Effective Field Theory

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Outline

Lecture I

- Principles, Operators, Power Counting, Matching, Loops, Using Equations of Motion, Renormalization and Decoupling
 Lecture II
- Summing Logarithms, α_s matching, HQET
 Weak Interactions at low energy, [power counting velocity NRQCD]
 Lecture III

Soft - Collinear Effective Theory

 $E \gg \Lambda_{\rm OCD}$

An effective field theory for energetic hadrons & jets

Non-relativistic QCD

Systems with Two Heavy Particles

eσ	e^+e^-	\rightarrow	positronium	(NRQED)
<u> </u>	pe^-	\rightarrow	Hydrogen	(NRQED)
	$b\overline{b},c\overline{c}$	\rightarrow	$\Upsilon, J/\Psi$	(NRQCD)
	$t\overline{t}$	\rightarrow	$e^+e^- \to t\bar{t}$	(NRQCD)
	NN	\rightarrow	deuteron	(few nucleon EFT)

Simplify p.c. and Implement multipole expansion

$$\triangleright P = (m, \mathbf{0}) + \mathbf{p} + k$$

$$m mv mv^{2}$$

$$\triangleright \mathbf{p} \text{ index}$$

$$\triangleright k = (k^{0}, \mathbf{k}) \text{ continuous}$$

$$\psi(x) = \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}}\psi_{\mathbf{p}}(x)$$

$$i\partial^{\mu}\psi_{\mathbf{p}}(x) \sim (mv^{2})\psi_{\mathbf{p}}(x)$$



 $\mathcal{O}(v^0)$ Kinetic Terms give

potential quarks $\psi, \chi \sim v^{3/2}$ soft gluons $A_s^{\mu} \sim v$ (scale μ_S)

ultrasoft gluons $A_{us}^{\mu} \sim v^2$ (scale μ_U)

(board)

• Can associate all powers of v with vertices

$$\delta = 5 + \sum_{k} (k-5)V_{k}^{P} + (k-8)V_{k}^{U} + (k-4)V_{k}^{S} - N_{s}$$

Power counting of operators implies power counting of states •

Coulombic Singularities \implies sum insertions of V_c



The rest of the NRQCD discussion can be found at the end of Lecture II on the website

Lecture III Outline

- d.o.f., SCET1 & SCET11, Lagrangians
- Label Operators, Gauge Invariance, Wilson Lines, RPI, multipole expansion
- Hard-Collinear and Ultrasoft-Collinear Factorization
- IR divergences, Running
- $B \to X_s \gamma$

SCET can be used for: B - decays by weak interactions: m_W $B \to X_{\mu} \ell \bar{\nu} \quad B \to D\pi$ $B \to K^* \gamma$ $B \to \rho \gamma$ $B \to X_s \gamma$ $B \to \pi \ell \bar{\nu}$ m_b $\begin{array}{ccc} B \to D^* \eta' & \begin{array}{ccc} B \to \rho \rho & B \to \pi \pi \\ & B \to K \pi \end{array} & \begin{array}{ccc} B \to \gamma \ell \bar{\nu} \end{array}$ The B is heavy, so many of its decay products m_c are energetic, EAny other QCD process with large energy transfer: $\Lambda_{
m QCD}$ $e^- p \to e^- X \qquad p \bar{p} \to X \ell^+ \ell^$ $e^-\gamma \to e^-\pi^0 \qquad \gamma^*M \to M' \qquad \Upsilon \to X\gamma$ m_s $e^+e^- \rightarrow \text{jets} \qquad e^+e^- \rightarrow J/\Psi X$ $m_{u,d}$

Degrees of Freedom eg. $(\pi) \rightarrow (D)$

Pion has: $p_{\pi}^{\mu} = (2.310 \,\text{GeV}, 0, 0, -2.306 \,\text{GeV}) = Q n^{\mu}$

 $Q \gg \Lambda_{\rm QCD}$ $n^{\mu} = (1, 0, 0, -1)$

Light - Cone coordinates: Basis vectors n^{μ} , \bar{n}^{μ} with $n^2 = 0$, $\bar{n}^2 = 0$, $n \cdot \bar{n} = 2$

$$p^{\mu} = \frac{n^{\mu}}{2}\bar{n} \cdot p + \frac{\bar{n}^{\mu}}{2}n \cdot p + p^{\mu}_{\perp} \qquad p^{+} \equiv n \cdot p, \qquad p^{-} \equiv \bar{n} \cdot p$$
$$g^{\mu\nu} = \frac{n^{\mu}\bar{n}^{\nu}}{2} + \frac{\bar{n}^{\mu}n^{\nu}}{2} + g^{\mu\nu}_{\perp} \qquad \text{eg. } \bar{n}^{\mu} = (1, 0, 0, 1)$$



eg. $B \to X_s \gamma$ $m_X^2 \sim m_B^2$ OPE in $1/m_b$ (not SCET) $m_X^2 \sim \Lambda^2$ not inclusive $m_X^2 \sim \Lambda Q$ n^{μ} R $\Lambda^2 \ll Q\Lambda \ll Q^2$ Jet constituents: $p^{\mu} \sim (\Lambda, Q, \sqrt{Q\Lambda}) \sim Q(\lambda^2, 1, \lambda)$ SCET Energetic jets usoft $p^{\mu} \sim \Lambda$ collinear $p_c^2 \sim Q\Lambda$, $\lambda = \sqrt{\Lambda/Q}$ $\frac{p^{\mu} = (+, -, \bot)}{Q(\lambda^2, 1, \lambda)} \quad p^{2} \quad \text{fields}$ $\frac{p^{\mu} = (+, -, \bot)}{Q^2 \lambda^2} \quad \xi_n, A_n^{\mu}$ modes collinear $Q(\lambda^2, \lambda^2, \lambda^2) = Q^2 \lambda^4$ q_{us}, A^{μ}_{us} usoft

What makes this EFT different?

 $p^2 = p^+p^- + p_\perp^2$

• Usually $m_1 \gg \Lambda$

 $\sum_{i=1}^{n} C_i(\mu, m_1) O_i(\mu, \Lambda)$

• In SCET constituent $p^- \sim m_b \sim E_{\pi}$

$$\int d\omega \ C(\omega) \ O(\omega)$$





Collinear Propagator

(board)

Power Counting for Collinear Fields

(board)

Currents

eg. $\bar{u} \Gamma b$ involves both collinear and ultrasoft objects

(board)

 $\bar{u} \Gamma b \implies \bar{\xi}_n W \Gamma h_v$



 $W = P \exp\left(ig \int_{-\infty}^{y} ds \,\bar{n} \cdot A_n(s\bar{n}^{\mu})\right)$

Interaction of modes: Offshell versus Onshell (board)

Separate Momenta (multipole expansion)



for labels

$$\mathcal{P}^{\mu}(\phi_{q_1}^{\dagger}\cdots\phi_{p_1}\cdots) = (p_1^{\mu}+\dots-q_1^{\mu}-\dots)(\phi_{q_1}^{\dagger}\cdots\phi_{p_1}\cdots)$$
$$i\partial^{\mu}e^{-ip\cdot x}\phi_n(x) = e^{-ip\cdot x}(\mathcal{P}^{\mu}+i\partial^{\mu})\phi_n(x)$$

• Labels are changed by collinear interactions q p p'

• Labels are preserved by ultrasoft interactions



Power Counting Summary

Туре	(p^+, p^-, p^\perp)	Fields	Field Scaling
collinear	$(\lambda^2, 1, \lambda)$	$\xi_{n,p}$	λ
		$(A_{n,p}^+, A_{n,p}^-, A_{n,p}^\perp)$	$(\lambda^2, 1, \lambda)$
soft	$(\lambda,\lambda,\lambda)$	$q_{s,p}$	$\lambda^{3/2}$
		$A^{\mu}_{s,p}$	λ
usoft	$(\lambda^2,\lambda^2,\lambda^2)$	q_{us}	λ^3
		A^{μ}_{us}	λ^2

Make kinetic terms order $\lambda^0 = \int d^4 X \quad \bar{\xi}_{n,p'} \frac{\bar{\eta}}{2} \left(in \cdot \partial + \dots \right) \xi_{n,p}$ $\lambda^0 = \lambda^{-4} \quad \lambda \qquad \lambda^2 \qquad \lambda$

• At leading power only λ^0 interactions are required **Power counting can be assigned to vertices** $\delta - 4 + \sum (k - 4)(V^c + V^s + V)$

$$\delta = 4 + \sum_{k} (k-4)(V_k^c + V_k^s + V_k^{sc}) + (k-8)V_k^{us}$$

LO SCET Lagrangian

$$\mathcal{L}_{QCD} = \bar{\psi} \, i \not D \, \psi \qquad \text{Write } \psi = \xi_n + \chi_{\bar{n}} \qquad \xi_n = \frac{\not n \, \bar{n} \, \bar{n}}{4} \, \psi$$

$$\mathcal{L} = (\bar{\chi}_{\bar{n}} + \bar{\xi}_n) \left[i \frac{\vec{n}}{2} \, n \cdot D + i \frac{\not n}{2} \, \bar{n} \cdot D + i \not D_\perp \right] (\xi_n + \chi_{\bar{n}}) \qquad \chi_{\bar{n}} = \frac{\vec{n} \, \bar{n} \, \ell}{4} \, \psi$$

$$= \left(\bar{\xi}_n \, \frac{\not n}{2} \, i n \cdot D \, \xi_n \right) + \left(\bar{\chi}_{\bar{n}} \, \frac{\not n}{2} \, i \bar{n} \cdot D \, \chi_{\bar{n}} \right) + \left(\bar{\xi}_n \, i \not D_\perp \, \chi_{\bar{n}} \right) + \left(\bar{\chi}_{\bar{n}} \, i \not D_\perp \, \xi_n \right)$$
e.o.m:
$$\frac{\delta}{\delta \bar{\chi}_{\bar{n}}} : \quad i \bar{n} \cdot D \chi_{\bar{n}} + \frac{\not n}{2} \, i \not D_\perp \xi_n = 0 \qquad \chi_{\bar{n}} = \frac{1}{i \bar{n} \cdot D} i \not D_\perp \frac{\not n}{2} \, \xi_n$$

$$\mathcal{L} = \bar{\xi}_n \left(i\bar{n} \cdot D + i \not D_\perp \frac{1}{i\bar{n} \cdot D} i \not D_\perp \right) \frac{n}{2} \xi_n$$

(board)

$$\mathcal{L}_{c}^{(0)} = \bar{\xi}_{n} \left\{ n \cdot i D_{us} + gn \cdot A_{n} + i \mathcal{D}_{\perp}^{c} \frac{1}{i\bar{n} \cdot D_{c}} i \mathcal{D}_{\perp}^{c} \right\} \frac{n}{2} \xi_{n}$$

That was tree level.

Use Symmetries

Power counting, Gauge symmetry, Discrete, Lorentz invariance (?)

Gauge symmetry

 $U(x) = \exp\left[i\alpha^A(x)T^A\right]$

need to consider U'scollinear $i\partial^{\mu}\mathcal{U}_{c}(x) \sim p_{c}^{\mu}\mathcal{U}_{c}(x) \leftrightarrow A_{n,q}^{\mu}$ which leave us in the EFTusoft $i\partial^{\mu}U_{us}(x) \sim p_{c}^{\mu}U_{us}(x) \leftrightarrow A_{us}^{\mu}$

Object	Collinear \mathcal{U}_c	Usoft U_{us}
ξ_n	$\mathcal{U}_c \ \xi_n$	$U_{us}\xi_n$.
gA_n^μ	$\mathcal{U}_c g A^{\mu}_n \mathcal{U}^{\dagger}_c + \mathcal{U}_c \big[i \mathcal{D}^{\mu}, \mathcal{U}^{\dagger}_c \big]$	$U_{us}gA^{\mu}_nU^{\dagger}_{us}$
W	$\mathcal{U}_c W$	$U_{us} W U_{us}^{\dagger}$
q_{us}	q_{us}	$U_{us} q_{us}$
gA^{μ}_{us}	gA^{μ}_{us}	$U_{us}gA^{\mu}_{us}U^{\dagger}_{us} + U_{us}[i\partial^{\mu}, U^{\dagger}_{us}]$
Y	Y	$U_{us}Y$

reconsider current (board)

Reparameterization Invariance (RPI)

 n, \bar{n} break Lorentz invariance, restored within collinear cone by RPI, three types

(I)
$$\begin{cases} n_{\mu} \to n_{\mu} + \Delta_{\mu}^{\perp} \\ \bar{n}_{\mu} \to \bar{n}_{\mu} \end{cases}$$
 (II)
$$\begin{cases} n_{\mu} \to n_{\mu} \\ \bar{n}_{\mu} \to \bar{n}_{\mu} + \varepsilon_{\mu}^{\perp} \end{cases}$$
 (III)
$$\begin{cases} n_{\mu} \to (1+\alpha) n_{\mu} \\ \bar{n}_{\mu} \to (1-\alpha) \bar{n}_{\mu} \end{cases}$$

eg. rules out

$$\bar{\xi}_n i D_\perp^\mu \, \frac{1}{i\bar{n}\cdot D} \, i D_\mu^\perp \, \frac{\bar{\eta}}{2} \xi_n$$

$$\mathcal{L}_{c}^{(0)} = \bar{\xi}_{n} \left\{ n \cdot iD_{us} + gn \cdot A_{n} + i \not\!\!\!D_{\perp}^{c} \frac{1}{i\bar{n} \cdot D_{c}} i \not\!\!\!D_{\perp}^{c} \right\} \frac{\hbar}{2} \xi_{n}$$

Wilson Coefficients and Hard - Collinear Factorization

 $C(\bar{\mathcal{P}},\mu)$: they depend on large momenta picked out by $\bar{\mathcal{P}} = \bar{n}\cdot\mathcal{P} \sim \lambda^0$

eg.
$$C(-\bar{\mathcal{P}},\mu) \left(\bar{\xi}_n W\right) \Gamma h_v = \left(\bar{\xi}_n W\right) \Gamma h_v C(\bar{\mathcal{P}}^{\dagger},\mu)$$

only the product is gauge invariant

Write

$$\left(\bar{\xi}_n W\right)\Gamma h_v C(\bar{\mathcal{P}}^{\dagger},\mu) = \int d\omega C(\omega,\mu) \left[\left(\bar{\xi}_n W\right) \delta(\omega - \bar{\mathcal{P}}^{\dagger})\Gamma h_v \right] = \int d\omega C(\omega,\mu) O(\omega,\mu)$$

In general:

 $f(i\bar{n} \cdot D_c) = Wf(\bar{\mathcal{P}})W^{\dagger}$ $= \int d\omega \ f(\omega) \ [W\delta(\omega - \bar{\mathcal{P}})W^{\dagger}]$ $= \int d\omega \ f(\omega) \ [W\delta(\omega - \bar{\mathcal{P}})W^{\dagger}]$ in collinear operator $p^2 \sim Q^2 \lambda^2$

hard-collinear factorization follows from properties of SCET operators Multipole Expansion and Ultrasoft - Collinear Factorization

Multipole Expansion:

(board)

$$\mathcal{L}_{c}^{(0)} = \bar{\xi}_{n} \left\{ n \cdot i D_{us} + gn \cdot A_{n} + i \not D_{\perp}^{c} \frac{1}{i \bar{n} \cdot D_{c}} i \not D_{\perp}^{c} \right\} \frac{\hbar}{2} \xi_{n}$$

usoft gluons have eikonal Feynman rules and induce eikonal propagators

Field Redefinition: (board)

 $\xi_n \to Y \xi_n \ , \ A_n \to Y A_n Y^{\dagger}$

$$Y(x) = P \exp\left(ig \int_{-\infty}^{0} ds \, n \cdot A_{us}(x+ns)\right)$$

 $n \cdot D_{us} Y = 0, Y^{\dagger} Y = 1$

gives:

$$\mathcal{L}_{c}^{(0)} = \bar{\xi}_{n} \left\{ n \cdot i D_{\mathrm{us}} + \dots \right\} \frac{\bar{\eta}}{2} \xi_{n} \to \bar{\xi}_{n} \left\{ n \cdot i D_{c} + i \mathcal{D}_{\perp}^{c} \frac{1}{i \bar{n} \cdot D_{c}} i \mathcal{D}_{\perp}^{c} \right\} \frac{\bar{\eta}}{2} \xi_{n}$$

Moves all usoft gluons to operators, simplifies cancellations

Field Theory gives the same results pre- and post- field redefinition, but the organization is different

Ultrasoft - Collinear Factorization:

eg1. $J = (\bar{\xi}_n W)_{\omega} \Gamma h_v \to (\bar{\xi}_n Y^{\dagger} Y W Y^{\dagger})_{\omega} \Gamma h_v = (\bar{\xi}_n W)_{\omega} \Gamma (Y^{\dagger} h_v)$

note: not upset by hard-collinear mometum fraction since ultrasoft gluons carry no hard momenta

(board) so usoft-collinear factorization is also simply a property of SCET

eg2. No ultrasoft fields

 $J = (\bar{\xi}_n W)_{\omega_1} \Gamma(W^{\dagger} \xi_n)_{\omega_2} \to (\bar{\xi}_n W)_{\omega_1} Y^{\dagger} Y \Gamma(W^{\dagger} \xi_n)_{\omega_2} = (\bar{\xi}_n W)_{\omega_1} \Gamma(W^{\dagger} \xi_n)_{\omega_2}$

color transparency

IR divergences, Matching, & Running $J^{\text{QCD}} = \bar{s} \Gamma b \qquad J^{\text{SCET}} = (\bar{\xi}_n W)_{\omega} \Gamma h_v \qquad \Gamma = \sigma^{\mu\nu}$



Running $\int d\omega \ C(\omega) \ O(\omega)$

$$\omega = m_b \text{ in } B \to X_s \gamma$$
counterterm
$$Z = 1 + \frac{\alpha_s}{3\pi} \left[\frac{1}{\epsilon^2} + \frac{2}{\epsilon} \ln\left(\frac{\mu}{\omega}\right) + \frac{5}{2\epsilon} \right]$$

RGE
$$\mu \frac{\partial}{\partial \mu} C(\mu) = -\frac{4\alpha_s(\mu)}{3\pi} \ln\left(\frac{\mu}{\omega}\right) C(\mu) + \dots$$

(board)

Endpoint $B \to X_s \gamma$

(board)

Endpoint $B \to X_s \gamma$

Optical Thm: $\Gamma \sim \text{Im} \int d^4x \ e^{-iq \cdot x} \langle B | T\{J^{\dagger}_{\mu}(x) J^{\mu}(0)\} | B \rangle$

$\leq \uparrow^q$		$\leq q$		$P_X^2 = m_B(m_B - 2E_\gamma)$
Z'	<i>S</i>	$-\infty$	standard OPE	$\sim m_B^2$
b	$\overrightarrow{P_X}$	b	endpoint region	$\sim m_B \Lambda_{QCD}$
$p_{\rm B}$			resonance region	$\sim \Lambda^2_{QCD}$

For EndPoint: $E_{\gamma} \gtrsim 2.2 \,\text{GeV}, X_s$ collinear, *B* usoft, $\lambda = \sqrt{\frac{\Lambda_{QCD}}{m_B}}$

Decay rate is given by factorized form

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_{\gamma}} = H(m_b,\mu) \int_{2E_{\gamma}-m_b}^{\bar{\Lambda}} dk^+ S(k^+,\mu) J(k^++m_b-2E_{\gamma},\mu)$$

Match:
$$\bar{s}\Gamma_{\mu}b \to e^{i(m_bv-\mathcal{P})\cdot x}C(\bar{\mathcal{P}})\bar{\xi}_{n,p}W\gamma^{\perp}_{\mu}P_Lh_v$$

$$T^{\mu}_{\mu} = \int d^4x \ e^{i(m_b \frac{\bar{n}}{2} - q) \cdot x} \ \left\langle B \left| T J^{\dagger}_{\text{eff}}(x) J_{\text{eff}}(0) \right| B \right\rangle \qquad \begin{array}{l} \text{label conservation} \\ \bar{\mathcal{P}} \to m_b \end{array}$$



$$T^{\mu}_{\mu} = |C(m_b)|^2 \int d^4x \, e^{i(m_b \frac{\bar{n}}{2} - q) \cdot x} \left\langle B \middle| T[\bar{h}_v Y](x) \left[Y^{\dagger} h_v \right](0) \middle| B \right\rangle$$
$$\times \left\langle 0 \middle| T[W^{(0)\dagger} \xi^{(0)}_n](x) [\bar{\xi}^{(0)}_n W^{(0)}](0) \middle| 0 \right\rangle \times [\Gamma_\mu \otimes \Gamma^\mu]$$



$$\operatorname{Im} T_{\mu}^{\mu} = |C(m_b)|^2 \int dl^+ S(l^+) \operatorname{Im} J_P(m_b - 2E_{\gamma} + l^+)$$

where $S(l^+) = \langle B | \bar{h}_v \delta(in \cdot D - l^+) h_v | B \rangle$





SCET is a field theory which:

- explains how these degrees of freedom communicate with each other, and with hard interactions
- organizes the interactions in a series expansion in
- provides a simple operator language to derive factorization theorems in fairly general circumstances
 - eg. unifies the treatment of factorization for exclusive and inclusive QCD processes
- new symmetry constraints
- scale separation & decoupling



 $\Lambda_{\rm QCD}$

How is SCET used?

- cleanly separate short and long distance effects in QCD
 - → derive new factorization theorems
 - → find universal hadronic functions, exploit symmetries & relate different processes
- model independent, systematic expansion
 - → study power corrections
- keep track of μ dependence

→ sum logarithms, reduce uncertainties

The End