New Tools for Understanding the Strong Interactions

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Carnegie Mellon / Pittsburgh
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Outline

- Effective Field Theory, QED, Hydrogen
- Introduction to QCD, $\alpha_s(\mu)$
- Soft-Collinear Effective Theory & Energetic Particles
- Weak Decays of B mesons
- Outlook

Introduction to QED

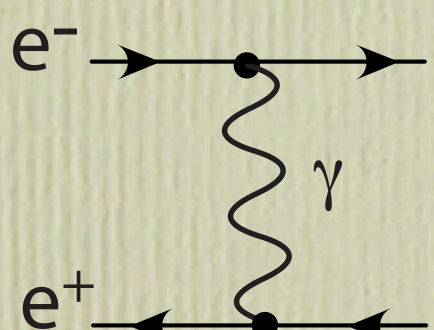
(quantum electromagnetism)

QED $\begin{cases} \text{Special Relativity:} & \text{spacetime,} \quad v \leq c \\ \text{Quantum Mechanics:} & \text{quantization,} \quad \Delta x \Delta p \geq \dfrac{\hbar}{2} \end{cases}$

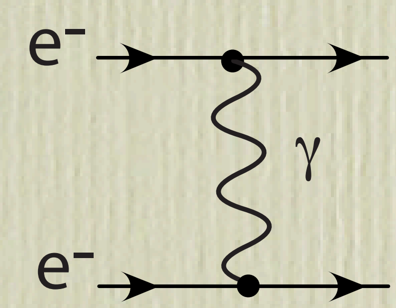
⇒ antiparticles, spin, gauge-theory

⇒ parameters: charge & masses

Interactions

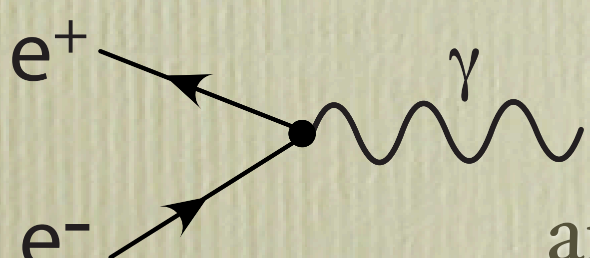


$$V = -\frac{e^2}{r}$$

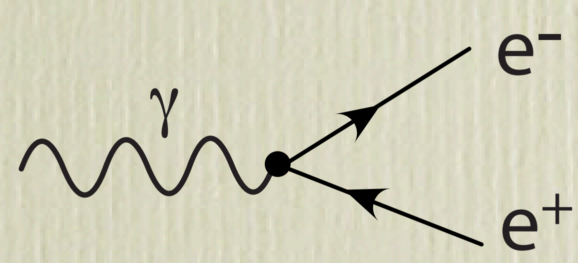


$$V = +\frac{e^2}{r}$$

two factors of the coupling



pair annihilation



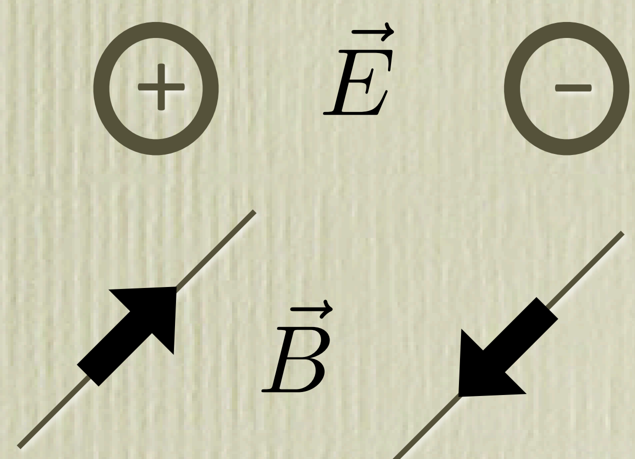
pair creation

The Standard Model Interactions

(leave out gravity and the higgs)

	Strong	Electromagnetism	Weak
	QCD	QED	
mediator:	gluons	photons	$W^{\pm},\ Z^0$
typical strength:	~ 1	$\sim 10^{-2}$	$\sim 10^{-6}$
range:	~ 1 fm	∞	$\frac{1}{m_W} \to \sim 10^{-3}$ fm

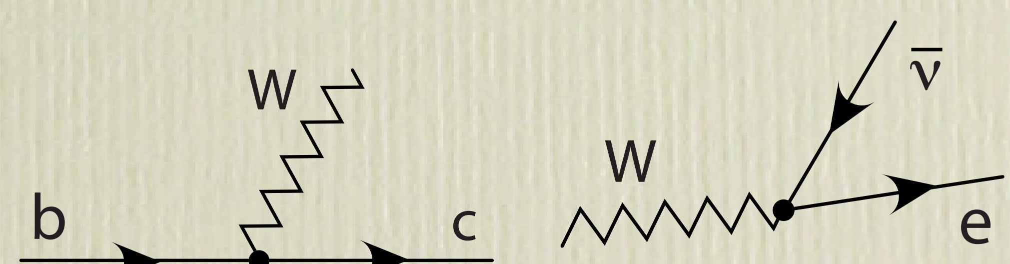
proton



$n \to pe\bar{\nu}$,

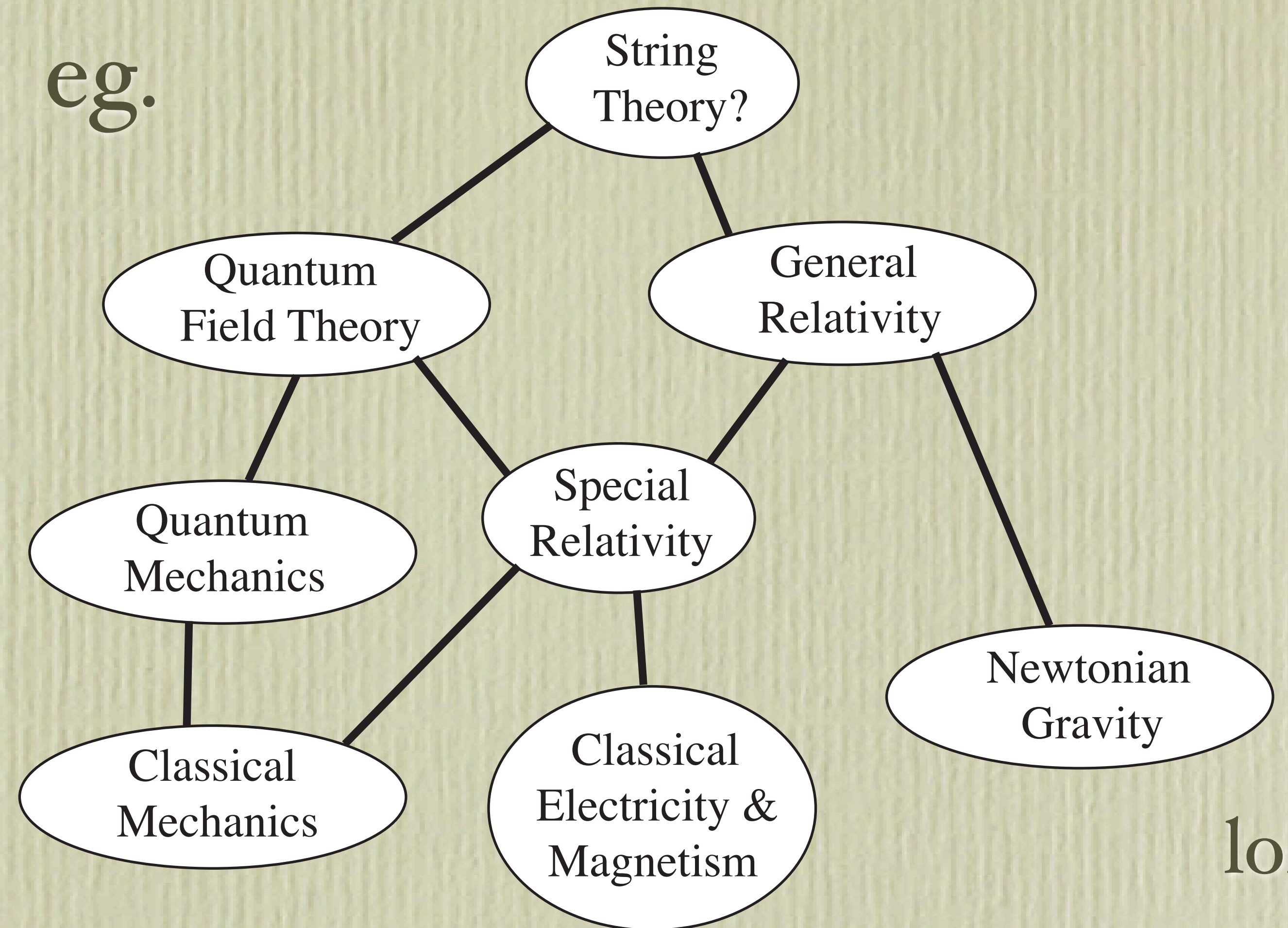
radioactive decay

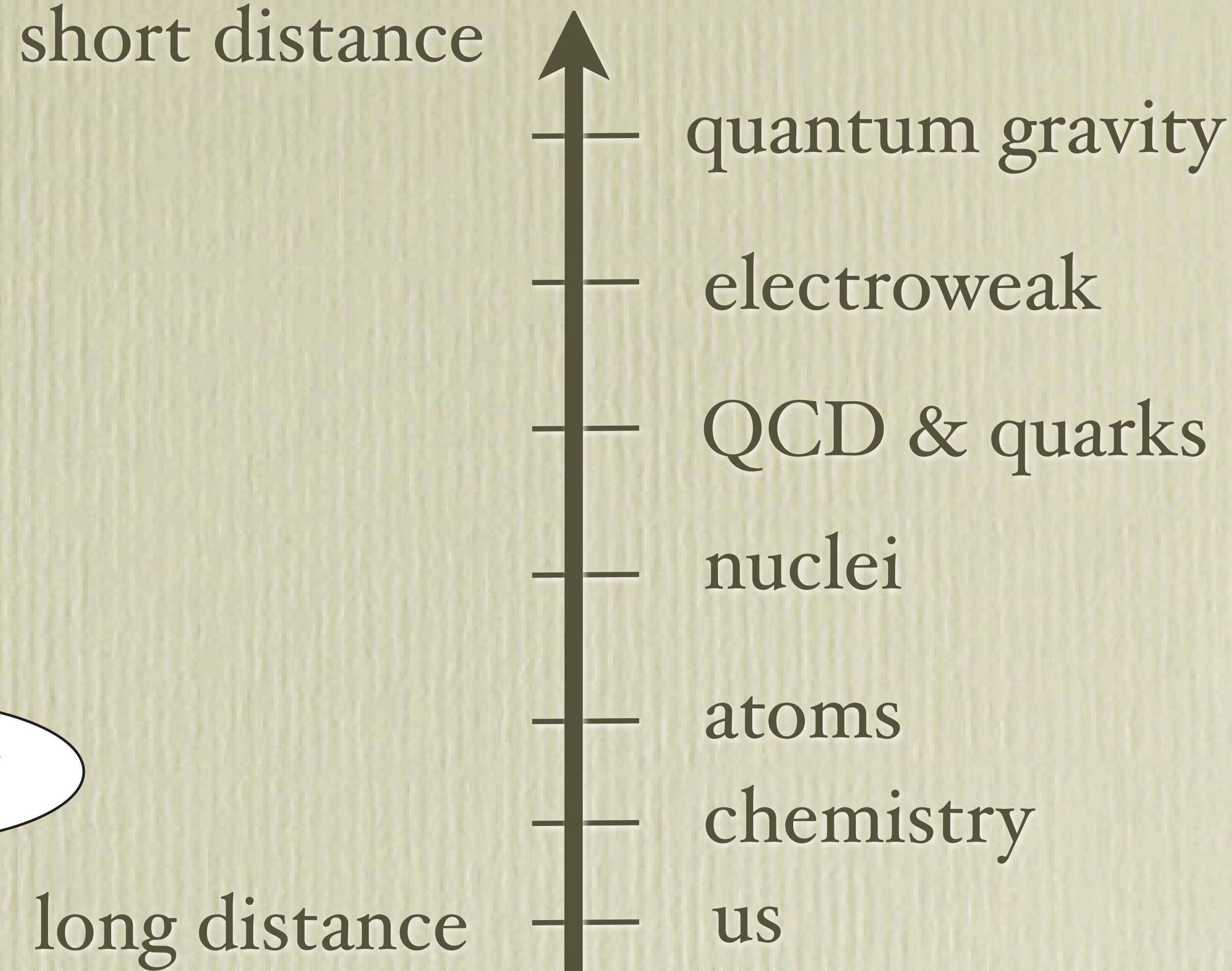
Other forces can (in principle) be derived from these



Physics compartmentalized

eg.





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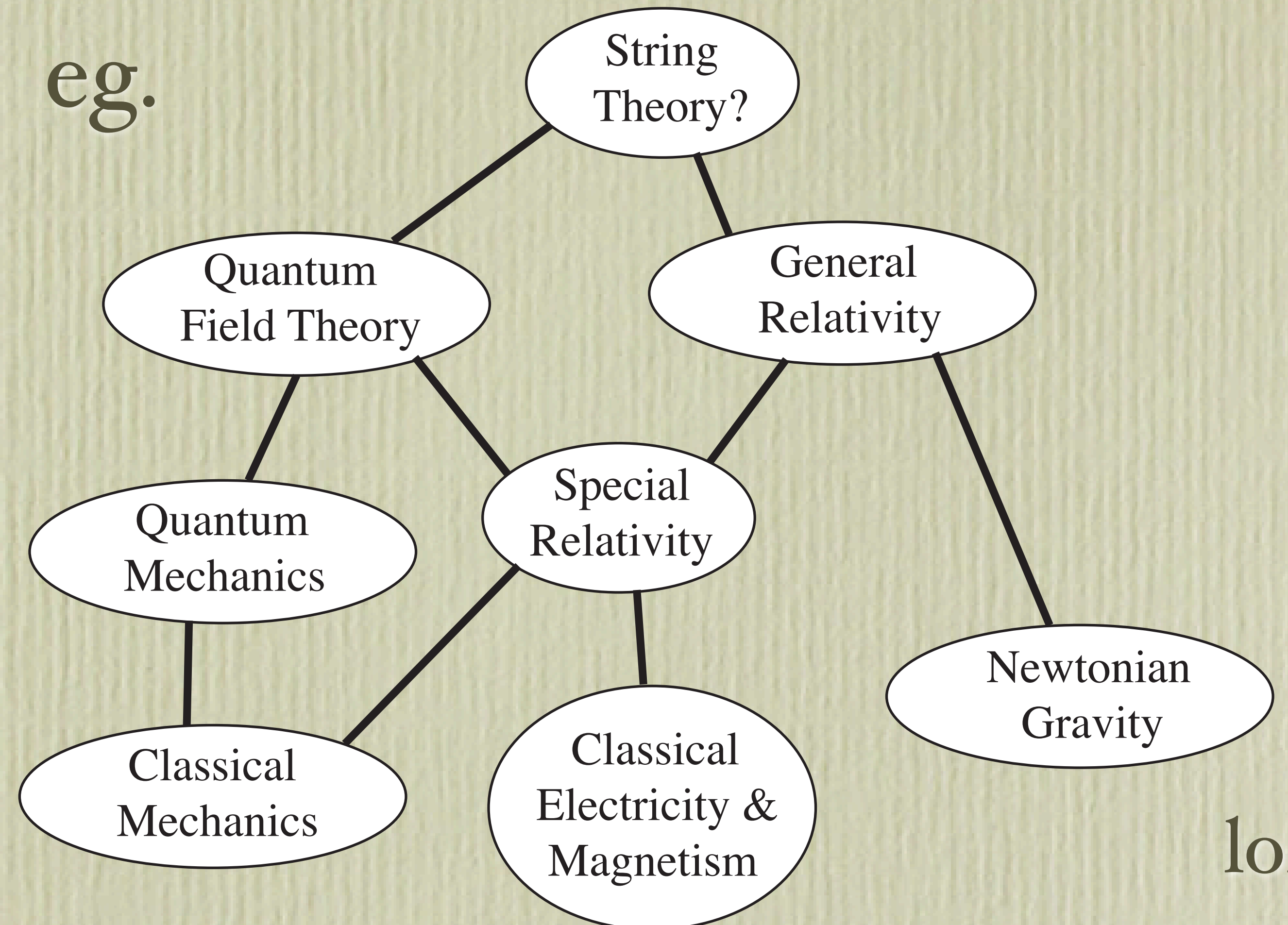
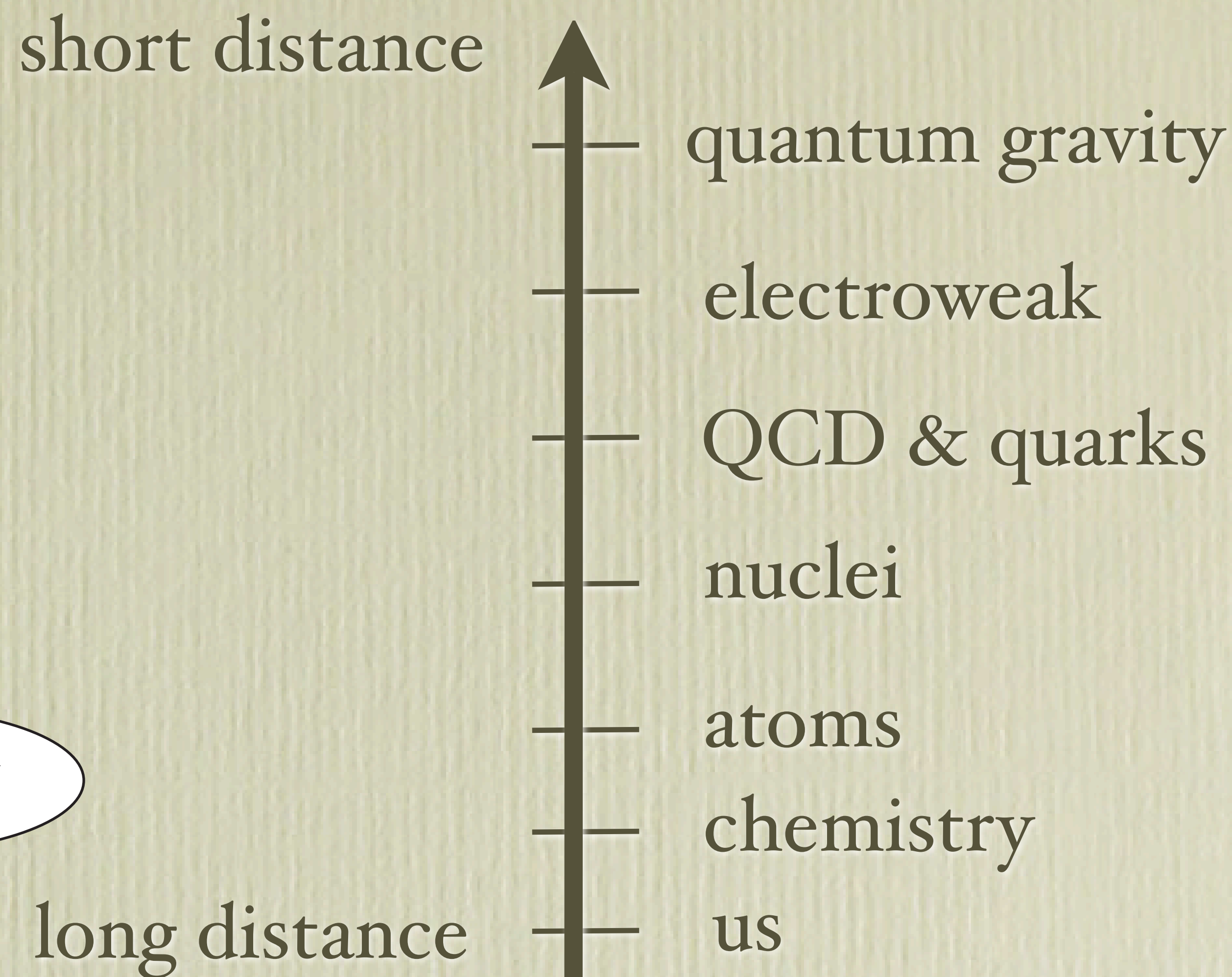
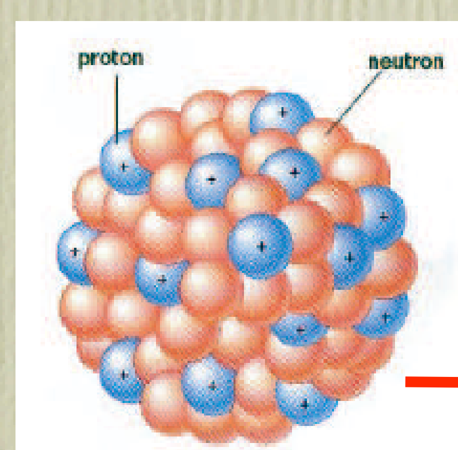




But, one doesn't need nuclear physics to build a boat

Generality
vs.
Precision



Dynamics at long distance does not depend on the details of what happens at short distance

In the quantum realm, $\lambda \sim \frac{1}{p}$, wavelength and momentum are related, so

Low energy interactions do not depend on the details of high energy interactions

Bad:

- we have to work harder to probe the interesting physics at short distances

Good:

- we can focus on the relevant interactions & degrees of freedom
- calculations are simpler

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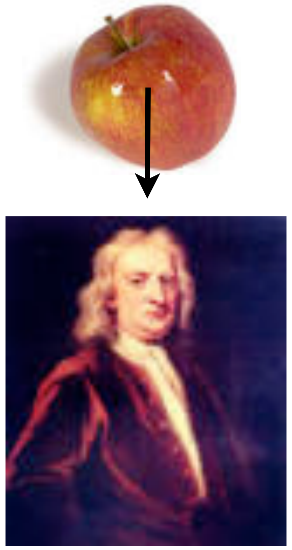

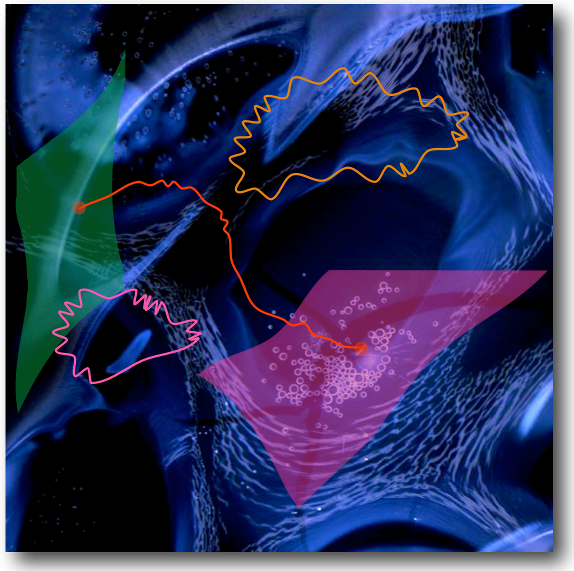

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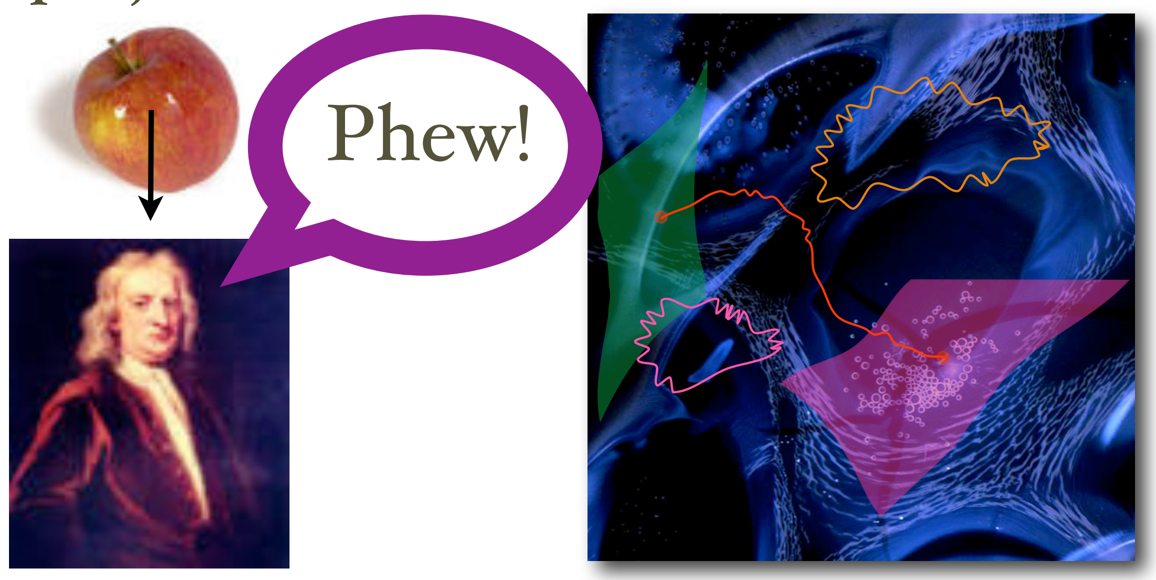
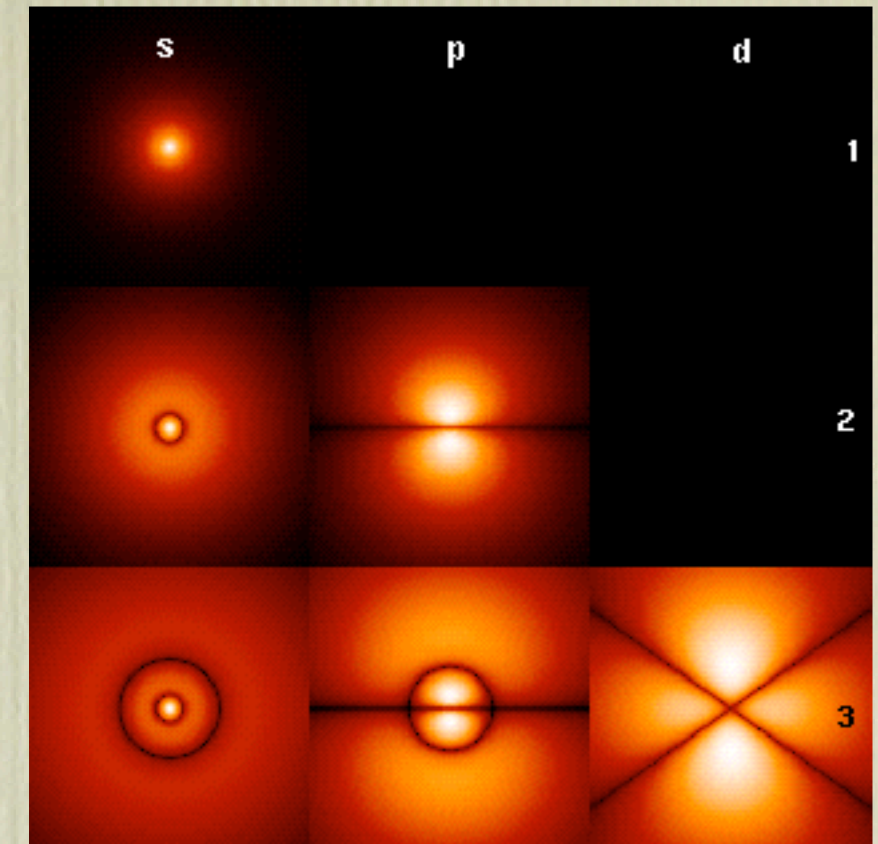


Example: Hydrogen

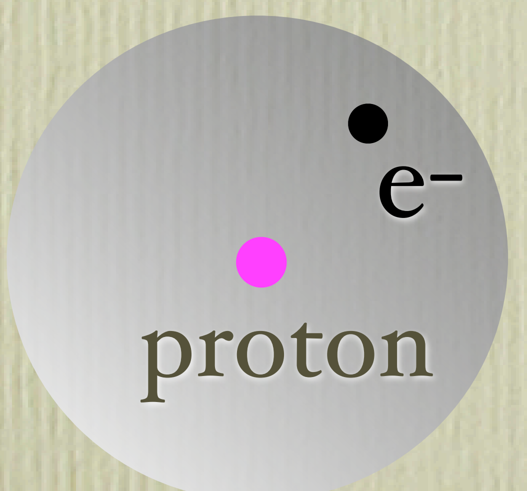


non-relativistic quantum mechanics

parameters:

mass	m_e
charges	$Q_e\ ,Q_p$
coupling	$\alpha = \dfrac{1}{137}$

degrees of freedom:



scales:

$$m_p = 938\,\mathrm{MeV} \qquad \longrightarrow \infty$$

$$m_e = 0.511\,\mathrm{MeV}$$

$$p \sim m_e\alpha = 3.7\,\mathrm{keV} \qquad \sim (a_{\mathrm{Bohr}})^{-1}$$

$$E_n = -\frac{m_e\alpha^2}{2n^2} = -\frac{13.6\,\mathrm{eV}}{n^2}$$

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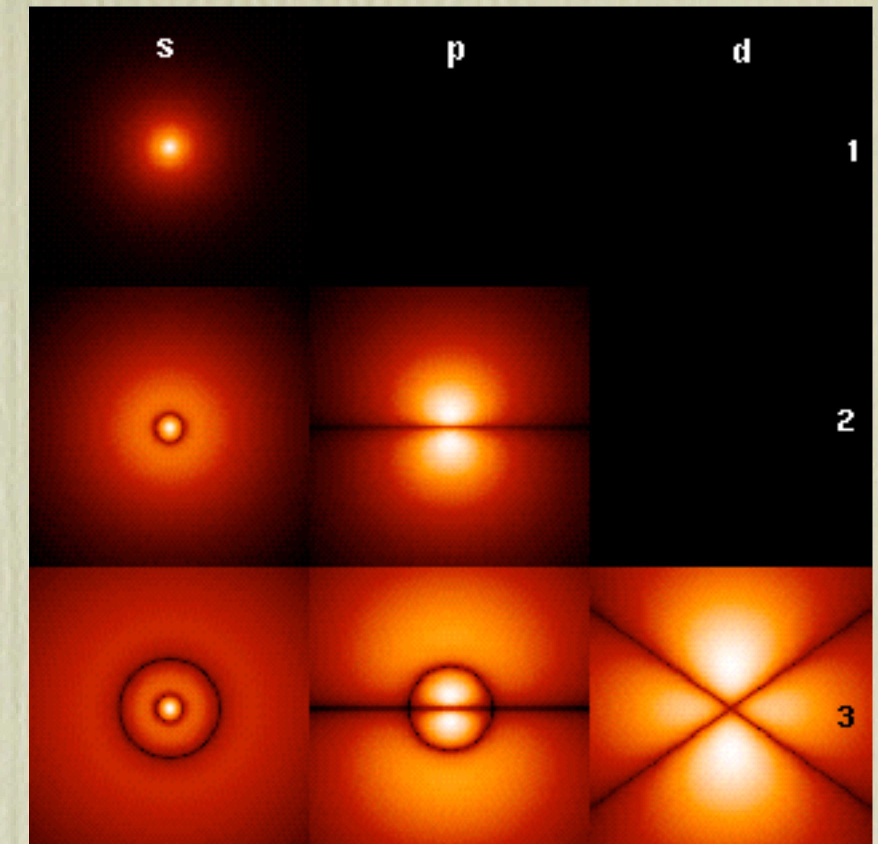

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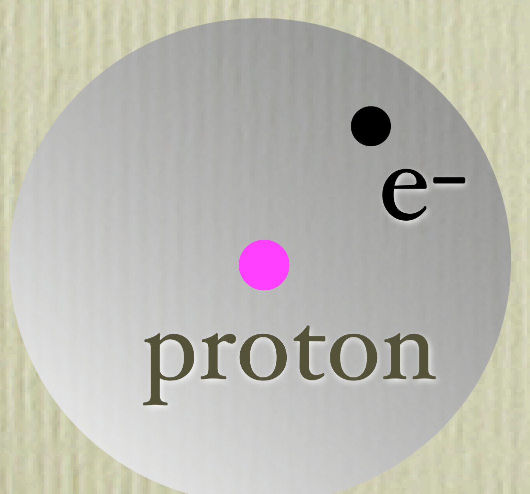

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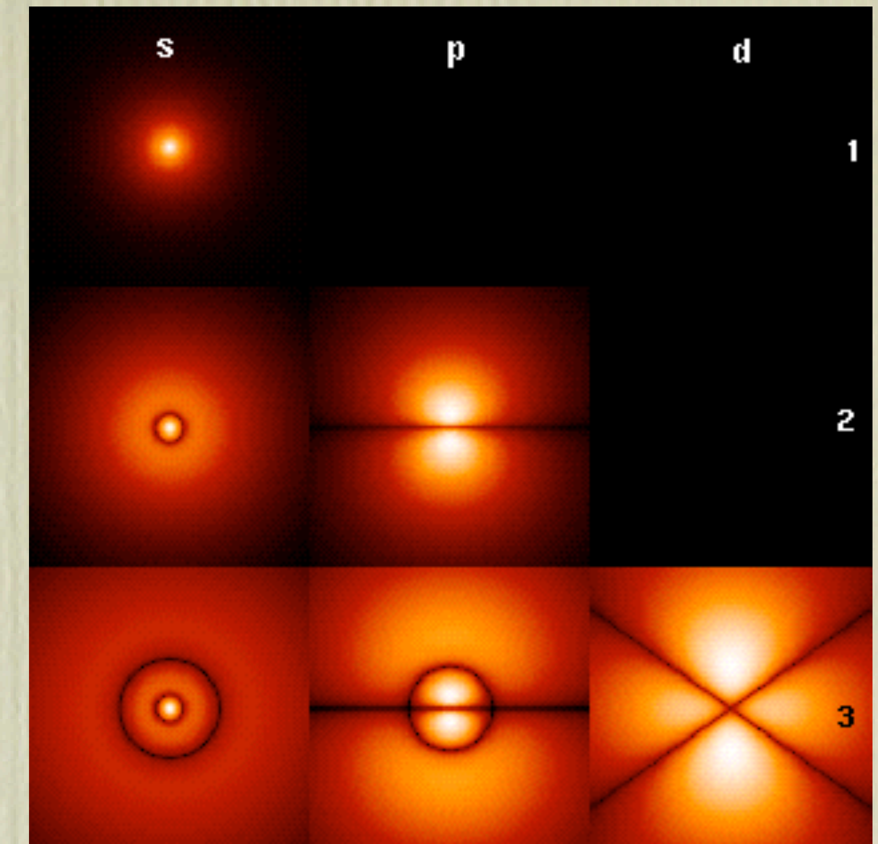

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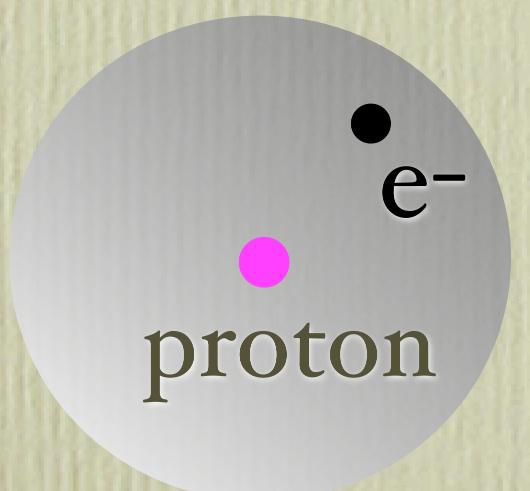

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+ corrections

Why not quarks? QCD? b-quark charge? e^+? weak force?

m_{proton} ? spin?

Effective Field Theory Idea

QED

short distance theory
is more general

expand in

$$\frac{p}{m_e},\ \frac{m_e}{m_p},\ \alpha$$

NRQED

long distance theory where
its easier to compute

$$H = H_0 + \sum_{m=1}^{\infty} \epsilon^m\ H_m$$

exact answer is irrelevant, work to
the desired level of precision

Nonrelativistic
Quantum
Mechanics

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Comments: Degrees of freedom can change

$$e^+ \longrightarrow \text{no} \ \ e^+$$

QCD, quarks $\longrightarrow$ proton

Effective Field Theory Idea

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Comments: Symmetries of QED constrain the form of NRQED

Charge conjugation ($e^+ \leftrightarrow e^-$)
Parity ($\vec{x} \to -\vec{x}$)
Time-Reversal ($t \to -t$)
Spin-Statistics Theorem

constrain the
H_m's

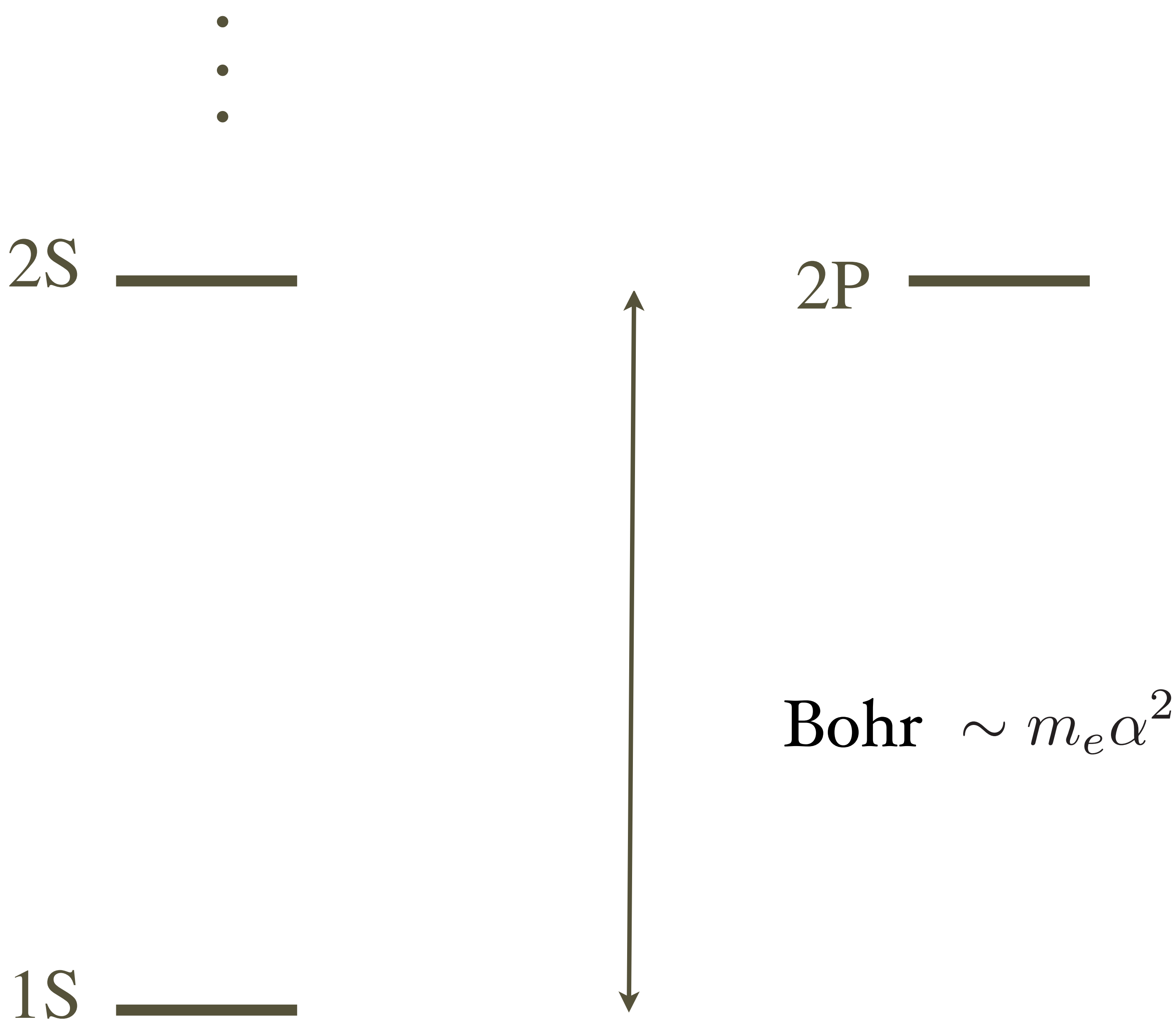
NRQED

Effective Field Theory for Non-relativistic bound states

nL_J

$F = J + S_p$

spectrum



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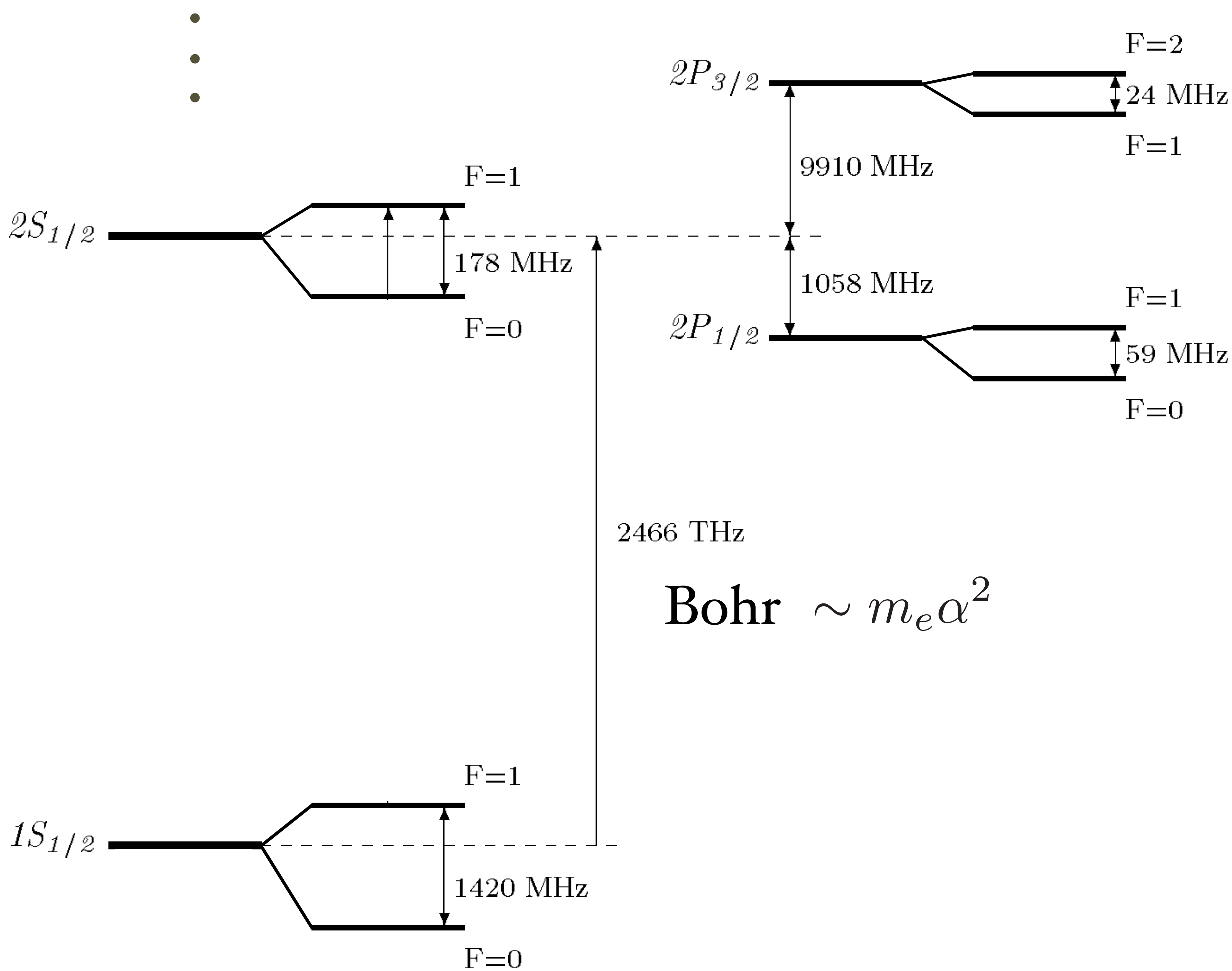

Bohr $\sim m_e \alpha^2$

NRQED

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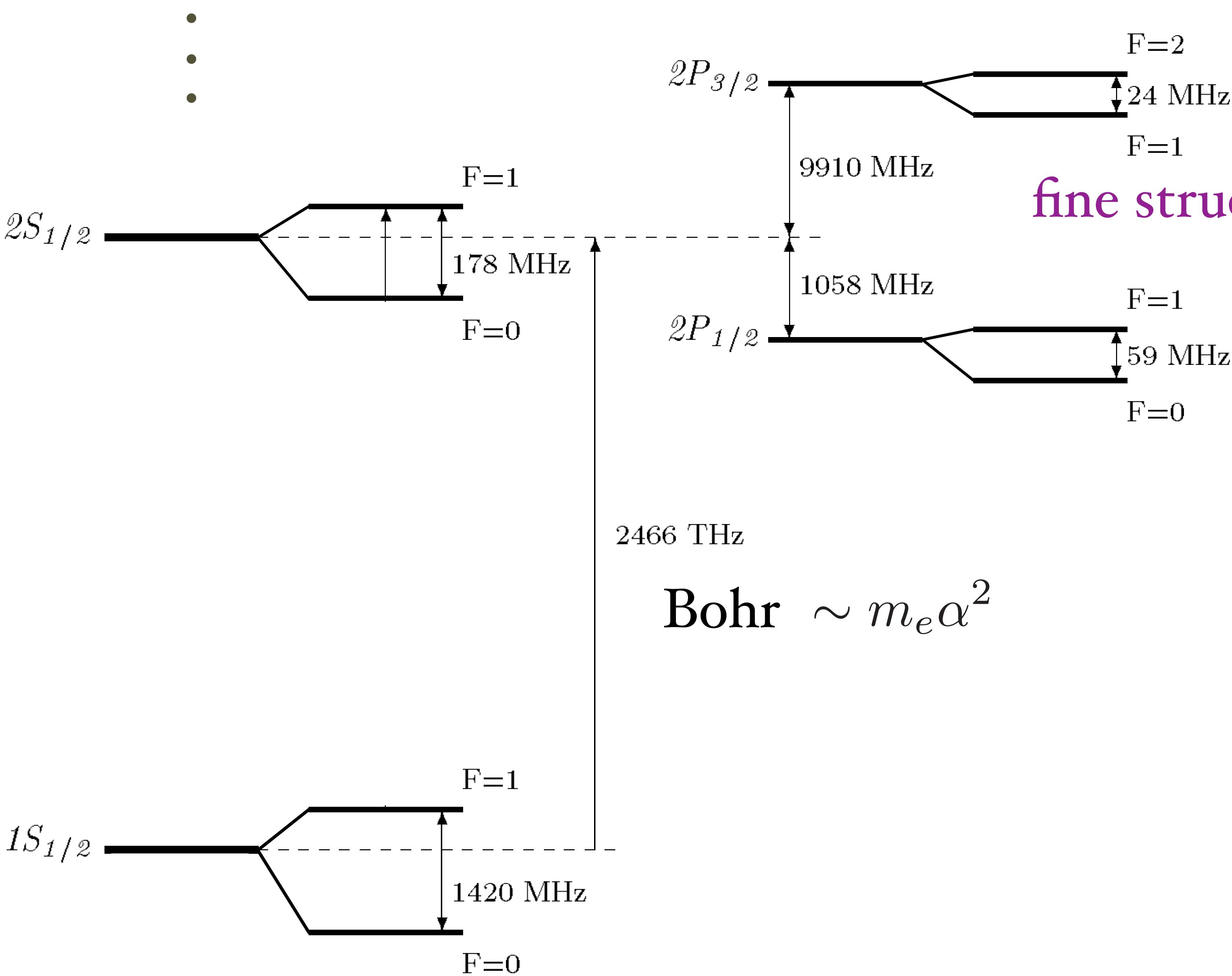

fine structure $\sim m_e \alpha^4$

Bohr $\sim m_e \alpha^2$

NRQED

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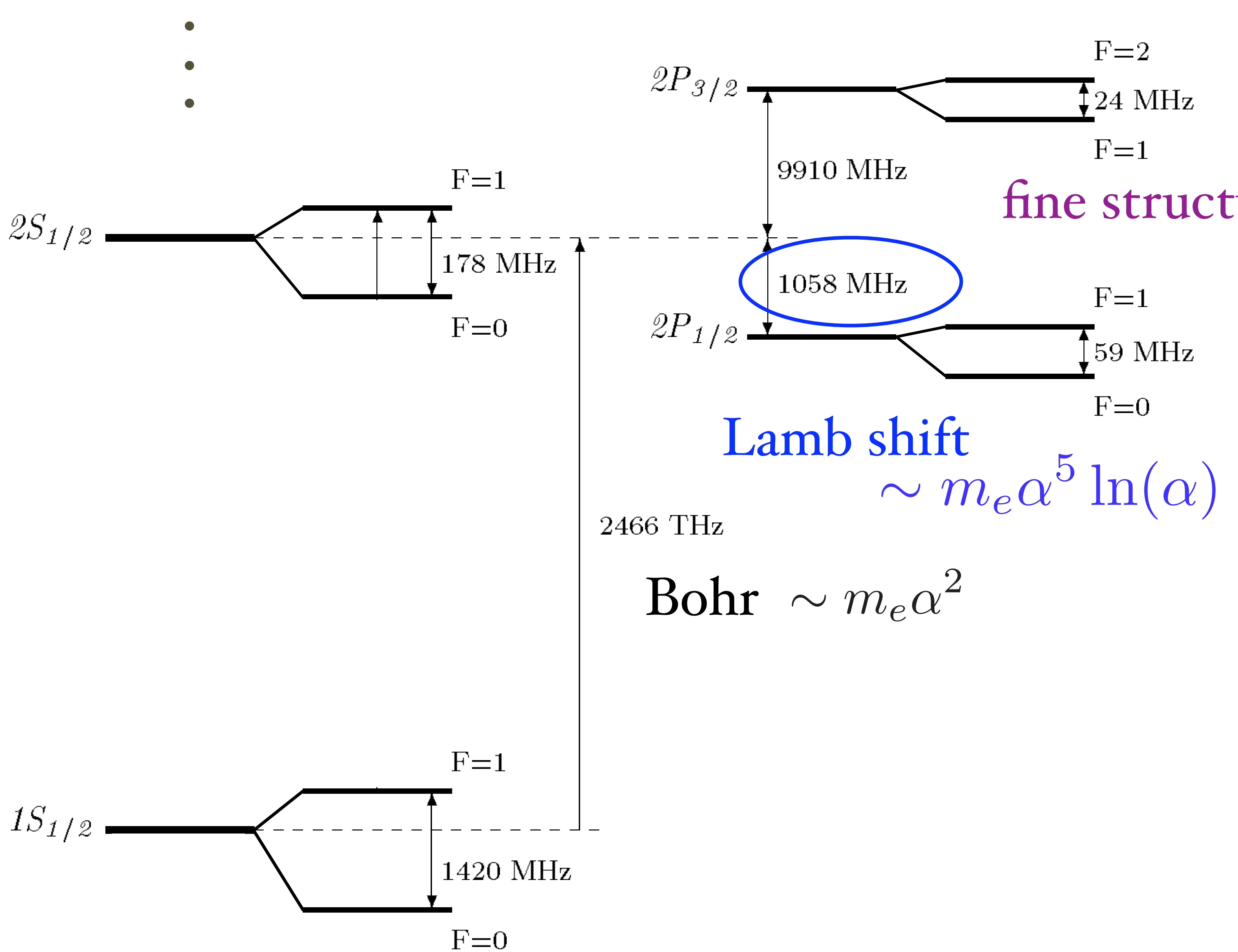

parameters fixed by QED

NRQED

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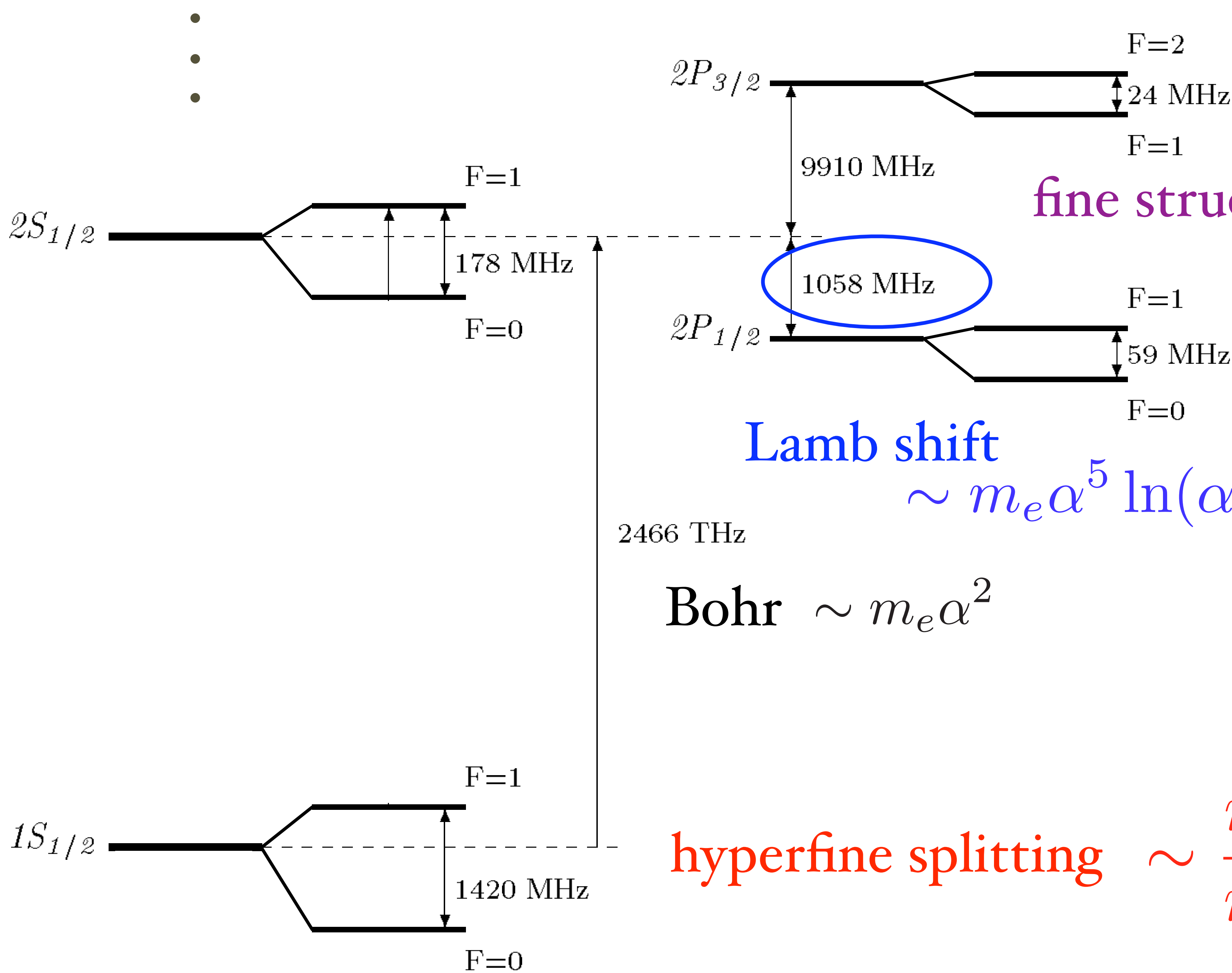


hyperfine

fine structure $\sim m_e \alpha^4$

hyperfine

Lamb shift

$\sim m_e \alpha^5 \ln(\alpha)$ ← parameters fixed by QED

Bohr $\sim m_e \alpha^2$

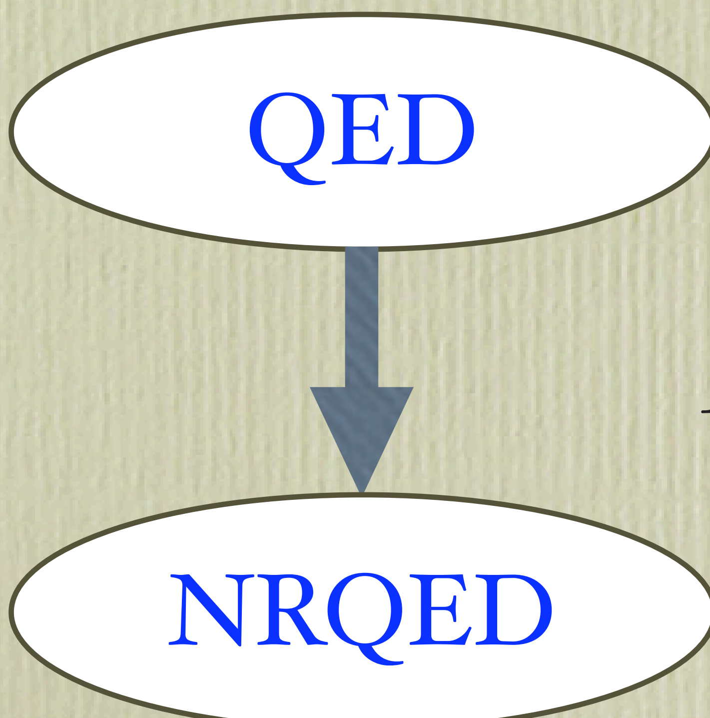
hyperfine splitting $\sim \frac{m_e^2}{m_p} \mu_e \mu_p \alpha^4$

QCD

Compute the H_m by "Matching"

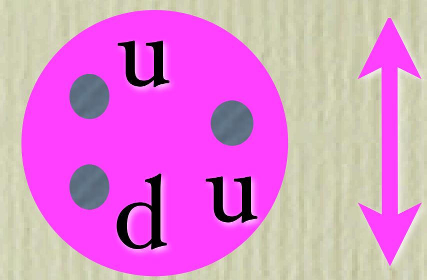
Relativity: $\dfrac{p^4}{8m_e^3} + \ldots$

QED: $\mu_e \,, \quad \vec{L}\cdot\vec{S} \quad , \ldots$ (coefficients determined by $\alpha \,, m_e$)



$$H = H_0 + \sum_{m=1}^{\infty} \epsilon^m \; H_m$$

What about quarks?



size $\sim 1\,\mathrm{fm} \rightarrow 200\,\mathrm{MeV} \gg p_\gamma$

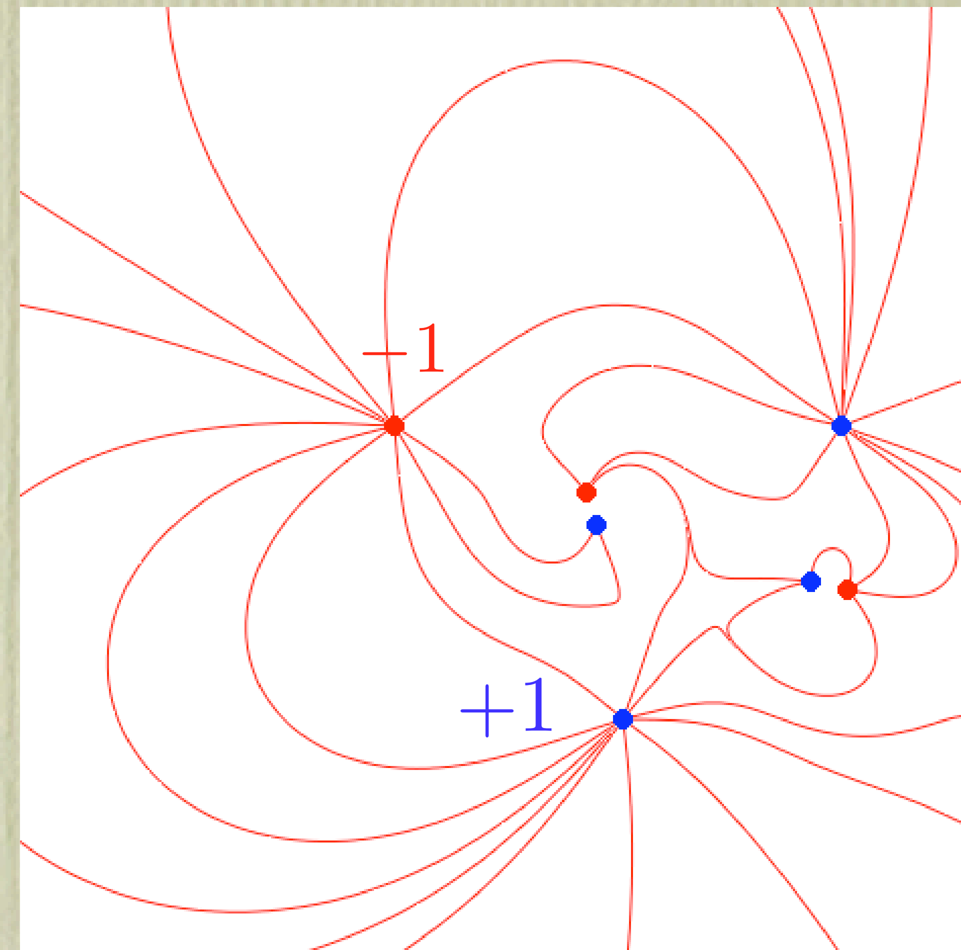
$Q_u = +2/3$

$Q_d = -1/3$

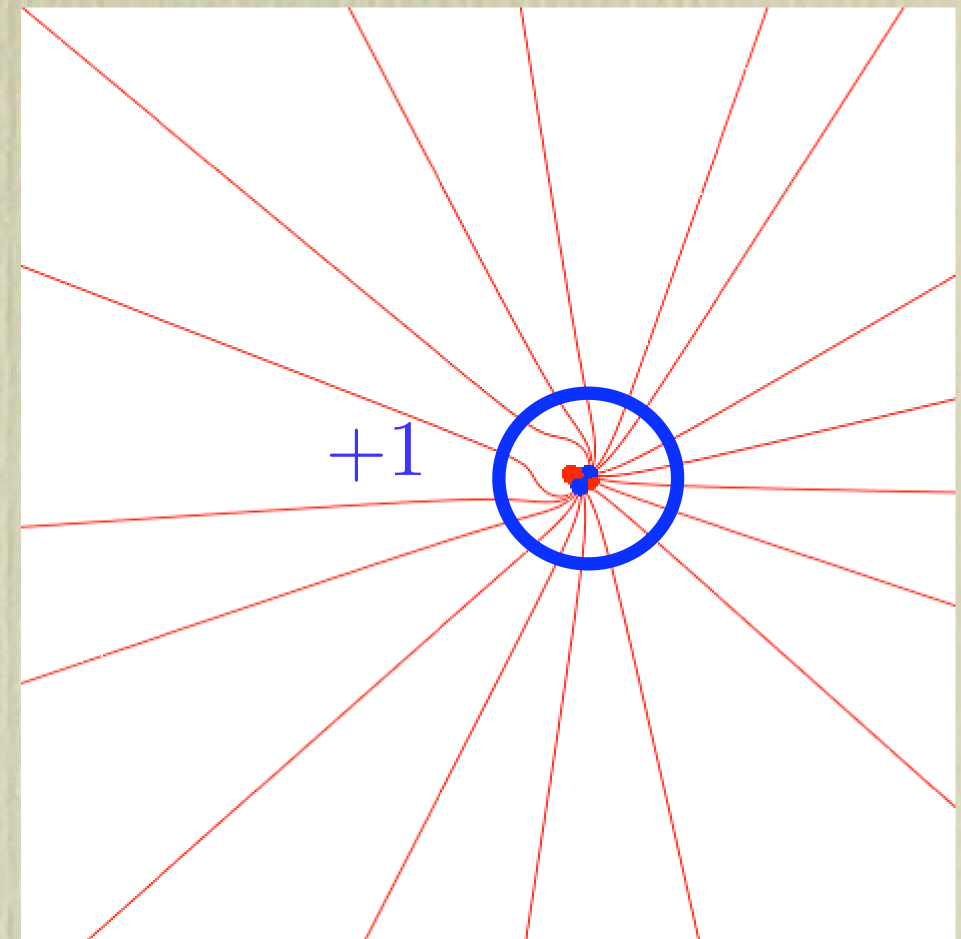
low momentum photons do not resolve the quarks, they see the proton charge

When matching couplings change too: $Q_{u,d} \rightarrow Q_p$

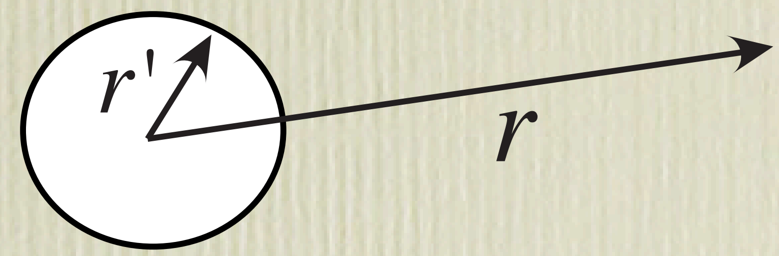
short distance



long distance



This is just an application of the multipole expansion, familiar from electromagnetism:

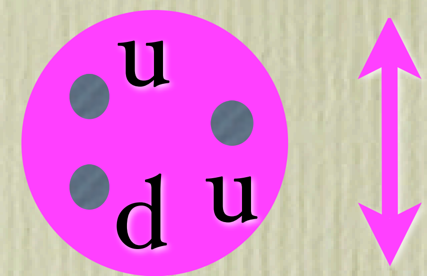


$$\mathcal{V}(\vec{r}) = \frac{1}{r}\underbrace{\int \rho\, d^3 r'}_{\text{total charge}} + \frac{1}{r^2}\int r' cos\theta \rho\, d^3 r' + \ldots$$

$$200\,\mathrm{MeV} \gg p_\gamma \Leftrightarrow r' \ll r$$

keV

What about quarks?



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When matching couplings change too: $Q_{u,d} \to Q_p$

other parameters: $m_p, \mu_p\ , \ldots$

in principle fixed by QCD, but it is more accurate to use experimental measurements

measure a parameter in one place, then use it in others!

= universality

Resolution μ

Resolution

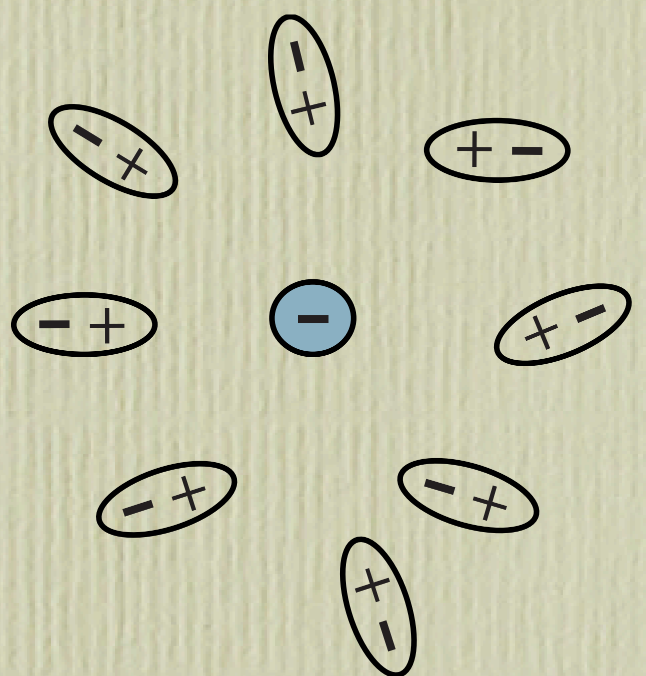
Resolution

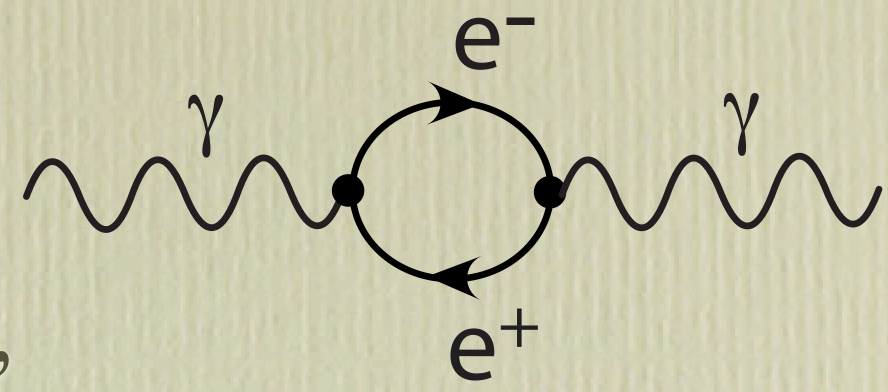
Resolution

Resolution

Resolution

Vacuum Polarization





$$\alpha = \frac{e^2}{4\pi}$$

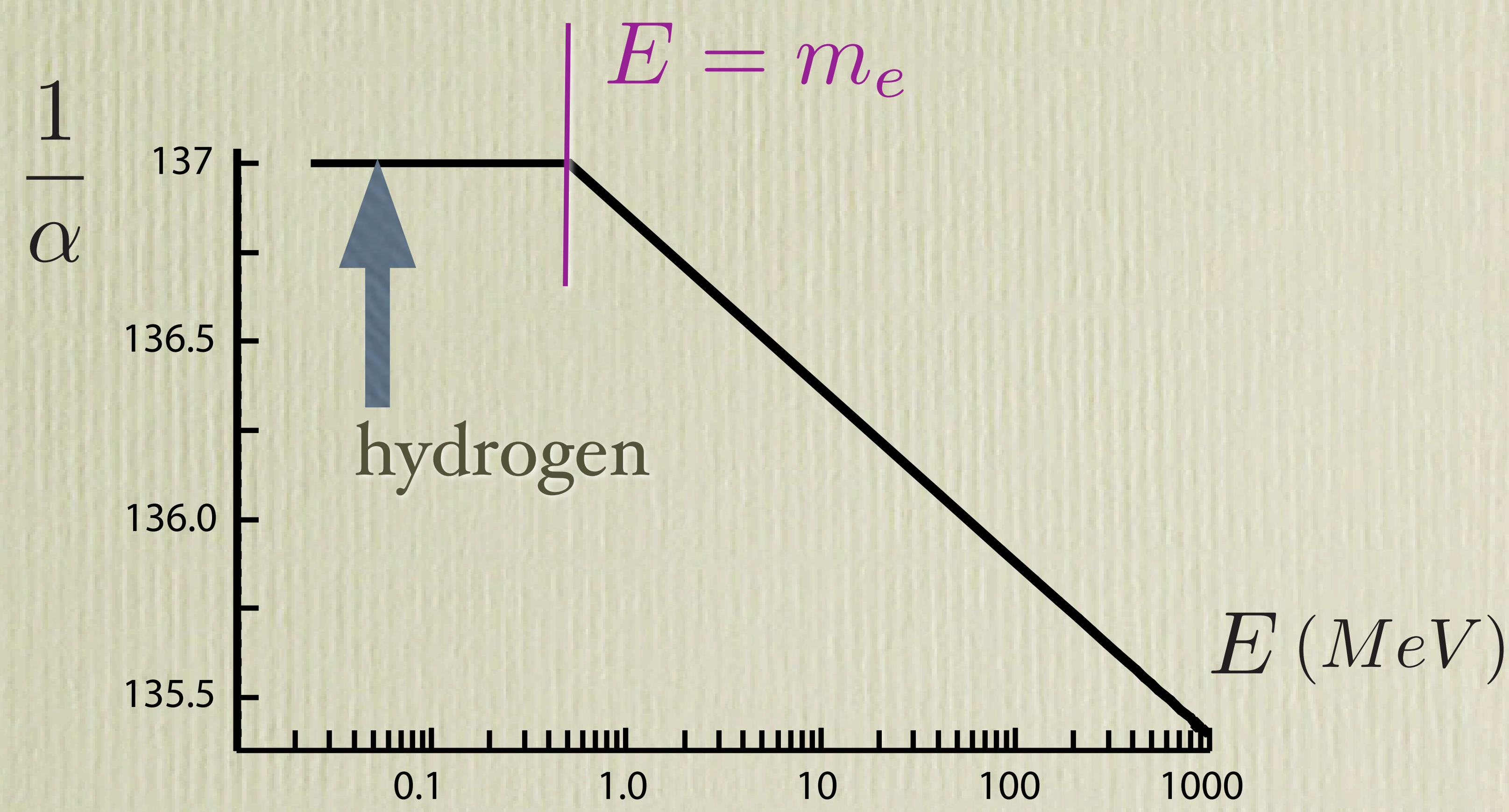
like a dielectric,
gives screening

coupling is
renormalized

resolution $\mu = E$

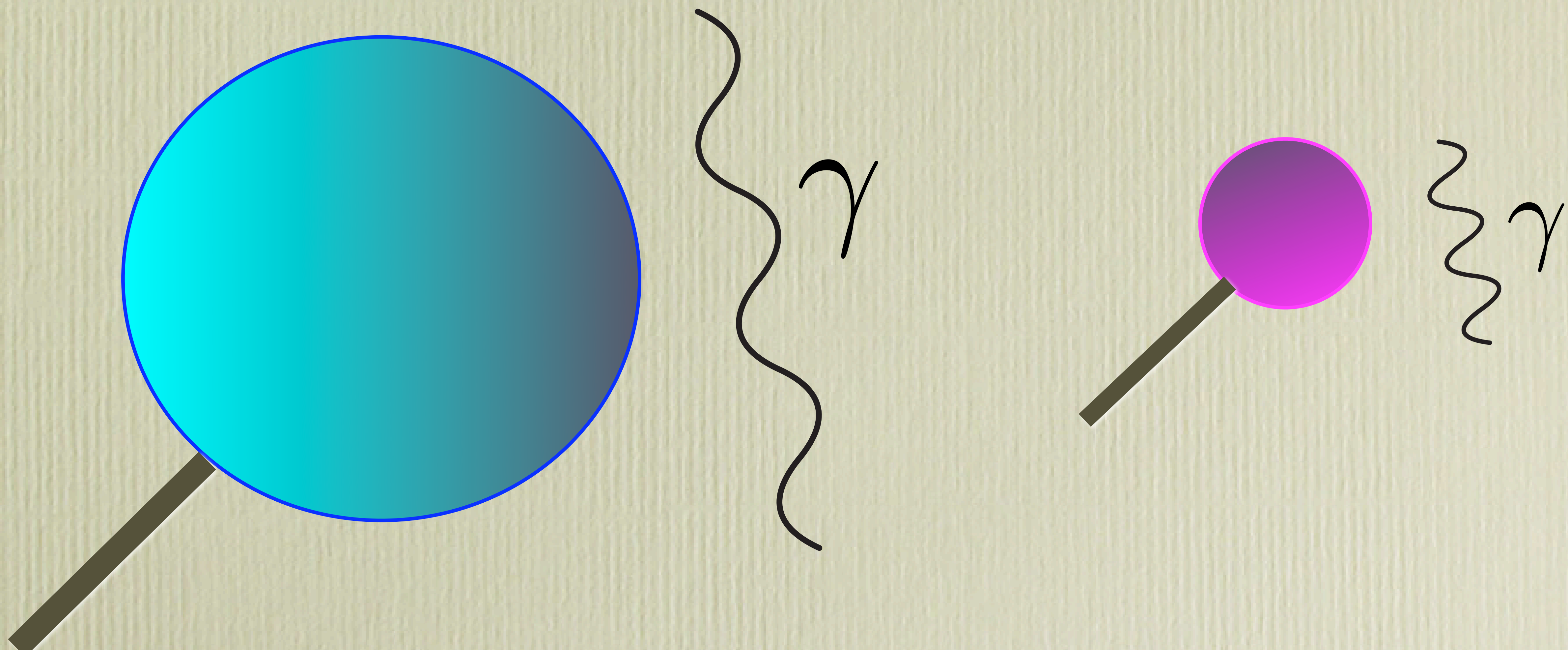
$$\mu \frac{d}{d\mu}\alpha(\mu) = \frac{2}{3\pi}\alpha^2(\mu)$$

at larger energy E, we
probe shorter distances
and see a larger charge



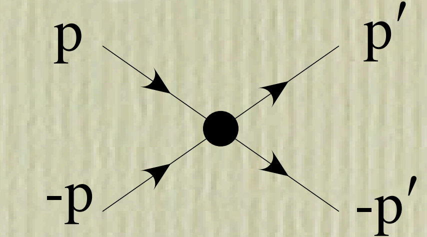
$$\alpha(E) = \frac{\alpha(0)}{1 - \frac{\alpha(0)}{3\pi}\ln\left(\frac{E^2}{m_e^2}\right)}$$

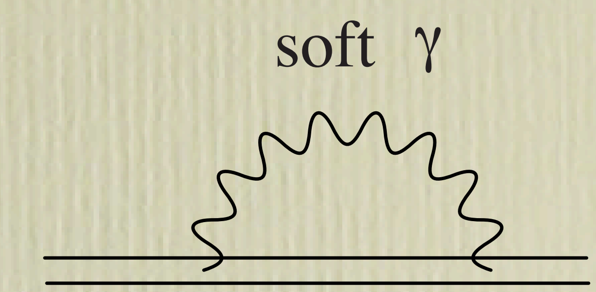
Long versus Short Distance



Lamb Shift in NRQED

Two parts:





i) effective potentials (short distance)

ii) radiation in the bound state (long distance)

$$\delta E_n = \left[\frac{4\alpha^2}{3m_e^2}|\psi_n(0)|^2 \ln\left(\frac{\mu}{m_e}\right)+\dots\right]+\left[\frac{1}{m_e^2}\sum_{k\neq n}|\langle n|\hat{p}|k\rangle|^2(E_k-E_n)\ln\left(\frac{\mu}{|E_n-E_k|}\right)+\dots\right]$$

μ dependence cancels, but allows us to give separate meaning to the two pieces

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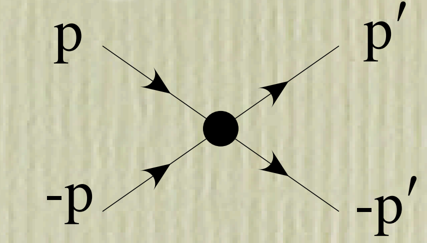

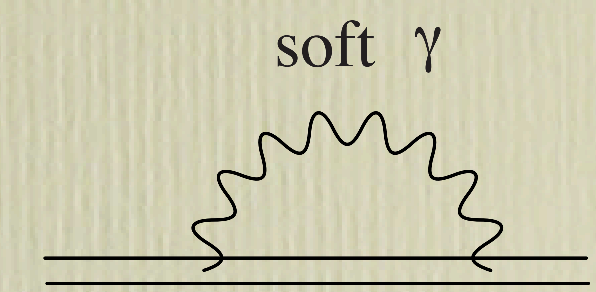

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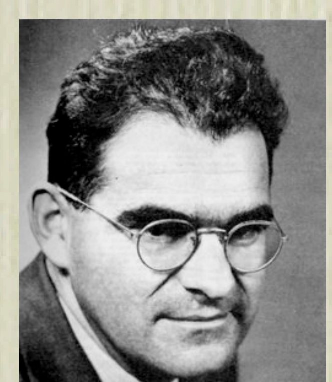
History:

- 1947 Bethe computed ii), with $\mu = m_e$

  large log: $\sim \ln\left(\frac{m_e}{m_e\alpha^2}\right) = -2\ln(\alpha)$

- 1949 French & Weisskopf
  Lamb & Kroll
  (Feynman, Schwinger)



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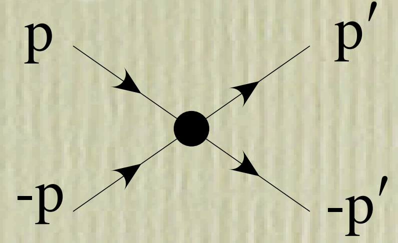

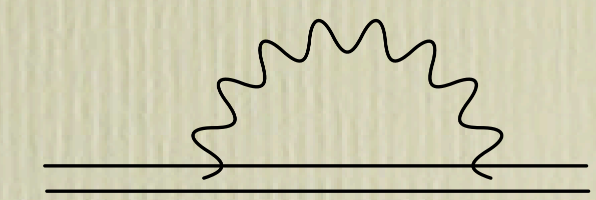

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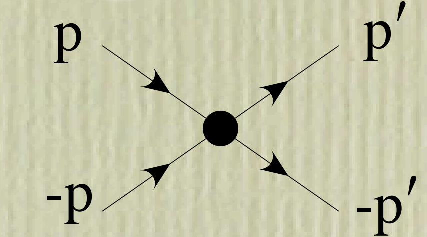
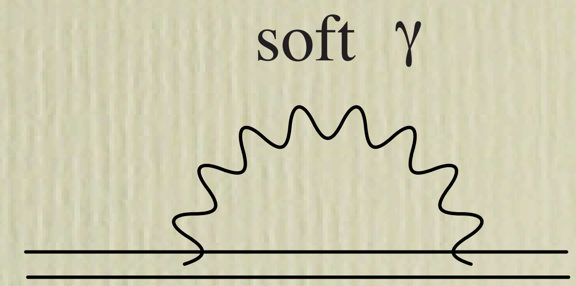




i) effective potentials (short distance)

ii) radiation in the bound state (long distance)

$$\delta E_n = \left[\frac{4\alpha^2}{3m_e^2}|\psi_n(0)|^2 \ln\left(\frac{\mu}{m_e}\right)+\ldots\right]+\left[\frac{1}{m_e^2}\sum_{k\neq n}|\langle n|\hat{p}|k\rangle|^2(E_k-E_n)\ln\left(\frac{\mu}{|E_n-E_k|}\right)+\ldots\right]$$

μ dependence cancels, but allows us to give separate meaning to the two pieces

History:

- 1947 Bethe computed ii), with $\mu = m_e$

  large log: $\sim \ln\left(\frac{m_e}{m_e\alpha^2}\right) = -2\ln(\alpha)$

  $\Delta E(2S-2P) = 1040\,\text{MHz}$ close to the $1058\,\text{MHz}$ answer

- 1949 French & Weisskopf, Lamb & Kroll (Feynman, Schwinger) computed i) in QED and combined with ii) $\Delta E(2S-2P) = 1051\,\text{MHz}$

The structure of QED logs can be derived from a non-relativistic renormalization group

Luke, Manohar, Rothstein, I.S.

$$E = \frac{p^2}{2m}$$

energy resolution μ_E

momentum resolution μ_p

$$\mu_E \sim \frac{\mu_p^2}{m}$$

Correction	Observable	System	Comparison
$\alpha^8 \ln^3 \alpha$	Lamb shift	H	agrees*
		μ^+e^-, e^+e^-	new
	(no h.f.s., no $\Delta\Gamma/\Gamma$)		
$\alpha^7 \ln^2 \alpha$	h.f.s.	H, μ^+e^-, e^+e^-	agrees
	Lamb shift	H, μ^+e^-, e^+e^-	agrees
$\alpha^3 \ln^2 \alpha$	$\Delta\Gamma/\Gamma$	e^+e^- ortho and para	agrees
$\alpha^6 \ln \alpha$	Lamb shift	H, μ^+e^-, e^+e^-	agrees
	h.f.s.	H, μ^+e^-, e^+e^-	agrees
$\alpha^2 \ln \alpha$	$\Delta\Gamma/\Gamma$	e^+e^- ortho and para	agrees
$\alpha^5 \ln \alpha$	Lamb shift	H, μ^+e^-, e^+e^-	agrees

all from one equation

LO anomalous dimension: $\alpha^4(\alpha \ln \alpha)^k$ stops at $k = 1$

NLO anomalous dimension: $\alpha^5(\alpha \ln \alpha)^k$ stops at $k = 3$

The structure of QED logs can be derived from a non-relativistic renormalization group

Luke, Manohar, Rothstein, I.S.

$$E = \frac{p^2}{2m}$$

energy resolution μ_E

momentum resolution μ_p

$$\mu_E \sim \frac{\mu_p^2}{m}$$

NRQED methods are also used for the non-logarithmic terms

		Expt.(MHz)	Theory(MHz)	Agree?
H	Lamb	1057.845(9)	1057.85(1)	$< r_p^2 >$
	h.f.s	1420.405751768(1)	1420.399(2)	$G_E,\, G_M$
$\mu^+ e^-$	h.f.s	4463.30278(5)	4463.30288(55)	m_e/m_μ
$e^+ e^-$	Lamb	13012.4(1)	13012.41(8)	agree
	h.f.s	203389.10(74)	203391.70(50)	3σ
	Γ_{para}	7990.9(1.7) μs^{-1}	7989.62(4) μs^{-1}	agree
	Γ_{ortho}	7.0404(13) μs^{-1}	7.03996(2) μs^{-1}	agree

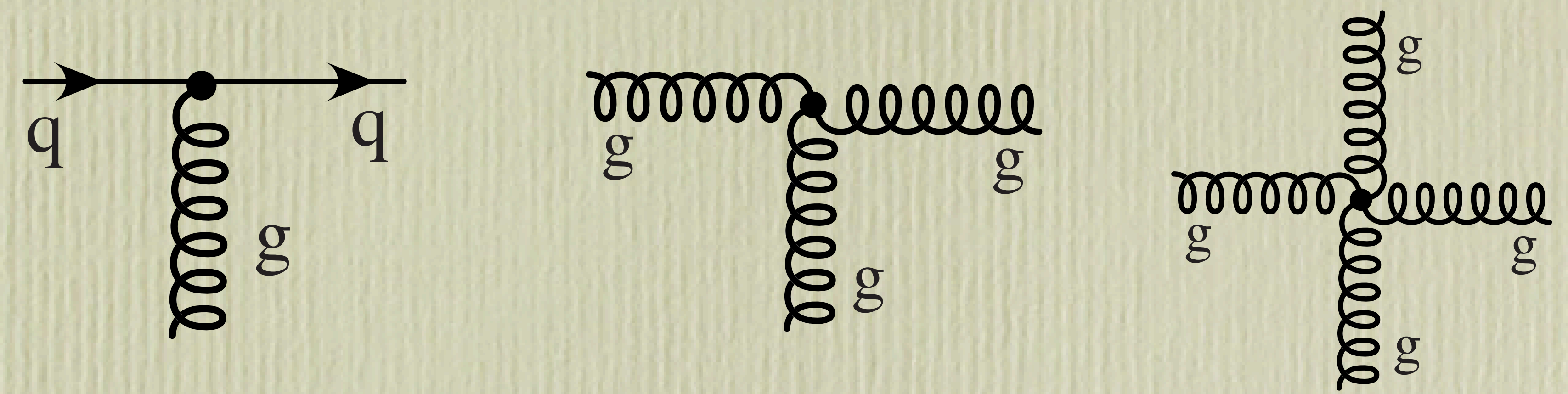
The ideas we've discussed in QED:

- resolution μ
- changes in degrees of freedom & couplings
- expansions, multiple scales
- universality

become even more crucial for QCD

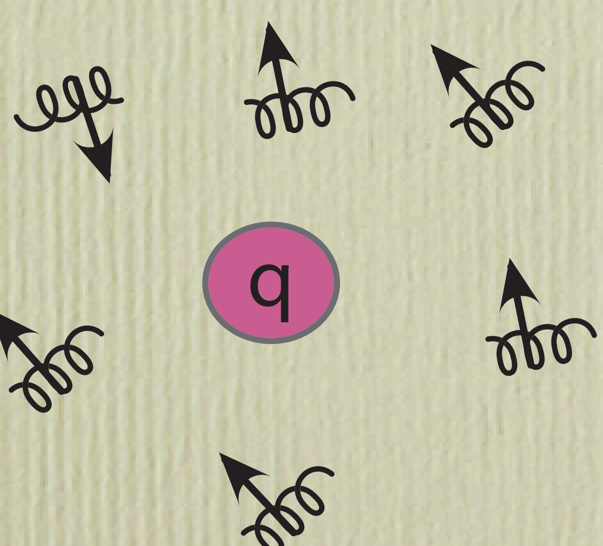
QCD Interactions are more complicated than QED:

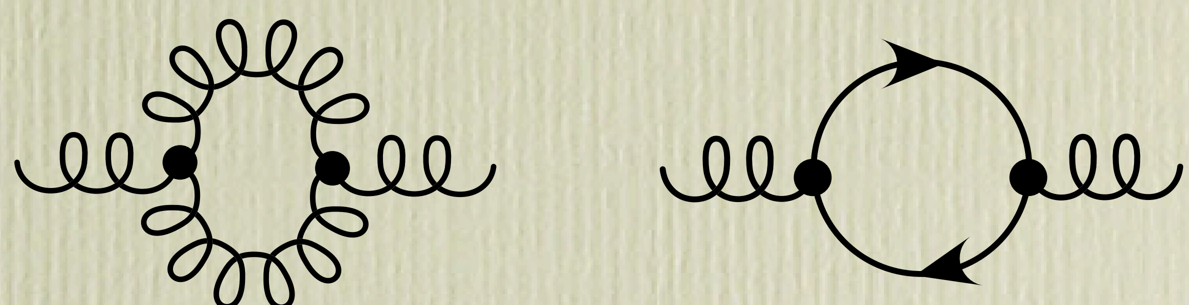
strong coupling: $g(\mu)$ $\qquad \alpha_s(\mu) = \frac{g(\mu)^2}{4\pi}$



these interactions involve the same coupling (gauge symmetry)

Vacuum response?





gluons have spin, carry color charge

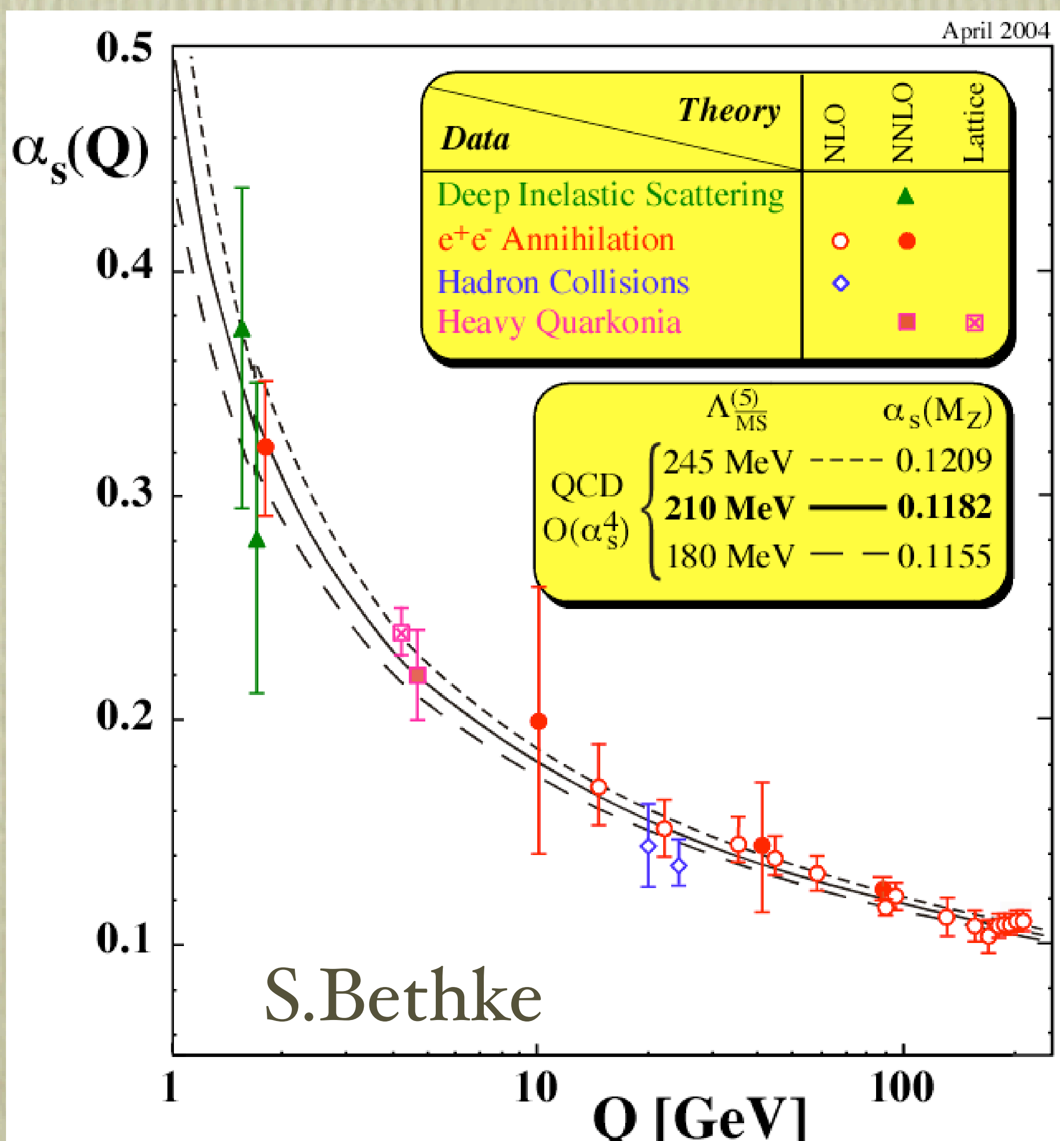
behave like a permanent magnet
anti-screen the charge

$$\beta(g) = \mu \frac{d}{d\mu} g(\mu) = -\frac{g(\mu)^3}{16\pi^2}\left(11 - \frac{2}{3}n_f\right) \quad < 0$$

In QCD, the coupling , $g(\mu)$, behaves in the opposite way to QED, it gets weaker at short distances

slope is negative

$$\alpha_s(\mu) = \frac{g(\mu)^2}{4\pi} \qquad \beta(g) = \mu \frac{d}{d\mu} g(\mu) < 0$$



Gross, Politzer, Wilczek

Nobel Prize, 2004

Asymptotic freedom

large $\mu = Q$, small α_s , free quarks

Infrared slavery

as $\mu = Q$ approaches a few 100 MeV $(r \rightarrow 1\,\text{fm})$, the coupling gets large

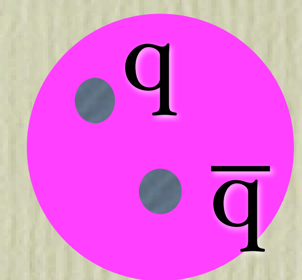
large change in the value

an expansion in $\alpha_s(\mu < 1\,\mathrm{GeV})$ is no good

→ coupling gets so strong that quarks never escape unless they form a color singlet (bound) state with other quarks, ie. they are confined

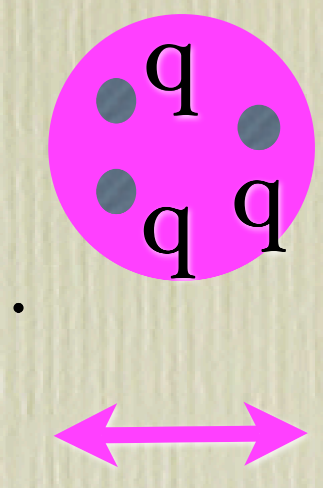
Mesons

$\pi,\ K,\ \rho,\ \ldots$



Baryons

$p,\ n,\ \Sigma,\ \Delta,\ \ldots$



$r = \Lambda_{\mathrm{QCD}}^{-1}$

degrees of freedom change

an expansion in $\alpha_s(\mu < 1\,\mathrm{GeV})$ is no good

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Mesons

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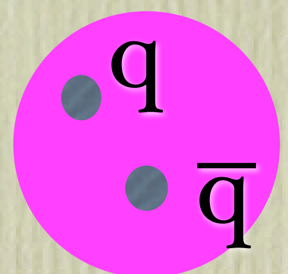
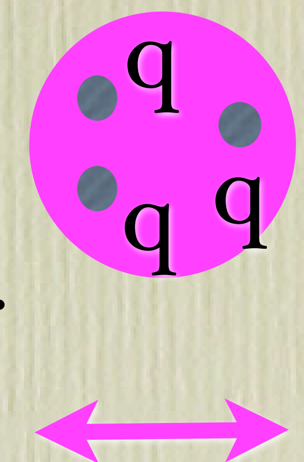


Baryons

$p,\ n,\ \Sigma,\ \Delta,\ \ldots$



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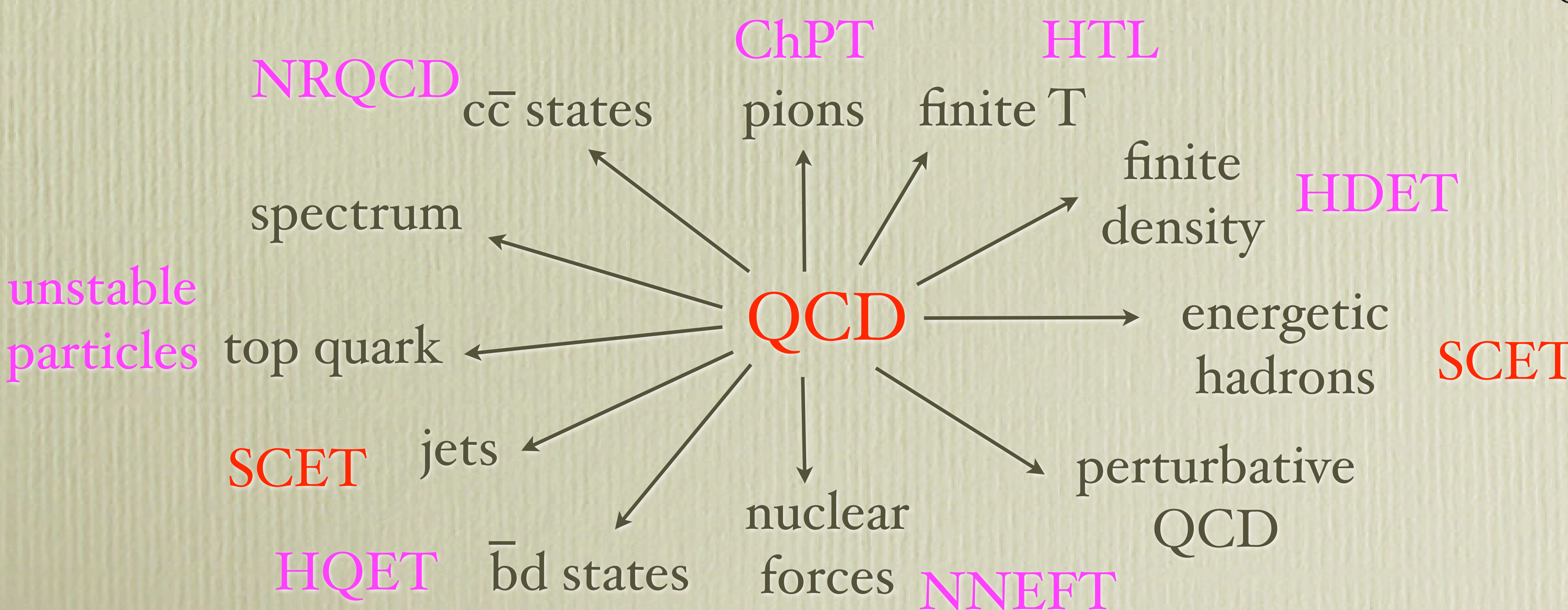

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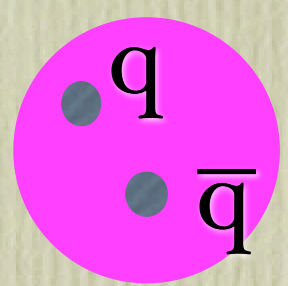
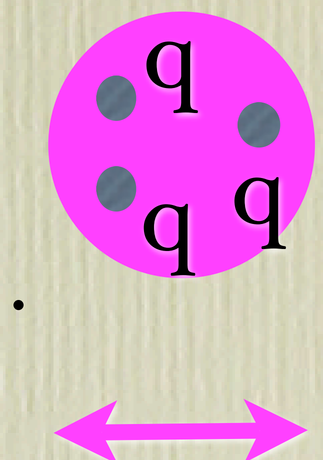
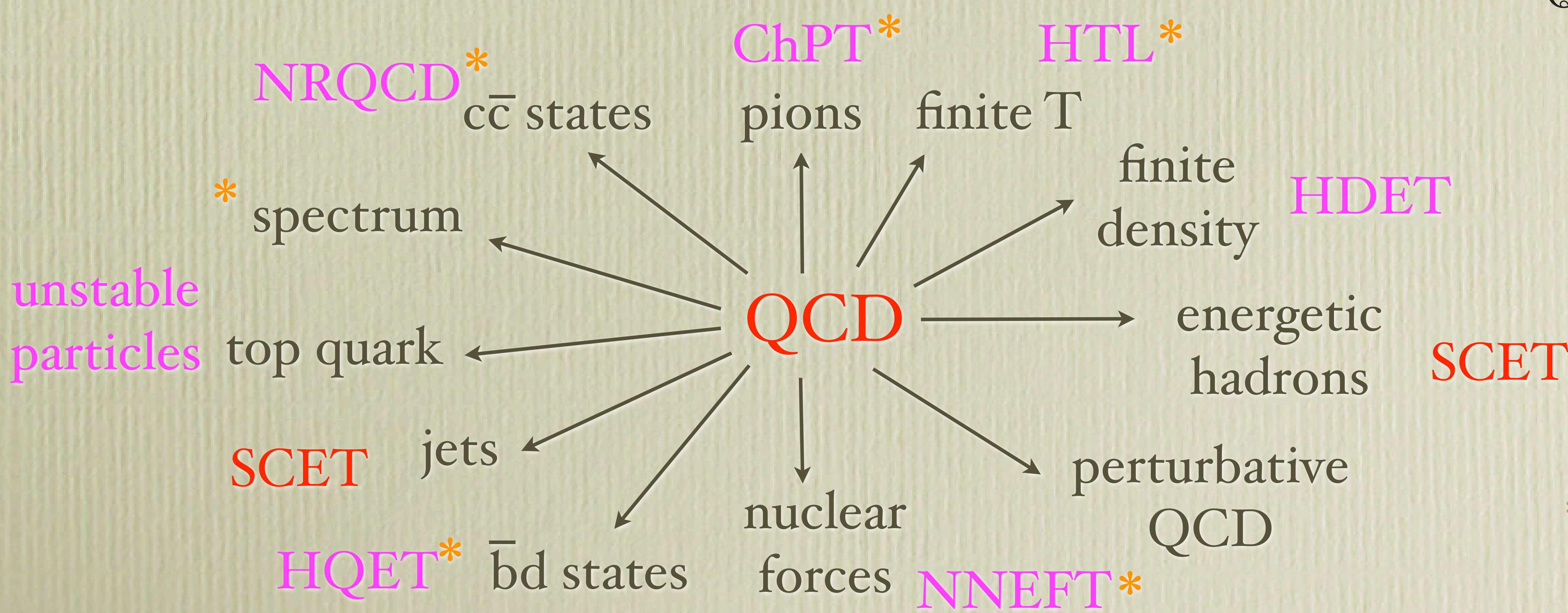


Baryons

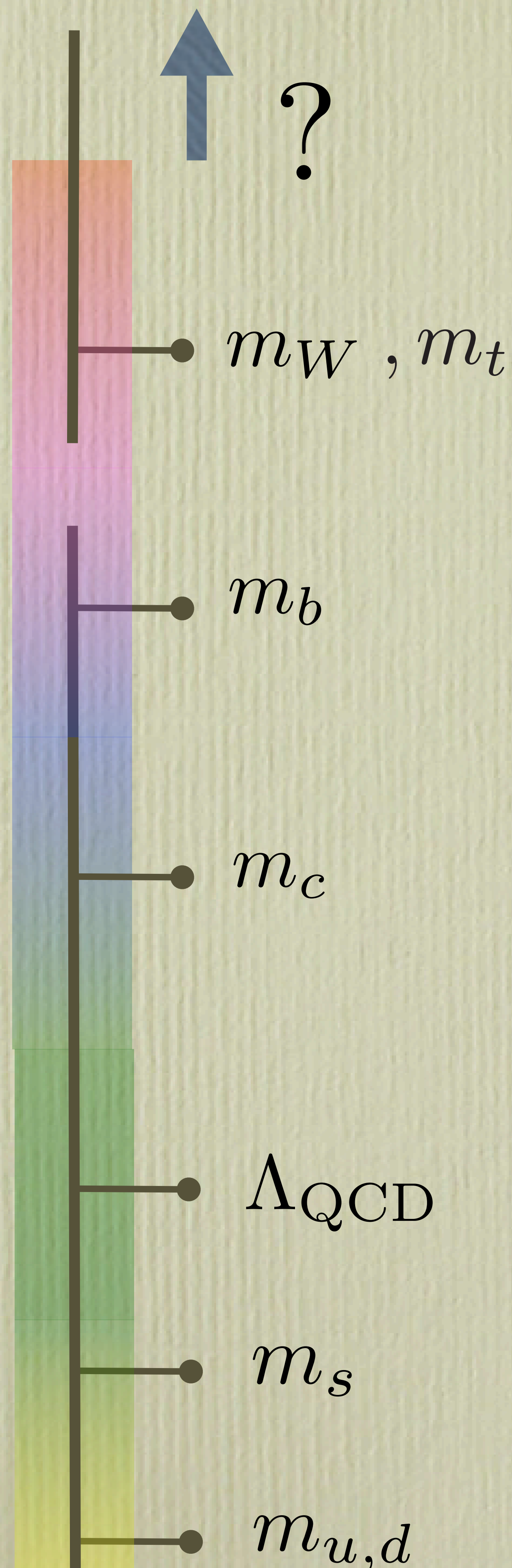
$p,\ n,\ \Sigma,\ \Delta,\ \ldots$



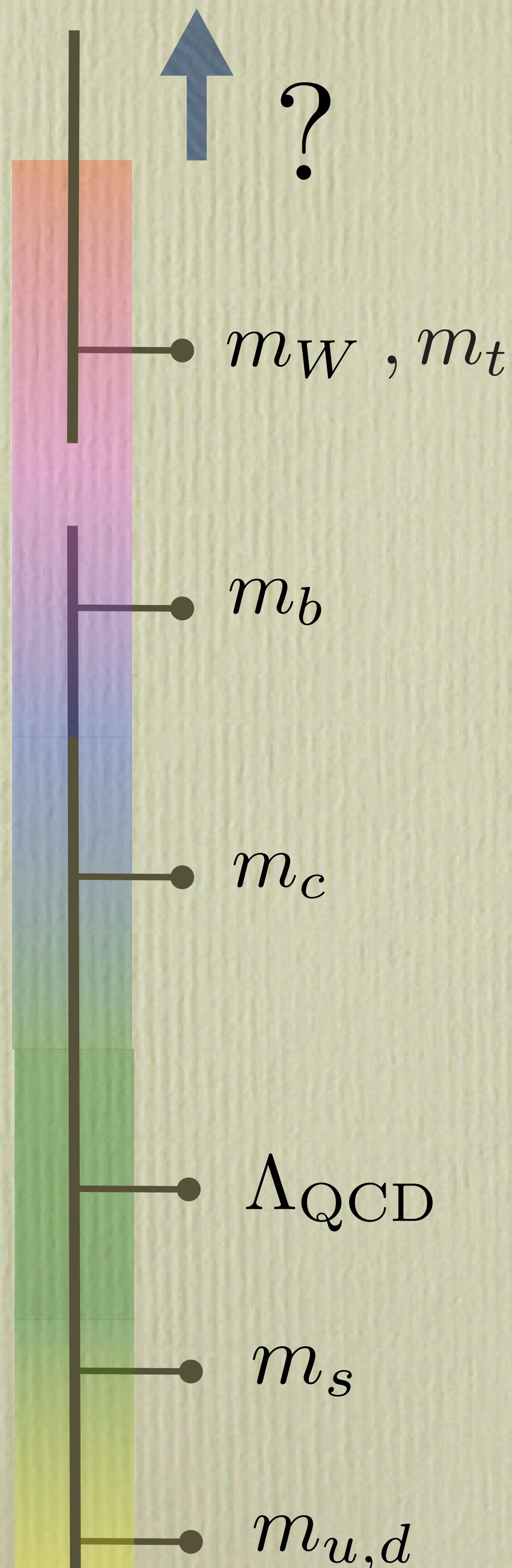
degrees of freedom change

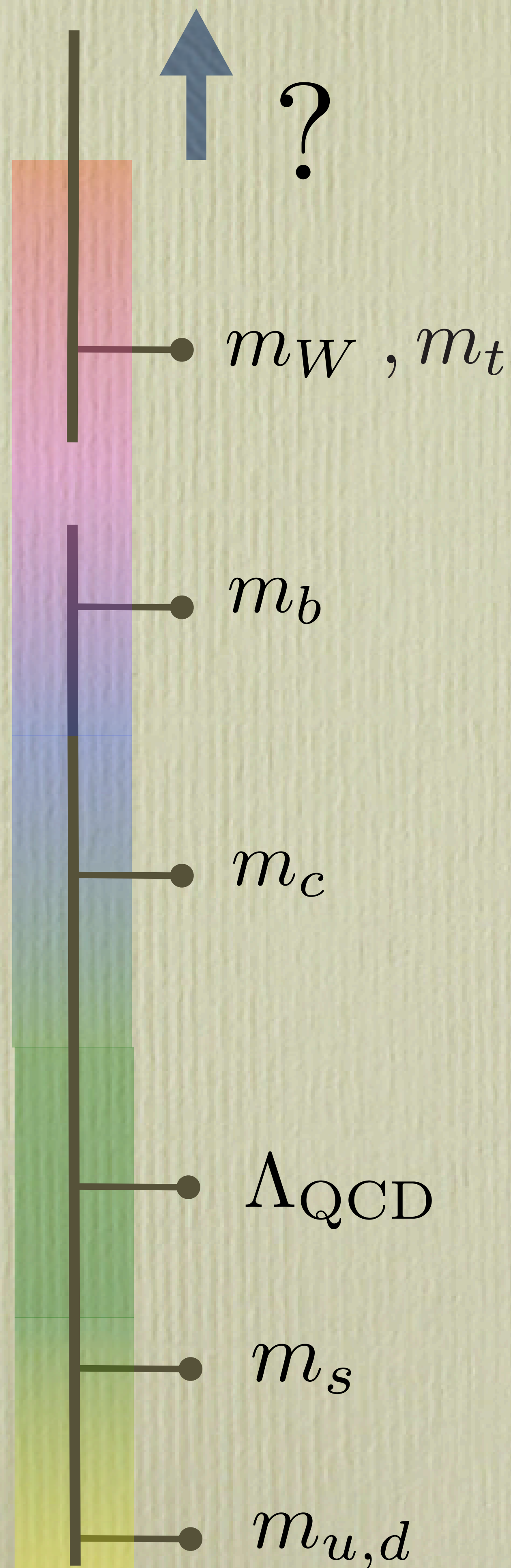


* Lattice QCD


?
$m_W \, , m_t$
m_b
m_c
Λ_{QCD}
m_s
$m_{u,d}$

Is there a "Hydrogen Atom" for QCD?





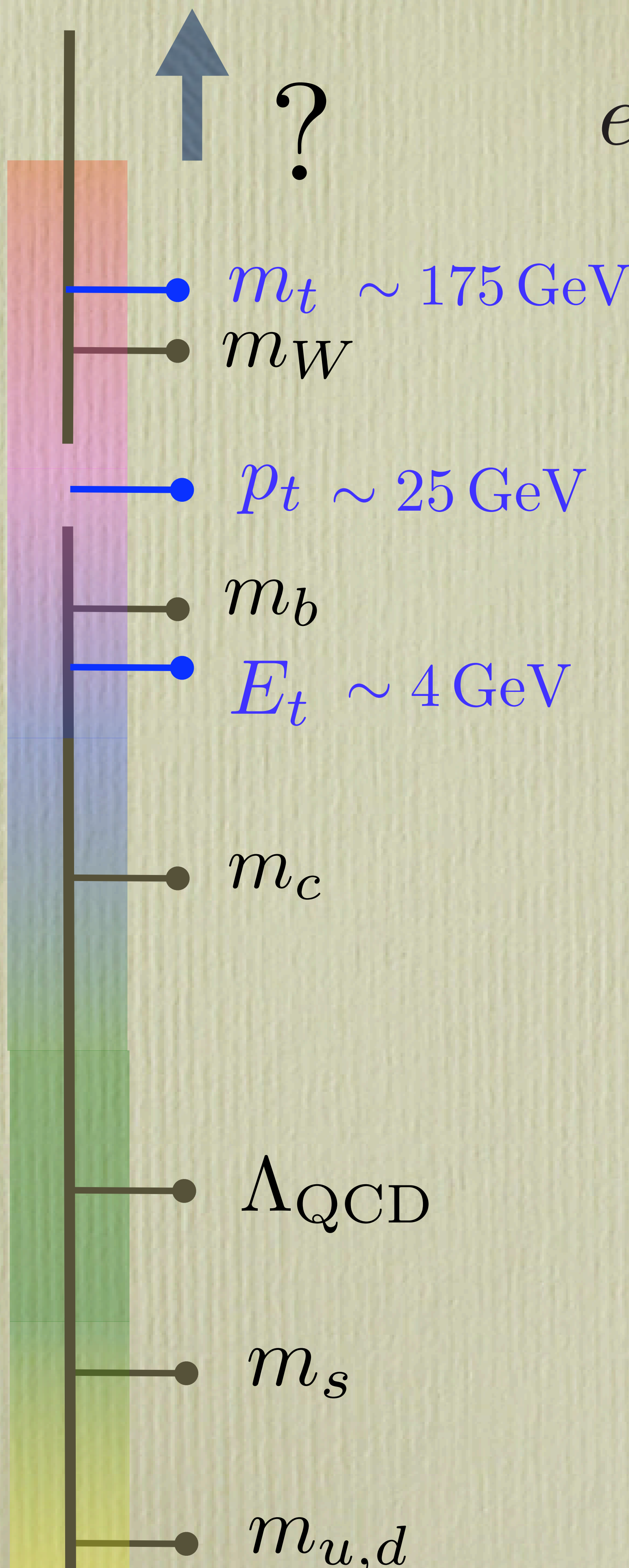
Is there a "Hydrogen Atom" for QCD?

candidates:

i) top quarks: $t\,\bar{t}$

ii) proton

iii) B mesons



$$e^+e^- \to t\bar{t}$$

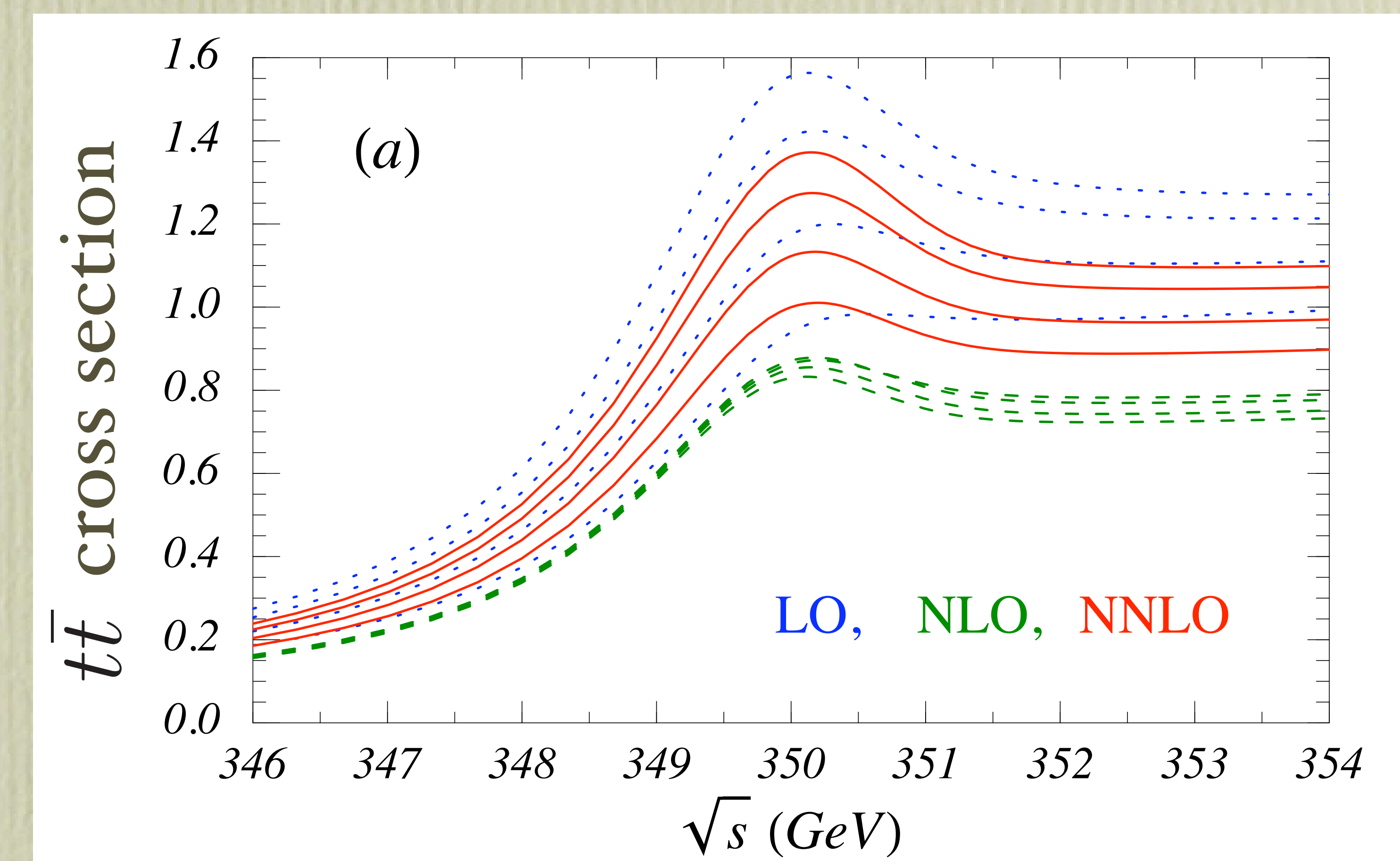
Nonrelativistic QCD bound states?

$$\Gamma_t = 1.4\,\mathrm{GeV} \gg \Lambda_{\mathrm{QCD}}$$

top decays before it hadronizes

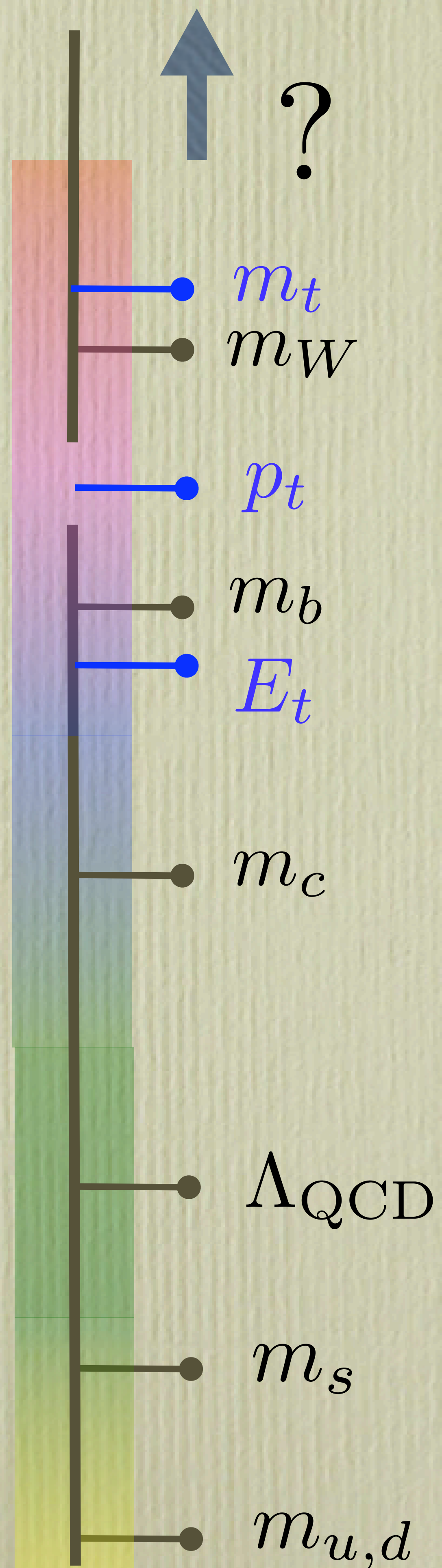
Coulombic, expansion in $\alpha_s(\mu)$:

$$\mathrm{LO} + \mathrm{NLO} + \mathrm{NNLO} + \ldots$$



vary μ

$$\mu = m_t, p_t, E_t?$$



$$e^+e^- \to t\bar{t}$$

Nonrelativistic QCD bound states?

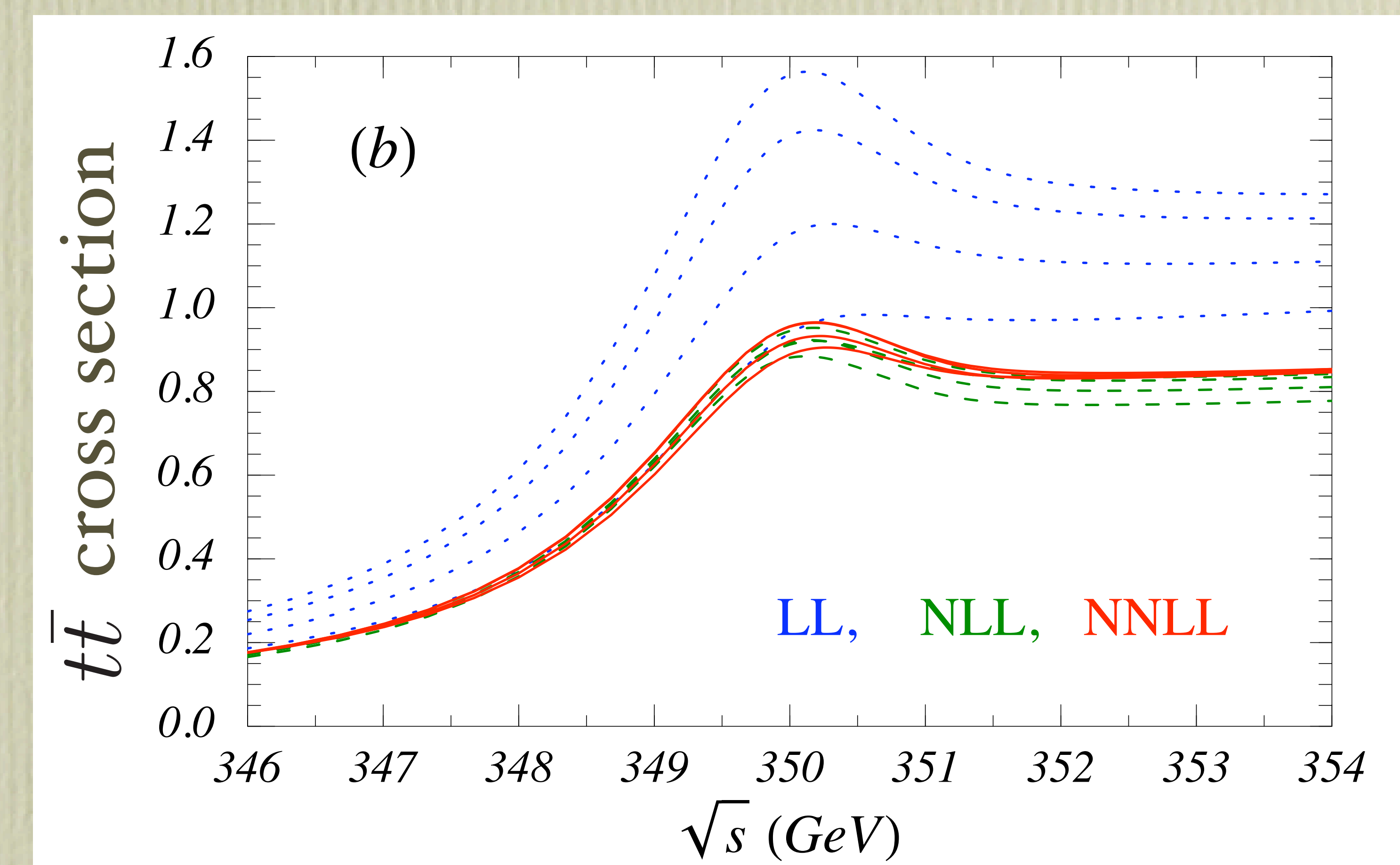
$$\Gamma_t = 1.4\,\mathrm{GeV} \gg \Lambda_{\mathrm{QCD}}$$

top decays before it hadronizes

$$\mu \frac{d}{d\mu} C_i(\mu) = \ldots$$

Determine the right scales

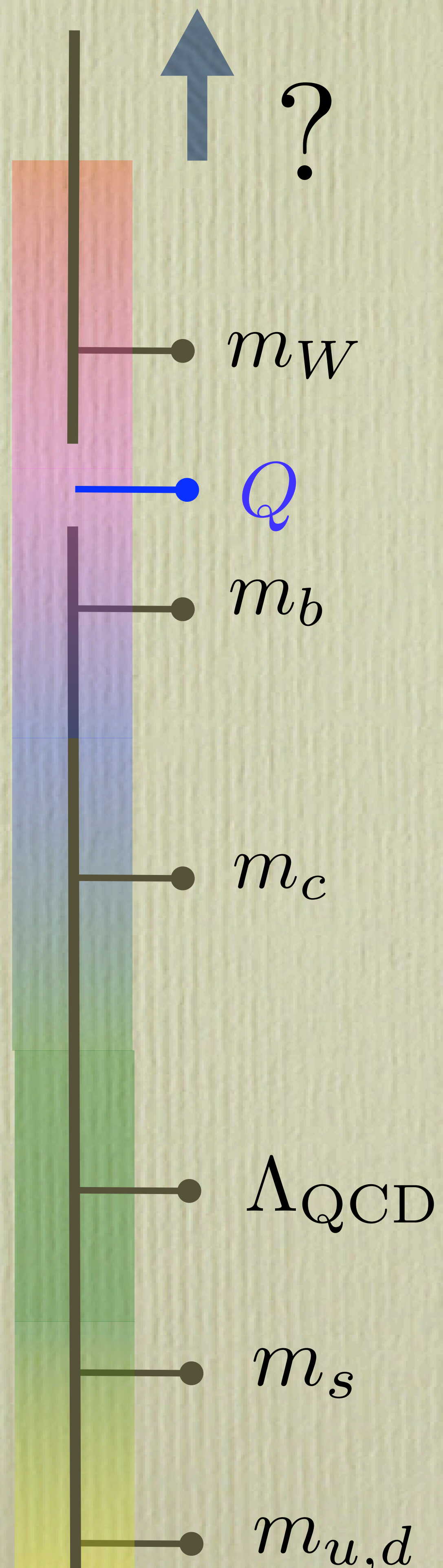
Hoang, Manohar, I.S., Teubner



vary μ

$\rightarrow$ m_t, y_t, Γ_t

Deep Inelastic Scattering on a proton



$$e^- p \to e^- X$$

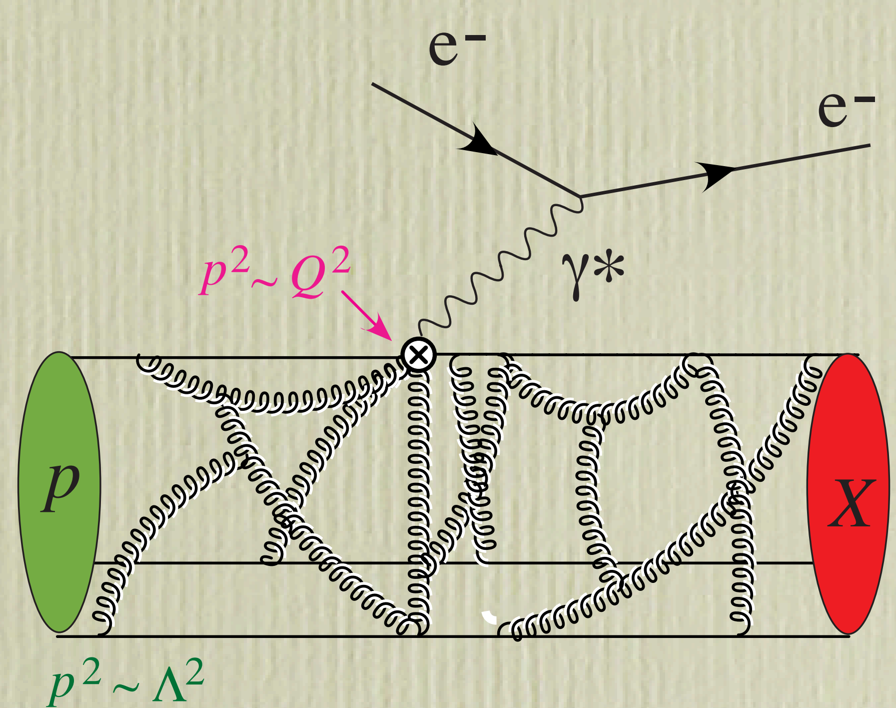
A factorization theorem

$$F_1(x,Q^2) = \frac{1}{x}\int_x^1 d\xi \; H(\xi/x, Q, \mu)\; f_{i/p}(\xi,\mu)$$

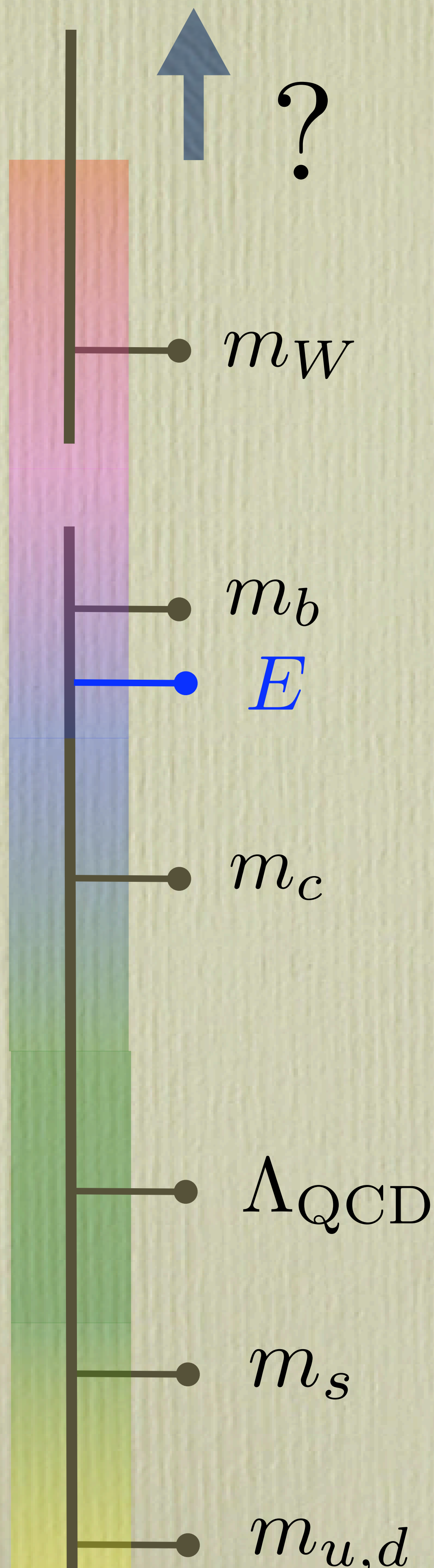
short distance process $p^2 \sim Q^2$

universal nonperturbative function

$p^2 \sim \Lambda^2_{\rm QCD}$

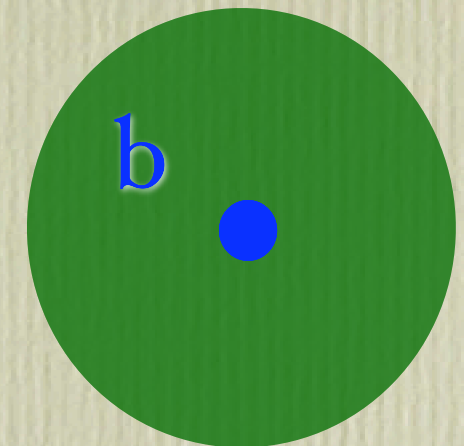


analogy: Bragg scattering of X-rays on a crystal, for this time scale the atoms are at rest



B-meson

$m_b \gg \Lambda_{\mathrm{QCD}}$



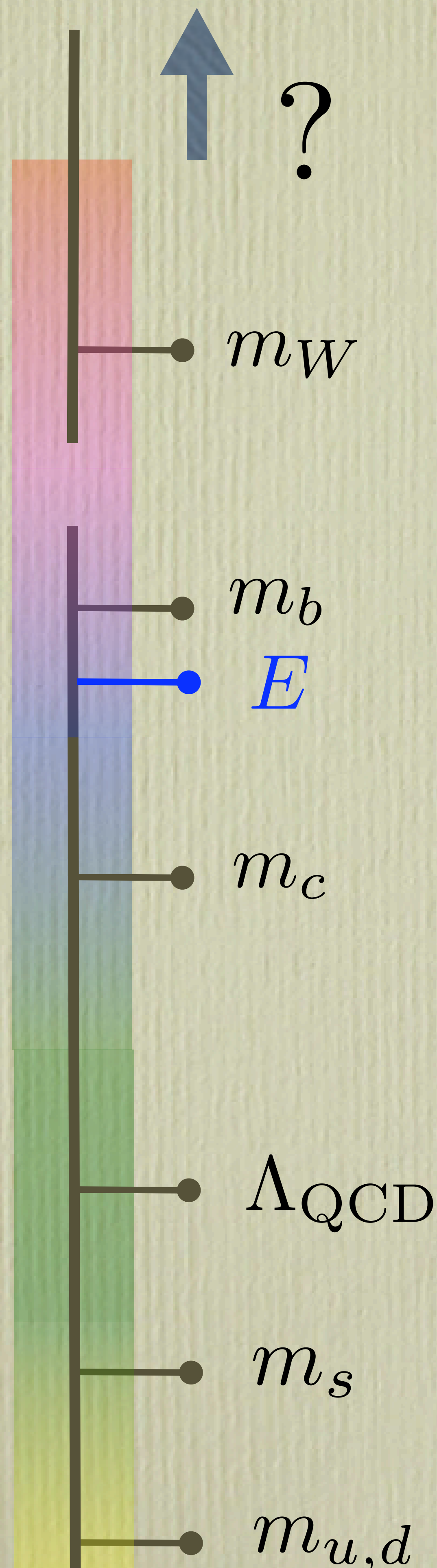
heavy quark symmetry

Isgur & Wise

Decay by weak interactions; long lived

Precision studies are sensitive to scales > $\mathrm{m_W}$

The B is heavy, so many of its decay products are energetic, E



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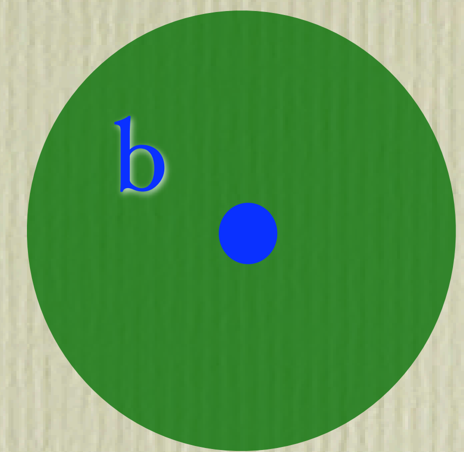


$m_b \gg \Lambda_{\rm QCD}$

heavy quark symmetry

Isgur & Wise

Decay by weak interactions; long lived

$B \to X_u \ell \bar{\nu}$ $\quad B \to D\pi$ $\quad B \to K^* \gamma$

$B \to \pi \ell \bar{\nu}$ $\quad B \to X_s \gamma$ $\quad B \to \rho\gamma$

$B \to D^* \eta'$ $\quad B \to \rho\rho$ $\quad B \to \pi\pi$ $\quad B \to K\pi$ $\quad B \to \gamma \ell \bar{\nu}$

Precision studies are sensitive to scales > m_W

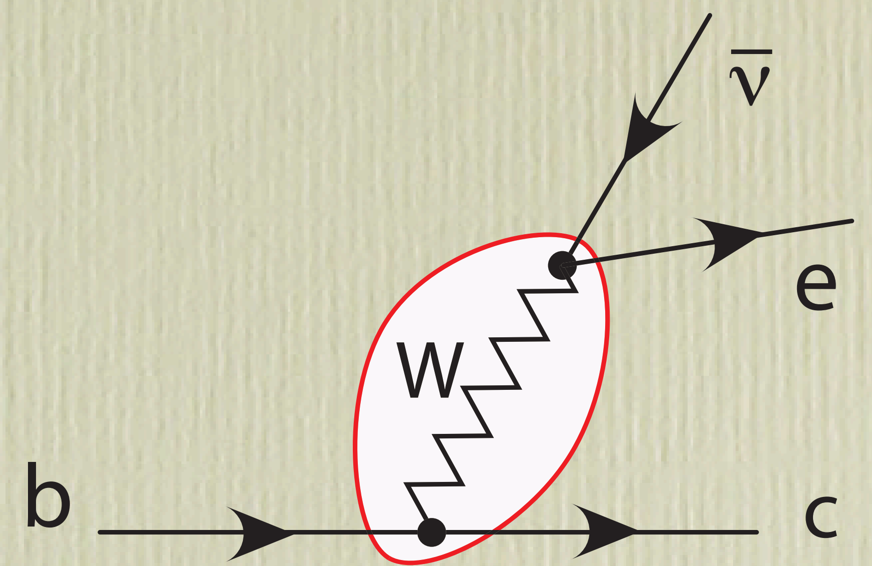
The B is heavy, so many of its decay products are energetic, E

eg. $B \to D\, e\, \bar{\nu}$, $\qquad M_W^2 \gg m_b^2 \gg \Lambda^2$

1) Short Distance

$\mu = m_W \simeq 80\,\mathrm{GeV}$

gluons perturbative



eg. $B \to D\, e\, \bar{\nu}$, $\qquad M_W^2 \gg m_b^2 \gg \Lambda^2$

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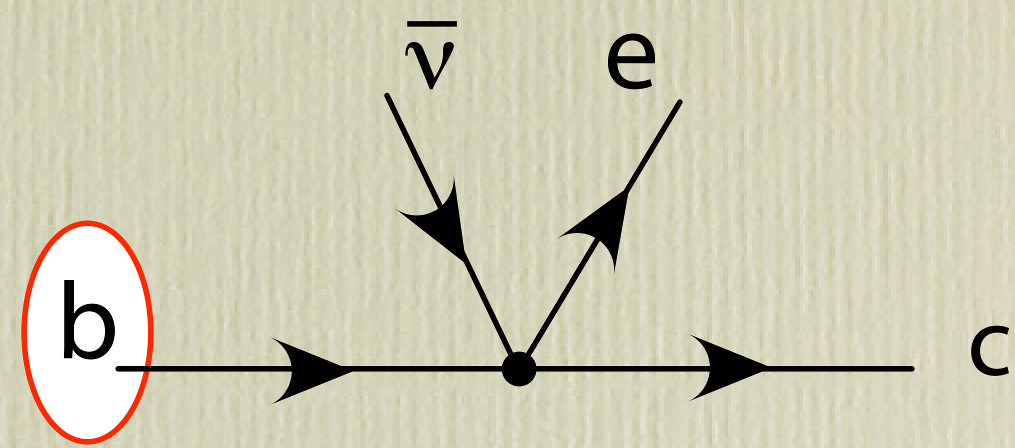
2) Intermediate Distance

$\mu = m_b \simeq 5\,\mathrm{GeV}$

gluons perturbative



eg. $B \to D\, e\, \bar{\nu}$, $\qquad M_W^2 \gg m_b^2 \gg \Lambda^2$

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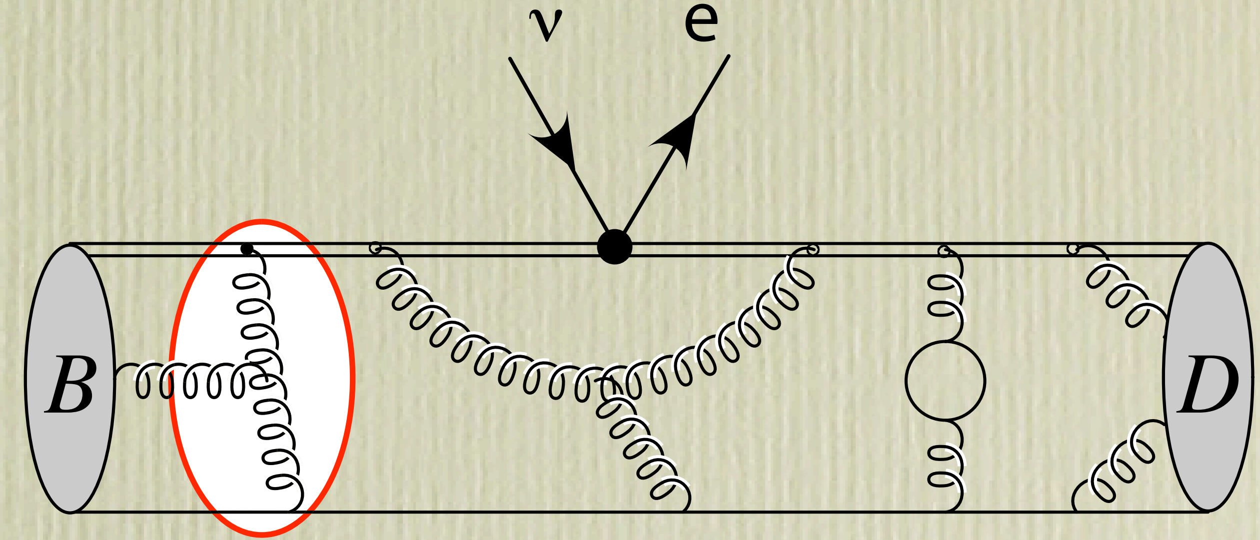
3) Long Distance

$\mu = \Lambda \simeq 0.5\,\mathrm{GeV}$

gluons nonperturbative



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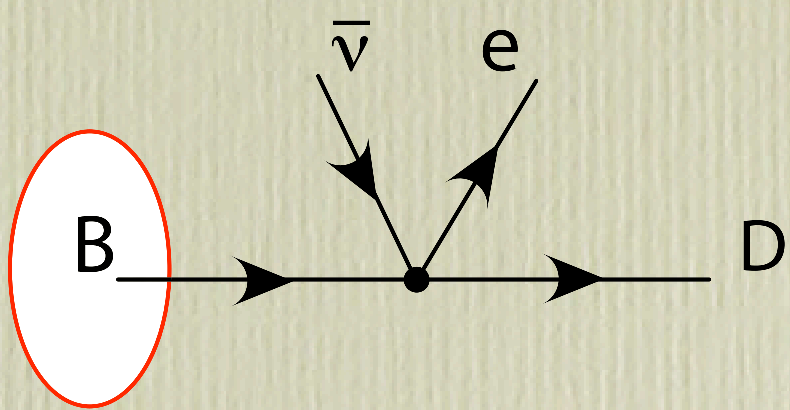
4) Very Long Distance

$\mu \ll \Lambda$

no gluons



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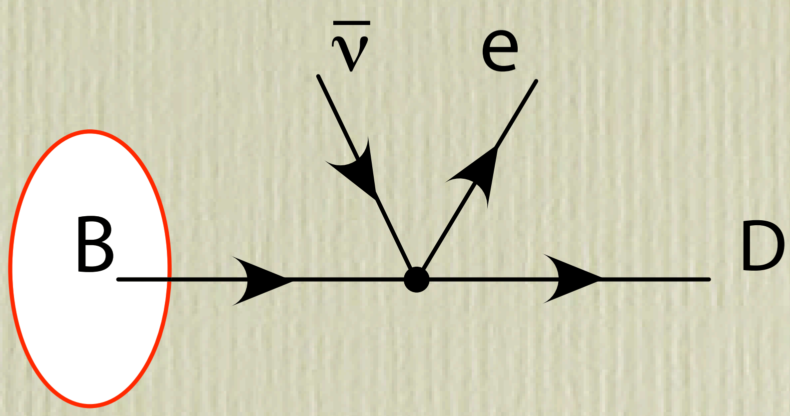


- Each of these pictures can be described by a field theory
- These theories can be matched together $H_1 \to H_2 \to H_3 \to H_4$
- At each μ we capture the most important physics

expansion parameters $\quad \dfrac{m_b^2}{m_W^2} \simeq \dfrac{1}{250}, \quad \alpha_s(m_b) \simeq 0.2, \quad \dfrac{\Lambda}{m_b} \simeq 0.1$

Soft - Collinear Effective Theory

Bauer, Pirjol, I.S.
Fleming, Luke

An effective field theory for energetic hadrons & jets

$$E \gg \Lambda_{\mathrm{QCD}}$$

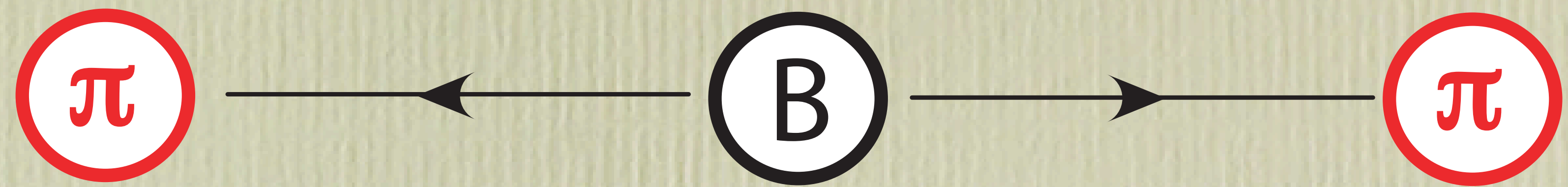
Analogy:

QED $\longleftrightarrow$ Quantum Mechanics (NRQED)

QCD $\longleftrightarrow$ SCET

Soft Collinear Effective Theory (SCET)

eg.

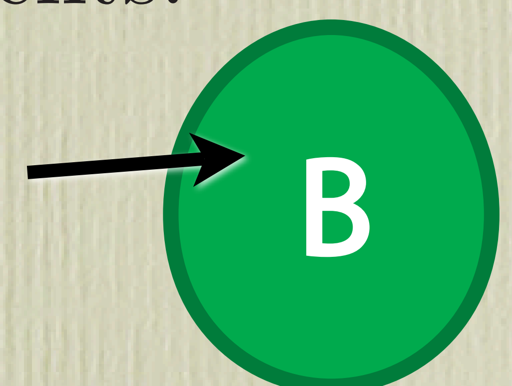


$$E_\pi = 2.6\,\mathrm{GeV} \gg \Lambda_{\mathrm{QCD}} \sim 0.3\,\mathrm{GeV}$$

$$m_B = 2E_\pi$$

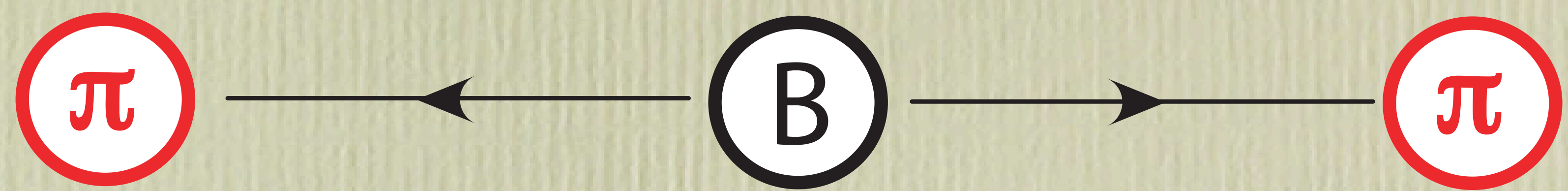
B has Soft constituents:

$p_s^\mu \sim \Lambda_{\mathrm{QCD}}$



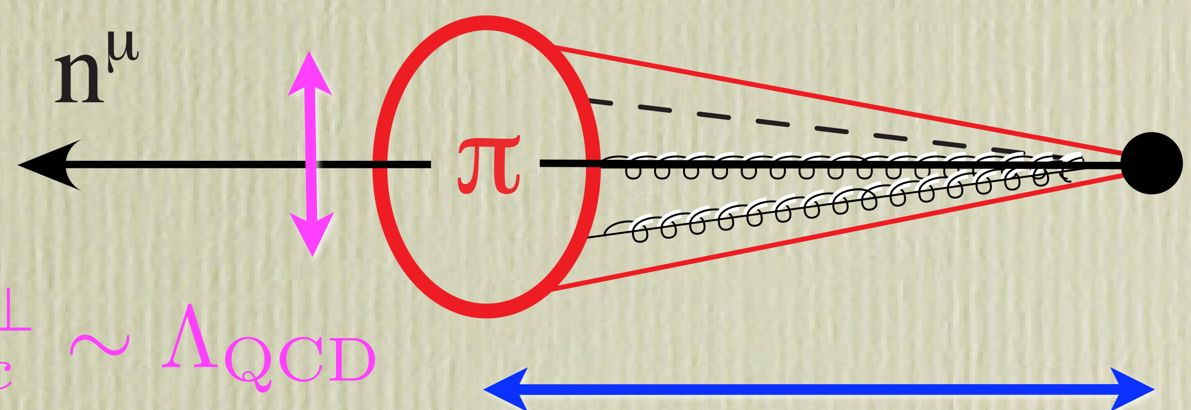
Soft Collinear Effective Theory (SCET)

eg.



$E_\pi = 2.6\,\mathrm{GeV} \gg \Lambda_{\mathrm{QCD}} \sim 0.3\,\mathrm{GeV}$ $\qquad$ $m_B = 2E_\pi$

π has Collinear constituents:

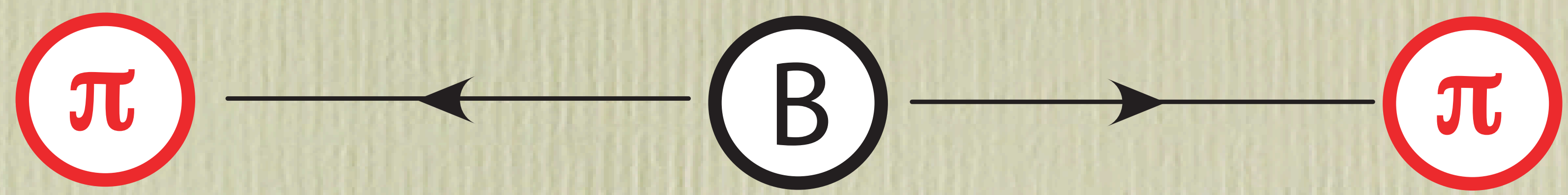


$p_c^{\perp} \sim \Lambda_{\mathrm{QCD}}$

$p_c^{\mathrm{longitudinal}} \sim m_B$

Soft Collinear Effective Theory (SCET)

eg.



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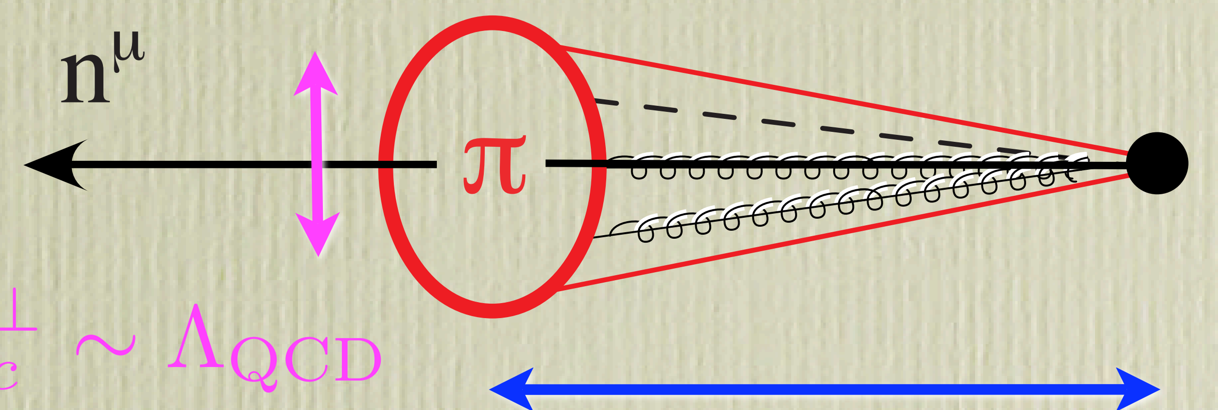

$p_c^\perp \sim \Lambda_{\mathrm{QCD}}$

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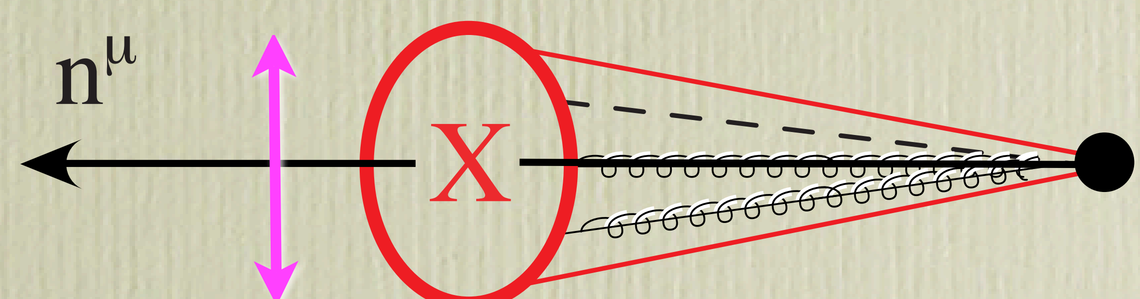
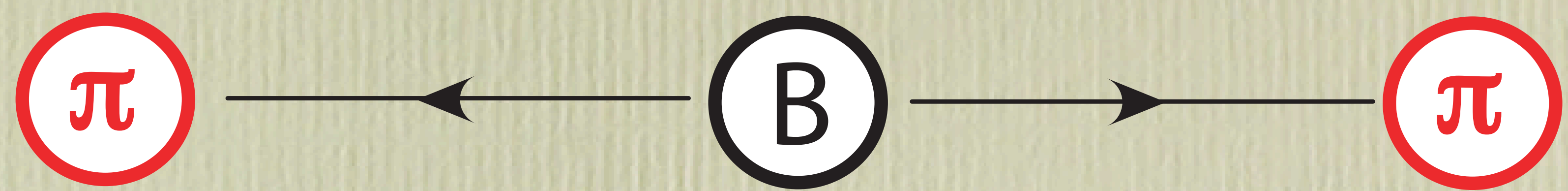


or replace π by a jet of many hadrons

$\Lambda_{\mathrm{QCD}} \ll p_c^\perp \ll m_B$

Soft Collinear Effective Theory (SCET)

eg.



$$E_\pi = 2.6\,\mathrm{GeV} \gg \Lambda_{\mathrm{QCD}} \sim 0.3\,\mathrm{GeV} \qquad m_B = 2E_\pi$$

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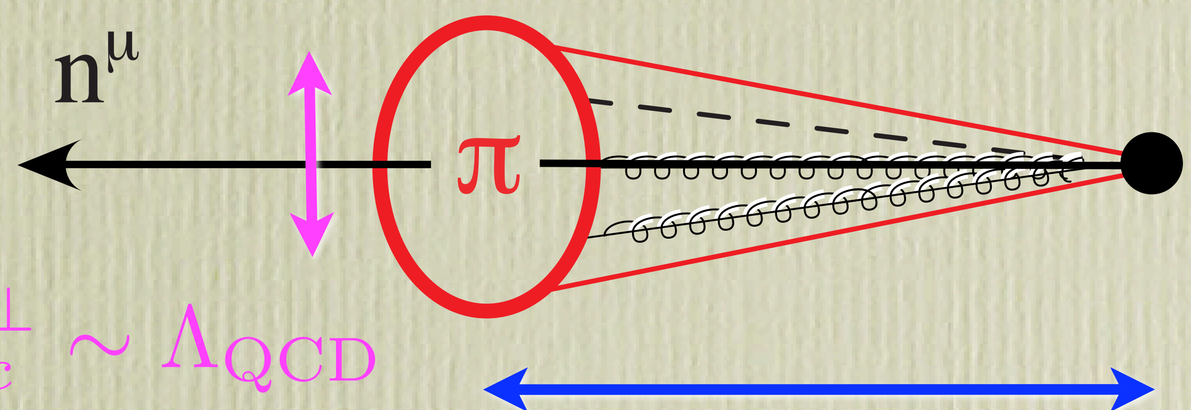
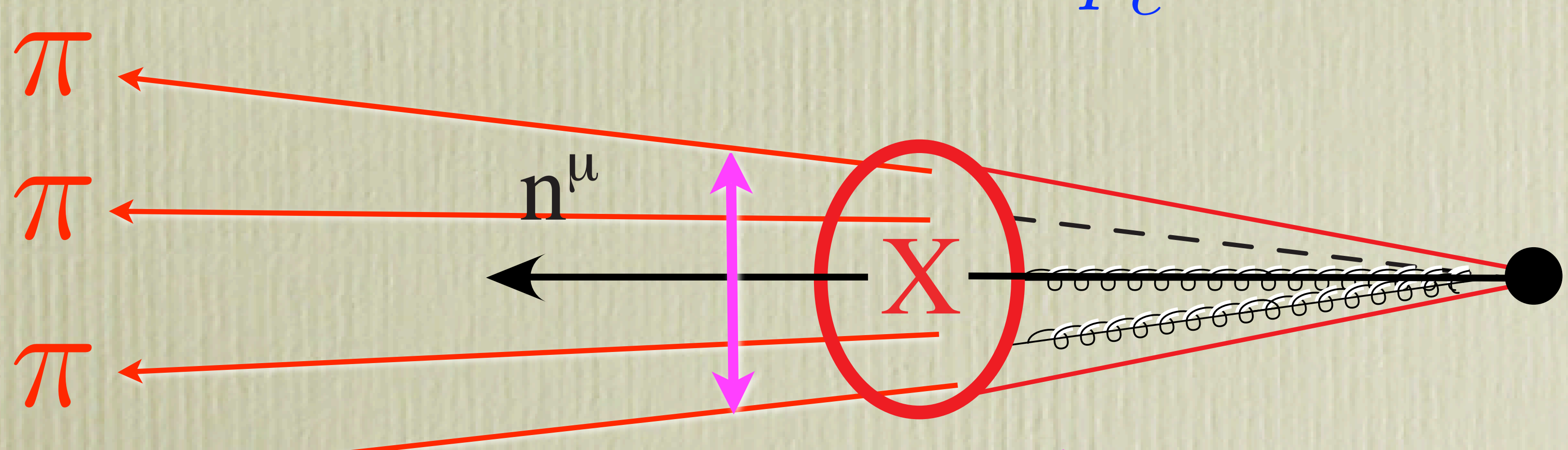


$p_c^\perp \sim \Lambda_{\mathrm{QCD}}$

$p_c^{\mathrm{longitudinal}} \sim m_B$

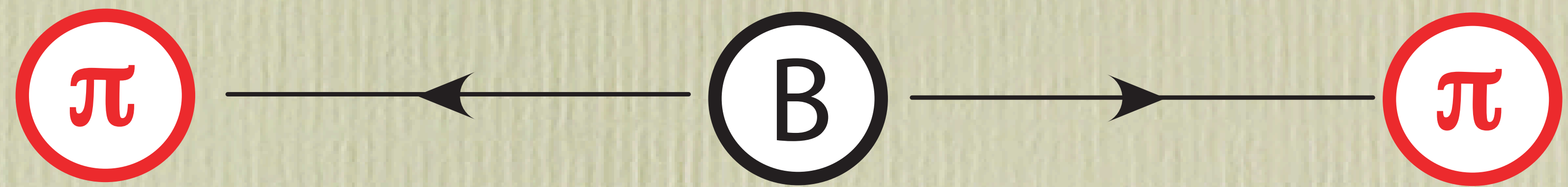


or replace π by a jet of many hadrons

$\Lambda_{\mathrm{QCD}} \ll p_c^\perp \ll m_B$

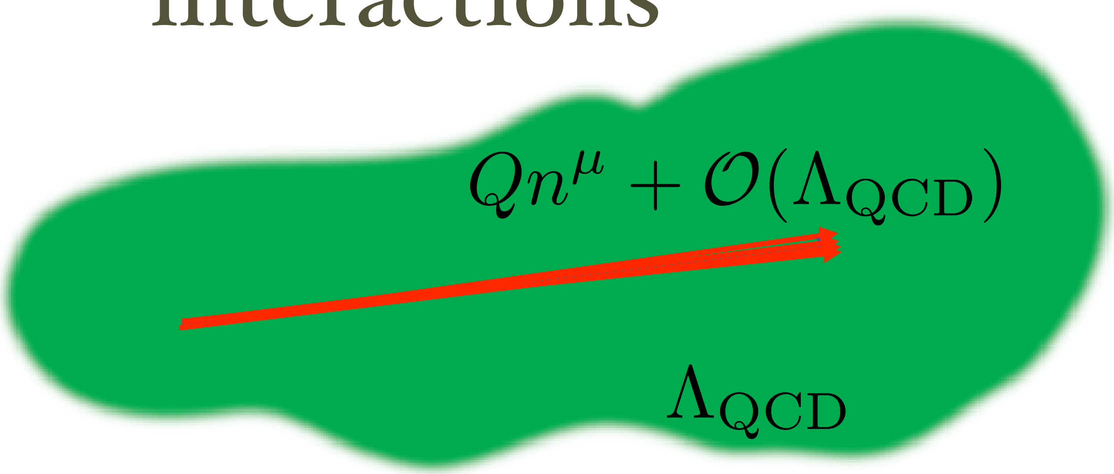
Soft Collinear Effective Theory (SCET)

eg.



$$E_\pi = 2.6\,\mathrm{GeV} \gg \Lambda_{\mathrm{QCD}} \sim 0.3\,\mathrm{GeV} \qquad m_B = 2E_\pi$$

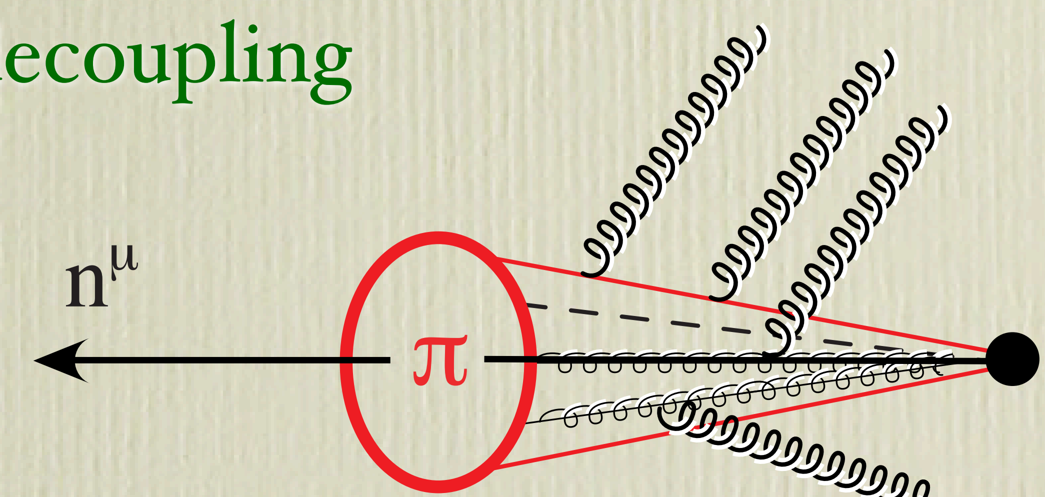
A field theory for
Soft & Collinear
interactions



organizes the interactions in a series expansion in $\dfrac{\Lambda_{\mathrm{QCD}}}{E}$

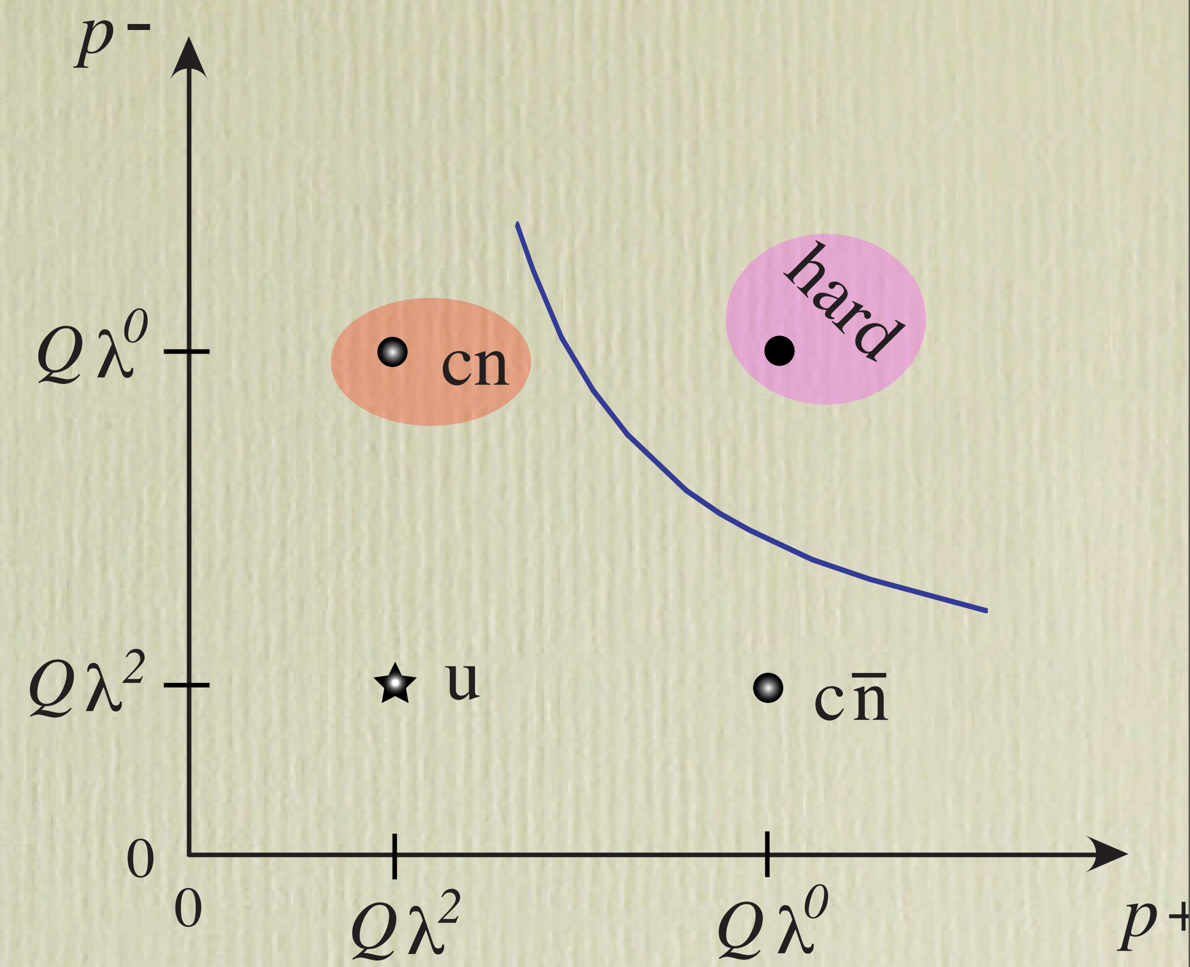
(analog of the non-relativistic expansion in Q.M.)

decoupling



SCET is a field theory which:

- explains how these degrees of freedom communicate with each other, and with hard interactions



$$F_1(x, Q^2) = \frac{1}{x} \int_x^1 d\xi \; H(\xi/x, Q, \mu) \; f_{i/p}(\xi, \mu)$$

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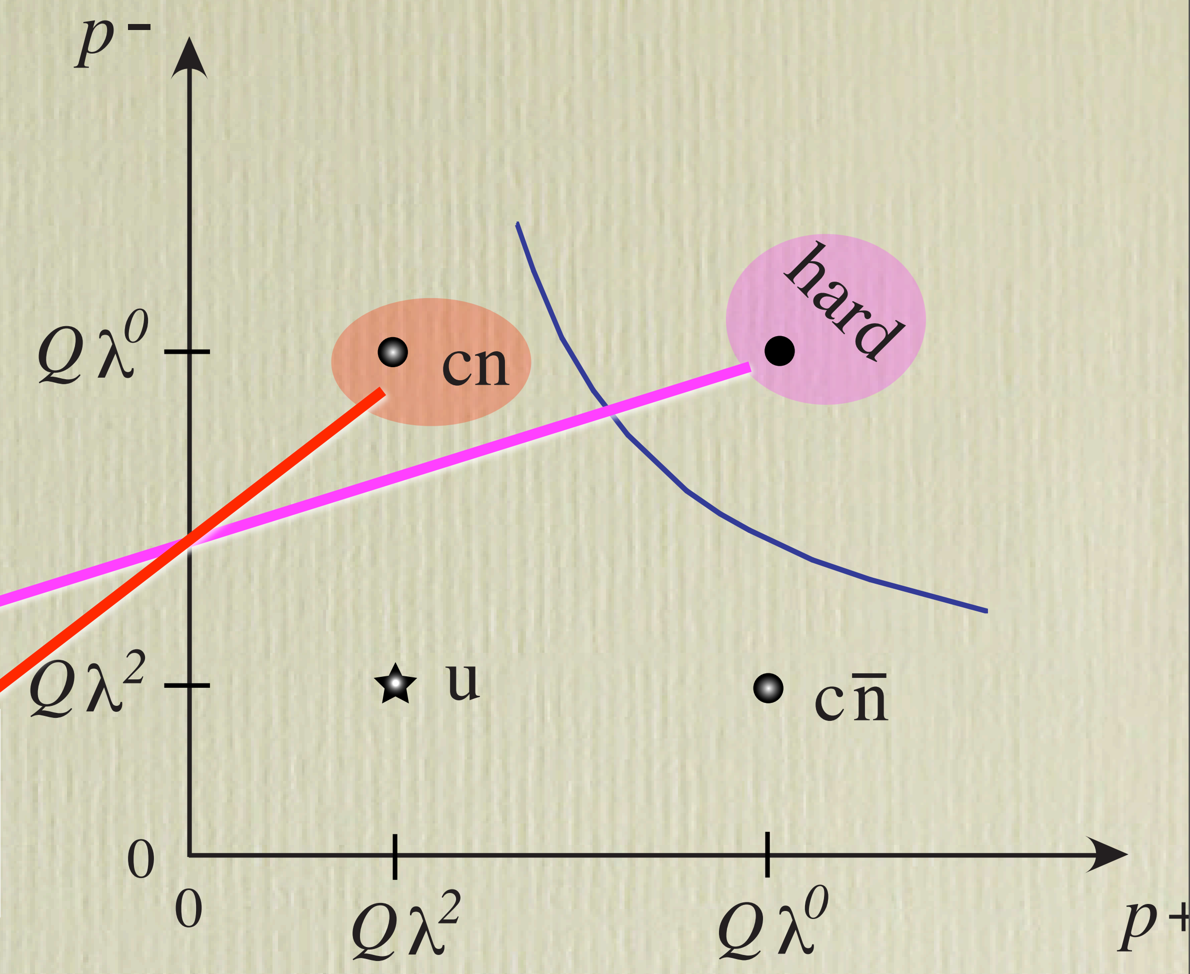

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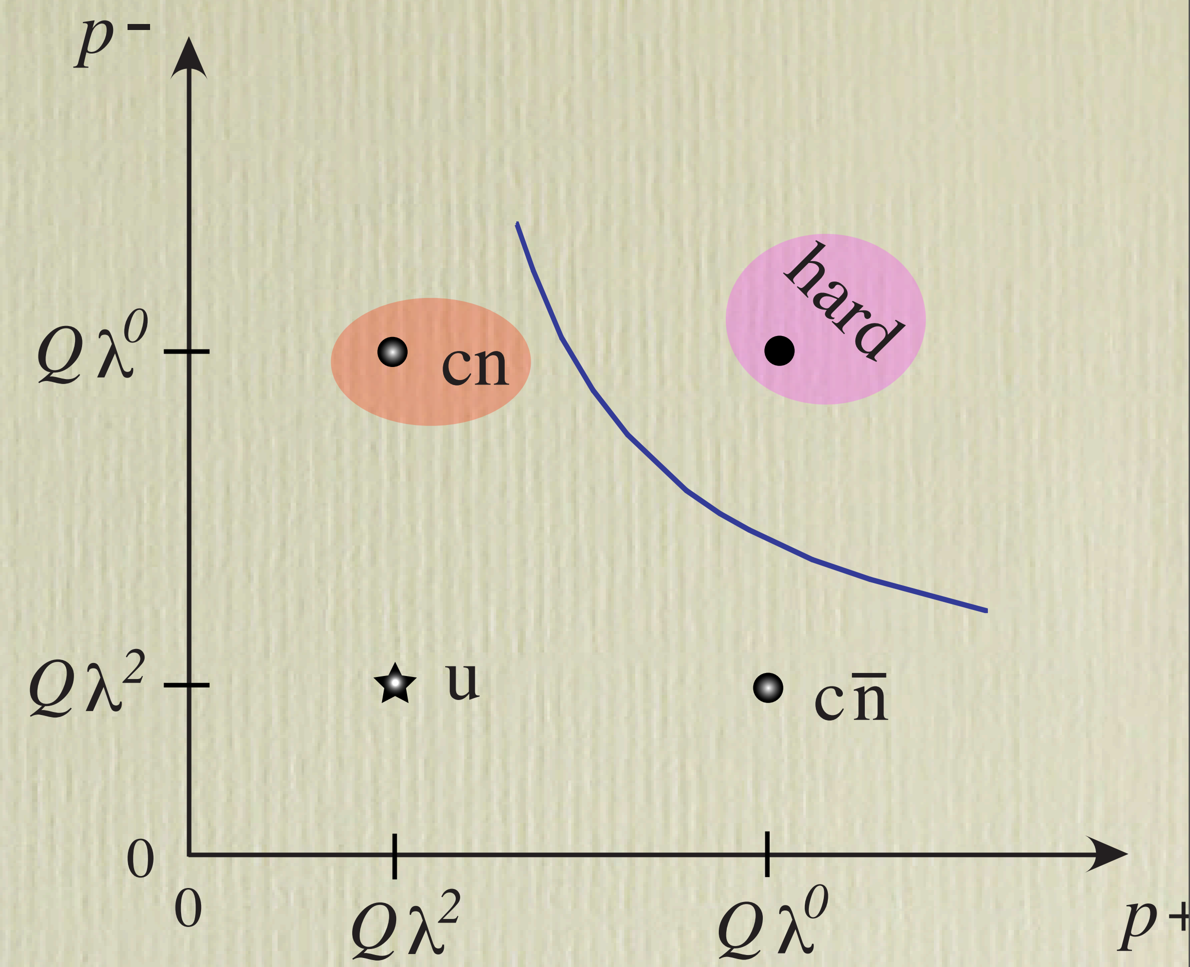

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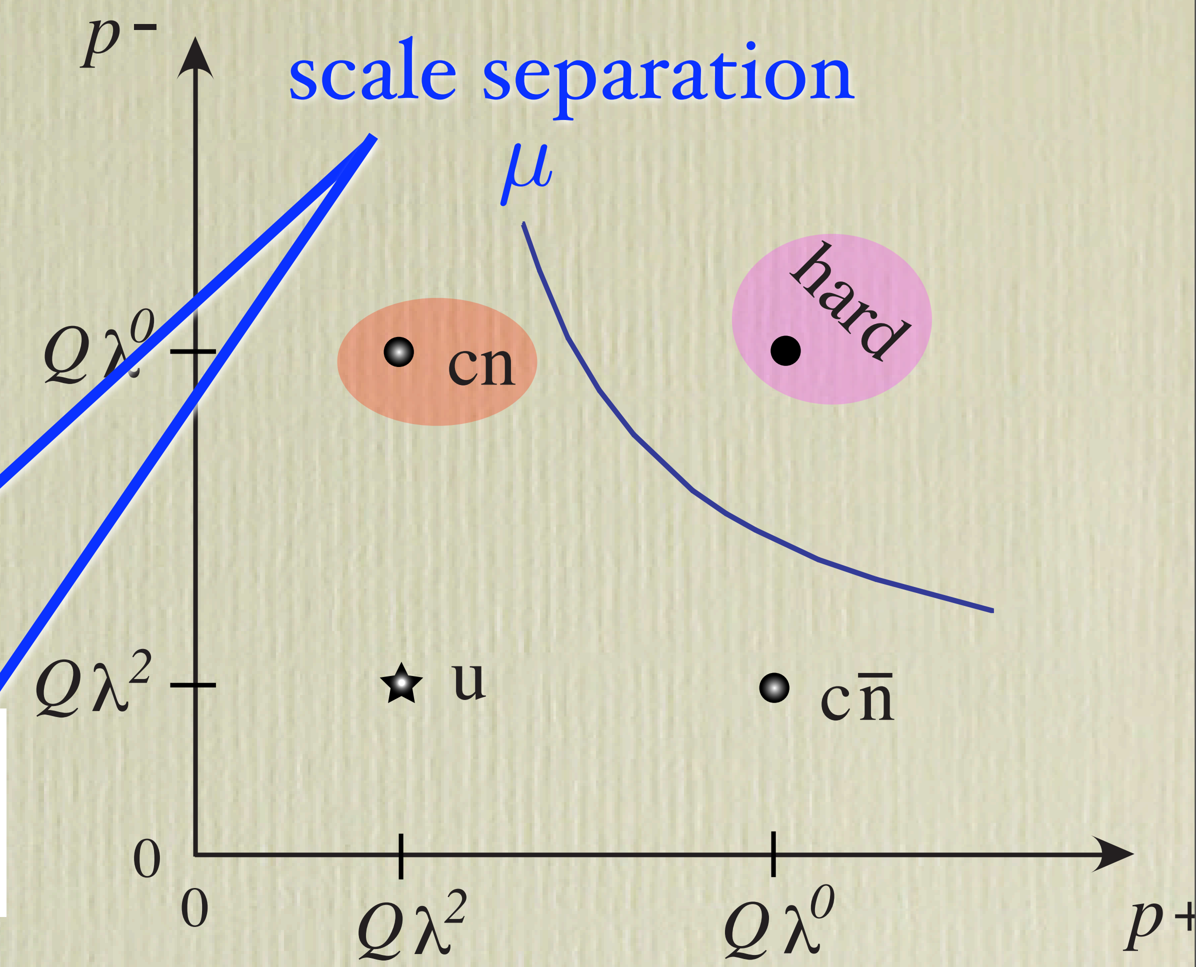

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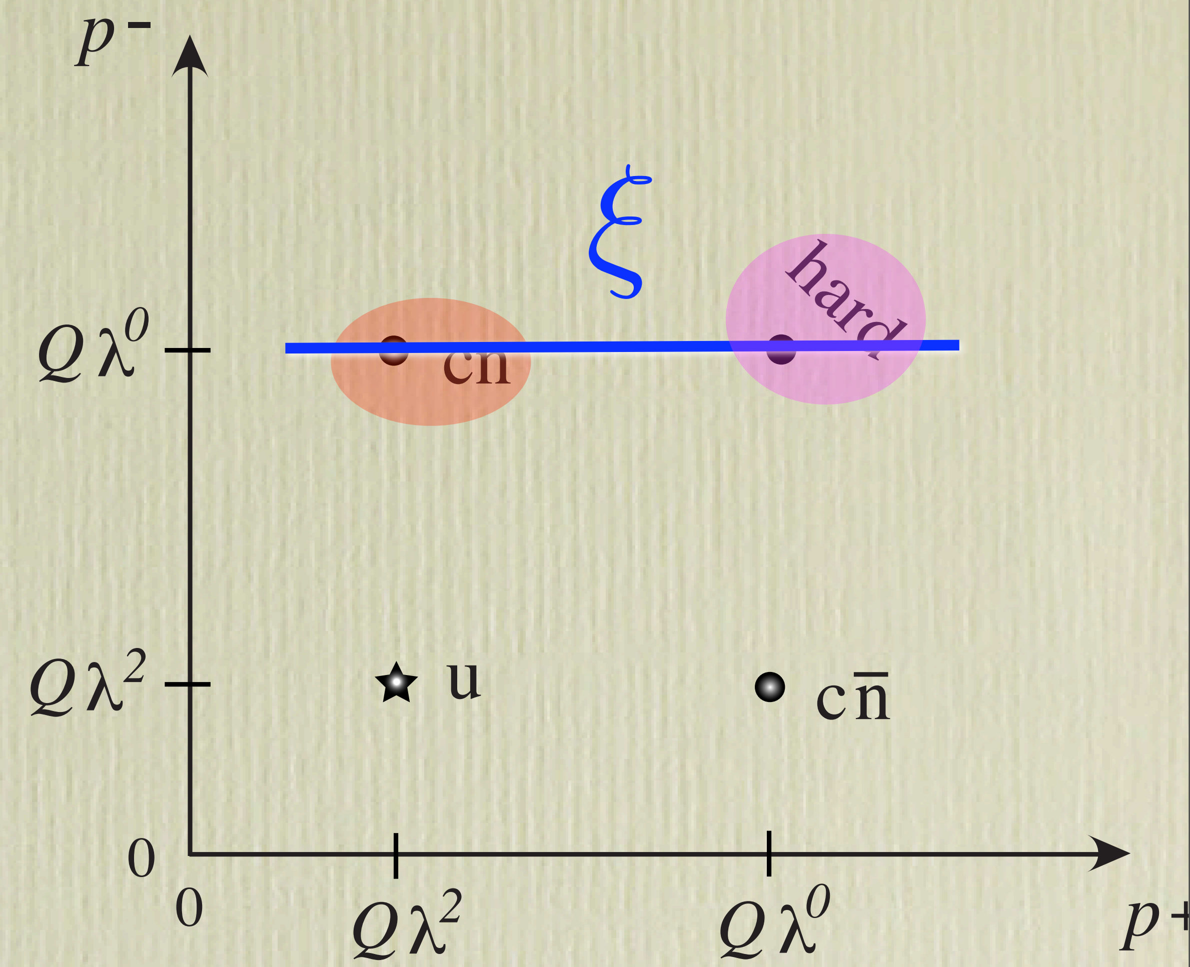

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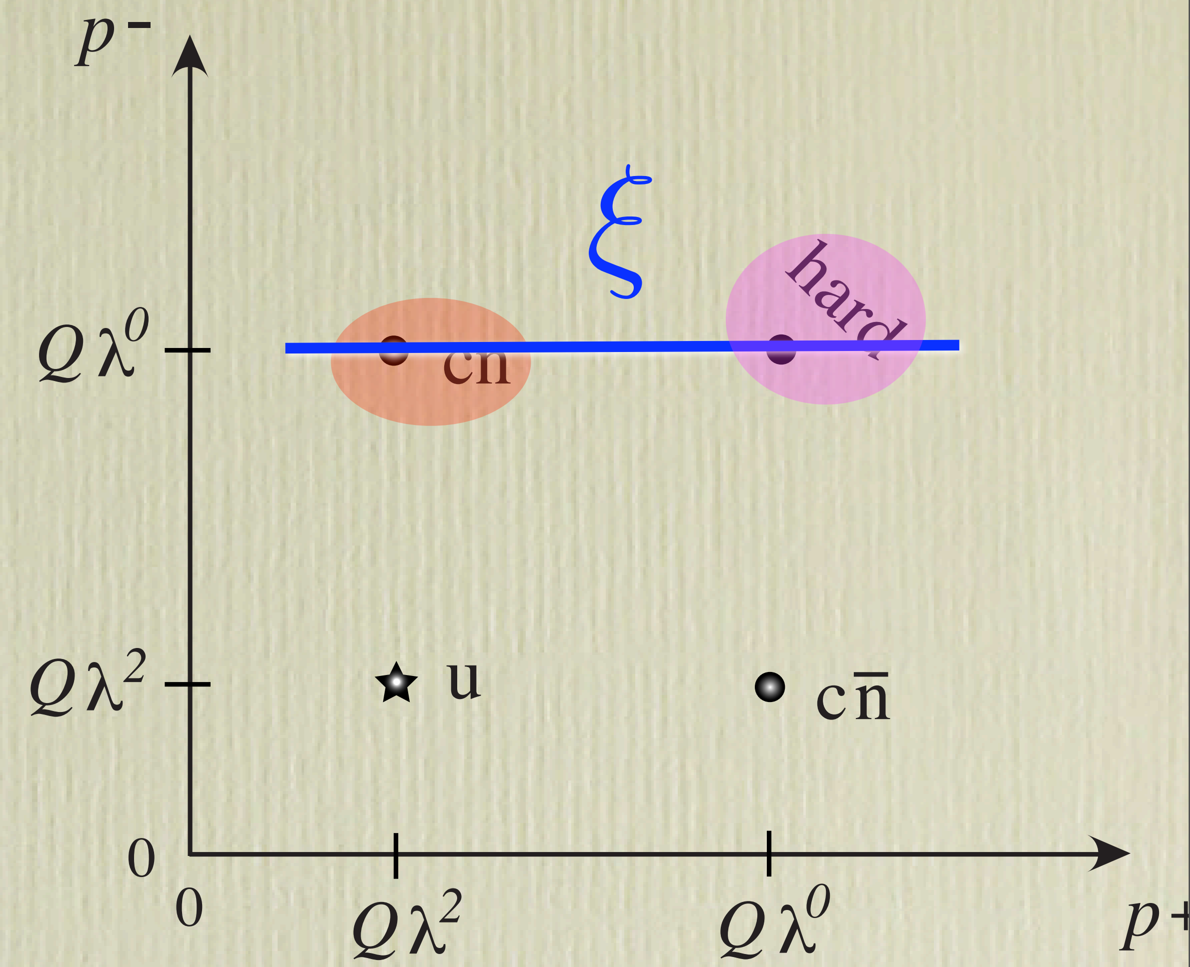


communicate by integrals

$$F_1(x,Q^2) = \frac{1}{x}\int_x^1 d\xi\ H(\xi/x,Q,\mu)\ f_{i/p}(\xi,\mu)$$

- provides a simple operator language to derive factorization theorems in fairly general circumstances

  eg. unifies the treatment of factorization for exclusive and inclusive QCD processes

- new symmetry constraints

How is SCET used?

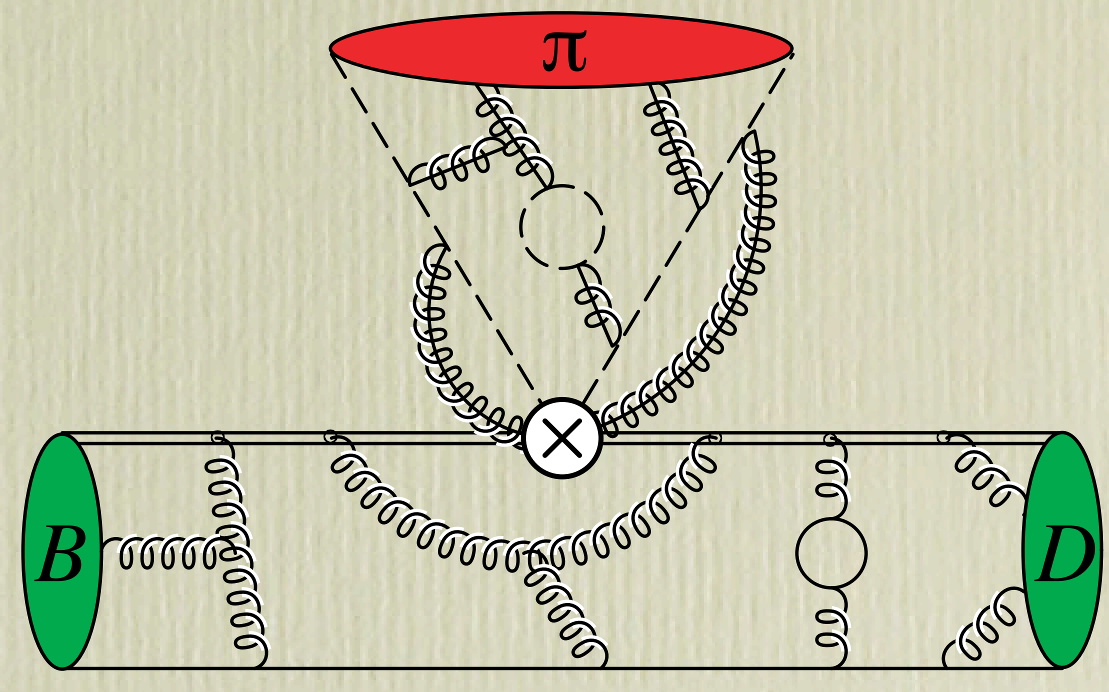
- cleanly separate short and long distance effects in QCD
  - ➡ derive new factorization theorems
  - ➡ find universal hadronic functions, exploit symmetries & relate different processes
- model independent, systematic expansion
  - ➡ study power corrections
- keep track of μ dependence
  - ➡ sum logarithms, reduce uncertainties

Factorization Example

$\bar{B}^0 \to D^+\pi^- \;\; , \;\; B^- \to D^0\pi^-$

B, D are soft , π collinear

$$\langle D\pi | H_{\rm weak} | B\rangle = N\xi(v\cdot v') \int_0^1 dx \; T(x,\mu) \; \phi_\pi(x,\mu)$$



SCET gives Universal functions
(analog of wavefunctions in Q.M.)

$\mathcal{L}_{\rm SCET} = \mathcal{L}_s^{(0)} + \mathcal{L}_c^{(0)}$ Factorization if $H_{\rm weak} = O_s \times O_c$

$\langle D^{(*)} | O_s | B\rangle = \xi(v\cdot v')$

$\langle \pi | O_c(x) | 0\rangle = f_\pi \phi_\pi(x)$

Calculate T, $\alpha_s(Q)$

$Q = E_\pi, m_b, m_c$

corrections will be $\Lambda/m_c \sim 30\%$

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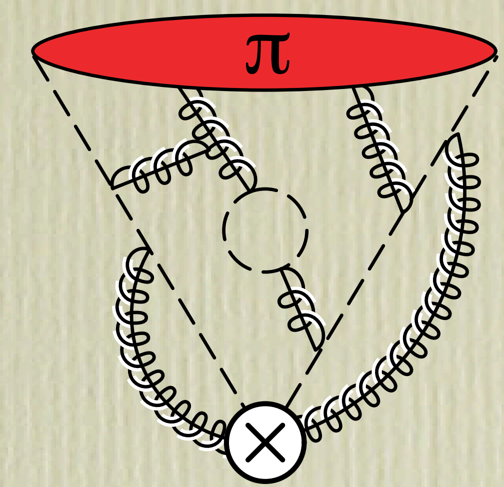

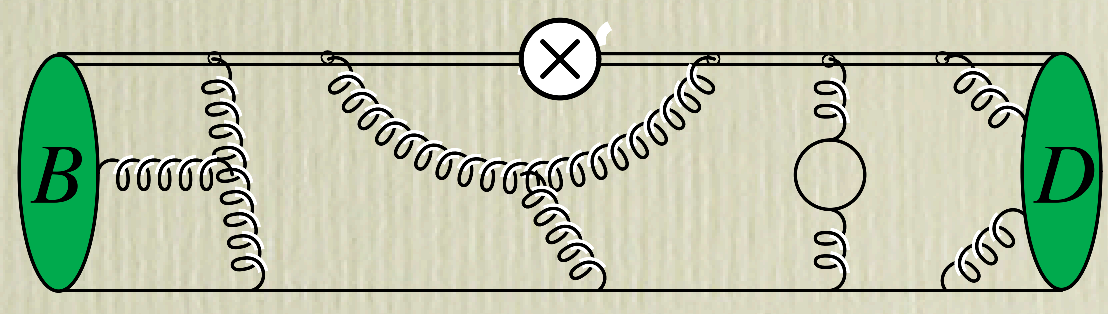

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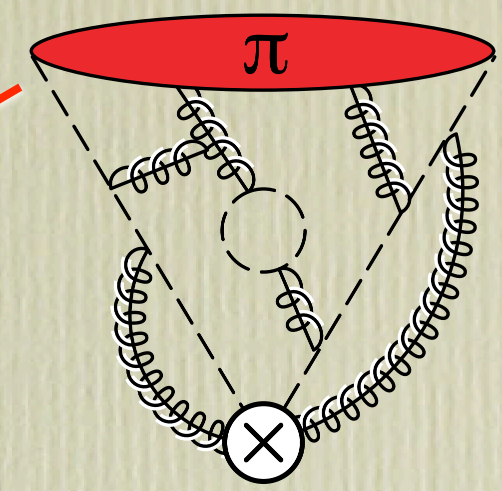

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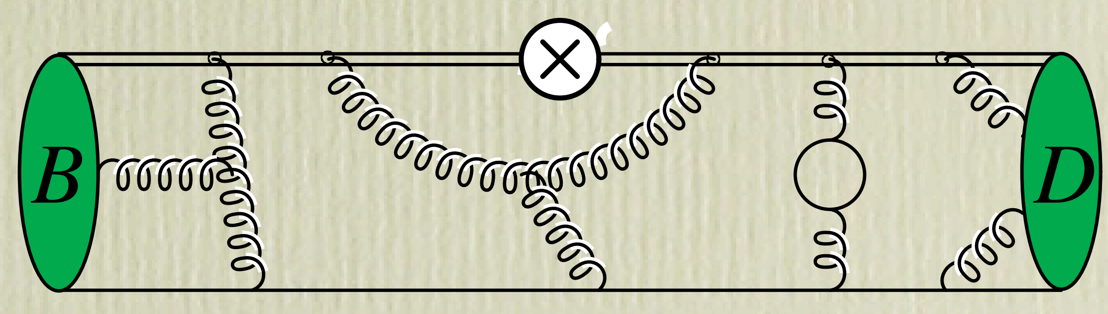

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Systematic Corrections

- Soft & Collinear start to Interact

Chay, Kim

Beneki, Chapovsky, Diehl, Feldmann

Bauer, Pirjol, I.S.

- Quark Mass Effects

Ligeti, Leibovich, Wise

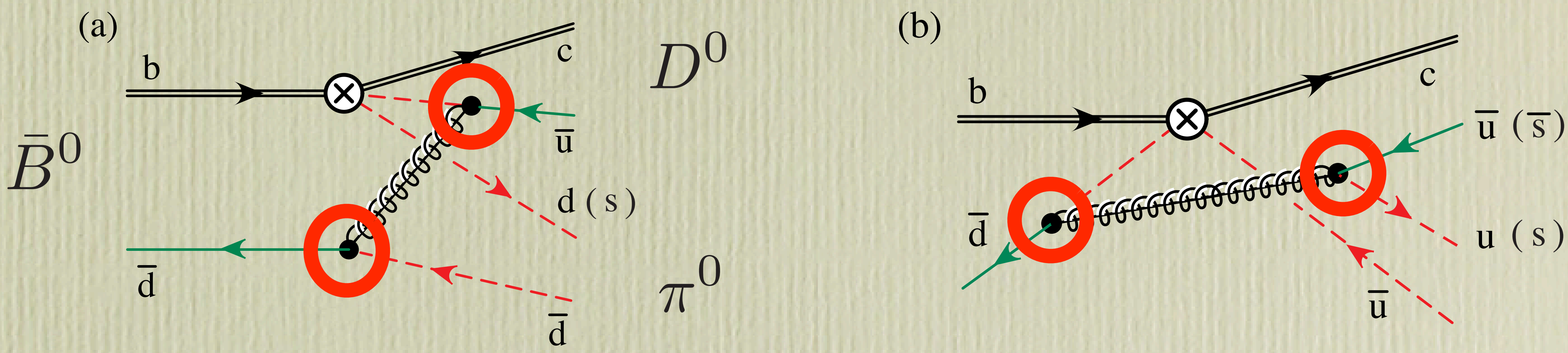
- At higher orders the description of the modes remains valid. However, we typically have more integrations and our results depend on new functions.

Color Suppressed Decays

Mantry, Pirjol, I.S.

$\bar{B}^0 \to D^0 \pi^0$ Intractable without SCET

subleading interaction



$$A_{00}^{D^{(*)}} = N_0^{(*)} \int dx\, dz\, dk_1^+ dk_2^+ \underbrace{T^{(i)}(z)}_{Q^2} \gg \underbrace{J^{(i)}(z,x,k_1^+,k_2^+)}_{Q\Lambda} \gg \underbrace{S^{(i)}(k_1^+,k_2^+)\, \phi_M(x)}_{\Lambda^2}$$

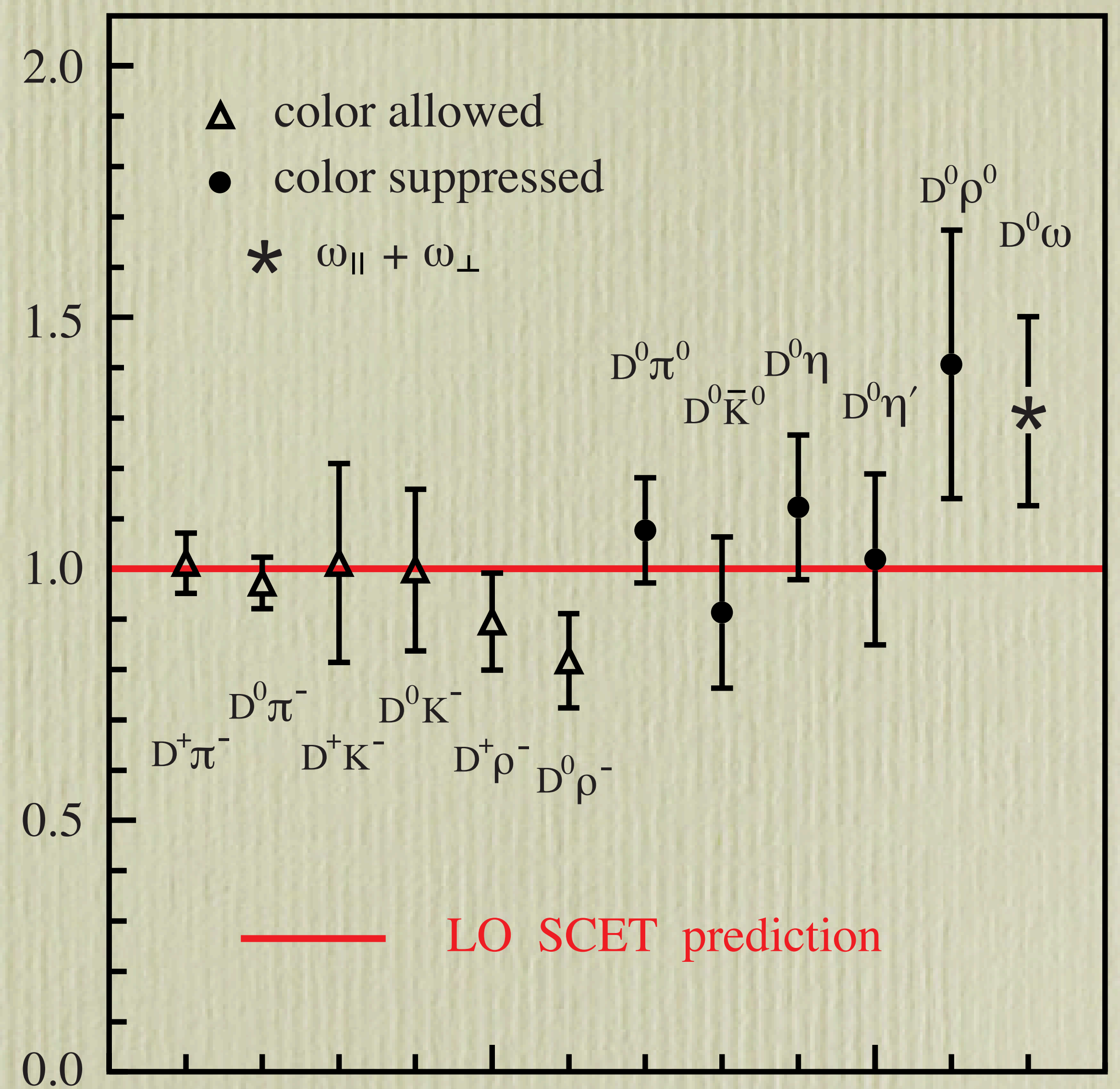
$Q = m_b, E_\pi, m_c$

prove S is same for D and D*

Comparison to Data

(Cleo, Belle, Babar)

$$\left|\frac{A(D^*M)}{A(D\,M)}\right|$$



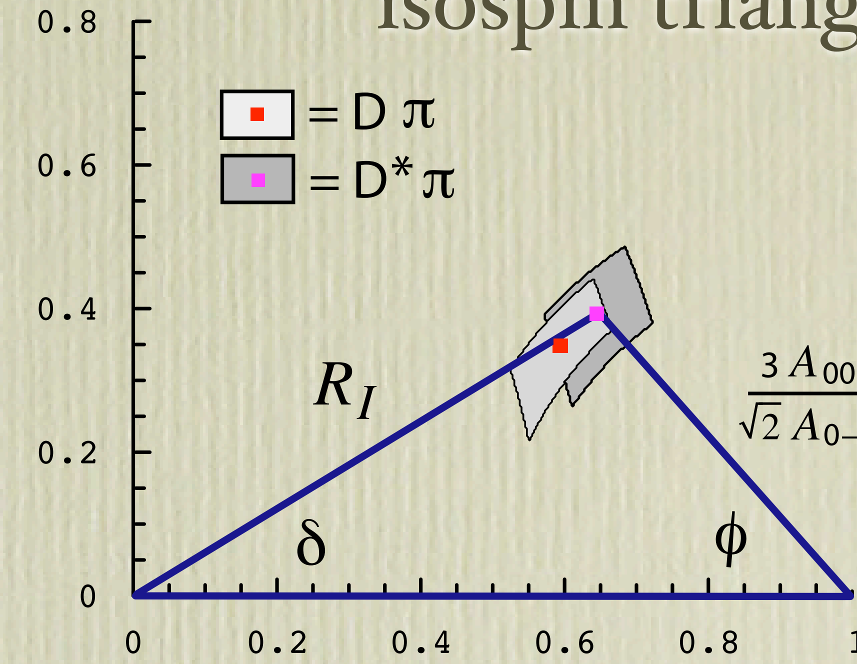
Extension to isosinglets:

Blechman, Mantry, I.S.

Extension to baryons (Λ_b) :

Leibovich, Ligeti, I.S., Wise

isospin triangle

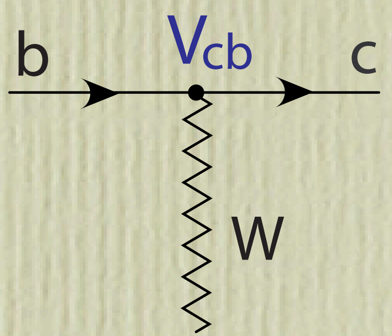


$$\delta(D\pi) = 30.4 \pm 4.8^\circ$$
$$\delta(D^*\pi) = 31.0 \pm 5.0^\circ$$

Not yet tested:

- $Br(D^*\rho^0_\parallel) \gg Br(D^*\rho^0_\perp)$, $Br(D^{*0}K^{*0}_\parallel) \sim Br(D^{*0}K^{*0}_\perp)$
- equal ratios $D^{(*)}K^*$, $D_s^{(*)}K$, $D_s^{(*)}K^*$; triangles for $D^{(*)}\rho$, $D^{(*)}K$

$B \to \pi\pi$ Decays & Weak Interactions



CKM Matrix

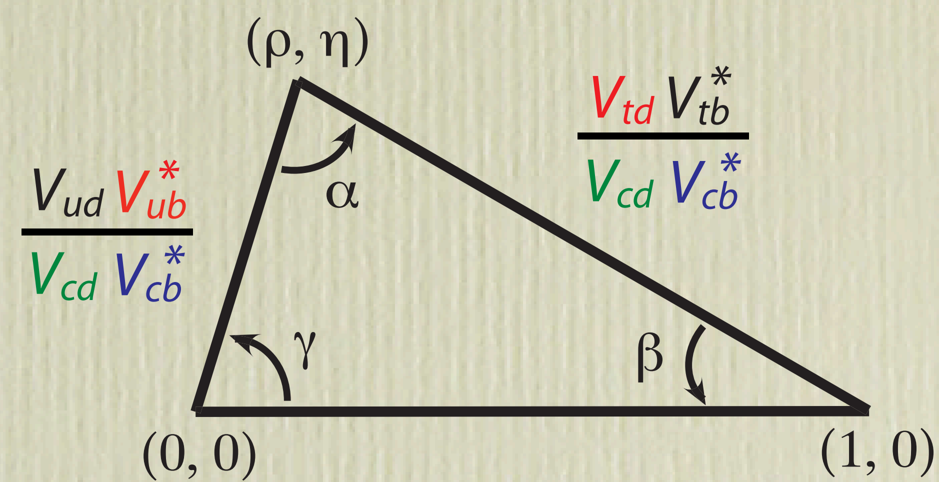
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Violate

C: exchange of particles & antiparticles

P: parity $\vec{x} \to -\vec{x}$

CP:



Can use CP-violating observables in $B \to \pi\pi$ to measure γ ,

but need to control QCD interactions

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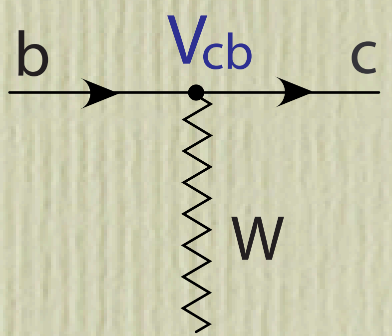

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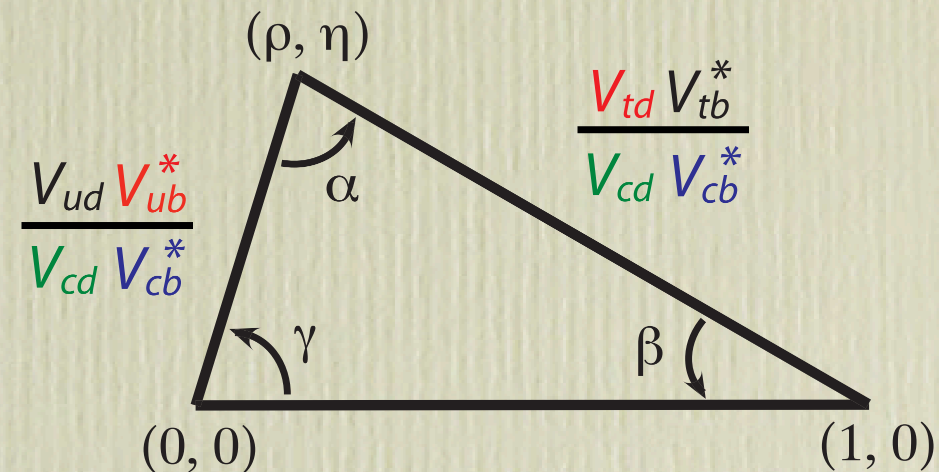

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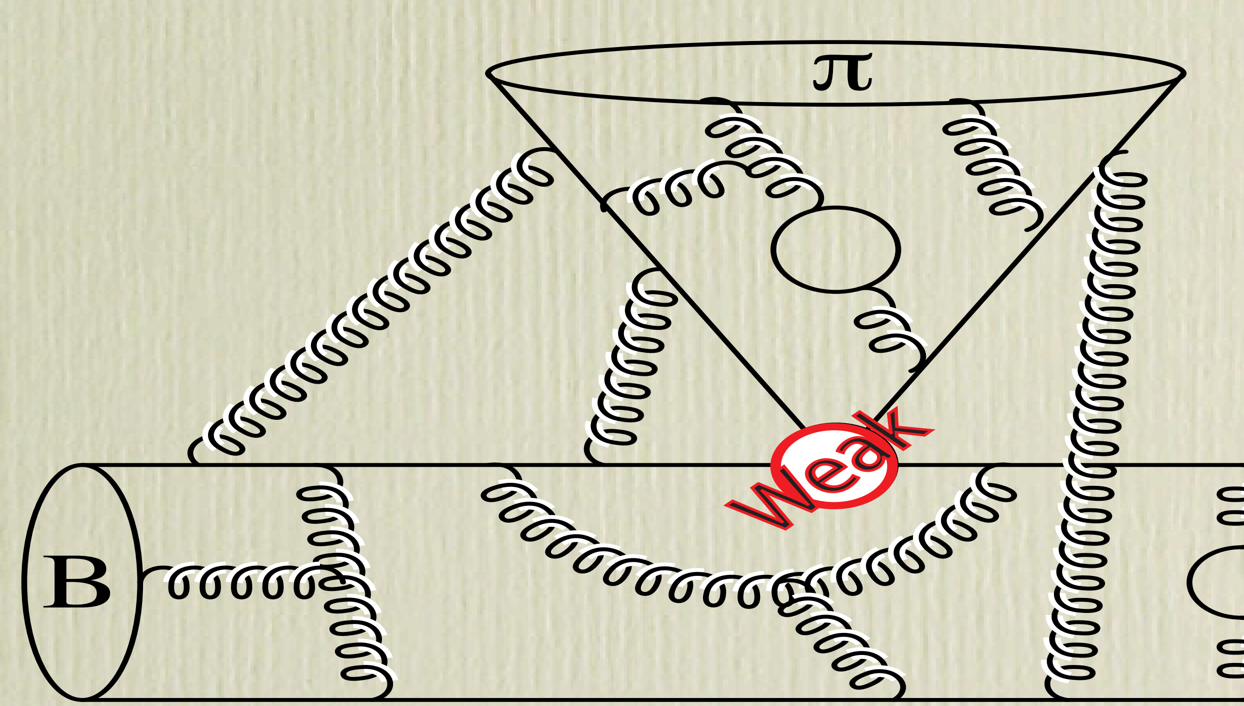
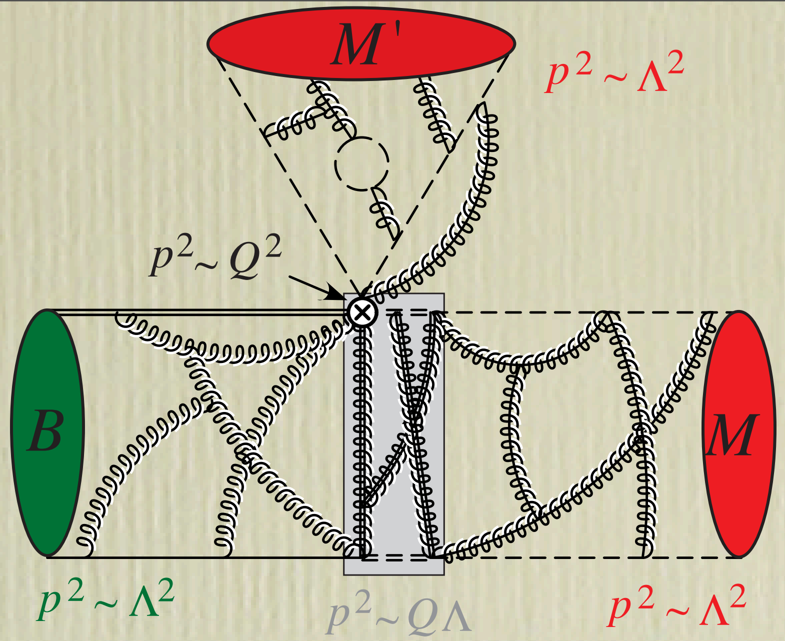


Factorization with SCET

Bauer, Pirjol, Rothstein, I.S.;

Beneke, Buchalla, Neubert, Sachrajda



Resolution $\mu = m_b$

Nonleptonic $\quad B \to M_1 M_2 \quad (\sim 120 \text{ channels})$

$$A(B \to M_1 M_2) = A^{c\bar{c}} + N \left\{ f_{M_2} \zeta^{BM_1} \int du\, T_{2\zeta}(u) \phi^{M_2}(u) + f_{M_2} \int du dz\, T_{2J}(u,z) \zeta_J^{BM_1}(z) \phi^{M_2}(u) + (1 \leftrightarrow 2) \right\}$$

Form Factors

$$f(E) = \int dz\, T(z,E)\, \zeta_J^{BM}(z,E)$$

$$+ C(E)\, \zeta^{BM}(E)$$

universality at $E\Lambda$

$B \to \pi \ell \bar{\nu}$,
$B \to K^* \ell^+ \ell^-$,
$B \to \rho\gamma$, ...

Resolution $\mu = \sqrt{E\Lambda}$, expansion in $\alpha_s(\sqrt{E\Lambda})$

$$\zeta_J^{BM}(z) = f_M f_B \int_0^1 dx \int_0^\infty dk^+ J(z,x,k^+,E) \phi_M(x) \phi_B(k^+)$$

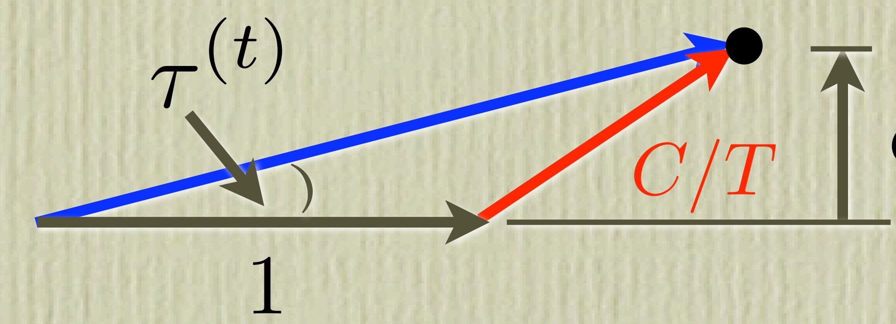
ζ^{BM} left as a form factor

$$\boxed{B \to \pi\pi}$$

$$\bar{B}^0 \to \pi^+\pi^-, \quad B^- \to \pi^0\pi^-, \ \bar{B}^0 \to \pi^0\pi^0,$$
$$B^0 \to \pi^+\pi^-, \quad B^0 \to \pi^0\pi^0$$

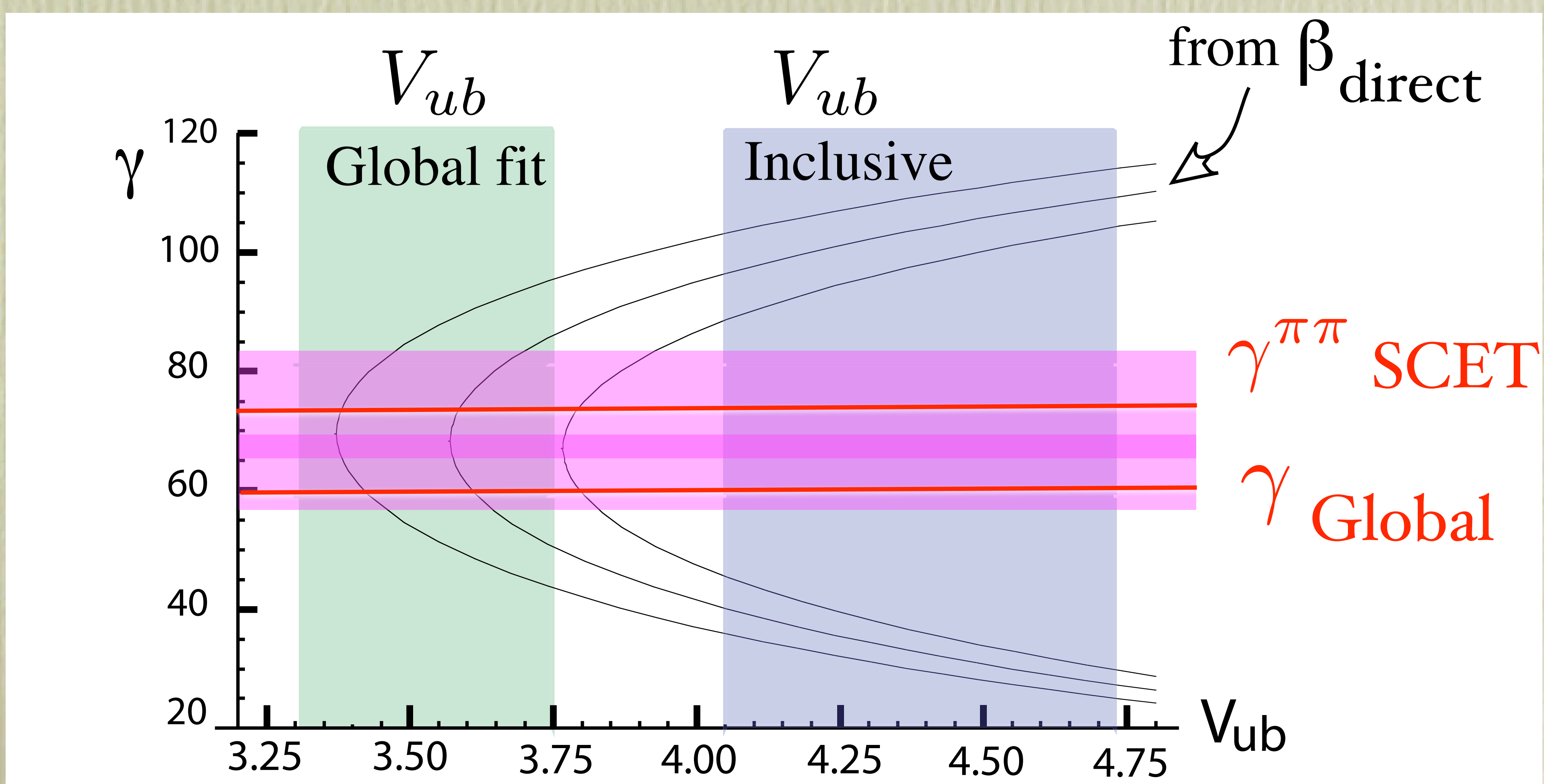
(Belle & Babar)

- $C_{\pi^0\pi^0} = -0.28 \pm 0.39$, uncertainty precludes measuring γ without input from QCD
- Factorization predicts a small relative phase for two amplitudes

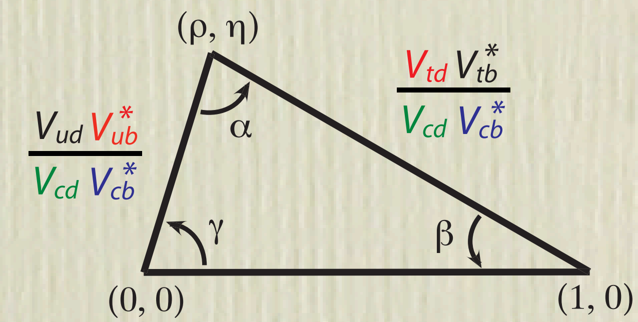


$$\epsilon \sim 0, \ \tau^{(t)} \sim 0 \qquad \frac{\Lambda_{\rm QCD}}{E_\pi} \ll 1$$

Bauer, Rothstein, I.S.



expt. errors dominate



B-decays with one Jet

$\boxed{B \to X_s \gamma}$

$$Br(B \to X_s\gamma)^{\text{expt}}_{E_\gamma > 1.6\,\text{GeV}} = (3.55 \pm 0.26) \times 10^{-4}$$

$$Br(B \to X_s\gamma)^{\text{theory}}_{E_\gamma > 1.6\,\text{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$$ Misiak et al.

-0.17 Becher, Neubert

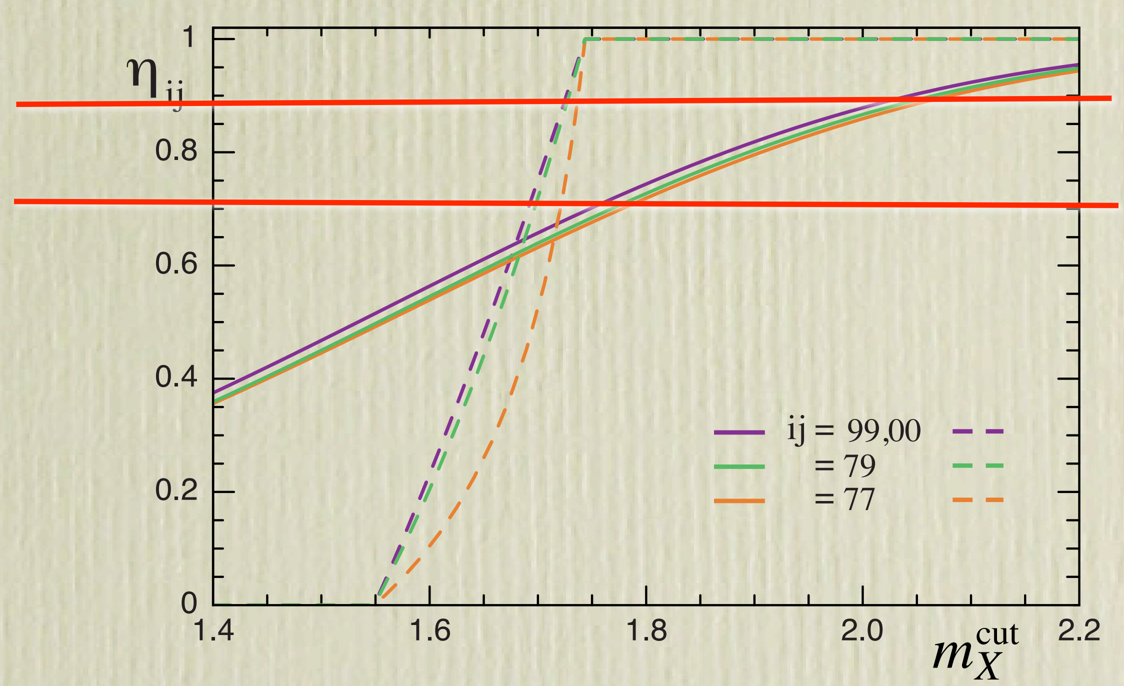
Cuts force the Xs to be jet-like and are important for comparison to the standard model

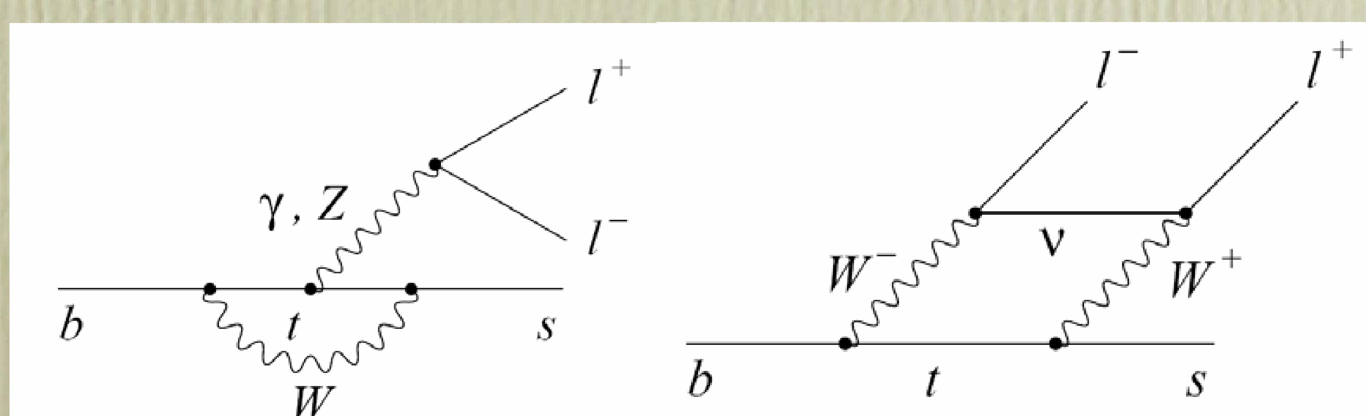
$\boxed{B \to X_s \ell^+ \ell^-}$

Again the cuts give a jet, and modify the standard model prediction

Lee, Ligeti, Stewart, Tackmann

10-30% reduction in the decay rate





SCET has been applied to many processes

Process	Non-Pert. functions	Utility
$\bar{B}^0 \to D^+\pi^-, \ldots$	$\xi(w)$, ϕ_π	study QCD
$\bar{B}^0 \to D^0\pi^0, \ldots$	$S(k_j^+)$, ϕ_π	study QCD
$B \to X_s^{endpt}\gamma$	$f(k^+)$	new physics, measure f
$B \to X_u^{endpt}\ell\nu$	$f(k^+)$	measure $\lvert V_{ub}\rvert$
$B \to \pi\ell\nu, \ldots$	$\phi_B(k^+)$, $\phi_\pi(x)$, $\zeta_\pi(E)$	measure $\lvert V_{ub}\rvert$, study QCD
$B \to \gamma\ell\nu$, $\gamma\ell^+\ell^-$	ϕ_B	measure ϕ_B, new physics
$B \to \pi\pi$, $K\pi$, $\ldots$	ϕ_B, ϕ_π, $\zeta_\pi(E)$	new physics, CP violation, γ
	$\phi_{\bar{K}}$, $\zeta_K(E)$	study QCD
$B \to K^*\gamma$, $\rho\gamma$	ϕ_B, ϕ_K, $\zeta_{K^*}^\perp(E)$	measure $\lvert V_{td}/V_{ts}\rvert$,
	ϕ_ρ, $\zeta_\rho^\perp(E)$	new physics
$B \to X_s\ell^+\ell^-$	$f(k^+)$	new physics
$e^-p \to e^-X$	$f_{i/p}(\xi)$, $f_{g/p}(\xi)$	study QCD , measure p.d.f's
$p\bar{p} \to X\ell^+\ell^-$	$f_{i/p}(\xi)$, $f_{g/p}(\xi)$	study QCD
$e^-\gamma \to e^-\pi^0$	ϕ_π	measure ϕ_π
$\gamma^*M \to M'$	ϕ_M, $\phi_{M'}$	study QCD
$e^+e^- \to j_1 + \text{jets}$	$\tilde{S}(k^+)$	event shapes & universality
$e^+e^- \to J/\Psi X$	$S^{(8,n)}(k^+)$	study QCD
$\Upsilon \to X\gamma$	$S^{(8,n)}(k^+)$	study QCD
$\vdots$	$\vdots$	$\vdots$

In Pittsburgh: C.Kim, A.Leibovich, I.Rothstein, A.Williamson, J.Zupan

Future

Who needs to understand QCD?

Babar, Belle

- For many channels, control of hadronic uncertainties is crucial to test standard model & look for new physics.

$$B \to X_s \ell^+ \ell^-, B \to \pi\pi, B \to K\pi, B \to \rho\pi, \ldots$$
$$B \to \rho\gamma, B \to K^*\gamma, B \to \phi K_s, B \to \eta' K_s$$

CDF, DØ

- Test standard model / new physics in $B_s, \Lambda_b, \ldots$
- Heavy quark production, jets, ...

Immediate future:

Babar, Belle

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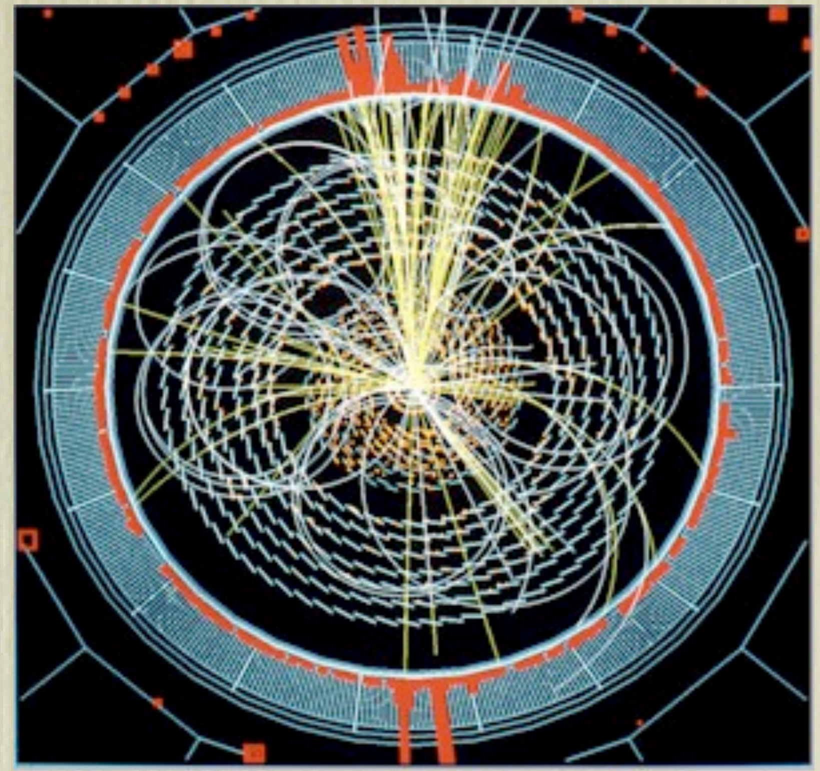
pp collider with $E_{cm} = 14\,\mathrm{TeV}$

scales: $m_W\,, m_t\,, E_T^{\mathrm{jet}}$

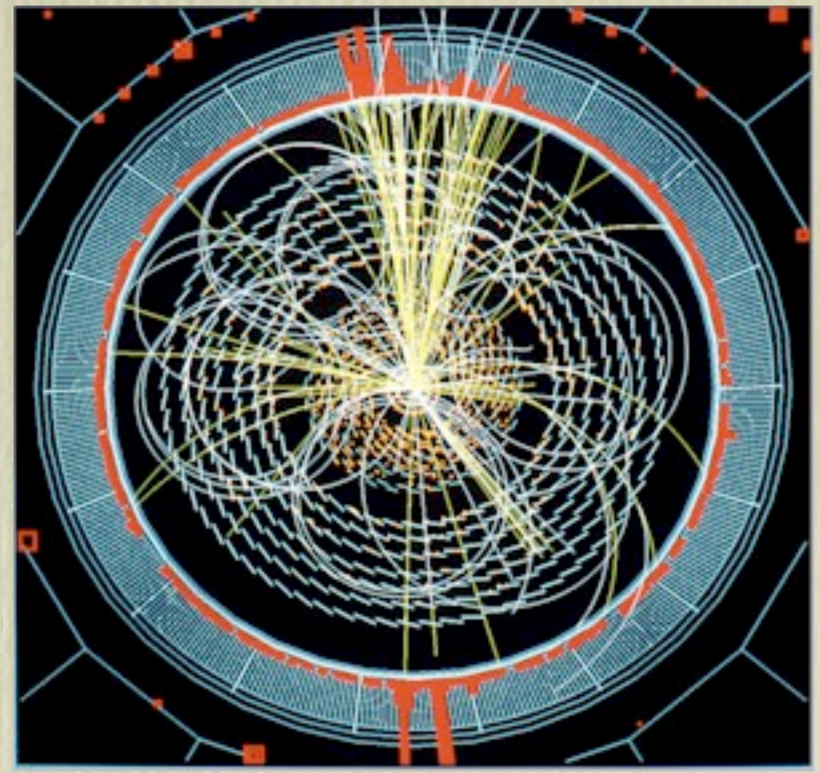
→ Energetic QCD (SCET)



Effective theory concepts will be helpful whether we're:

- exploring QCD,
- computing precision standard model cross sections (resolution scales or summation of logs),
- or puzzling out signals of unexplored particle physics

LHC era:



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- exploring QCD,
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Concluding Remarks

- QED fundamental parameters & precision quantum field theory
- QCD today is as rich & diverse as ever

  many subfields which focus on different degrees of freedom
  and different relevant interactions

- SCET a new approach to derive factorization theorems
  and treat power corrections for energetic hadrons & jets

  Nonleptonic B-decays

  ➡ predictions for the size of amplitudes

  ➡ universal hadronic parameters, strong phases

  ➡ γ (or α) from individual $B \to M_1 M_2$ channels

- A lot of theory and phenomenology left to study ...