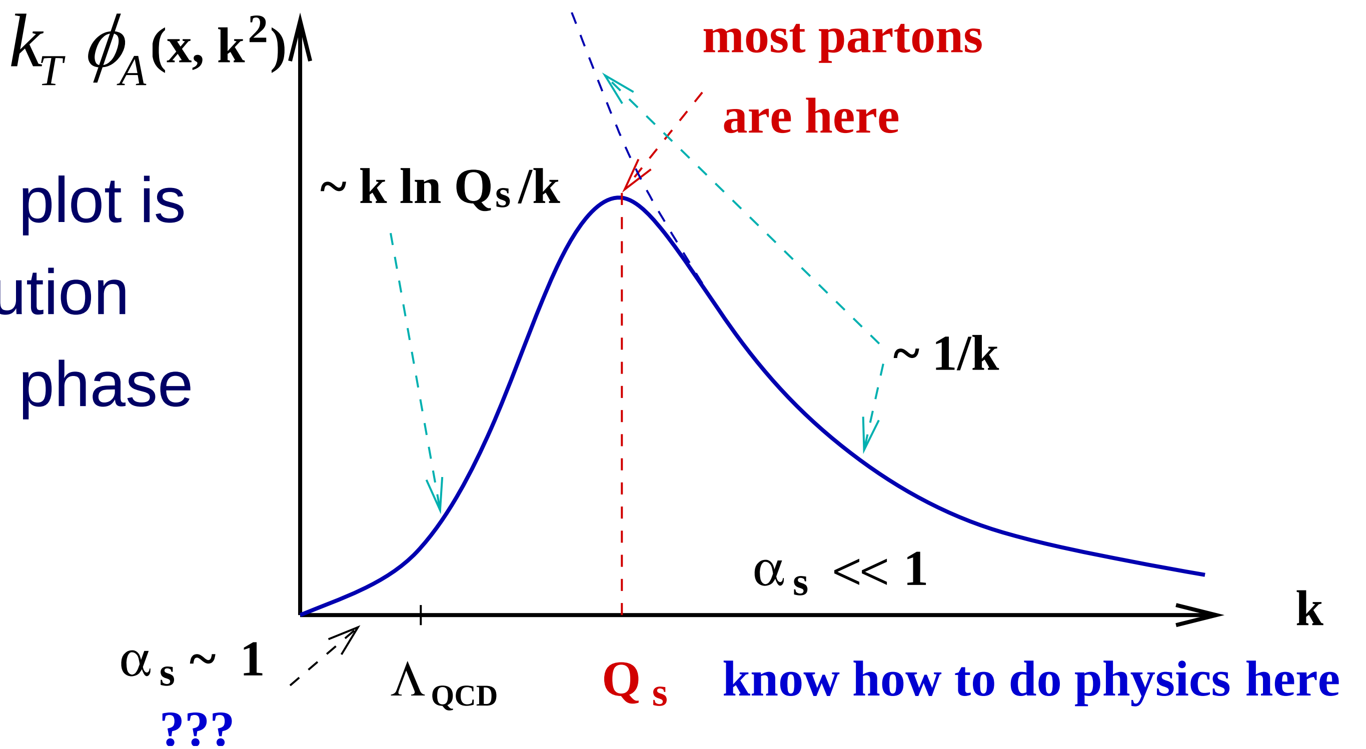


- Note a change in concept: instead of writing an evolution equation a la DGLAP, we can simply write down a closed expression for the distribution of gluons. The calculation is non-perturbative (classical).

- Gluon field is $A_\mu \sim 1/g$, which is what one would expect for a classical field: gluon fields are strong!

Classical Gluon Distribution

A good object to plot is the gluon distribution multiplied by the phase space k_T:



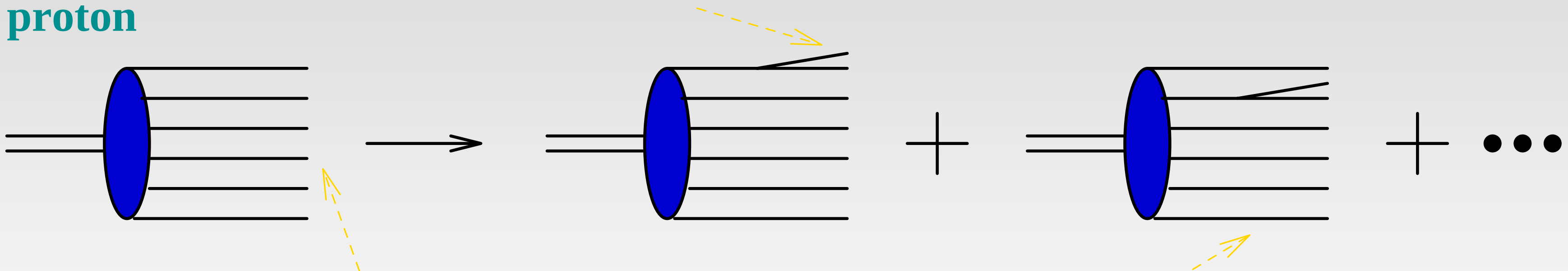
⇨ Most gluons in the nuclear wave function have transverse momentum of the order of $k_T \sim Q_S$ and $Q_S^2 \sim A^{1/3}$

⇨ We have a small coupling description of the whole wave function in the classical approximation.

BFKL Equation

Balitsky, Fadin, Kuraev, Lipatov '78

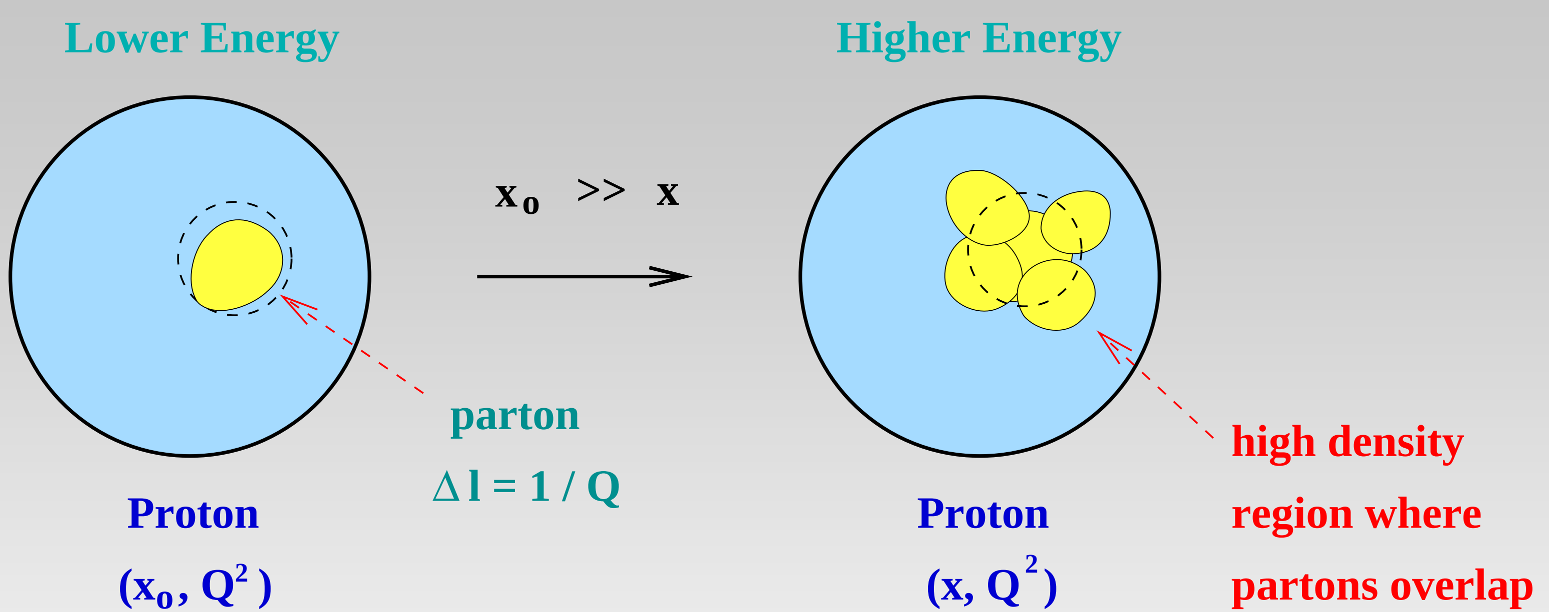
The powers of the parameter α ln s <u>without</u> multiple rescatterings are resummed by the BFKL equation. Start with N particles in the proton's wave function. As we increase the energy a new particle can be emitted by either one of the N particles. The number of newly emitted particles is proportional to N.



The BFKL equation for the number of partons N reads:

$$\frac{\partial}{\partial \ln(1/x)} N(x,Q^2) = \alpha_S K_{BFKL} \otimes N(x,Q^2)$$

BFKL Equation as a High Density Machine



- But can parton densities rise forever? Can gluon fields be infinitely strong? Can the cross sections rise forever?
- No! There exists a black disk limit for cross sections, which we know from Quantum Mechanics: for a scattering on a disk of radius R the total cross section is bounded by

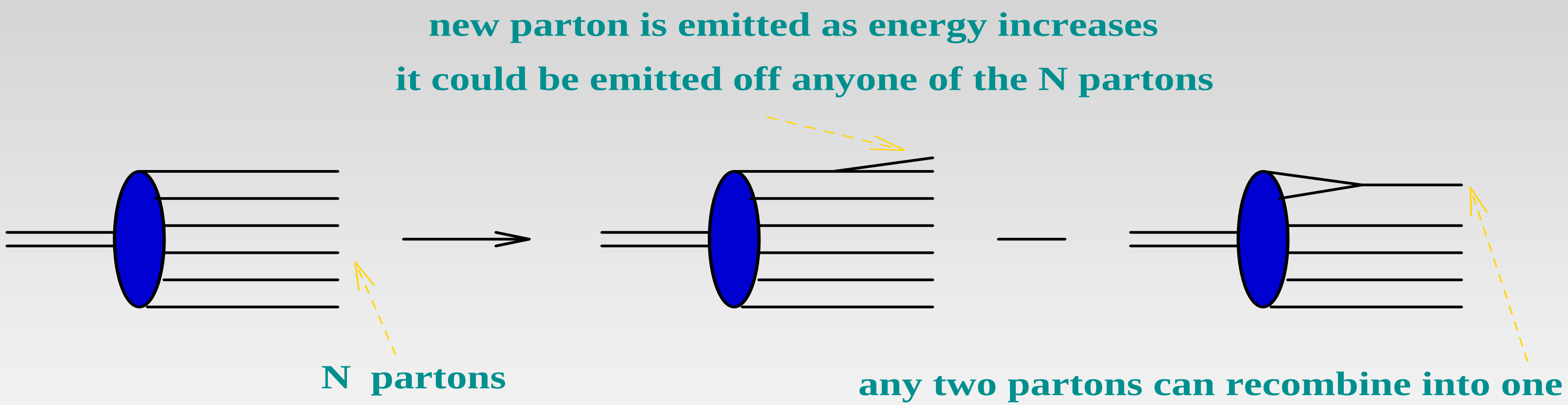
$$\sigma_{total} \le 2\pi R^2$$

- As energy increases BFKL evolution produces more partons, roughly of the same size. The partons overlap each other creating areas of very high density.
- Number density of partons, along with corresponding cross sections grows as a power of energy

$$N \sim s^{\Delta}$$

Nonlinear Equation

At very high energy parton recombination becomes important. Partons not only split into more partons, but also recombine. Recombination reduces the number of partons in the wave function.

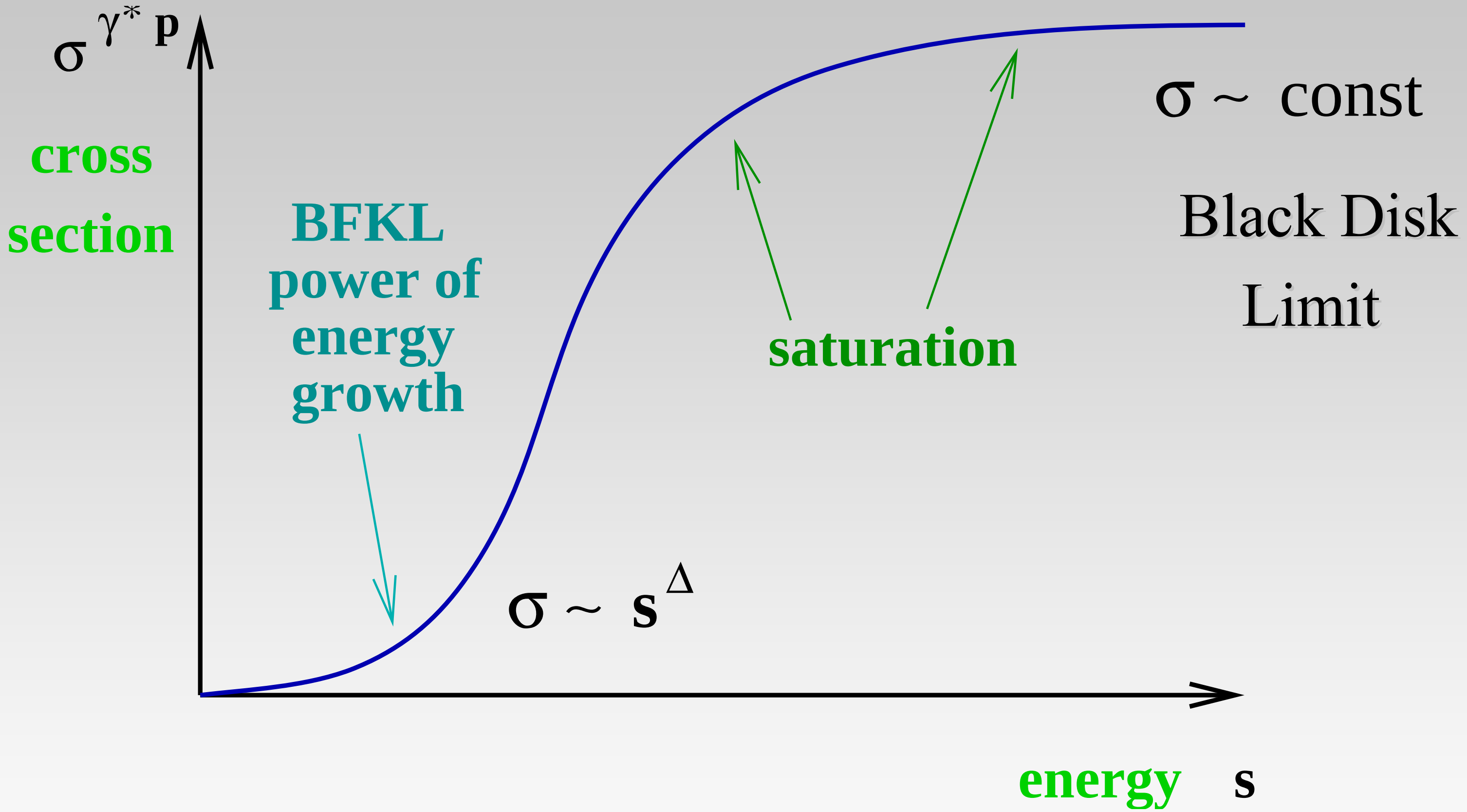


$$\frac{\partial N(x,k^2)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes N(x,k^2) - \alpha_s [N(x,k^2)]^2$$

Number of parton pairs ~ N^2

I. Balitsky '96 (effective lagrangian)
Yu. K. '99 (large N_C QCD)

Nonlinear Equation: Saturation

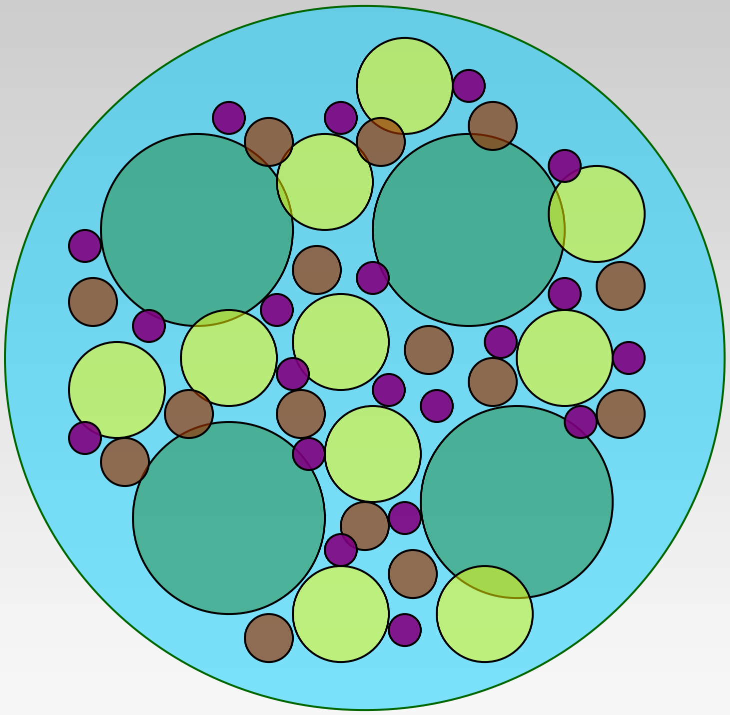


Gluon recombination tries to reduce the number of gluons in the wave function. At very high energy recombination begins to compensate gluon splitting. Gluon density reaches a limit and does not grow anymore. So do total DIS cross sections. Unitarity is restored!

Nonlinear Evolution at Work

✓ First partons are produced overlapping each other, all of them about the same size.

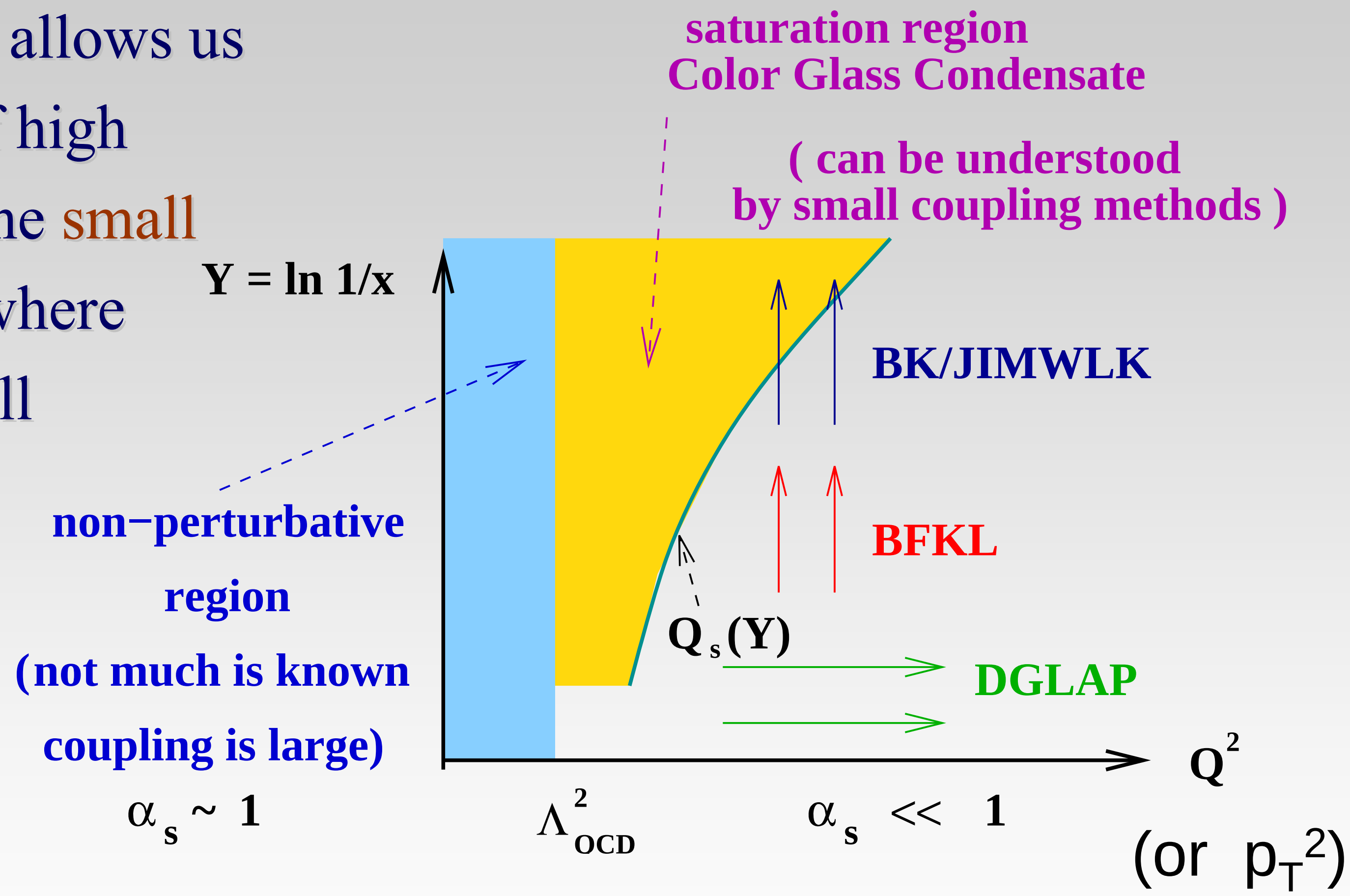
✓ When some critical density is reached no more partons of given size can fit in the wave function. The proton starts producing smaller partons to fit them in.



Color Glass Condensate

Game Plan of High Energy QCD

Saturation physics allows us to study regions of high parton density in the small coupling regime, where calculations are still under control!



Transition to saturation region is characterized by the <u>saturation scale</u>

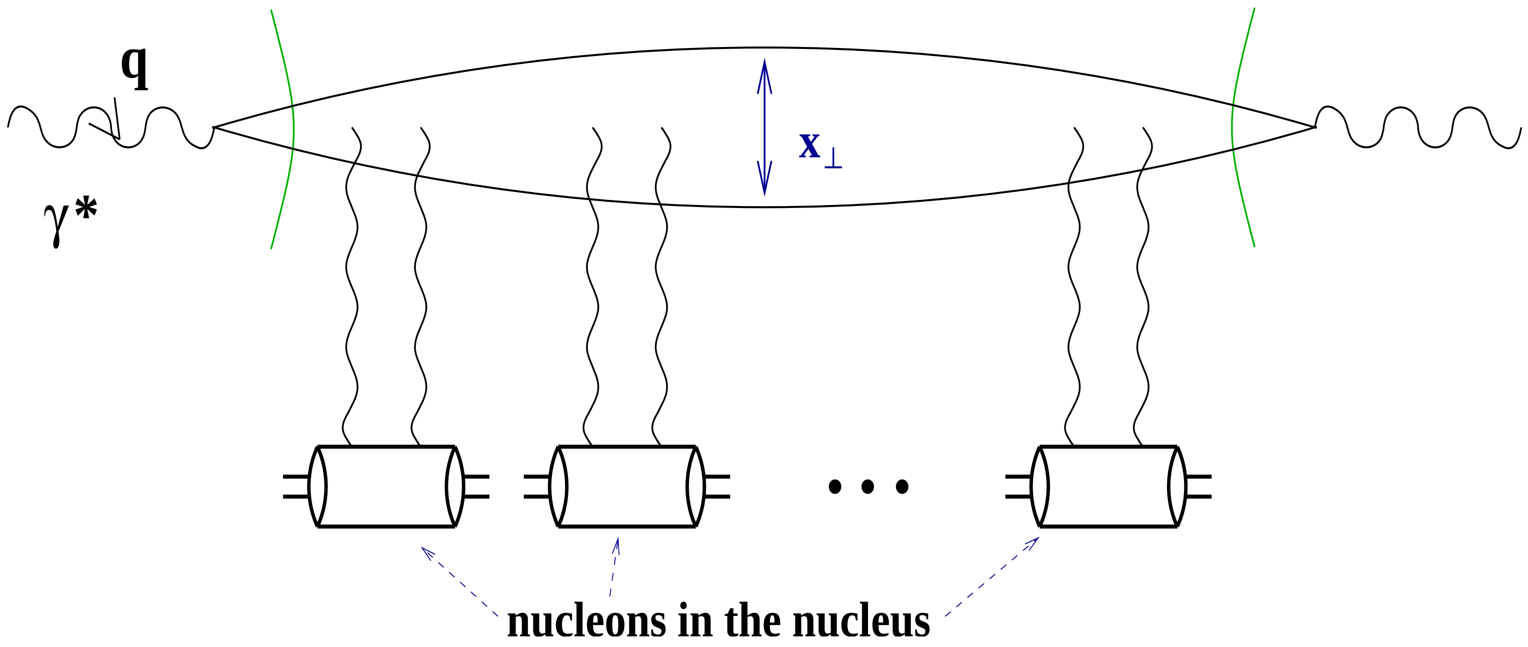
The Little That We (Probably) Know Already

How to Calculate Observables

- Start by finding the classical field of the McLerran-Venugopalan model.
- Continue by including the quantum corrections of the BK/JIMWLK evolution equations.

Dipole Models in DIS

The DIS process in the rest frame of the target is shown below. It factorizes into



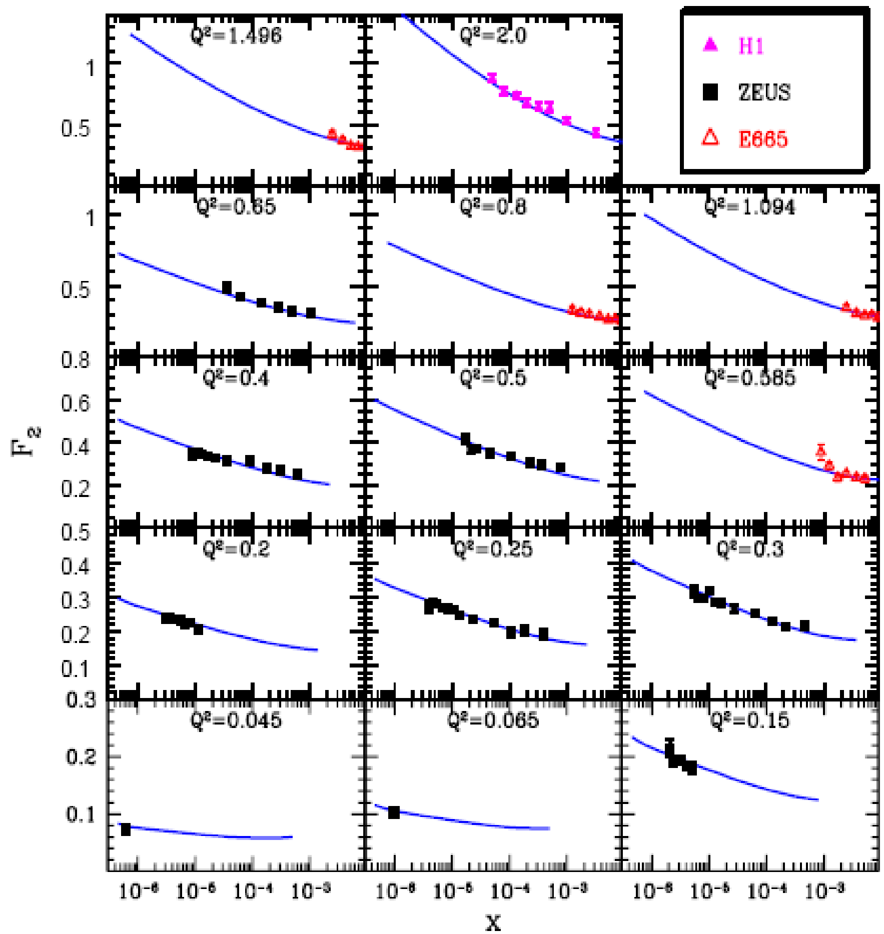
$$\sigma_{tot}^{\gamma^* A}(x_{Bj}, Q^2) = \Phi^{\gamma^* \to q\bar{q}} \otimes N(x_\perp, Y = \ln(1/x_{Bj}))$$

QCD dynamics is all in N.

HERA DIS Results

Most of HERA DIS data is well-described by dipole models based on CGC/saturation physics. This is particularly true in the **low-x low-Q region**, where DGLAP-based pdf's fail.

from Gotsman, Levin, Lublinsky, Maor '02

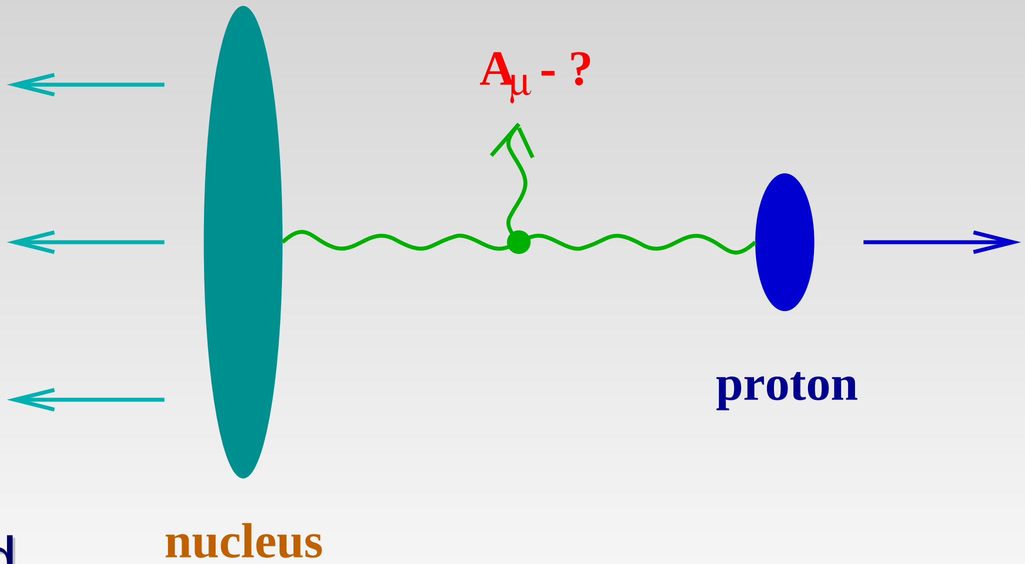


Gluon Production in Proton-Nucleus Collisions (pA): Classical Field

To find the gluon production cross section in pA one has to solve the same classical Yang-Mills equations

$$D_\nu F^{\mu\nu} = J^\mu$$

for two sources – proton and nucleus.



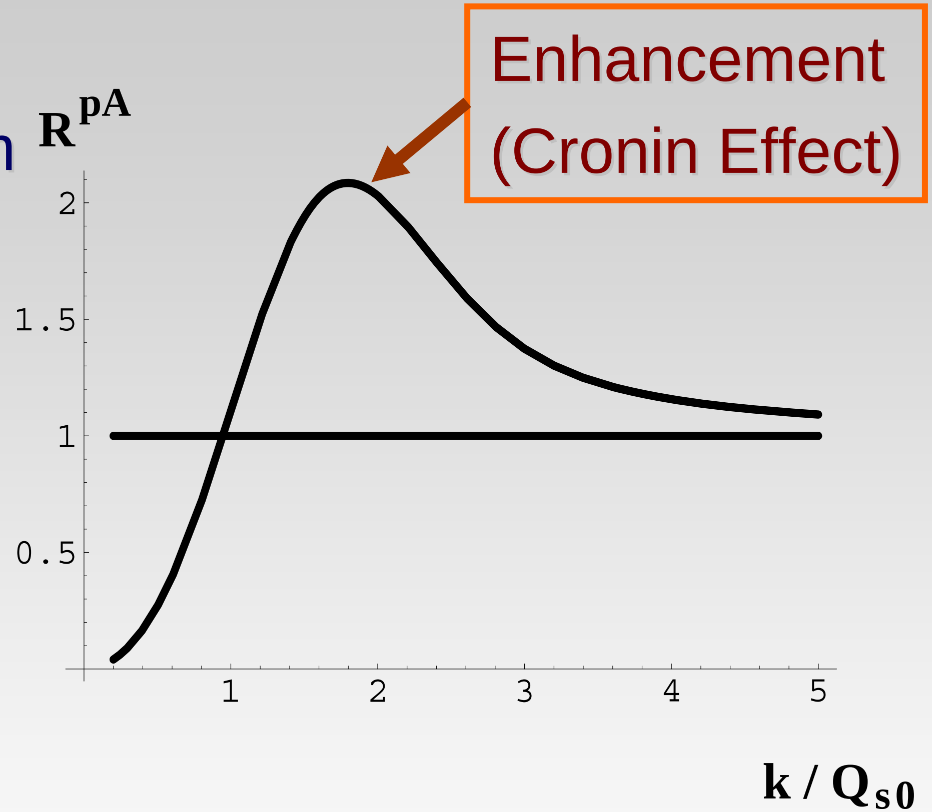
Yu. K., A.H. Mueller in '98

Gluon Production in pA: Classical Field

To understand how the gluon production in pA is different from independent superposition of *A* proton-proton (pp) collisions one constructs the quantity

$$R^{pA} = \frac{E \frac{d\sigma^{pA}}{d^3k}}{A \times E \frac{d\sigma^{pp}}{d^3k}}$$

which is = 1 for independent superposition of sub-collisions.

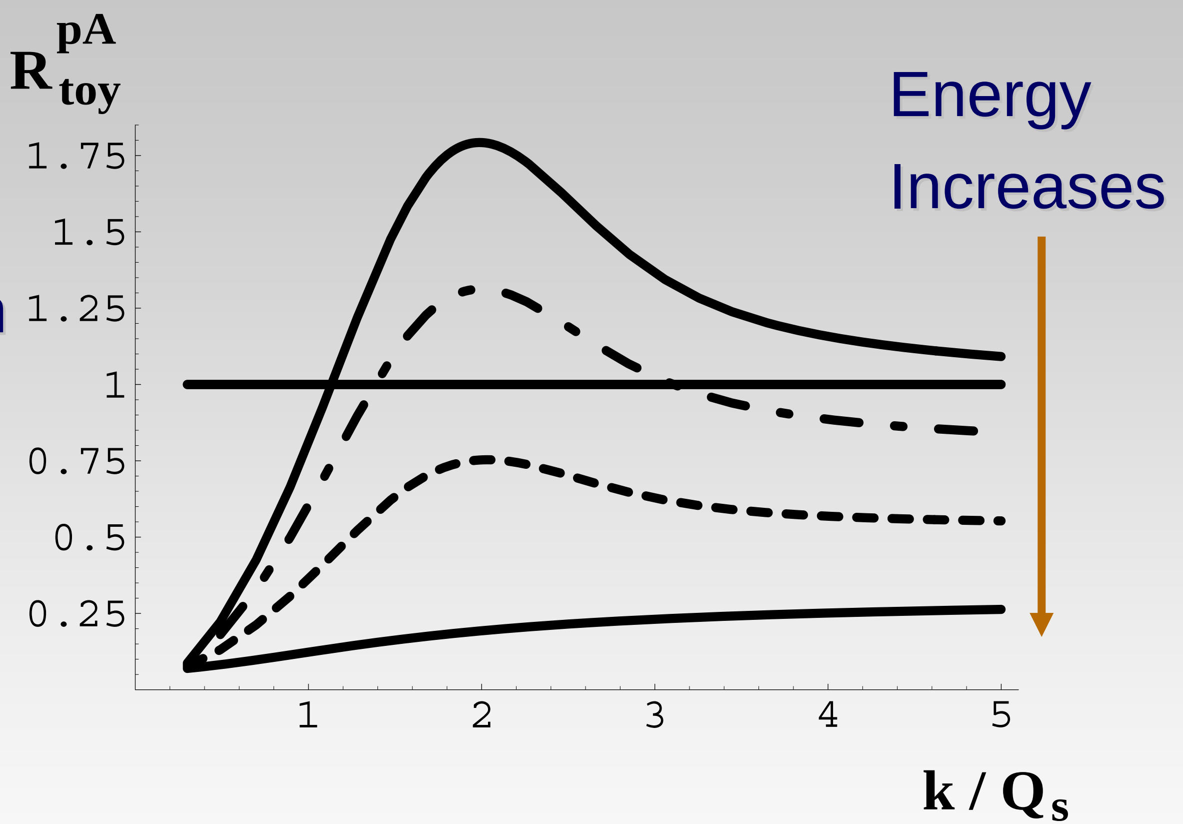


The quantity R^{pA} plotted for the classical solution.

Nucleus pushes gluons to higher transverse momentum!

Gluon Production in pA: BK Evolution

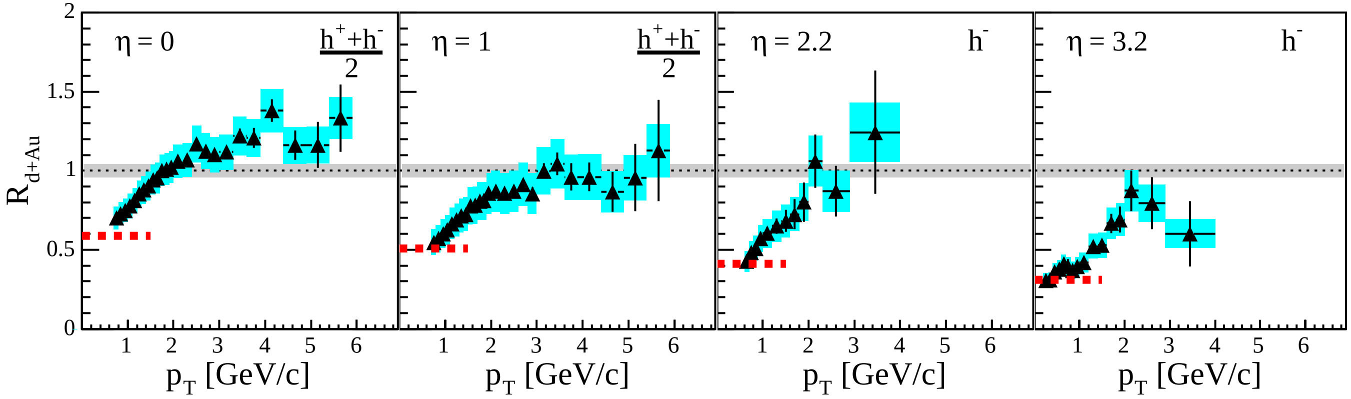
Including quantum corrections to gluon production cross section in pA using BK/JIMWLK evolution equations introduces suppression in R^{pA} with increasing energy!



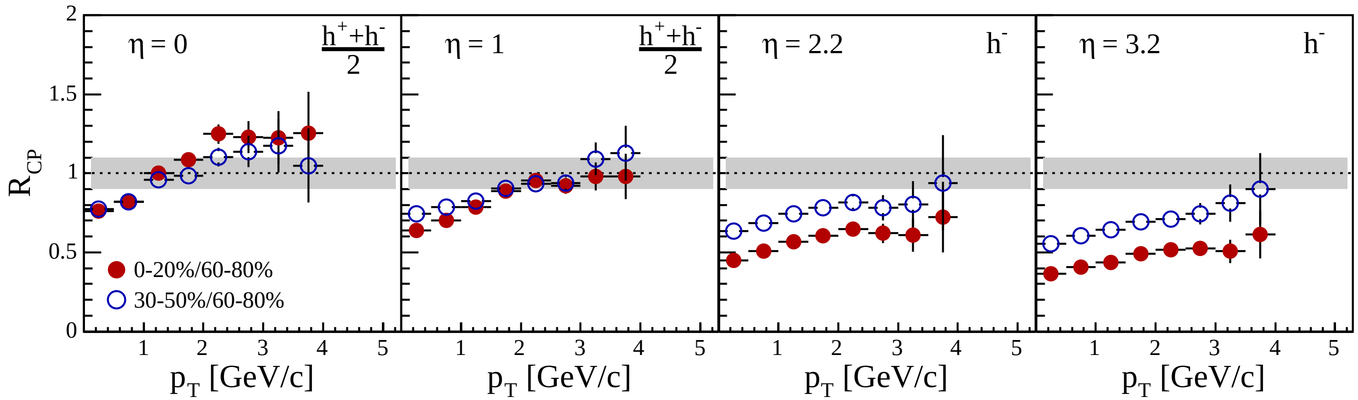
The plot is from D. Kharzeev, Yu. K., K. Tuchin '03
(see also Kharzeev, Levin, McLerran, '02 – original prediction, Albacete, Armesto, Kovner, Salgado, Wiedemann, '03)

R_{dAu} at different rapidities

R^{dAu}



R^{CP} – central to peripheral ratio



Most recent data from BRAHMS Collaboration nucl-ex/0403005

Our prediction of suppression was confirmed!

(indeed quarks have to be included too to describe the data)

What We May Know Already

- Saturation/CGC effects appear to manifest themselves at $x \sim 10^{-3}$ and p_T up to 3.5 GeV for gold nuclei at RHIC via breakdown of naïve factorization.
- Saturation-based dipole models are hugely successful in describing HERA data, especially in the low-x low-Q region where DGLAP-based pdf’s fail.
- eRHIC is almost certainly going to probe deep into the saturation region.
- EM probes would be more convincing: no fragmentation effects there.

How EIC can solve many of the puzzles

EIC

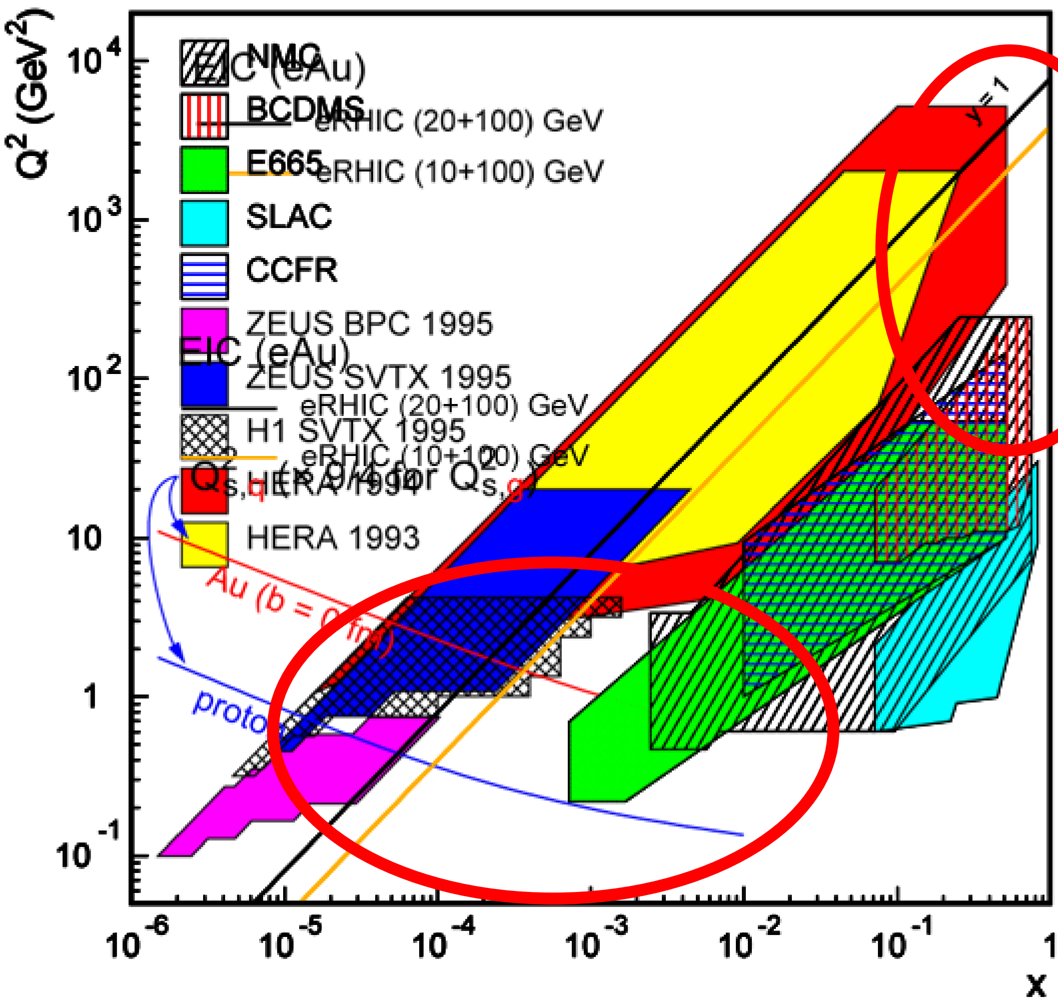
- EIC/eRHIC would produce dedicated data on nuclear structure functions and would allow one to answer many questions in small-x physics.
- DIS on a nucleus with atomic number A would allow to test the physics equivalent to that of DIS on the proton at

  $x_{proton} = x_{nucleus} / A.$

$$Q_S^2 \sim \left(\frac{A}{x} \right)^{1/3}$$

- This is a much lower effective x!

eA Landscape and a new Electron Ion Collider



The x, Q^2 plane looks well mapped out – doesn't it?

Except for ℓ+A (νA) many of those with small A and very low statistics

Electron Ion Collider (EIC):

E_e = 10 GeV (20 GeV)

E_A = 100 GeV

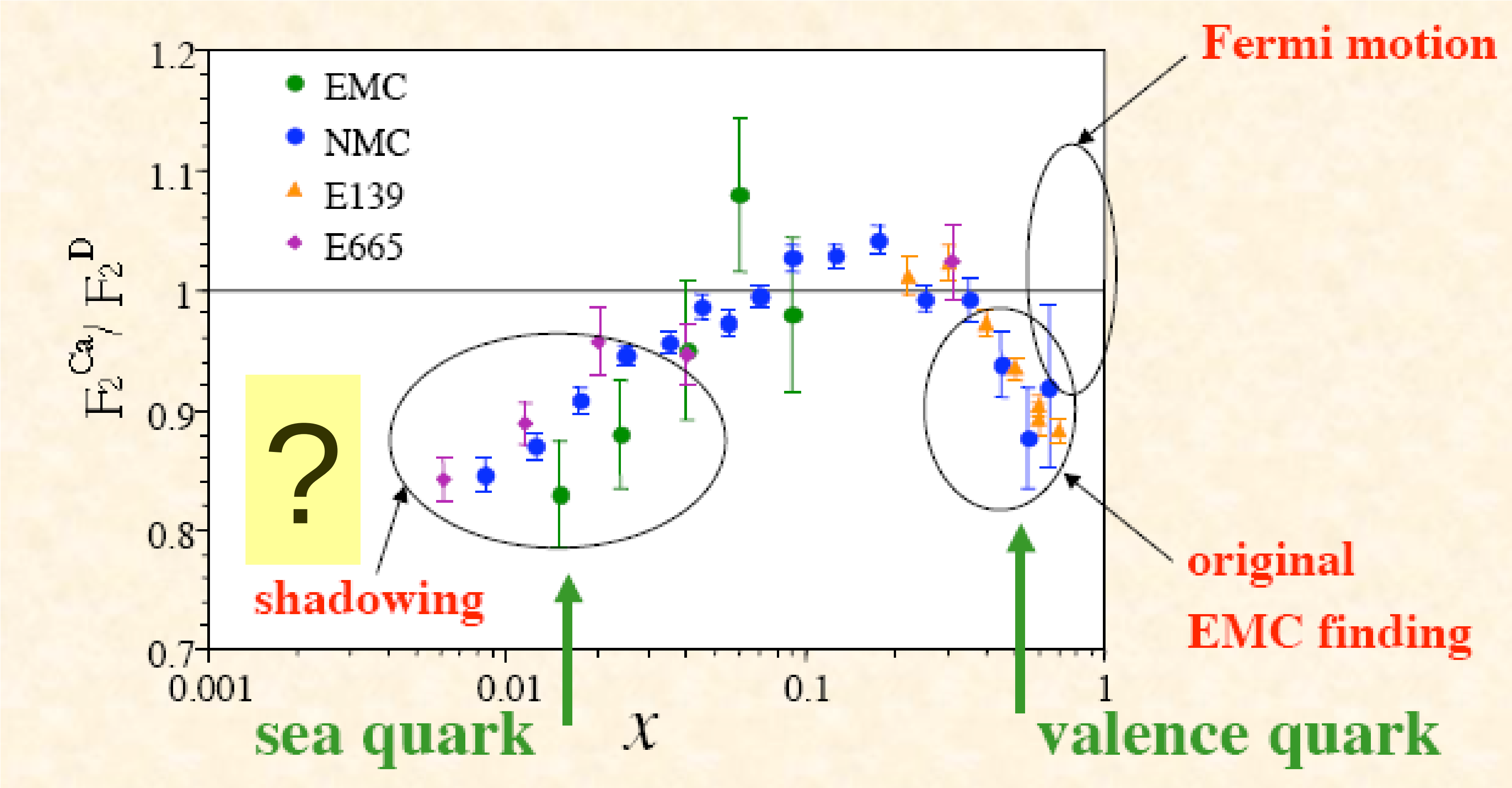
$\sqrt{s_{eN}}$ = 63 GeV (90 GeV)

High $L_{eAu} \sim 6 \cdot 10^{30}$ cm^{-2} s^{-1}

Terra incognita: small-x, $Q \approx Q_s$

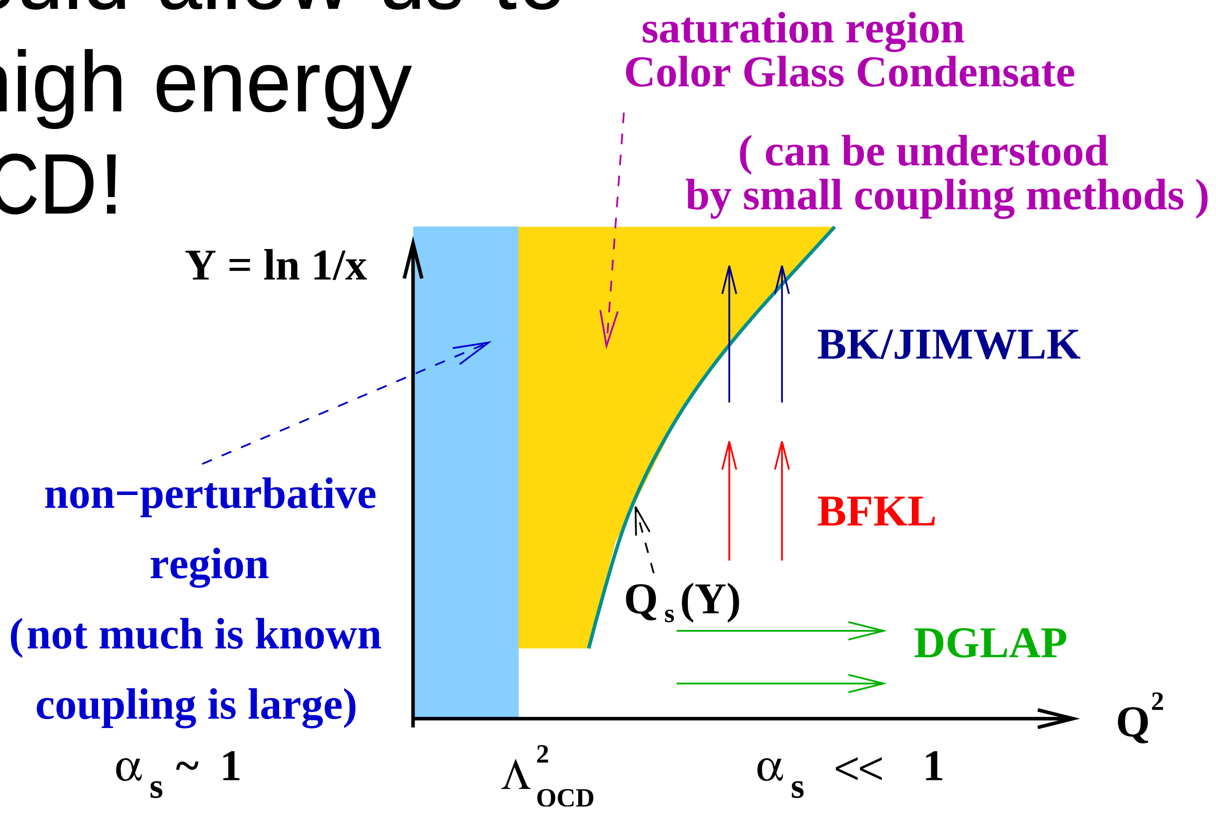
high-x, large Q^2

What Happens to F_2 at Small-x?



EIC

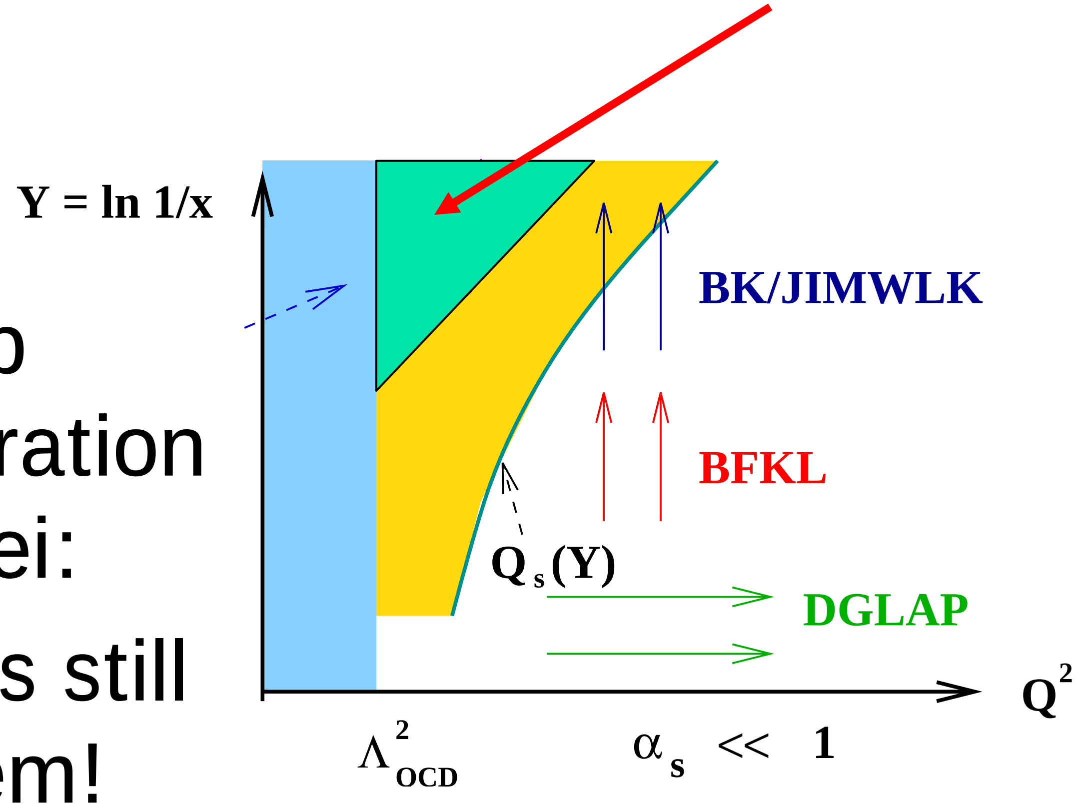
- EIC/eRHIC would allow us to map out the high energy behavior of QCD!



Open Theoretical Questions

Pomeron Loops

[Here Be Ploops?]



- Important deep inside the saturation region for nuclei:
- Resummation is still an open problem!

Higher Order Corrections

- To have the predictions of BK/JIMWLK under control one needs to understand higher order corrections.

$$\frac{\partial N(x,k^2)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes N(x,k^2) - \alpha_s [N(x,k^2)]^2$$

- Recently there has been some progress on running coupling corrections. $\alpha_S(???)$
- NLO is still an open question.

Marriage with DGLAP

- Can we make BK/JIMWLK equations match smoothly onto DGLAP to have the right physics at very large Q^2 ? Still an open problem.

Conclusions: Big Questions

- What is the nature of glue at high density?
  - How do strong fields appear in hadronic or nuclear wave functions at high energies?
  - What are the appropriate degrees of freedom?
  - How do they respond to external probes or scattering?
  - Is this response universal (ep,pp,eA,pA,AA)?

An Electron Ion Collider (EIC) can provide definitive answers to these questions.