# Effective Field Theory 

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## Outline

## Lecture I

- Principles, Operators, Power Counting, Matching, Loops,

Using Equations of Motion, Renormalization and Decoupling

## Lecture II

- Summing Logarithms, $\alpha_{s}$ matching, HQET

Weak Interactions at low energy, [power counting velocity NRQCD]

## Lecture III

## Soft-Collinear Effective Theory

An effective field theory for energetic hadrons \& jets

$$
E \gg \Lambda_{\mathrm{QCD}}
$$

Non-relativistic QCD

## Systems with Two Heavy Particles

| eg. | $e^{+} e^{-}$ | $\rightarrow$ | positronium | (NRQED) |
| :--- | :---: | :--- | :--- | :--- |
| $p e^{-}$ | $\rightarrow$ | Hydrogen | (NRQED) |  |
| $b \bar{b}, c \bar{c}$ | $\rightarrow$ | $\Upsilon, J / \Psi$ | (NRQCD) |  |
| $t \bar{t}$ | $\rightarrow$ | $e^{+} e^{-} \rightarrow t \bar{t}$ | (NRQCD) |  |
| $N N$ | $\rightarrow$ | deuteron | (few nucleon EFT) |  |

- $E=p^{2} /(2 m) \sim v^{2}$, count powers of $v \quad$ (and $\alpha_{s}$ )


## Momentum Regions

integrate these out
potential: $m v^{2} m v$
soft:
$m v$
$m v$
ultrasoft: $m v^{2} m v^{2}$
ptnl gluons are not propagating $\psi, \chi$
radiative corrections, binding $\quad A_{s}^{\mu}, q_{s}$
need multipole expansion
$A_{u s}^{\mu}$

## Simplify p.c. and Implement multipole expansion

$$
\triangleright P=(m, \mathbf{0})+\mathbf{p}+k
$$

## $m \quad m v \quad m v^{2}$

$\triangleright \mathbf{p}$ index
$\triangleright k=\left(k^{0}, \mathbf{k}\right)$ continuous

$$
\begin{aligned}
& \psi(x)=\sum_{\mathbf{p}} e^{i \mathbf{p} \cdot \mathbf{x}} \psi_{\mathbf{p}}(x) \\
& i \partial^{\mu} \psi_{\mathbf{p}}(x) \sim\left(m v^{2}\right) \psi_{\mathbf{p}}(x)
\end{aligned}
$$


$\mathcal{O}\left(v^{0}\right)$ Kinetic Terms give potential quarks $\psi, \chi \sim v^{3 / 2}$ soft gluons $\quad A_{s}^{\mu} \sim v \quad$ (scale $\mu_{S}$ ) ultrasoft gluons $A_{u s}^{\mu} \sim v^{2} \quad$ (scale $\mu_{U}$ )
(board)

- Can associate all powers of $v$ with vertices

$$
\delta=5+\sum_{k}(k-5) V_{k}^{P}+(k-8) V_{k}^{U}+(k-4) V_{k}^{S}-N_{s}
$$

- Power counting of operators implies power counting of states

Coulombic Singularities $\Longrightarrow$ sum insertions of $V_{c}$


1


$$
\left(\alpha_{s} / v\right)
$$

$$
\left(\alpha_{s} / v\right)^{2}
$$

Coulombic: $v \sim \alpha_{s}$

The rest of the NRQCD discussion can be found at the end of Lecture II on the website

## Lecture III Outline

- d.o.f., SCETı \& SCETir, Lagrangians
- Label Operators, Gauge Invariance, Wilson Lines, RPI, multipole expansion
- Hard-Collinear and Ultrasoft-Collinear Factorization
- IR divergences, Running
- $B \rightarrow X_{s} \gamma$

B-decays by weak interactions:
$m_{W}$

$$
\begin{array}{ccl}
B \rightarrow X_{u} \ell \bar{\nu} & B \rightarrow D \pi & B \rightarrow K^{*} \gamma \\
B \rightarrow \pi \ell \bar{\nu} & B \rightarrow X_{s} \gamma & B \rightarrow \rho \gamma
\end{array}
$$

$m_{b}$
E

$$
\begin{array}{cc}
B \rightarrow D^{*} \eta^{\prime} & B \rightarrow \rho \rho \quad B \rightarrow \pi \pi \\
B \rightarrow K \pi & B \rightarrow \gamma \ell \bar{\nu}, ~
\end{array}
$$

The B is heavy, so many of its decay products are energetic, $E$

Any other QCD process with large energy transfer: $\Lambda_{\mathrm{QCD}}$
$m_{s}$

- $m_{u, d}$

$$
\begin{array}{lc}
e^{-} p \rightarrow e^{-} X & p \bar{p} \rightarrow X \ell^{+} \ell^{-} \\
e^{-} \gamma \rightarrow e^{-} \pi^{0} & \gamma^{*} M \rightarrow M^{\prime} \quad \Upsilon \rightarrow X \gamma \\
e^{+} e^{-} \rightarrow \text { jets } & e^{+} e^{-} \rightarrow J / \Psi X
\end{array}
$$

## Degrees of Freedom

eg.


Pion has: $\quad p_{\pi}^{\mu}=(2.310 \mathrm{GeV}, 0,0,-2.306 \mathrm{GeV})=Q n^{\mu}$

$$
Q \gg \Lambda_{\mathrm{QCD}} \quad n^{\mu}=(1,0,0,-1)
$$

Light-Cone coordinates: Basis vectors $n^{\mu}, \bar{n}^{\mu}$ with $n^{2}=0, \bar{n}^{2}=0, n \cdot \bar{n}=2$

$$
\begin{array}{rlr}
p^{\mu}=\frac{n^{\mu}}{2} \bar{n} \cdot p+\frac{\bar{n}^{\mu}}{2} n \cdot p+p_{\perp}^{\mu} & p^{+} \equiv n \cdot p, \quad p^{-} \equiv \bar{n} \cdot p \\
g^{\mu \nu}=\frac{n^{\mu} \bar{n}^{\nu}}{2}+\frac{\bar{n}^{\mu} n^{\nu}}{2}+g_{\perp}^{\mu \nu} & \text { eg. } \bar{n}^{\mu}=(1,0,0,1)
\end{array}
$$

Collinear constituents:

$$
p_{c}^{\mu}=\left(p^{+}, p^{-}, p^{\perp}\right) \sim\left(\frac{\Lambda^{2}}{Q}, Q, \Lambda\right) \sim Q\left(\lambda^{2}, 1, \lambda\right) \quad \lambda=\frac{\Lambda}{Q}
$$

Just a boost of

$$
\left(p^{+}, p^{-}, p^{\perp}\right) \sim(\Lambda, \Lambda, \Lambda)
$$

Soft Constituents

$$
p_{s}^{\mu}=\left(p^{+}, p^{-}, p^{\perp}\right) \sim(\Lambda, \Lambda, \Lambda) \quad \mathrm{D} \text { and }
$$

$\mathrm{SCET}_{\mathrm{II}} \quad$ Energetic hadrons

$$
\lambda=\frac{\Lambda}{Q}
$$

| modes | $p^{\mu}=(+,-, \perp)$ | $p^{2}$ | fields |
| :---: | :---: | :---: | :---: |
| collinear | $Q\left(\lambda^{2}, 1, \lambda\right)$ | $Q^{2} \lambda^{2}$ | $\xi_{n}, A_{n}^{\mu}$ |
| soft | $Q(\lambda, \lambda, \lambda)$ | $Q^{2} \lambda^{2}$ | $q_{s}, A_{s}^{\mu}$ |

$$
\begin{array}{ll}
\text { eg. } B \rightarrow X_{S} \gamma & m_{X}^{2} \sim m_{B}^{2} \text { OPE in } 1 / m_{b}(\text { not SCET }) \\
& m_{X}^{2} \sim \Lambda^{2} \text { not inclusive }
\end{array}
$$

$$
\begin{aligned}
& m_{X}^{2} \sim \Lambda Q \\
& \Lambda^{2} \ll Q \Lambda \ll Q^{2}
\end{aligned}
$$



Jet constituents: $p^{\mu} \sim(\Lambda, Q, \sqrt{Q \Lambda}) \sim Q\left(\lambda^{2}, 1, \lambda\right)$
SET $_{\text {I }}$ Energetic jets

$$
\begin{array}{ll}
\text { usoft } & p^{\mu} \sim \Lambda \\
\text { collinear } & p_{c}^{2} \sim Q \Lambda, \lambda \lambda=\sqrt{\Lambda / Q}
\end{array}
$$

$$
\begin{array}{cccc}
\text { modes } & p^{\mu}=(+,-, \perp) & p^{2} & \text { fields } \\
\hline \text { collinear } & Q\left(\lambda^{2}, 1, \lambda\right) & Q^{2} \lambda^{2} & \xi_{n}, A_{n}^{\mu}
\end{array}
$$

usoft

$$
Q\left(\lambda^{2}, \lambda^{2}, \lambda^{2}\right) \quad Q^{2} \lambda^{4} \quad q_{u s}, A_{u s}^{\mu}
$$

What makes this EFT different?

- Usually $m_{1} \gg \Lambda$

$$
\sum_{i=1}^{n} C_{i}\left(\mu, m_{1}\right) O_{i}(\mu, \Lambda)
$$



- In SCET constituent $p^{-} \sim m_{b} \sim E_{\pi}$

$$
\int d \omega C(\omega) O(\omega)
$$



## Collinear Propagator

## (board)

Power Counting for Collinear Fields

(board)

## Currents

eg. $\bar{u} \Gamma b$ involves both collinear and ultrasoft objects

$$
\begin{gathered}
\bar{u} \Gamma b \longrightarrow \bar{\xi}_{n} W \Gamma h_{v} \\
W=P \exp \left(i g \int_{-\infty}^{y} d s \bar{n} \cdot A_{n}\left(s \bar{n}^{\mu}\right)\right)
\end{gathered}
$$

Interaction of modes: Offshell versus Onshell
(board)

## Separate Momenta (multipole expansion)

## label residual

HQET

$$
P^{\mu}=m_{b} v^{\mu}+k^{\mu}
$$

$$
h_{v}(x)
$$

## SCAT

$$
\begin{aligned}
P^{\mu}= & p^{\mu}+ \\
& \imath_{(1, \lambda)}
\end{aligned}
$$

Collinear Quarks
$\triangleright \psi(x) \rightarrow \sum_{p} e^{-i p \cdot x} \xi_{n, p}(x)$
$\triangleright \nsim \xi_{n, p}=0$
$\triangleright \partial^{\mu} \xi_{n, p} \sim\left(Q \lambda^{2}\right) \xi_{n, p}$
usual derivative
$h_{v}(x)$
$\xi_{n, p}(x)$

$Q \lambda^{2}$

## Introduce Label Operator

$$
\begin{aligned}
& \mathcal{P}^{\mu}\left(\phi_{q_{1}}^{\dagger} \cdots \phi_{p_{1}} \cdots\right)=\left(p_{1}^{\mu}+\ldots-q_{1}^{\mu}-\cdots\right)\left(\phi_{q_{1}}^{\dagger} \cdots \phi_{p_{1}} \cdots\right) \\
& i \partial^{\mu} e^{-i p \cdot x} \phi_{p}(x)=e^{-i p \cdot x}\left(\mathcal{P}^{\mu}+i \partial^{\mu}\right) \phi_{p}(x)
\end{aligned}
$$

- Labels are changed by collinear interactions

- Labels are preserved by ultrasoft interactions



## Power Counting Summary

| Type | $\left(p^{+}, p^{-}, p^{\perp}\right)$ | Fields | Field Scaling |
| :--- | :---: | :---: | :---: |
| collinear | $\left(\lambda^{2}, 1, \lambda\right)$ | $\xi_{n, p}$ | $\lambda$ |
|  |  | $\left(A_{n, p}^{+}, A_{n, p}^{-}, A_{n, p}^{\perp}\right)$ | $\left(\lambda^{2}, 1, \lambda\right)$ |
| soft | $(\lambda, \lambda, \lambda)$ | $q_{s, p}$ | $\lambda^{3 / 2}$ |
|  |  | $A_{s, p}^{\mu}$ | $\lambda$ |
| usoft | $\left(\lambda^{2}, \lambda^{2}, \lambda^{2}\right)$ | $q_{u s}$ | $\lambda^{3}$ |
|  |  | $A_{u s}^{\mu}$ | $\lambda^{2}$ |

Make kinetic terms order $\lambda^{0} \quad \int d^{4} X \quad \bar{\xi}_{n, p^{\prime}} \frac{\vec{n}}{2}(i n \cdot \partial+\ldots) \xi_{n, p}$

$$
\lambda^{0}=\begin{array}{llll}
\lambda^{-4} & \lambda & \lambda^{2} & \lambda
\end{array}
$$

- At leading power only $\lambda^{0}$ interactions are required

Power counting can be assigned to vertices

$$
\delta=4+\sum_{k}(k-4)\left(V_{k}^{c}+V_{k}^{s}+V_{k}^{s c}\right)+(k-8) V_{k}^{u s}
$$

## LO SCET Lagrangian

$$
\begin{aligned}
& \mathcal{L}_{\mathrm{QCD}}=\bar{\psi} i D \psi \quad \text { Write } \psi=\xi_{n}+\chi_{\bar{n}} \\
& \xi_{n}=\frac{\mu \pi}{4} \psi \\
& \left.\mathcal{L}=\left(\bar{x}_{n}+\bar{\xi}_{n}\right)\left[\frac{i}{2} n \cdot D+i \frac{\phi}{2} \bar{n} \cdot D+i D_{D}\right]\right]\left(\xi_{n}+\chi_{n}\right) \\
& \chi_{\bar{n}}=\frac{\bar{\pi} \eta \eta}{4} \psi
\end{aligned}
$$

e.o.m: $\frac{\delta}{\delta \bar{\chi}_{\bar{n}}}: \quad i \bar{n} \cdot D \chi_{\bar{n}}+\frac{\vec{n}}{2} i D_{\perp} \xi_{n}=0$ $\chi_{\bar{n}}=\frac{1}{i \bar{n} \cdot D} i \not D_{\perp} \frac{\vec{n}}{2} \xi_{n}$

$$
\mathcal{L}=\bar{\xi}_{n}\left(i \bar{n} \cdot D+i \not D_{\perp} \frac{1}{i \bar{n} \cdot D} i \not D_{\perp}\right) \frac{\ddot{n}}{2} \xi_{n}
$$

## (board)

$$
\mathcal{L}_{c}^{(0)}=\bar{\xi}_{n}\left\{n \cdot i D_{u s}+g n \cdot A_{n}+i D_{\perp}^{c} \frac{1}{i \bar{n} \cdot D_{c}} i D_{\perp}^{c}\right\} \frac{\hbar}{2} \xi_{n}
$$

That was tree level.
Use Symmetries
Power counting, Gauge symmetry, Discrete, Lorentz invariance (?)

Gauge symmetry $\quad U(x)=\exp \left[i \alpha^{A}(x) T^{A}\right]$
need to consider U's which leave us in the EFT
collinear $\quad i \partial^{\mu} \mathcal{U}_{c}(x) \sim p_{c}^{\mu} \mathcal{U}_{c}(x) \leftrightarrow A_{n, q}^{\mu}$
usoft $\quad i \partial^{\mu} U_{u s}(x) \sim p_{c}^{\mu} U_{u s}(x) \leftrightarrow A_{u s}^{\mu}$

| Object | Collinear $\mathcal{U}_{c}$ | Usoft $U_{u s}$ |
| :---: | :---: | :---: |
| $\xi_{n}$ | $\mathcal{U}_{c} \xi_{n}$ | $U_{u s} \xi_{n}$ |
| $g A_{n}^{\mu}$ | $\mathcal{U}_{c} g A_{n}^{\mu} \mathcal{U}_{c}^{\dagger}+\mathcal{U}_{c}\left[i \mathcal{D}^{\mu}, \mathcal{U}_{c}^{\dagger}\right]$ | $U_{u s} g A_{n}^{\mu} U_{u s}^{\dagger}$ |
| $W$ | $\mathcal{U}_{c} W$ | $U_{u s} W U_{u s}^{\dagger}$ |
| $q_{u s}$ | $q_{u s}$ | $U_{u s} q_{u s}$ |
| $g A_{u s}^{\mu}$ | $g A_{u s}^{\mu}$ | $U_{u s} g A_{u s}^{\mu} U_{u s}^{\dagger}+U_{u s}\left[i \partial^{\mu}, U_{u s}^{\dagger}\right]$ |
| $Y$ | $Y$ | $U_{u s} Y$ |

## Reparameterization Invariance (RPI)

$n, \bar{n}$ break Lorentz invariance, restored within collinear cone by RPI, three types
(I) $\left\{\begin{array}{l}n_{\mu} \rightarrow n_{\mu}+\Delta_{\mu}^{\perp} \\ \bar{n}_{\mu} \rightarrow \bar{n}_{\mu}\end{array}\right.$
(II) $\left\{\begin{array}{l}n_{\mu} \rightarrow n_{\mu} \\ \bar{n}_{\mu} \rightarrow \bar{n}_{\mu}+\varepsilon_{\mu}^{\perp}\end{array}\right.$
(III) $\left\{\begin{array}{l}n_{\mu} \rightarrow(1+\alpha) n_{\mu} \\ \bar{n}_{\mu} \rightarrow(1-\alpha) \bar{n}_{\mu}\end{array}\right.$
eg. rules out $\quad \bar{\xi}_{n} i D_{\perp}^{\mu} \frac{1}{i \bar{n} \cdot D} i D_{\mu}^{\perp} \frac{\vec{t}}{2} \xi_{n}$

$$
\mathcal{L}_{c}^{(0)}=\bar{\xi}_{n}\left\{n \cdot i D_{u s}+g n \cdot A_{n}+i \not D_{\perp}^{c} \frac{1}{i \bar{n} \cdot D_{c}} i \not D_{\perp}^{c}\right\} \frac{\hbar \hbar}{2} \xi_{n}
$$

## Wilson Coefficients and Hard - Collinear Factorization

$C(\overline{\mathcal{P}}, \mu)$ : they depend on large momenta picked out by $\overline{\mathcal{P}}=\bar{n} \cdot \mathcal{P} \sim \lambda^{0}$

$$
\text { eg. } \quad C(-\overline{\mathcal{P}}, \mu)\left(\bar{\xi}_{n} W\right) \Gamma h_{v}=\underbrace{\left(\bar{\xi}_{n} W\right)}_{\begin{array}{r}
\text { only the product } \\
\text { is gauge invariant }
\end{array}} \Gamma h_{v} C\left(\overline{\mathcal{P}}^{\dagger}, \mu\right)
$$

## Write

$$
\left(\bar{\xi}_{n} W\right) \Gamma h_{v} C\left(\overline{\mathcal{P}}^{\dagger}, \mu\right)=\int d \omega C(\omega, \mu)\left[\left(\bar{\xi}_{n} W\right) \delta\left(\omega-\overline{\mathcal{P}}^{\dagger}\right) \Gamma h_{v}\right]=\int d \omega C(\omega, \mu) O(\omega, \mu)
$$

## In general:

$$
f\left(i \bar{n} \cdot D_{c}\right)=W f(\overline{\mathcal{P}}) W^{\dagger}
$$

$$
=\int d \omega f(\omega)\left[W \delta(\omega-\overline{\mathcal{P}}) W^{\dagger}\right]
$$

hard-collinear factorization follows from properties of SCET operators
hard coefficient $p^{2} \sim Q^{2} \quad \begin{array}{r}\text { in collinear } \\ \text { operator }\end{array} p^{2} \sim Q^{2} \lambda^{2}$

## Multipole Expansion and Ultrasoft - Collinear Factorization

Multipole Expansion:

$$
\mathcal{L}_{c}^{(0)}=\bar{\xi}_{n}\left\{n \cdot i D_{u s}+g n \cdot A_{n}+i D_{\perp}^{c} \frac{1}{i \bar{n} \cdot D_{c}} i D_{\perp}^{c}\right\} \frac{\hbar}{2} \xi_{n}
$$

(board)
Field Redefinition: (board)

$$
\begin{aligned}
& Y(x)=P \exp \left(i g \int_{-\infty}^{0} d s n \cdot A_{u s}(x+n s)\right) \\
& n \cdot D_{u s} Y=0, Y^{\dagger} Y=1
\end{aligned}
$$

gives:

$$
\mathcal{L}_{c}^{(0)}=\bar{\xi}_{n}\left\{n \cdot i D_{\mathrm{us}}+\ldots\right\} \frac{\vec{p}}{2} \xi_{n} \rightarrow \bar{\xi}_{n}\left\{n \cdot i D_{c}+i D_{\perp} \frac{1}{i \bar{n} \cdot D_{c}} i D_{\perp}\right\} \frac{\vec{n}}{2} \xi_{n}
$$

Moves all usoft gluons to operators, simplifies cancellations

Field Theory gives the same results pre- and post- field redefinition, but the organization is different

Ultrasoft - Collinear Factorization:

$$
\text { eg1. } J=\left(\bar{\xi}_{n} W\right)_{\omega} \Gamma h_{v} \rightarrow\left(\bar{\xi}_{n} Y^{\dagger} Y W Y^{\dagger}\right)_{\omega} \Gamma h_{v}=\left(\bar{\xi}_{n} W\right)_{\omega} \Gamma\left(Y^{\dagger} h_{v}\right)
$$

note: not upset by hard-collinear mometum fraction since ultrasoft gluons carry no hard momenta
(board)
so usoft-collinear factorization is also simply a property of SCET
eg2. No ultrasoft fields

$$
J=\left(\bar{\xi}_{n} W\right)_{\omega_{1}} \Gamma\left(W^{\dagger} \xi_{n}\right)_{\omega_{2}} \rightarrow\left(\bar{\xi}_{n} W\right)_{\omega_{1}} Y^{\dagger} Y \Gamma\left(W^{\dagger} \xi_{n}\right)_{\omega_{2}}=\left(\bar{\xi}_{n} W\right)_{\omega_{1}} \Gamma\left(W^{\dagger} \xi_{n}\right)_{\omega_{2}}
$$

## IR divergences, Matching, \& Running

$$
J^{\mathrm{QCD}}=\bar{s} \Gamma b \quad J^{\mathrm{SCET}}=\left(\bar{\xi}_{n} W\right)_{\omega} \Gamma h_{v} \quad \Gamma=\sigma^{\mu \nu}
$$

QCD »


IR regulator $p^{2} \neq 0$ for s-quark $1 / \epsilon_{\mathrm{IR}}$ for b-quark
sum $=-\frac{\alpha_{s}}{3 \pi}\left[\ln ^{2}\left(\frac{-p^{2}}{m_{b}^{2}}\right)+\frac{3}{2} \ln \left(\frac{-p^{2}}{m_{b}^{2}}\right)+\frac{1}{\epsilon_{\mathrm{IR}}}+2 \ln \left(\frac{\mu^{2}}{m_{b}^{2}}\right)+\right.$ constants $]$

SCET

c)

d)

sum $=\quad-\frac{\alpha_{s}}{3 \pi}\left[\ln ^{2}\left(\frac{-p^{2}}{m_{b}^{2}}\right)+\frac{3}{2} \ln \left(\frac{-p^{2}}{m_{b}^{2}}\right)+\frac{1}{\epsilon_{\mathrm{IR}}}\right.$ same IR

$$
\left.-\frac{1}{\epsilon_{\mathrm{UV}}^{2}}-\frac{5}{2 \epsilon_{\mathrm{UV}}}-\frac{2}{\epsilon_{\mathrm{UV}}} \ln \left(\frac{\mu}{m_{b}}\right)-2 \ln ^{2}\left(\frac{\mu}{m_{b}}\right)-\frac{3}{2} \ln \left(\frac{\mu^{2}}{m_{b}^{2}}\right)+\text { constants }\right]
$$

Running $\quad \int d \omega C(\omega) O(\omega)$

$$
\omega=m_{b} \text { in } B \rightarrow X_{s} \gamma
$$

counterterm

$$
Z=1+\frac{\alpha_{s}}{3 \pi}\left[\frac{1}{\epsilon^{2}}+\frac{2}{\epsilon} \ln \left(\frac{\mu}{\omega}\right)+\frac{5}{2 \epsilon}\right]
$$

RGE

$$
\mu \frac{\partial}{\partial \mu} C(\mu)=-\frac{4 \alpha_{s}(\mu)}{3 \pi} \ln \left(\frac{\mu}{\omega}\right) C(\mu)+\ldots
$$

(board)

Endpoint $B \rightarrow X_{s} \gamma$
(board)

## Endpoint $B \rightarrow X_{s} \gamma$

Optical Thm: $\quad \Gamma \sim \operatorname{Im} \int d^{4} x e^{-i q \cdot x}\langle B| T\left\{J_{\mu}^{\dagger}(x) J^{\mu}(0)\right\}|B\rangle$


|  | $P_{X}^{2}=$ |
| :--- | :---: |
| standard OPE | $\left(m_{B}-2 E_{\gamma}\right)$ |
| endpoint region | $\sim m_{B}^{2}$ |
| resonance region | $\sim m_{B} \Lambda_{Q C D}$ |
|  | $\sim \Lambda_{Q C D}^{2}$ |

For EndPoint: $\quad E_{\gamma} \gtrsim 2.2 \mathrm{GeV}, X_{s}$ collinear, $B$ usoft, $\quad \lambda=\sqrt{\frac{\Lambda_{Q C D}}{m_{B}}}$
Decay rate is given by factorized form

$$
\frac{1}{\Gamma_{0}} \frac{d \Gamma}{d E_{\gamma}}=H\left(m_{b}, \mu\right) \int_{2 E_{\gamma}-m_{b}}^{\bar{\Lambda}} d k^{+} S\left(k^{+}, \mu\right) J\left(k^{+}+m_{b}-2 E_{\gamma}, \mu\right)
$$

Match: $\quad \bar{s} \Gamma_{\mu} b \rightarrow e^{i\left(m_{b} v-\mathcal{P}\right) \cdot x} C(\overline{\mathcal{P}}) \bar{\xi}_{n, p} W \gamma_{\mu}^{\perp} P_{L} h_{v}$

$$
T_{\mu}^{\mu}=\int d^{4} x e^{i\left(m_{b} \frac{\tilde{n}}{2}-q\right) \cdot x}\langle B| T J_{\mathrm{eff}}^{\dagger}(x) J_{\mathrm{eff}}(0)|B\rangle
$$

label conservation $\overline{\mathcal{P}} \rightarrow m_{b}$

Factor usoft: $\quad \bar{\xi}_{n} W \Gamma_{\mu} h_{v}=\bar{\xi}_{n}^{(0)} W^{(0)} \Gamma_{\mu} Y_{n}^{\dagger} h_{v}$

$$
\begin{gathered}
T_{\mu}^{\mu}=\left|C\left(m_{b}\right)\right|^{2} \int d^{4} x e^{i\left(m_{b} \frac{\bar{n}}{2}-q\right) \cdot x}\langle B| T\left[\bar{h}_{v} Y\right](x)\left[Y^{\dagger} h_{v}\right](0)|B\rangle \\
\times\langle 0| T\left[W^{(0) \dagger} \xi_{n}^{(0)}\right](x)\left[\bar{\xi}_{n}^{(0)} W^{(0)}\right](0)|0\rangle \times\left[\Gamma_{\mu} \otimes \Gamma^{\mu}\right]
\end{gathered}
$$

## Convolution:

 $\langle 0| \cdots|0\rangle \longrightarrow J_{P}\left(k_{+}\right)$
## Define Fourier Transforms

$$
\operatorname{Im} T_{\mu}^{\mu}=\left|C\left(m_{b}\right)\right|^{2} \int d l^{+} S\left(l^{+}\right) \operatorname{Im} J_{P}\left(m_{b}-2 E_{\gamma}+l^{+}\right)
$$

$$
\text { where } \quad S\left(l^{+}\right)=\langle B| \bar{h}_{v} \delta\left(i n \cdot D-l^{+}\right) h_{v}|B\rangle
$$




SCET is a field theory which:

- explains how these degrees of freedom communicate with each other, and with hard interactions
- organizes the interactions in a series expansion in $\frac{\Lambda_{\mathrm{QCD}}}{E}$
- provides a simple operator language to derive factorization theorems in fairly general circumstances
eg. unifies the treatment of factorization for exclusive and inclusive QCD processes
- new symmetry constraints
- scale separation \& decoupling



## How is SCET used?

- cleanly separate short and long distance effects in QCD
$\rightarrow$ derive new factorization theorems
$\rightarrow$ find universal hadronic functions, exploit symmetries \& relate different processes
- model independent, systematic expansion
$\rightarrow$ study power corrections
- keep track of $\mu$ dependence
$\rightarrow$ sum logarithms, reduce uncertainties

The End

