Effective Field Theory

Iain Stewart
MIT

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Outline

Lecture I

- Principles, Operators, Power Counting , Matching, Loops,
  Using Equations of Motion, Renormalization and Decoupling

Lecture II

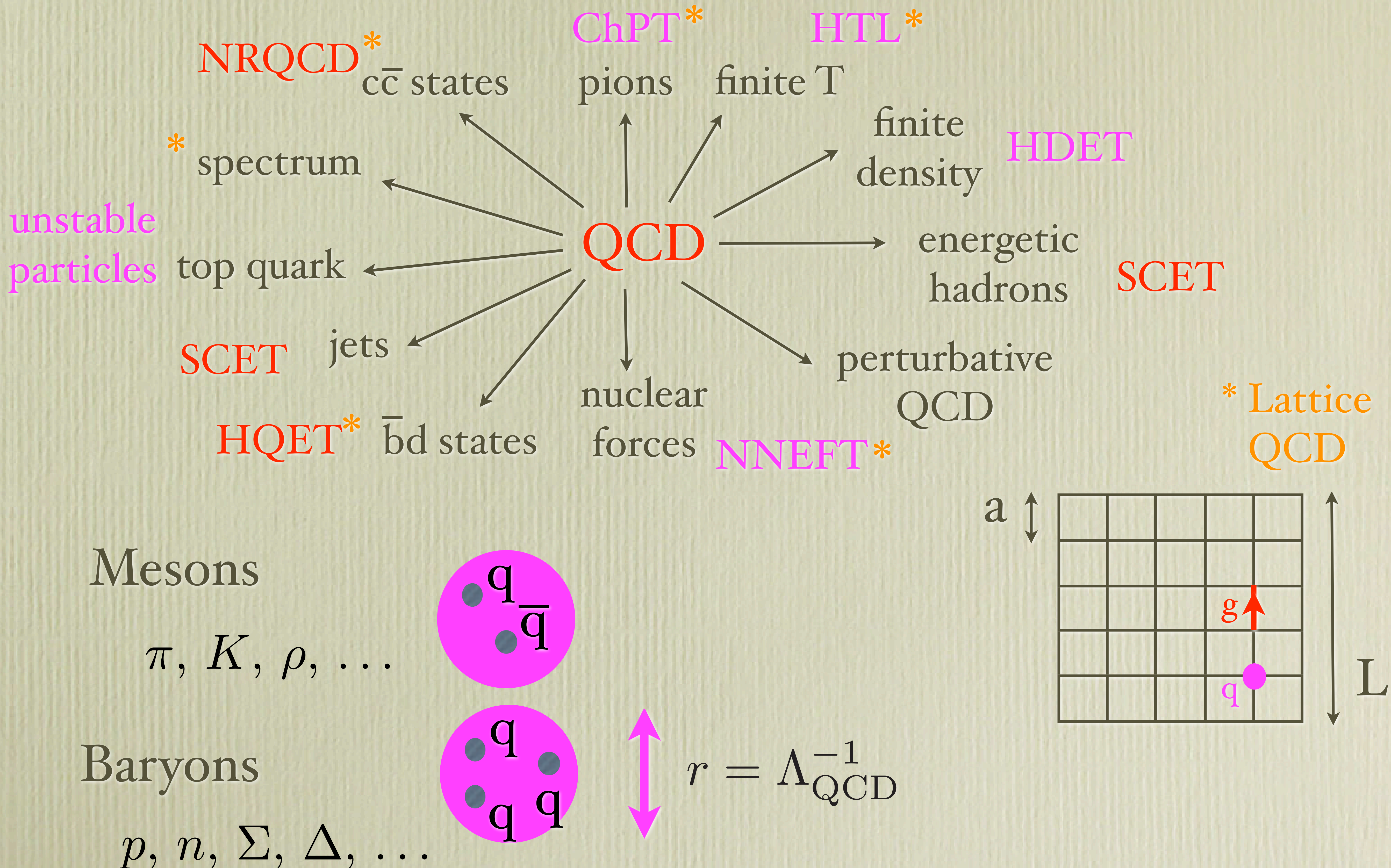
Summing Large Logarithms, α_s matching,
power counting without mass dimension

- Weak Interactions at low energy
- Heavy Quark Effective Theory
  (An Effective Theory for Static Sources)
- Non-relativistic QCD and QED

Lecture III

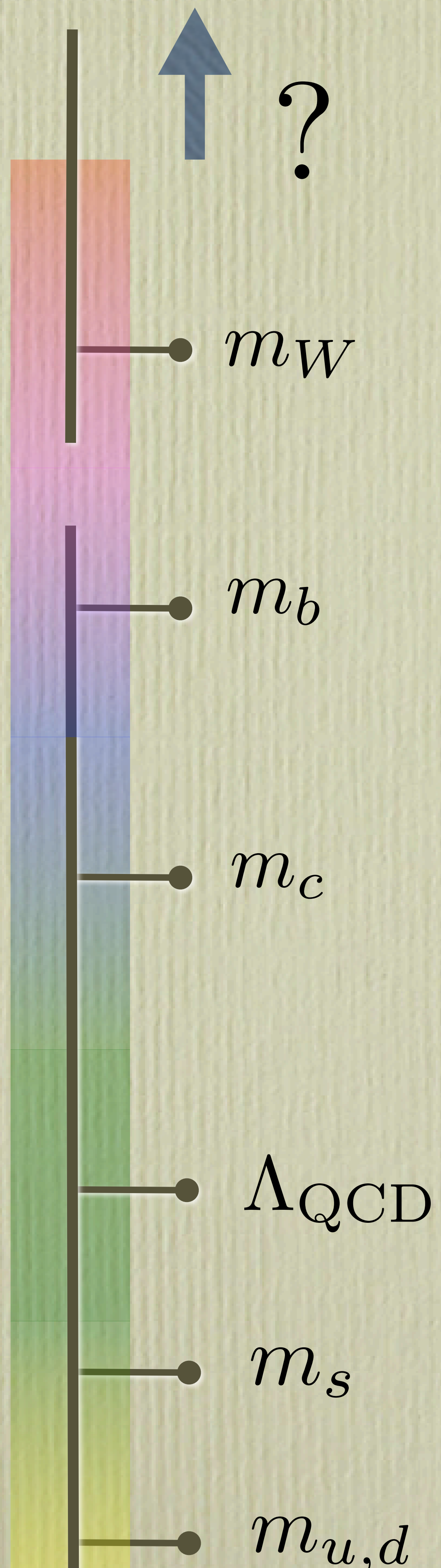
Soft - Collinear Effective Theory

Effective Field Theories of QCD



Lecture II

Weak Interactions at Low Energy



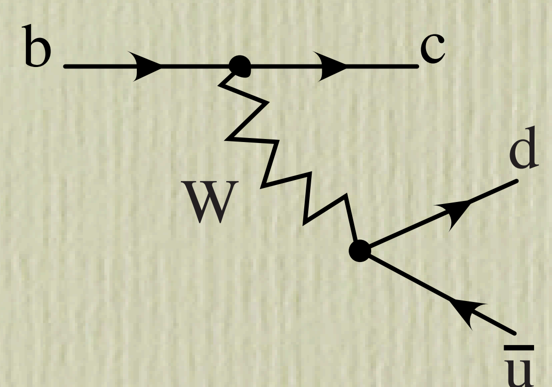
eg. $b \to c\bar{u}d$

(Matching & Running)

$$m_W \gg m_b$$

1) Tree level matching

$k = p_b - p_c = p_u + p_d \quad \sim$ masses

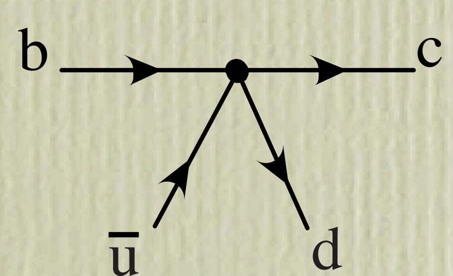


$$= \left(\frac{ig_2}{\sqrt{2}}\right)^2 V_{cb} V_{ud}^* (-i) \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_W^2}\right) \frac{1}{k^2 - m_W^2}$$
$$\times \left[\bar{u}^c \gamma_\mu P_L u^b\right] \left[\bar{u}^d \gamma_\nu P_L v^u\right]$$

expand

$G_F = \sqrt{2} g_2^2 / (8 m_W^2)$

$$= -i \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[\bar{u}^c \gamma_\mu P_L u^b\right] \left[\bar{u}^d \gamma^\mu P_L v^u\right] + \ldots$$



demand the same for this Feynman rule to get

$$H_F = -\mathcal{L}_F = \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \left[\bar{c} \gamma_\mu P_L b\right] \left[\bar{d} \gamma^\mu P_L u\right]$$

2) Most General Operator Basis

$b \to c\bar{u}d$

$$H_F = \frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \Big[C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu) \Big]$$

- at $\mu = m_W$ can treat b,c,d,u as massless to get C's, massless perturbative QCD does not change chirality $\Rightarrow$ odd # of γ 's

  reduce to one gamma matrix

  $\bar{c}\gamma^\mu P_L b$

- Two color structures give an overall singlet

$$O_1 = [\bar{c}\gamma_\mu P_L b][\bar{d}\gamma^\mu P_L u]$$
$$O_2 = [\bar{c}^\beta \gamma_\mu P_L b_\alpha][\bar{d}^\alpha \gamma^\mu P_L u_\beta]$$

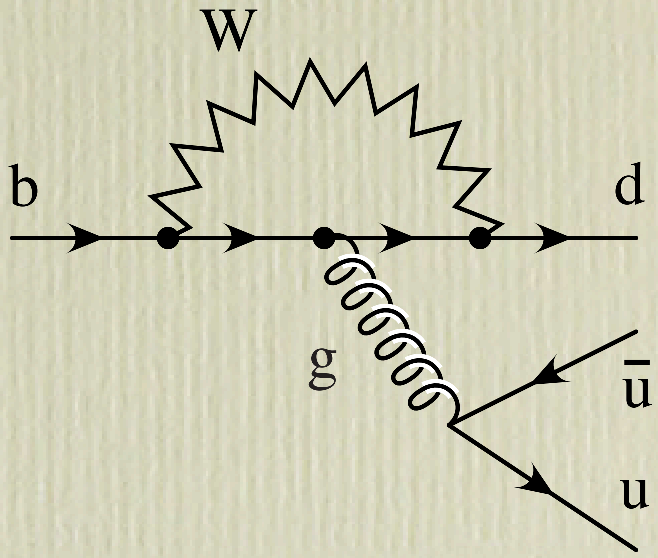
Tree level matching

$$C_1 = 1 \ + \mathcal{O}(\alpha_s(m_W))$$
$$C_2 = 0 \ + \mathcal{O}(\alpha_s(m_W))$$

Important: Matching is indep. of choice of states (& IR regulator) as long as same choice is made in both theories

We used free quark states, but result is valid for use with bound states, eg. $B \to D\pi$

Exercise 4: Construct a complete basis of four quark operators in the EFT for the case where two of the flavors are the same (you can ignore the photon and Z)

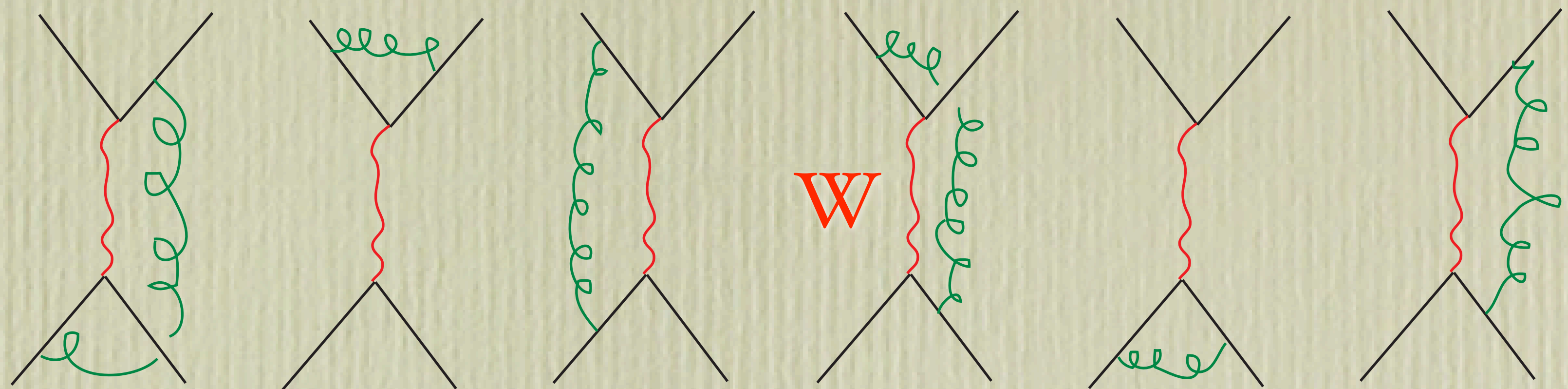


3) Renormalization of EFT at one-loop

(board)

4) Comparing matrix elements at one-loop

full theory



$$A^{\rm full} = \left[1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2}\right] S_1 + \frac{3}{N_c} \frac{\alpha_s}{4\pi} \ln \frac{m_W^2}{-p^2} S_1 + (S_2 \text{ terms, finite terms})$$

$$\langle O_1 \rangle = \left[1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2}\right] S_1 + \frac{3}{N_c} \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2} S_1 \; + (S_2 \text{ terms, finite terms})$$

- EFT computation is easier
- In EFT $m_W \to \infty$, so m_W's become μ's (cutoffs)
- $\ln(-p^2)$ terms agree, so IR agrees (a check that EFT has right d.o.f.)

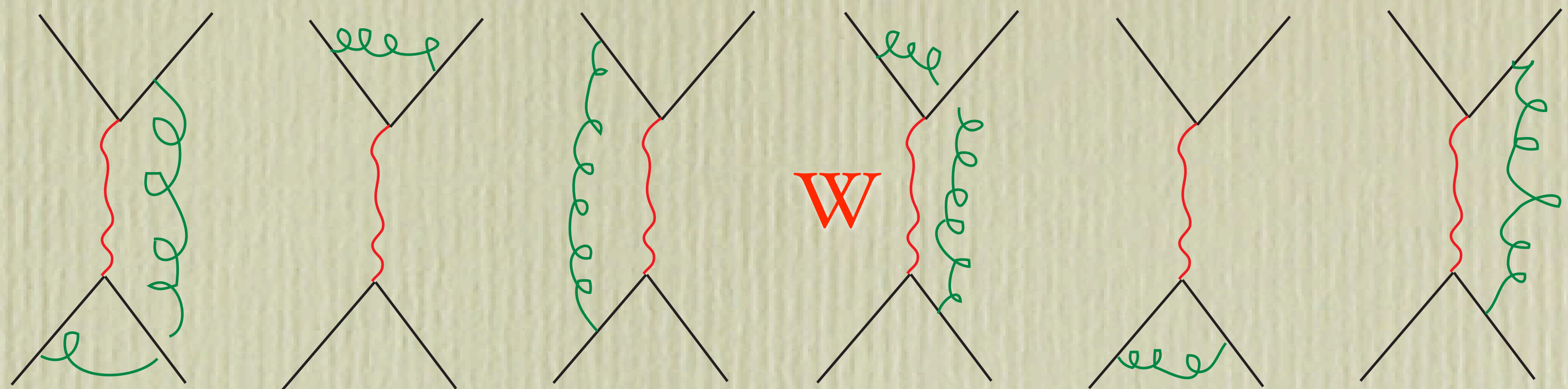
C's are determined by difference, $A^{\rm full} - \langle O_1 \rangle$
and are independent of the IR regulator

(they do depend on the scheme choosen in the EFT)

$$C_i(\mu) = \# \frac{\alpha_s}{\pi} \ln \frac{\mu^2}{m_W^2} + \# \frac{\alpha_s}{\pi}$$

4) Comparing matrix elements at one-loop

full theory



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- split $C(\mu)O(\mu)$ is full theory = short distance $*$ long distance

$$\ln \frac{m_W^2}{-p^2} = \ln \frac{m_W^2}{\mu^2} + \ln \frac{\mu^2}{-p^2}$$

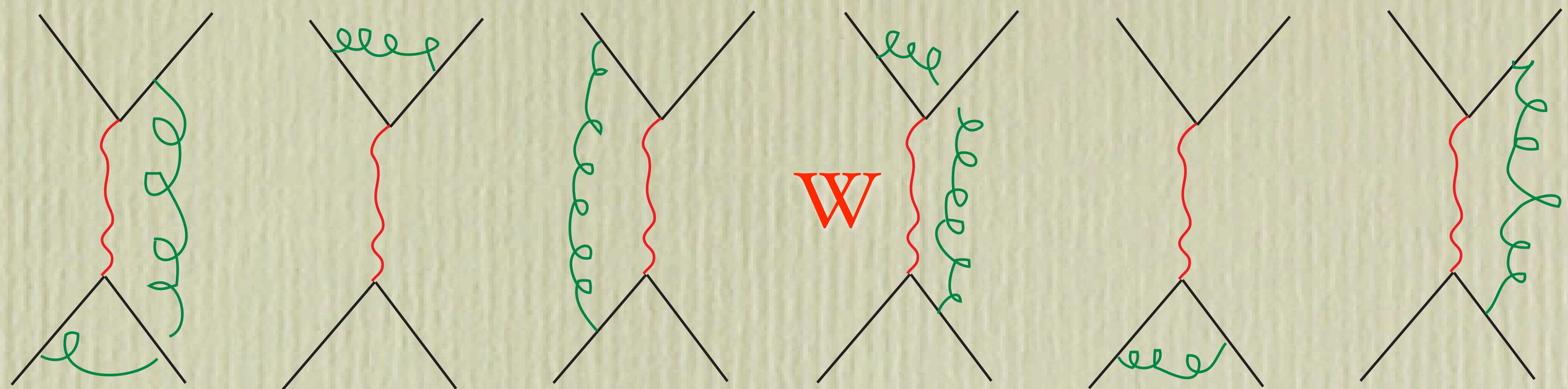
$$\left(1 + \alpha_s \ln \tfrac{m_W^2}{-p^2}\right) = \left(1 + \alpha_s \ln \tfrac{m_W^2}{\mu^2}\right) * \left(1 + \alpha_s \ln \tfrac{\mu^2}{-p^2}\right)$$

renormalization scale in EFT acts like a “factorization” scale

- one-loop in full theory has less information, ie. misses C’s on RHS and no mixing through RGE
- order by order in α_s the $\ln(\mu)'s$ in $C(\mu)$ and $\langle O(\mu)\rangle$ cancel

4) Comparing matrix elements at one-loop

full theory



$$A^{\text{full}} = \left[1 + 2C_F \frac{\alpha_s}{4\pi} \ln \frac{\mu^2}{-p^2}\right] S_1 + \frac{3}{N_c} \frac{\alpha_s}{4\pi} \ln \frac{m_W^2}{-p^2} S_1 + (S_2 \text{ terms, finite terms})$$

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- EFT scheme dependence cancels between $C(\mu)$ and $\langle O(\mu) \rangle$

eg. NLL

$$C(\mu) = \left[1 + \frac{\alpha_s(\mu)}{4\pi} J\right] \underbrace{\left(\frac{\alpha_s(m_W)}{\alpha_s(\mu)}\right)^{\frac{\gamma^{(0)}}{2\beta_0}}}_{\text{scheme independent}} \left[1 + \frac{\alpha_s(m_W)}{4\pi} K\right]$$

scheme dependent
cancels with $\langle O(\mu) \rangle$

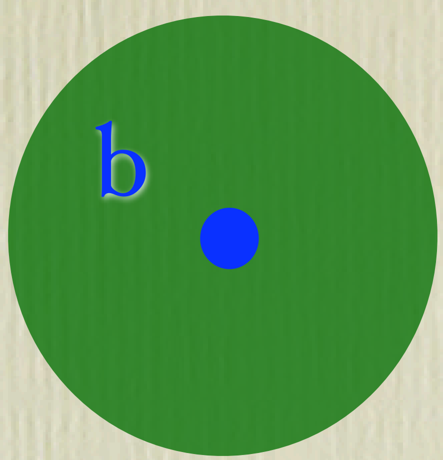
scheme independent,
a cancellation between
matching and anom.dim.

Heavy Quark Effective Theory

A low energy EFT for heavy particles that are not removed from the theory (static sources that perturbations can cause to wiggle)

$$v^\mu = (1,0,0,0)$$

B-meson



$$m_b \gg \Lambda_{\rm QCD}$$

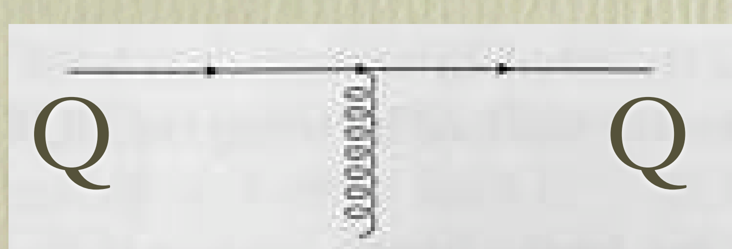
Want to describe fluctuations of heavy quark Q, due to lighter degrees of freedom.

- At LO, light d.o.f. have QCD Lagrangian
- $\lim\limits_{m_Q\to\infty} \mathcal{L}_{\rm QCD} = \lim\limits_{m_Q\to\infty} \bar{Q}(i\not{D} - m_Q)Q$?

Propagator: $p^\mu = m_Q v^\mu + k^\mu \qquad k^\mu \sim \Lambda_{\rm QCD}$

$$\frac{\not{p}+m_Q}{p^2-m_Q^2+i\epsilon} = \frac{m_Q(\not{v}+1)+\not{k}}{2m_Q\, v\cdot k + k^2 + i\epsilon} = \frac{1+\not{v}}{2}\,\frac{1}{v\cdot k + i\epsilon} + \mathcal{O}(1/m_Q)$$

Vertex:



$$\frac{(1+\not{v})}{2}\,\gamma^\mu\,\frac{(1+\not{v})}{2} = \underbrace{\frac{(1+\not{v})}{2}\,\frac{(1-\not{v})}{2}}_{0}\gamma^\mu + \frac{(1+\not{v})}{2}\,v^\mu \to v^\mu$$

$$-ig\gamma^\mu T^A = -igv^\mu T^A$$

$$\boxed{\mathcal{L}_{\rm HQET} = \bar{Q}_v iv\cdot D Q_v\,, \qquad \frac{(1+\not{v})}{2}Q_v = Q_v}$$

Direct Derivation

$$\frac{(1+\not{v})}{2}Q_v = Q_v \qquad \not{v}Q_v = Q_v$$

change variables $Q(x) = e^{-im_Q v\cdot x}\left[Q_v(x) + B_v(x)\right]$

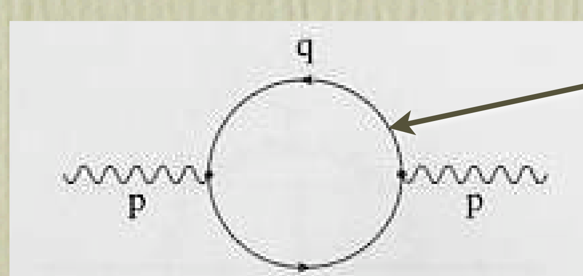
$$\frac{(1-\not{v})}{2}B_v = B_v \qquad \not{v}B_v = -B_v$$

$$\begin{aligned}
\mathcal{L}_{\rm QCD} &= [\bar{Q}_v + \bar{B}_v]e^{im_Q v\cdot x}\{\not{v} iv\cdot D + i\not{D}_T - m_Q\}e^{-im_Q v\cdot x}[Q_v + B_v] \\
&= [\bar{Q}_v + \bar{B}_v]\Big\{(\not{v}-1)m_Q + \not{v} iv\cdot D + i\not{D}_T\Big\}[Q_v + B_v] \\
&= \bar{Q}_v(iv\cdot D)Q_v - \bar{B}_v(iv\cdot D + 2m_Q)B_v + \bar{Q}_v(i\not{D}_T)B_v + \bar{B}_v(i\not{D}_T)Q_v
\end{aligned}$$

So far we've done nothing to QCD

- Take Q_v external particles, then as $m_Q \to \infty$ the B_v particles (ie. anti-particles) decouple

$$\frac{1+\gamma_0}{2}U_{\rm Dirac} = \begin{pmatrix} \psi_v \\ 0 \end{pmatrix}$$



offshell by $2m_Q$

Surviving term is HQET Lagrangian at LO

Comments

$$\mathcal{L}_{\rm HQET} = \bar{Q}_v iv\cdot D Q_v\,, \qquad \frac{(1+\not{v})}{2} Q_v = Q_v$$

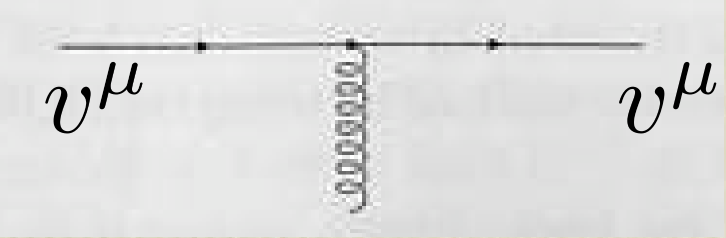
1) Antiparticles are integrated out, number of heavy quarks is preserved, a U(1) symmetry

2) Heavy Quark Spin-Flavor Symmetry U(2 N_Q)

- no flavor (m_Q) dependence
- no dependence on remaining two spin components

3) Velocity v^μ is preserved by low energy QCD interactions

"velocity superselection rule"



4) Power Counting in $1/m_Q$ is now simple!

$$Q_v(x) \sim e^{-ik\cdot x} \qquad i\partial^\mu Q_v(x) \sim \Lambda_{\rm QCD} Q_v(x)$$

all powers of $1/m_Q$ appear in prefactors

$$\mathcal{L}_{\rm HQET} = \mathcal{L}^{(0)}_{\rm HQET} + \sum_{n=1}^{\infty} \frac{1}{m_Q^n} \mathcal{L}^{(n)}_{\rm HQET} \qquad J_{\rm HQET} = J^{(0)}_{\rm HQET} + \sum_{n=1}^{\infty} \frac{1}{m_Q^n} J^{(n)}_{\rm HQET}$$

$$\mathcal{L}_{\rm QCD} = \bar{Q}_v(iv\cdot D)Q_v - \bar{B}_v(iv\cdot D + 2m_Q)B_v + \bar{Q}_v(i\not{D}_T)B_v + \bar{B}_v(i\not{D}_T)Q_v$$

Integrating out the quadratic B_v field at tree level:

$$\frac{\delta\mathcal{L}}{\delta\bar{B}_v} = 0 \qquad (iv\cdot D + 2m_Q)B_v = i\not{D}_T Q_v \qquad B_v = \frac{1}{(iv\cdot D + 2m_Q)} i\not{D}_T Q_v$$

$$\mathcal{L}_{\rm HQET} = \bar{Q}_v(iv\cdot D)Q_v - \frac{c_k}{2m_Q}\bar{Q}_v D_T^2 Q_v - \frac{c_F}{4m_Q}\bar{Q}_v g\sigma_{\mu\nu}G^{\mu\nu}Q_v + \ldots$$

kinetic chromomagnetic

6) This was tree level. If we use symmetry instead we find the same two operators but with Wilson coefficients:

Power counting, Gauge symmetry, Discrete, Lorentz invariance (?)

in rest frame, have rotations, but boosts are broken by $v^\mu = (1,0,0,0)$

$c_k = 1$

Restored by a **Reparameterization Invariance** (board)

Applications

- Spectroscopy (board)

$$m_B = m_b + \overline{\Lambda} - \frac{\lambda_1}{2m_b} - \frac{3\lambda_2(m_b)}{2m_b}$$
$$m_{B^*} = m_b + \overline{\Lambda} - \frac{\lambda_1}{2m_b} + \frac{\lambda_2(m_b)}{2m_b}$$
$$m_D = m_c + \overline{\Lambda} - \frac{\lambda_1}{2m_c} - \frac{3\lambda_2(m_c)}{2m_c}$$
$$m_{D^*} = m_c + \overline{\Lambda} - \frac{\lambda_1}{2m_c} + \frac{\lambda_2(m_c)}{2m_c}$$

- Form factor relations in Exclusive Decays (next,board)

eg. $J^{\mathrm{QCD}}(0) = \bar{q}\Gamma Q \;=\; \bar{q}\Gamma Q_v \quad \text{“=”} \quad \mathrm{tr}\big(\, X\,\Gamma\,\mathcal{H}_v\,\big)$

where $\mathcal{H}_v \equiv \frac{(1+\not{v})}{2}\Big[P^*_{v\mu}\gamma^\mu + iP_v\gamma_5\Big]$

under HQS $\quad Q_v \to D(R)Q_v$, $\quad \mathcal{H}_v \to D(R)\mathcal{H}_v$

- Perturbative Corrections (board)

- Inclusive Decays $B \to X_c \ell \bar{\nu}_\ell$

$$\Gamma = c^{(0)} + \frac{0}{m_b} + c^{(2a)}\frac{\lambda_1}{m_b^2} + c^{(2b)}\frac{\lambda_2}{m_b^2} + \dots$$

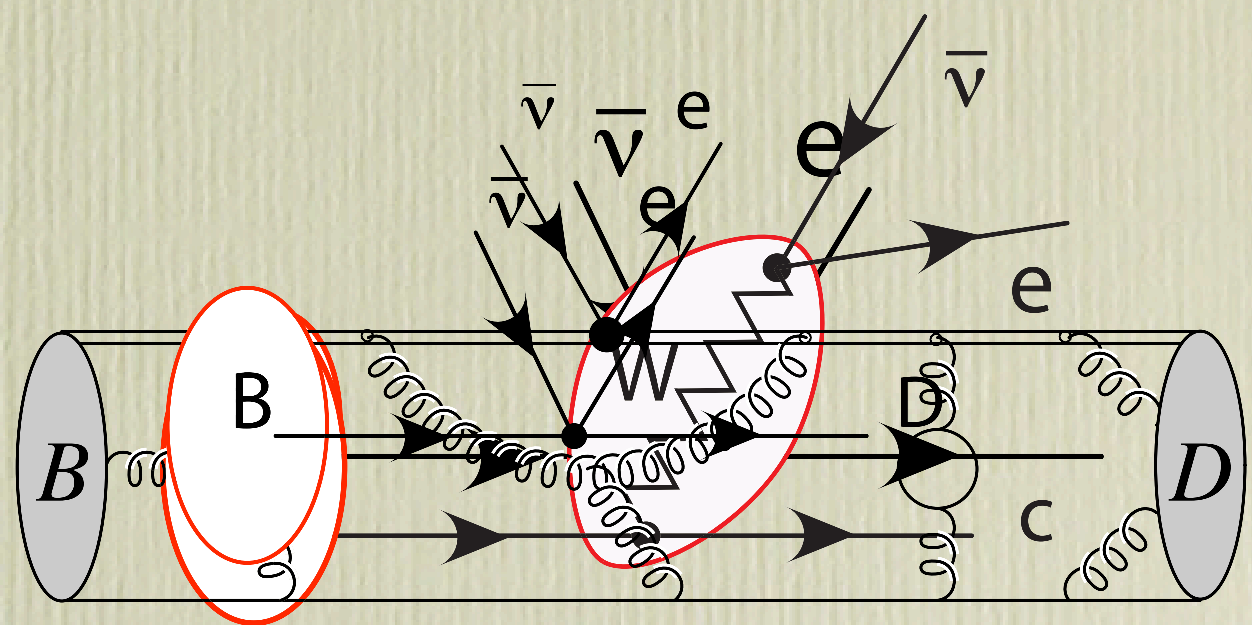
eg. $B \to D\, e\, \bar{\nu}$, $\qquad M_W^2 \gg m_b^2 \gg \Lambda^2$

expansion parameters $\qquad \dfrac{m_b^2}{m_W^2} \simeq \dfrac{1}{250}\,, \quad \alpha_s(m_b) \simeq 0.2, \quad \dfrac{\Lambda}{m_b} \simeq 0.1$

1) Short Distance

$\mu \simeq m_W \simeq 80\,\mathrm{GeV}$

gluons perturbative



Non-relativistic QCD

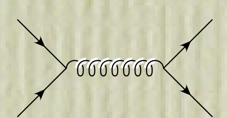
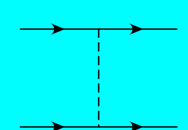
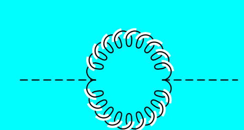
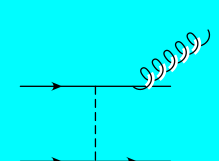
Systems with Two Heavy Particles

eg.			
e^+e^-	$\rightarrow$	positronium	(NRQED)
pe^-	$\rightarrow$	Hydrogen	(NRQED)
$b\bar{b}, c\bar{c}$	$\rightarrow$	$\Upsilon, J/\Psi$	(NRQCD)
$t\bar{t}$	$\rightarrow$	$e^+e^- \rightarrow t\bar{t}$	(NRQCD)
NN	$\rightarrow$	deuteron	(few nucleon EFT)

- $E = p^2/(2m) \sim v^2$, count powers of v $\quad$ (and α_s)

treat $mv^2 \gg \Lambda_{\rm QCD}$

Momentum Regions

	k^0	**k**			
hard:	m	m		integrate these out	
potential:	mv^2	mv		ptnl gluons are not propagating	ψ, χ
soft:	mv	mv		radiative corrections, binding	A_s^μ, q_s
ultrasoft:	mv^2	mv^2		need multipole expansion	A_{us}^μ

$\mathcal{O}(v^0)$ Kinetic Terms give

potential quarks	$\psi, \chi \sim v^{3/2}$	
soft gluons	$A_s^\mu \sim v$	(scale μ_S)
ultrasoft gluons	$A_{us}^\mu \sim v^2$	(scale μ_U)

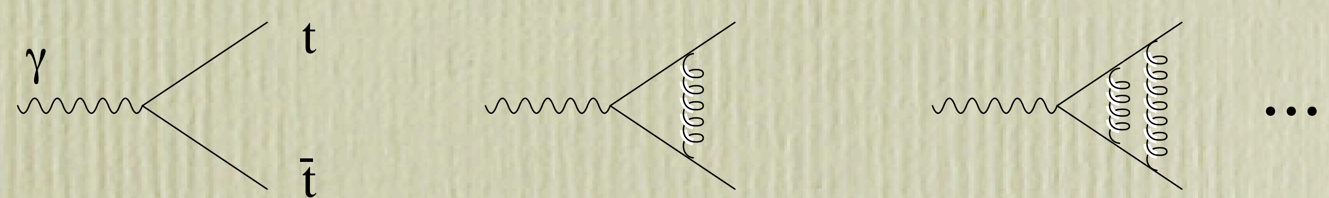
(board)

- Can associate all powers of v with vertices

$$\delta = 5 + \sum_k (k-5)V_k^P + (k-8)V_k^U + (k-4)V_k^S - N_s$$

- Power counting of operators implies power counting of states

Coulombic Singularities $\Longrightarrow$ sum insertions of V_c



1 $\quad (\alpha_s/v)$ $\quad (\alpha_s/v)^2$

Coulombic: $v \sim \alpha_s$

Want an expansion:

$$\sum_n \left(\frac{\alpha_s}{v}\right)^n \Big[\underset{\text{LO}}{1}, \ \underset{\text{NLO}}{\{v, \alpha_s\}}, \ \underset{\text{NNLO}}{\{v^2, \alpha_s v, \alpha_s^2\}}, \ \ldots \Big]$$

Simplify p.c. and Implement multipole expansion

▷ $P = (m, \mathbf{0}) + \mathbf{p} + k$

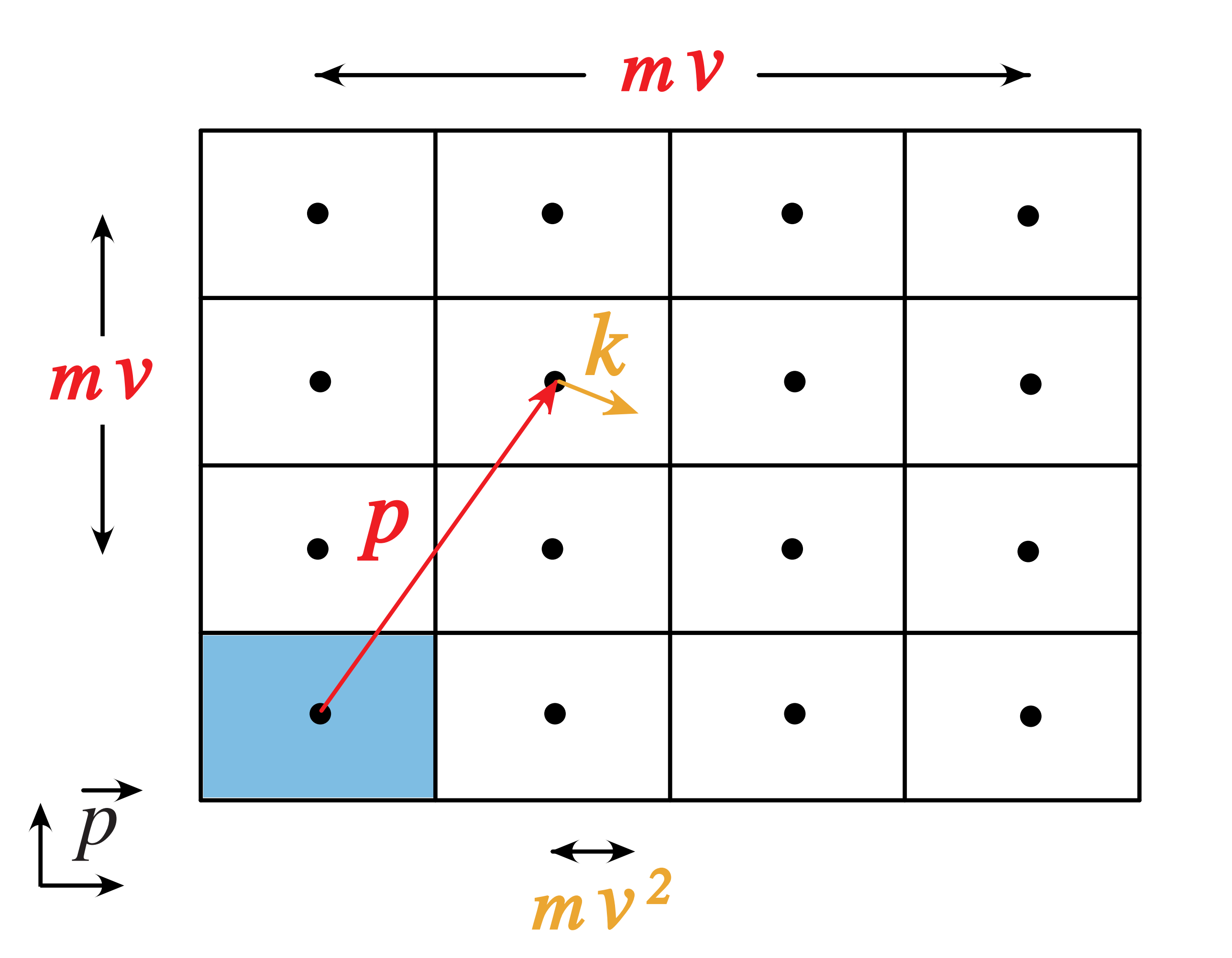
$\qquad m \qquad mv \qquad mv^2$

▷ $\mathbf{p}$ index

▷ $k = (k^0, \mathbf{k})$ continuous

$$\psi(x) = \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} \psi_{\mathbf{p}}(x)$$

$$i\partial^\mu \psi_{\mathbf{p}}(x) \sim (mv^2)\psi_{\mathbf{p}}(x)$$



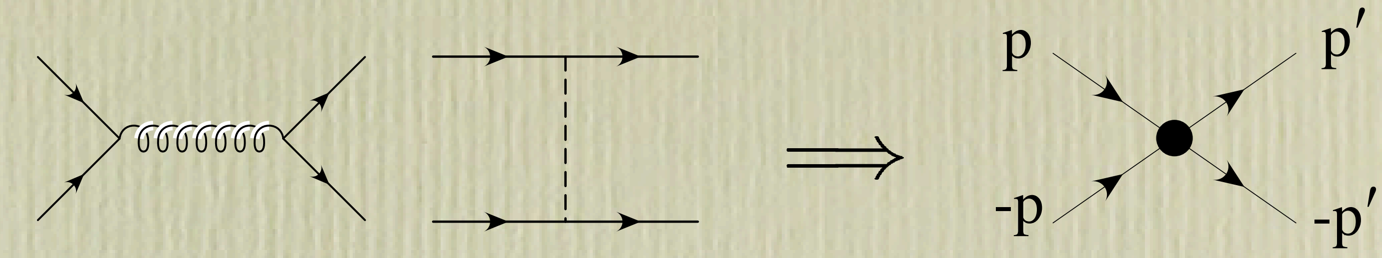
- Can associate all powers of v with vertices

$$\delta = 5 + \textstyle\sum_k (k-5)V_k^P + (k-8)V_k^U + (k-4)V_k^S - N_s$$

(discussion)

$$\mathcal{L}^{\mathbf{NRQCD}} = \mathcal{L}^{\text{ultrasoft}} + \mathcal{L}^{\text{potential}} + \mathcal{L}^{\text{soft}}$$

Potentials



$$\mathcal{L} = -\sum_{\mathbf{p},\mathbf{p}'} V(\mathbf{p},\mathbf{p}')\ {\psi_{\mathbf{p}'}}^{\dagger}\,\psi_{\mathbf{p}}\,{\chi_{-\mathbf{p}'}}^{\dagger}\,\chi_{-\mathbf{p}} \qquad (\mathbf{k}=\mathbf{p}'-\mathbf{p})$$

$$V^{(-1)} = \frac{\mathcal{V}_c}{\mathbf{k}^2}$$

Coulomb: k=4, k-1 = -1 $\quad V_4^p = 1$

$$V^{(0)} = \frac{\mathcal{V}_k\,\pi^2}{m|\mathbf{k}|}$$

$$\delta = 5 + \sum_k (k-5)V_k^P + (k-8)V_k^U + (k-4)V_k^S - N_s$$

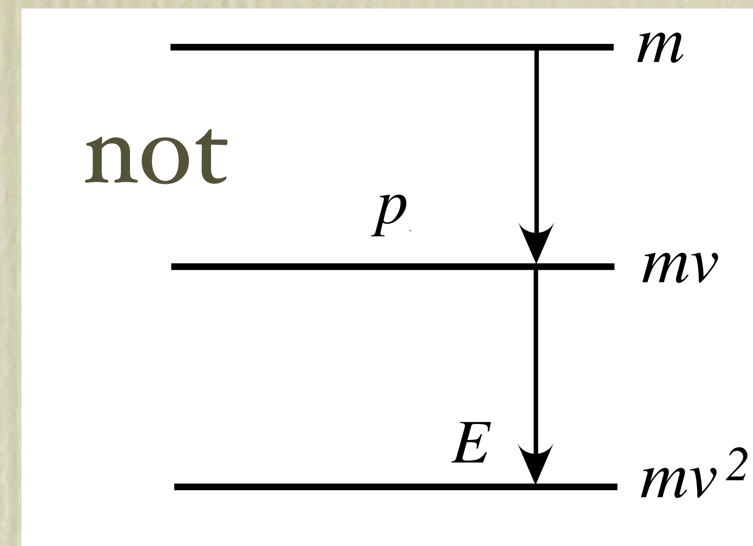
Ultrasoft Lagrangian

(board)

$$\delta = 5 + \sum_k (k-5)V_k^P + (k-8)V_k^U + (k-4)V_k^S - N_s$$

Renormalization $\quad \mu = m \to mv \to mv^2 \;\; ?$

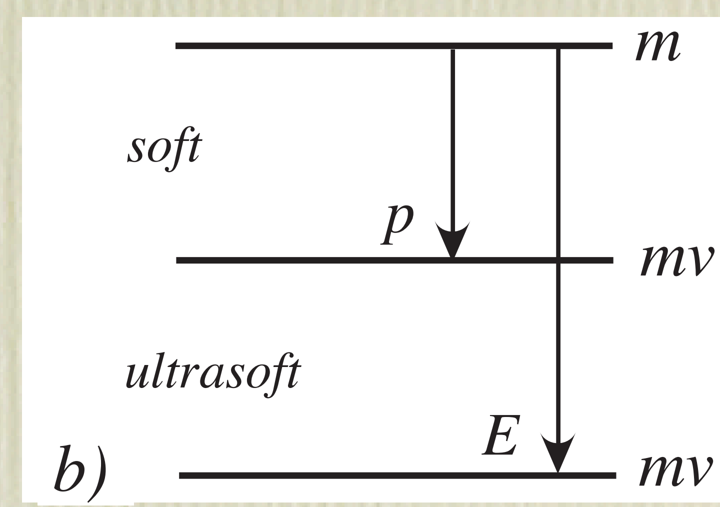
- problematic since scales are coupled, $E = p^2/m$
- $E \lesssim \Lambda \quad \leftrightarrow \quad p \lesssim \sqrt{m\Lambda}$



Power counting implies $mv^2 \leftrightarrow mv$ correlation

in dimensional regularization

- $\psi \sim (mv)^{3/2-\epsilon}$, $A_s^\mu \sim (mv)^{1-\epsilon}$, $A_{us}^\mu \sim (mv^2)^{1-\epsilon}$
- implies we need $\mu_U = \mu_S^2/m \sim mv^2$, so take $\mu_U = m\nu^2$, $\mu_S = m\nu$
- uniquely fixes powers of μ_U, μ_S in operators



run: $\quad \nu = 1 \to v_0 \sim v$

Why bother?

- renormalize the field theory consistenty
- would like to know at what scales to evaluate coefficients, $\alpha_s(m)$, $\alpha_s(mv)$, $\alpha_s(mv^2)$ + others: currents $c_{1,2}$, potentials $\mathcal{V}_i$
- equivalent to summing logs $\alpha_s \ln\left(\frac{m}{mv}\right)$, $\alpha_s \ln\left(\frac{m}{mv^2}\right)$

eg. Logs in Binding energies

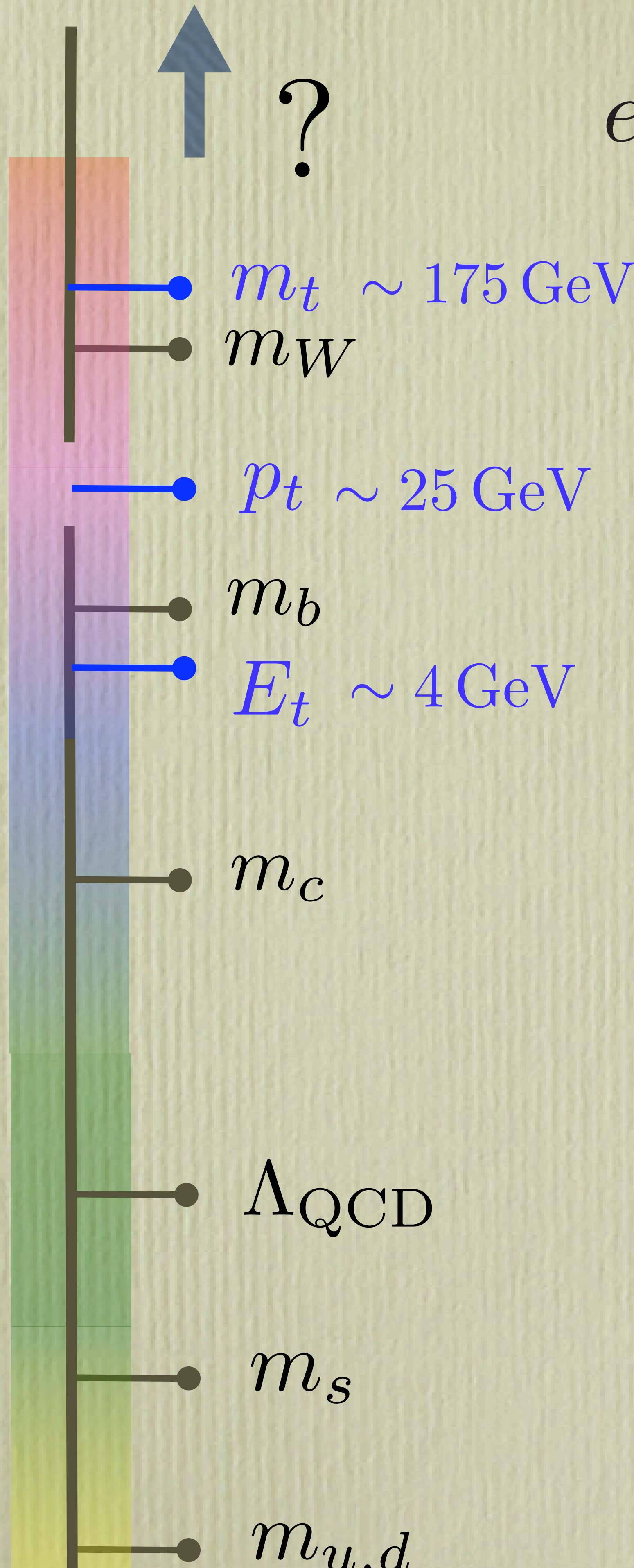
$\nu \frac{d}{d\nu} = \mu_S \frac{\partial}{\partial \mu_S} + 2\mu_U \frac{\partial}{\partial \mu_U}$

LL anomalous dimension $\alpha^4(\alpha \ln \alpha)^k$

NLL anomalous dimension $\alpha^5(\alpha \ln \alpha)^k$

$\alpha^8 \ln^3 \alpha$	Lamb	H	agree*
		μ^+e^-, e^+e^-	new
	(no h.f.s.)		
$\alpha^4 \ln^3 \alpha$	(no $\Delta\Gamma/\Gamma$)		
$\alpha^7 \ln^2 \alpha$	Lamb	H, μ^+e^-, e^+e^-	agree
	h.f.s.	H, μ^+e^-, e^+e^-	agree
$\alpha^3 \ln^2 \alpha$	$\Delta\Gamma/\Gamma$	e^+e^- ortho and para	agree
$\alpha^6 \ln \alpha$	Lamb	H, μ^+e^-, e^+e^-	agree
	h.f.s.	H, μ^+e^-, e^+e^-	agree
$\alpha^2 \ln \alpha$	$\Delta\Gamma/\Gamma$	e^+e^- ortho and para	agree

- Notation: μ^+e^- indicates agreement for $1/m_\mu$ (recoil) terms



$$e^+e^- \to t\bar{t}$$

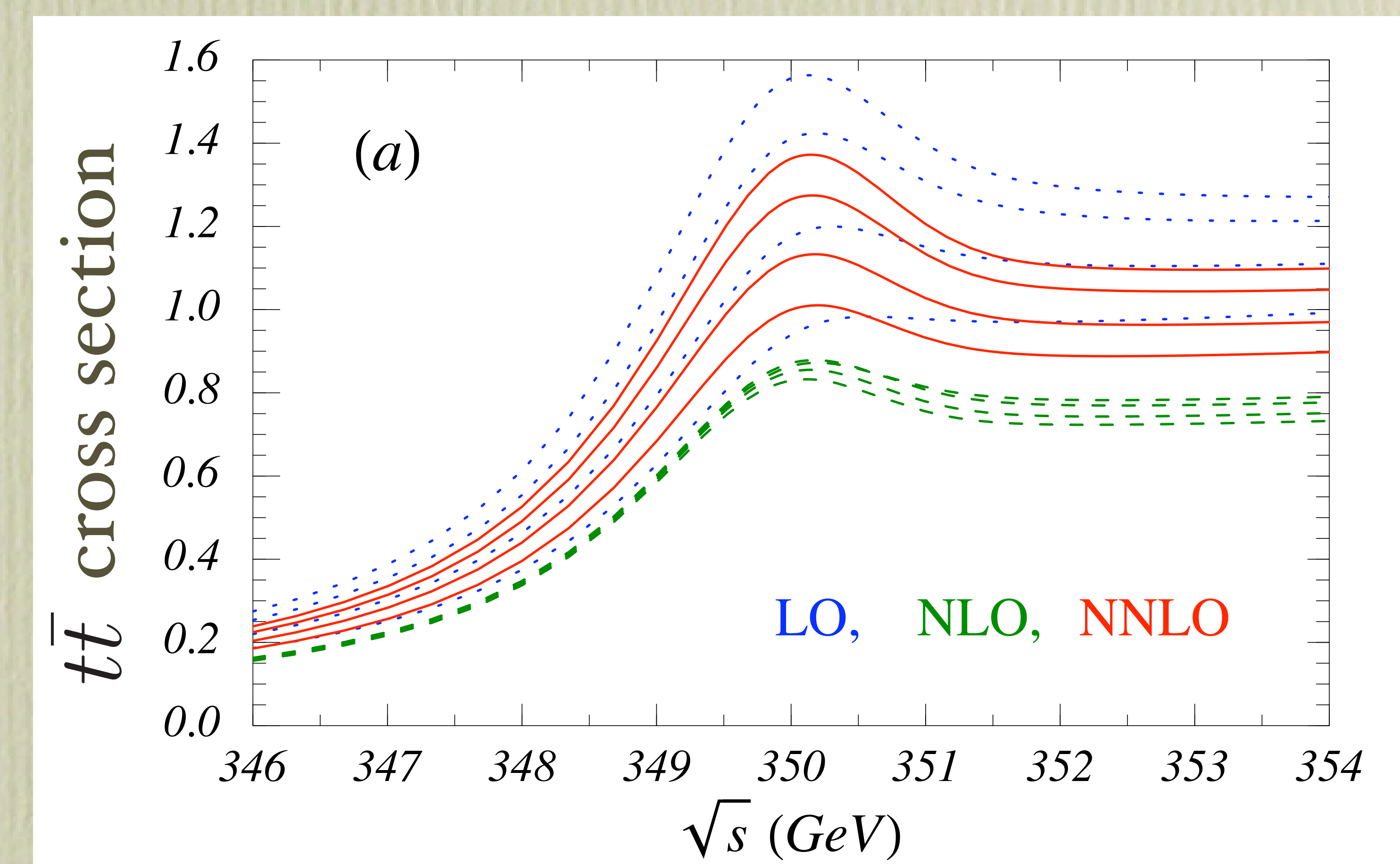
Nonrelativistic QCD bound states?

$$\Gamma_t = 1.4\,\mathrm{GeV} \gg \Lambda_{\mathrm{QCD}}$$

top decays before it hadronizes

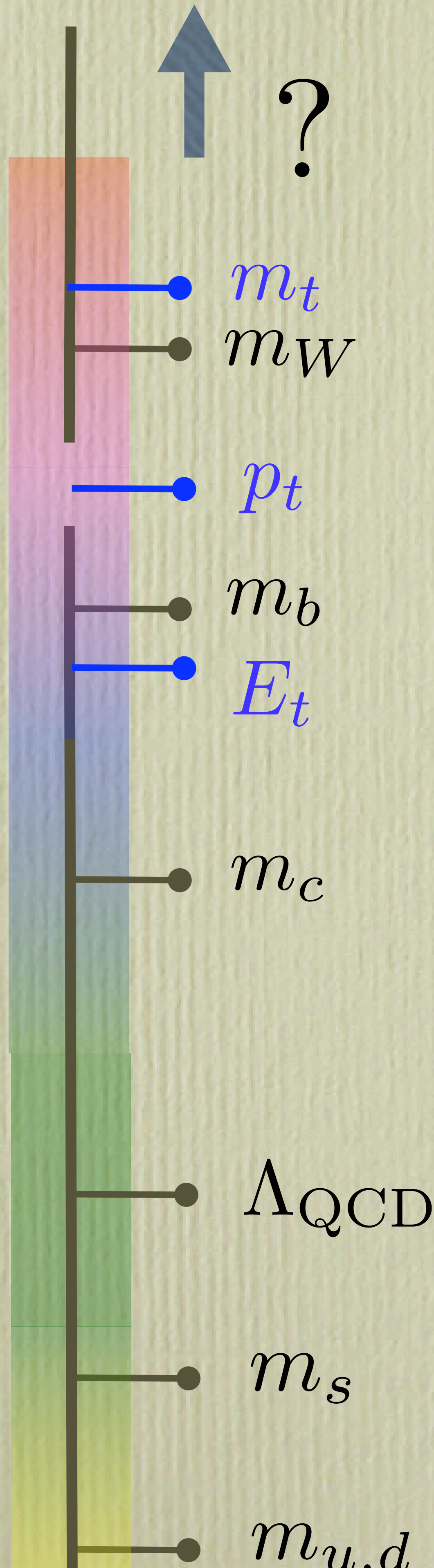
Coulombic, expansion in $\alpha_s(\mu)$:

$$\mathrm{LO} + \mathrm{NLO} + \mathrm{NNLO} + \ldots$$



vary μ

$\mu = m_t, p_t, E_t$?



Nonrelativistic QCD bound states?

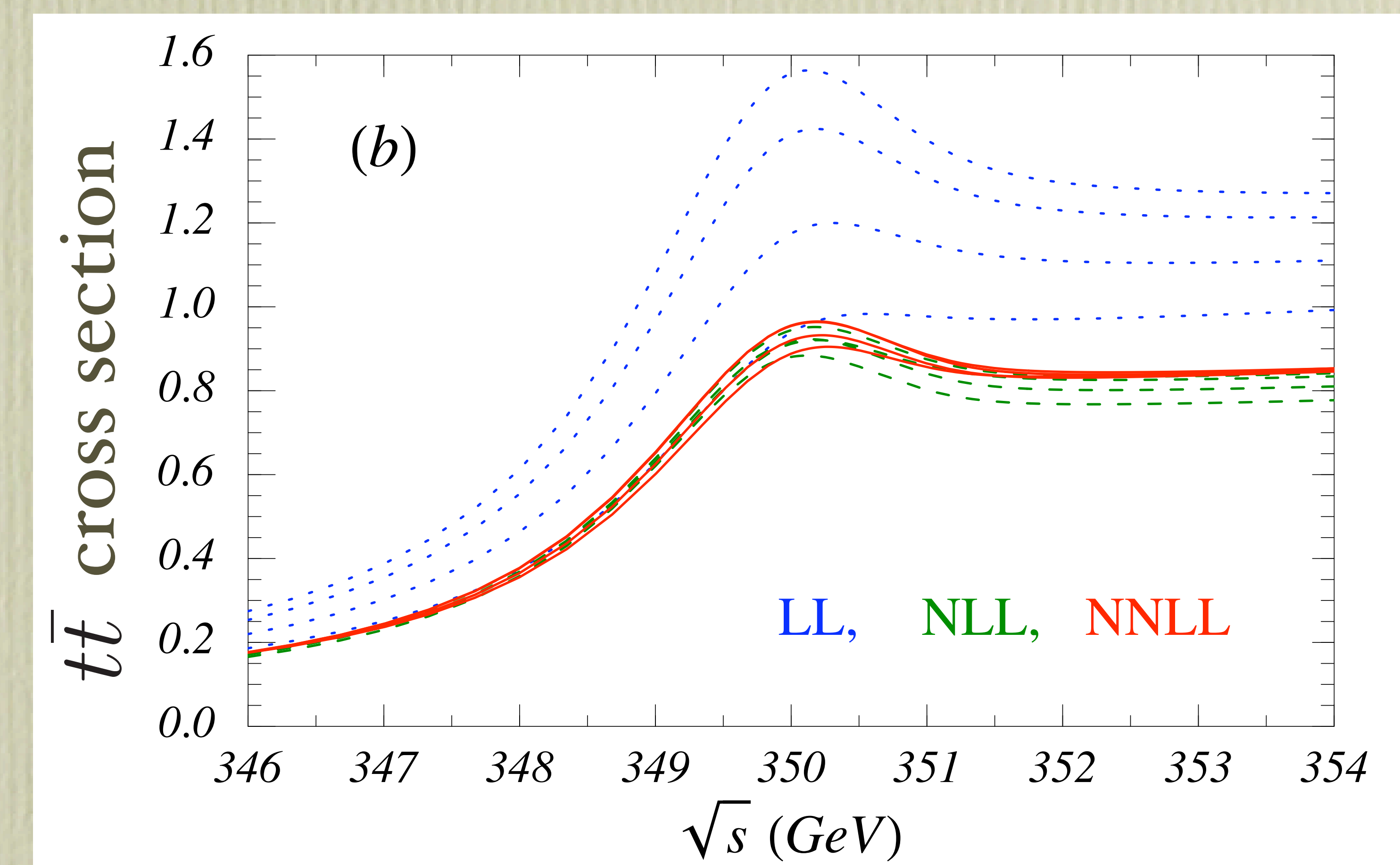
$$e^+e^- \to t\bar{t}$$

$$\Gamma_t = 1.4\,\mathrm{GeV} \gg \Lambda_{\mathrm{QCD}}$$

top decays before it hadronizes

$$\mu \frac{d}{d\mu} C_i(\mu) = \ldots$$

Determine the right scales



vary μ

$\rightarrow m_t, y_t, \Gamma_t$