Effective Field Theory

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The 19'th Taiwan Spring School on
Particles and Fields
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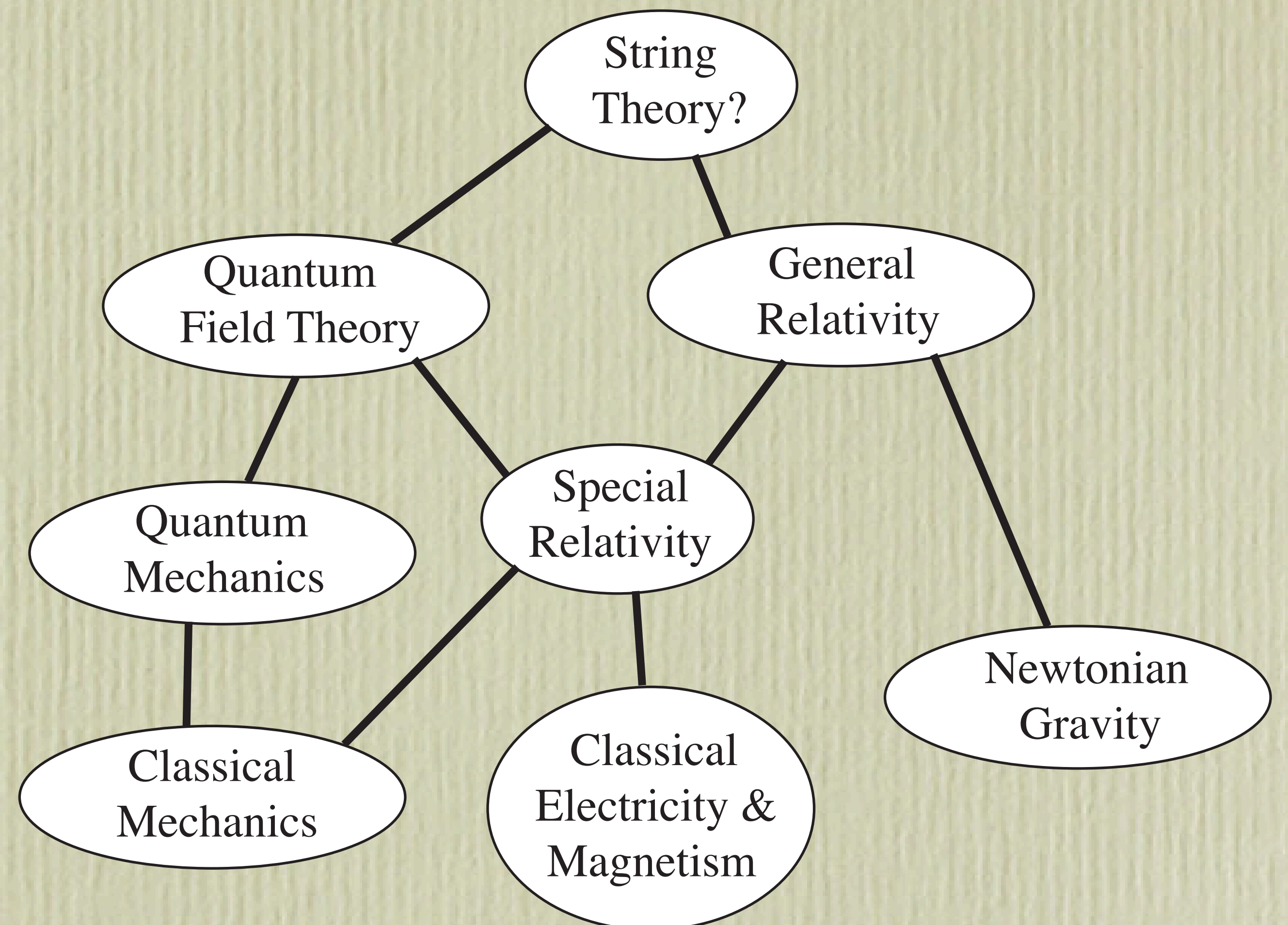
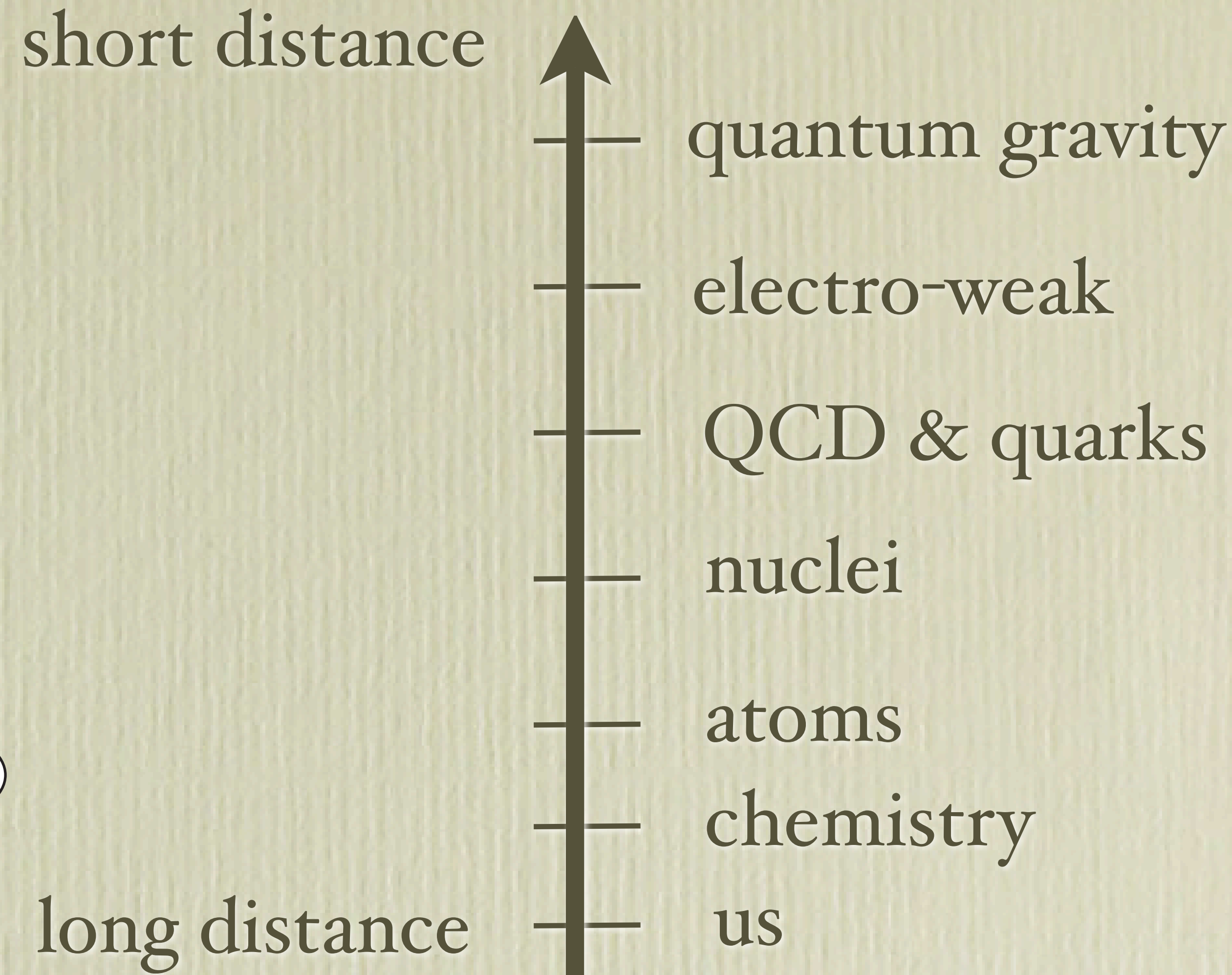
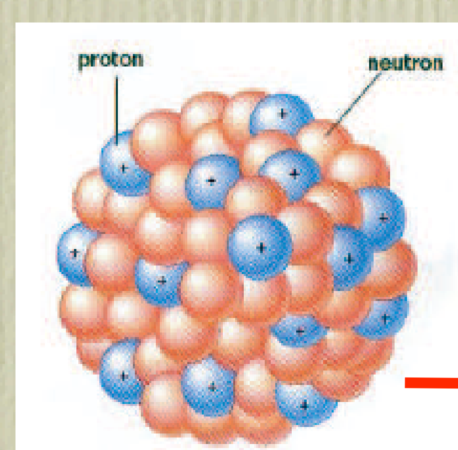
Physics compartmentalized





But, one doesn't need nuclear physics to build a boat

Generality
vs.
Precision



Dynamics at long distance do not depend on the details of what happens at short distance

In the quantum realm, $\lambda \sim \frac{1}{p}$, wavelength and momentum are related, so

Low energy interactions do not depend on the details of high energy interactions

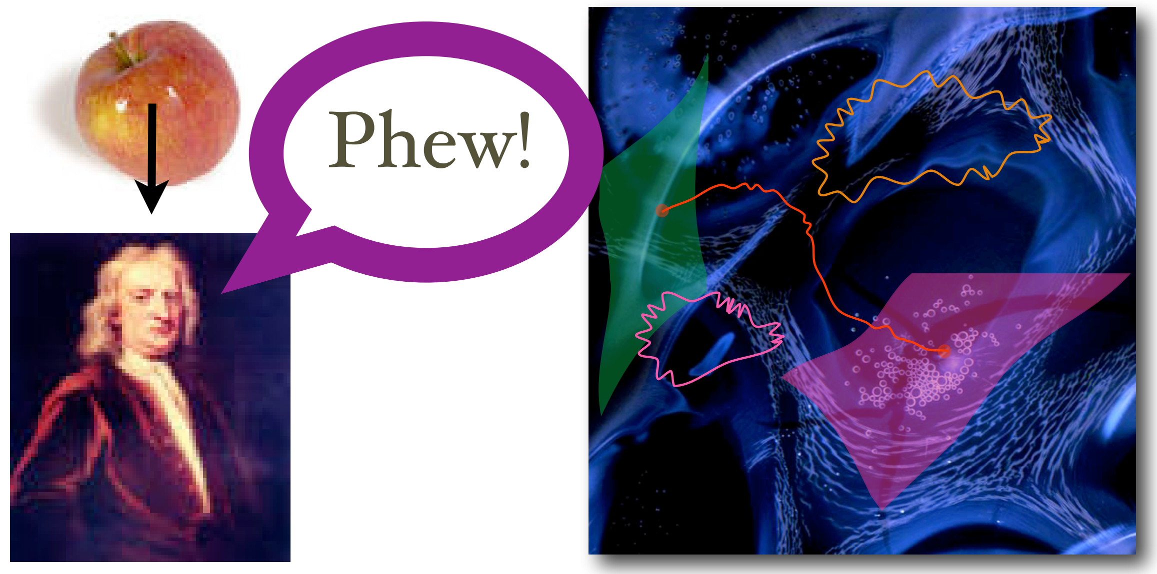
Good:

- we can focus on the relevant interactions & degrees of freedom
- calculations are simpler

Bad:

- we have to work harder to probe the interesting physics at short distances

Newton didn't need quantum gravity for projectile motion



Outline

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Bottom

Lecture I

- Principles, Operators, Power Counting ,

NRQED, Why the Sky is Blue, Weak Decays

- Operator Counting, Matching, Using Equations of Motion
- Quantum Loops, Renormalization and Decoupling
- Summing Large Logarithms

Lecture II

QCD, α_s matching, Heavy Quark Effective Theory
(An Effective Theory for Static Sources)

Lecture III

Soft - Collinear Effective Theory

Masses

particle	mass
γ	$< 6 \times 10^{-17}\,\mathrm{eV}$
gluon	0 (theory)
$\nu_e,\, \nu_\mu,\, \nu_\tau$	$\Delta m^2_{sun} = 8 \times 10^{-5}\,\mathrm{eV}^2$
	$\Delta m^2_{atm} = 2 \times 10^{-3}\,\mathrm{eV}^2$
e	$0.511\,\mathrm{MeV}$
u quark	$\sim 4\,\mathrm{MeV}$
d quark	$\sim 7\,\mathrm{MeV}$
μ	$106\,\mathrm{MeV}$
s quark	$\sim 120\,\mathrm{MeV}$
c quark	$\sim 1.4\,\mathrm{GeV}$
τ	$1.78\,\mathrm{GeV}$
b quark	$\sim 4.5\,\mathrm{GeV}$
W boson	$80.4\,\mathrm{GeV}$
Z boson	$91.2\,\mathrm{GeV}$
higgs	$> 114\,\mathrm{GeV}$
t quark	$174\,\mathrm{GeV}$

$$m_{\mathrm{proton}} \sim 1\,\mathrm{GeV}$$

Lecture I

EFT Concepts

1) The ingredients in any EFT: Relevant degrees of freedom, symmetries, scales & power counting

2) Renormalization: Meaning of parameters

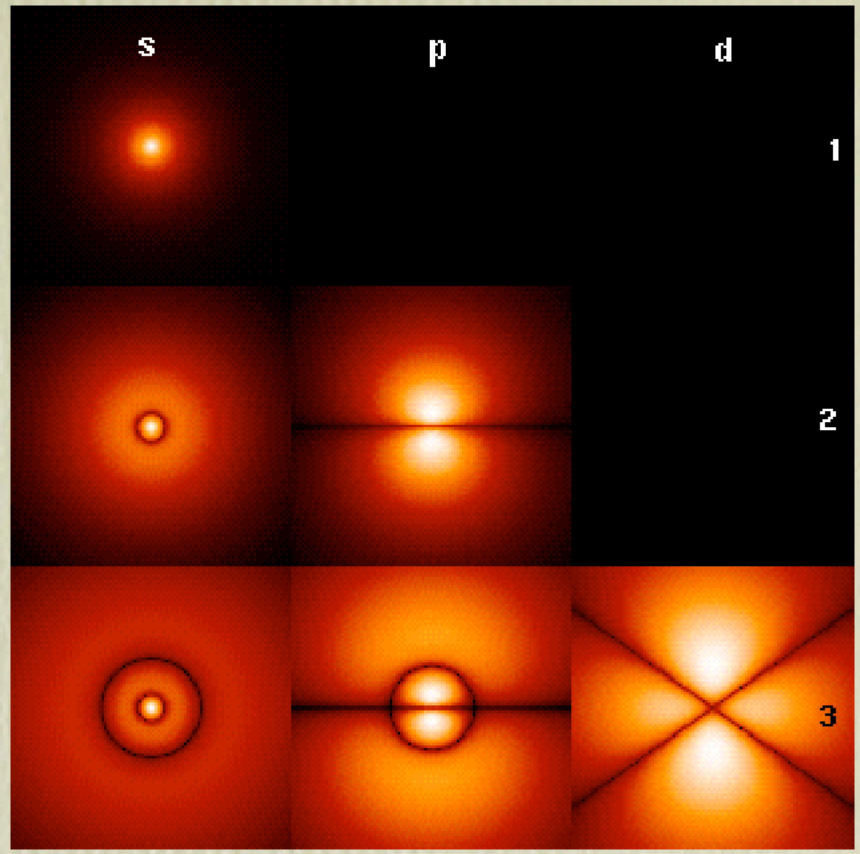
3) Decoupling: Effects from Heavy Particles are suppressed

4) Matching: How we can encode dynamics of one theory into another

5) Running: Connecting physics at different momentum scales

Example: Hydrogen

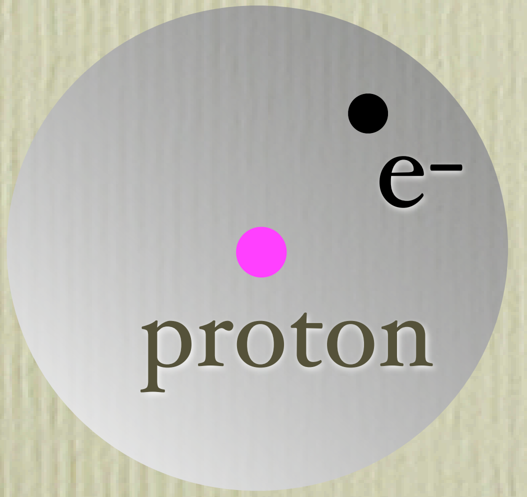
non-relativistic quantum mechanics



parameters:

mass	m_e
charges	$Q_e\,, Q_p$
coupling	$\alpha = \frac{1}{137}$

degrees of freedom:



scales:

$$m_p = 938\,\text{MeV} \quad \longrightarrow \infty$$

$$m_e = 0.511\,\text{MeV}$$

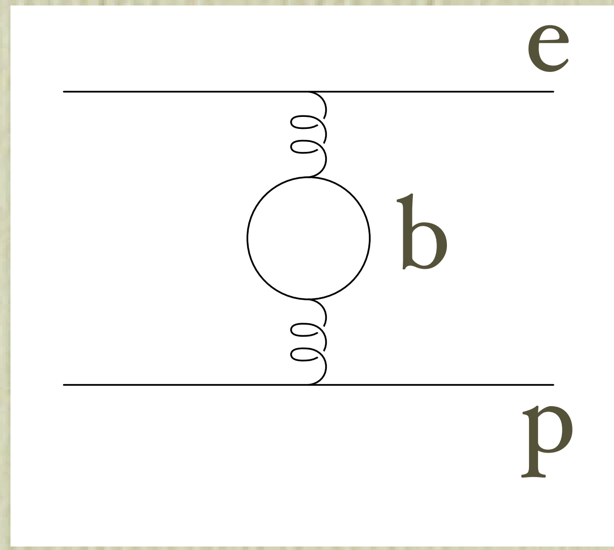
$$p \sim m_e\alpha = 3.7\,\text{keV} \quad \sim (a_{\text{Bohr}})^{-1}$$

$$E_n = -\frac{m_e\alpha^2}{2n^2} = -\frac{13.6\,\text{eV}}{n^2}$$

+ corrections

Why not quarks? QCD? b-quark charge? e^+? weak force?
m_{proton}? spin?

- $\mathcal{L}(p, e^-, \gamma, b; \alpha, m_b) = \mathcal{L}(p, e^-, \gamma; \alpha') + \mathcal{O}\Big(\frac{p^2}{m_b^2}\Big)$ suppressed

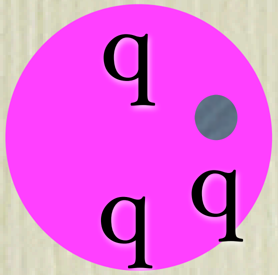


coupling changed, it runs: $\alpha(0) = \frac{1}{137}$

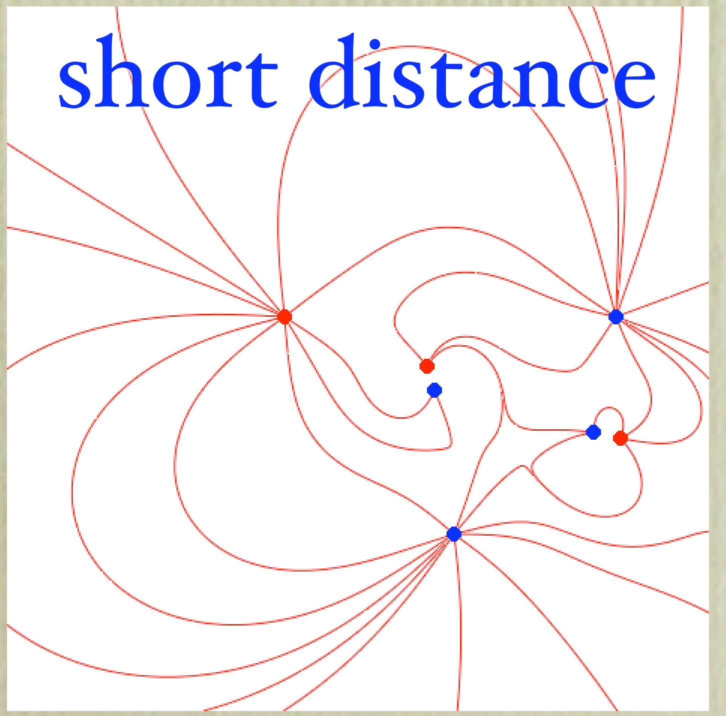
$\alpha(m_W) = \frac{1}{128}$

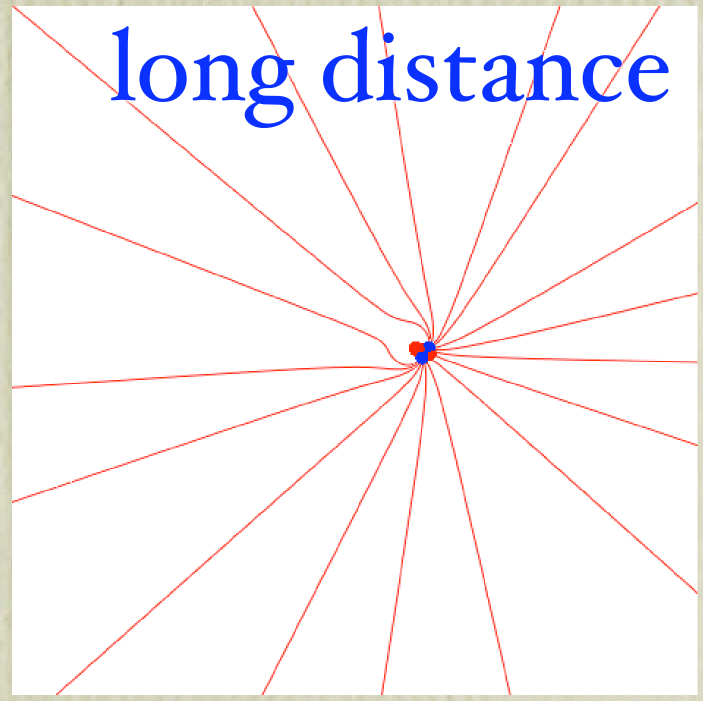
- Insensitive to quarks in proton:

$p_\gamma \sim m_e \alpha \ll (\text{proton size})^{-1} \sim \Lambda_{\text{QCD}} \sim 200\,\text{MeV}$



$r = \Lambda_{\text{QCD}}^{-1}$





- Insensitive to proton mass:

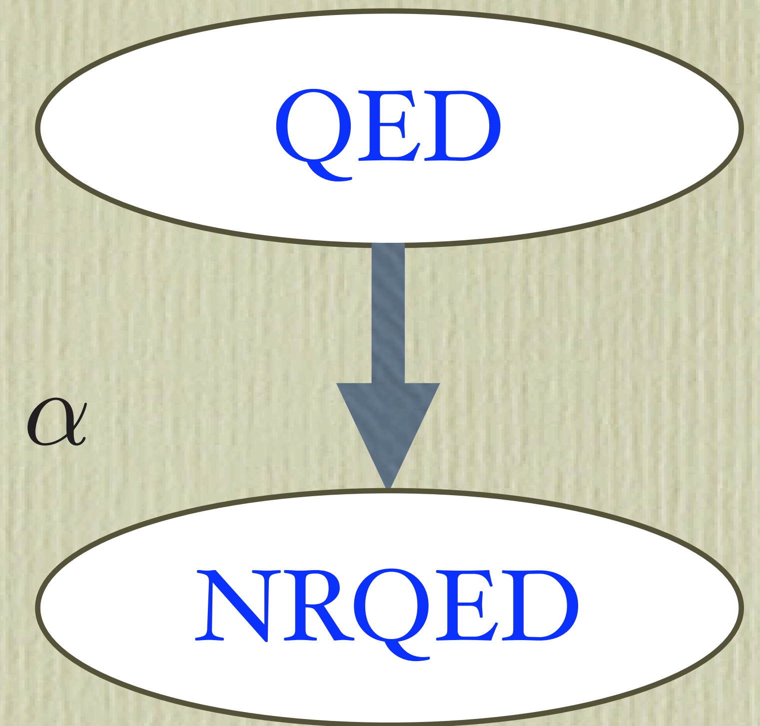
$m_e \alpha \ll m_p \sim 1\,\mathrm{GeV}$ (static proton suffices)

hyperfine splitting $\sim \dfrac{m_e^2}{m_p}\,\mu_e \mu_p\,\alpha^4$

from QCD
but universal

Experiment for m_p, μ_p is more accurate that QCD computations

- Non-relativistic Lagrangian $\quad m_e\alpha \ll m_e \qquad$ (no e^+)



short distance theory is more general

expand in $\dfrac{p}{m_e}, \dfrac{m_e}{m_p}, \alpha$

$$\mathcal{L}^{\text{NRQED}} = \mathcal{L}_0 + \sum_{n=1}^{\infty} \epsilon^n \, \mathcal{L}_n$$

long distance theory where its easier to compute

exact answer is irrelevant, work to the desired level of precision

Leading Order

$$\mathcal{L}_0 = \psi^\dagger\left(i\partial^0 - \frac{\nabla^2}{2m_e} + \ldots\right)\psi + \Psi^\dagger i\partial^0\Psi + \text{“V”}(\psi^\dagger\psi)(\Psi^\dagger\Psi)$$

- Non-relativistic Lagrangian $m_e\alpha \ll m_e$ (no e^+)

QED

short distance theory
is more general

expand in

$\frac{p}{m_e}, \frac{m_e}{m_p}, \alpha$

$\downarrow$

$$\mathcal{L}^{\text{NRQED}} = \mathcal{L}_0 + \sum_{n=1}^{\infty} \epsilon^n \, \mathcal{L}_n$$

NRQED

long distance theory where
its easier to compute

exact answer is irrelevant, work to the desired level of precision

Symmetries of QED constrain the form of NRQED:

Charge conjugation ($e^+ \leftrightarrow e^-$)
Parity ($\vec{x} \longrightarrow -\vec{x}$)
Time-Reversal ($t \longrightarrow -t$)
Gauge Symmetry
Spin-Statistics Theorem

$\Longrightarrow$ constrain the $\mathcal{L}_n$'s

Why $\mathcal{L}_{\rm QED} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$?   Why $\mathcal{L}_{\rm SM}$?

Use: Observed d.o.f.

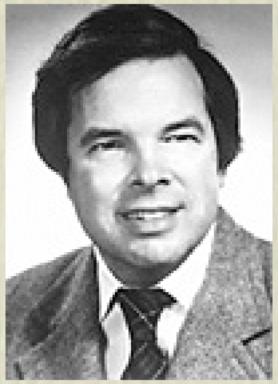
Symmetries
- Lorentz Invariance
- Gauge Invariance $SU(3) \times SU(2) \times U(1)$
- Unitarity (Hermitian $\mathcal{L}$)

Renormalizability (absorb divergences from loops in finite # of parameters)

this was not in our list!

Modern Definition: renormalizable order by order in power expansion

$$\mathcal{L} = \mathcal{L}_0 + \sum_{n=1}^{\infty} \epsilon^n \, \mathcal{L}_n$$



Ken Wilson

Power Counting in Mass Dimension

$\longleftrightarrow$ Dimension of Operators

(Marginal, Irrelevant, Relevant Operators)

Consider $$S[\phi] = \int d^d x \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 - \frac{\tau}{6!}\phi^6\right)$$

(board)

eg. Standard Model

$C = i\gamma_2\gamma_0$ Left-handed Lepton doublet $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$

$$\mathcal{L} = \mathcal{L}^{\text{dim-4}} + \frac{1}{\Lambda^{\text{new}}}\left[L_L^{T\,i} C \epsilon_{ij} H^j H^\ell \epsilon_{\ell k} L_L^k\right] + \mathcal{L}^{\text{dim-6}} + \ldots$$

Higgs SU(2) doublet

Exercise 1: Show that this is most general dim-5 operator that is consistent with the symmetries of the Standard Model

When the Higgs gets a vev this gives a small Majorana Neutrino Mass

eg. Why the Sky is Blue

Low energy scattering of photons from neutral atoms in their ground state $E_\gamma \ll \Delta E \sim m_e\alpha^2 \ll a_0^{-1} \sim m_e\alpha \ll M_{\rm atom}$

- Let $v^\mu = (1,0,0,0)$, atoms are static $\mathcal{L} = \phi_v^\dagger\, i\partial^0\, \phi_v = \phi_v^\dagger\, iv\cdot\partial\, \phi_v$

  ϕ_v is field which destroys an atom, mass dim. $[\phi_v] = 3/2$

- Interactions are constrained by gauge invariance, parity & charge conjugation. Consider

  $$\mathcal{L}^{\rm int} = \tau_1\ \phi_v^\dagger\, \phi_v F_{\mu\nu}F^{\mu\nu} + \tau_2\ \phi_v^\dagger\, \phi_v v^\lambda F_{\lambda\mu} v_\sigma F^{\sigma\mu}$$

  $[F_{\mu\nu}] = 2$ so $[\tau_1] = [\tau_2] = -3$

  even number of $F_{\mu\nu}$'s

  no $\tilde{F}_{\mu\nu}$ here

  $\partial^\mu F_{\mu\nu} = 0,\ v^\mu\partial_\mu\phi_v = 0$

- Very low energy photons do not probe inside the atom, so expect cross section to depend on size of atom: $\tau_1 \sim \tau_2 \sim a_0^3$

  $[\sigma] = -2$

  $\sigma \propto |A|^2 \sim \tau_i^2$

  so $\sigma \propto E_\gamma^4\, a_0^6$

  blue light is scattered stronger than red light

Exercise 2:

Consider QED for photon momenta much less than the mass of the electron.

By constructing an appropriate operator in a low energy EFT, estimate the cross section for $\gamma\gamma \to \gamma\gamma$ with 10 keV photons. Include factors of e in your estimate, by considering which QED graphs generate your operator.

Can we really use equations of motion to simplify operators?

$$\partial^\mu F_{\mu\nu} = 0, \; v^\mu \partial_\mu \phi_v = 0$$

eg. $\quad p^2\tau \ll 1$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \lambda\phi^4 + \tau g_1\phi^6 + \tau g_2\phi^3\partial^2\phi$$

e.o.m. $\quad \partial^2\phi = -m^2\phi - -4\lambda\phi^3$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \lambda'\phi^4 + \tau g_1'\phi^6$$

$$\lambda' = \lambda + m^2 g_2\tau$$

$$g_1' = g_1 - 4\lambda g_2$$

(board)

Exercise 3: Demonstrate the equivalence for tree level 6-pt functions at $\mathcal{O}(\tau)$, pre- and post- the use of the equations of motion.

Regularization and Renormalization

Regularization: How we cutoff UV infinities in loop integrals

Renormalization: How we pick a scheme to give definite meaning to parameters of a theory

- Computations are easier if our regulator preserve symmetries

and

preserves power counting by not mixing up terms of different order in the expansion

Dimensional Regularization $\int d^d p \;= \int dp\, p^{d-1}\, d\Omega_d \qquad d = 4 - 2\epsilon$

Linearity: $\int d^d p\, [a f(p) + b f(p)] = a \int d^d p\; f(p) + b \int d^d p\; g(p)$

Scaling: $\int d^d p\, f(sp) = s^{-d} \int d^d p\; f(p)$

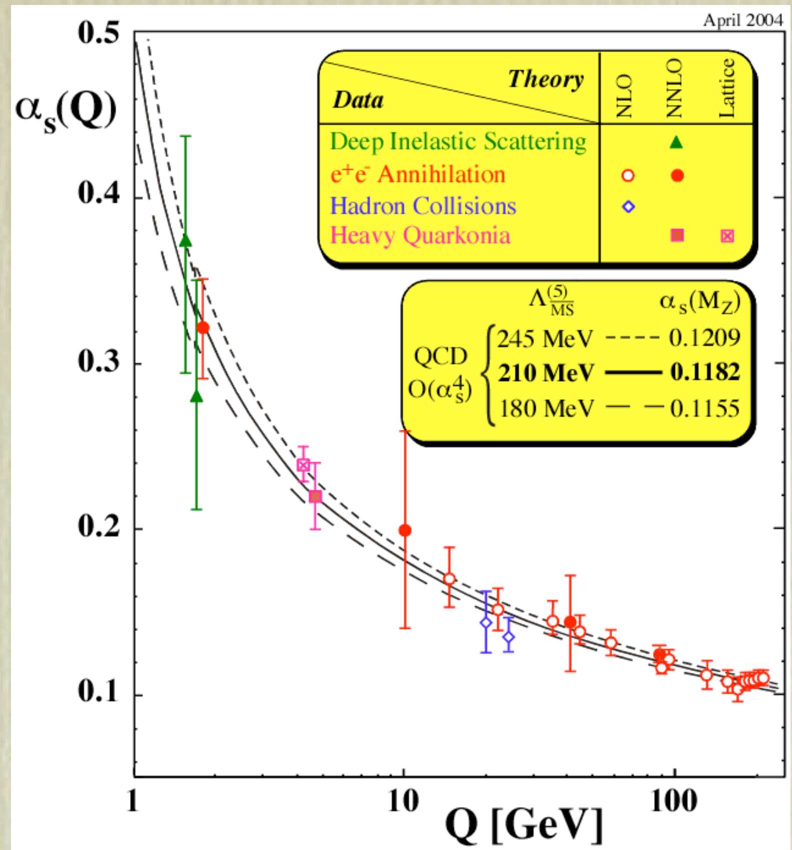
Translation: $\int d^d p\, f(p+q) = \int d^d p\; f(p)$

$$\int d^d p (p^2)^\alpha = \begin{cases} 0 \text{ for } \alpha < 4 \text{ and } \alpha > 4 & \text{no power divergences} \\ \frac{i}{16\pi^2}\left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\right) \;= 0 \text{ for } \alpha = 4 & \longleftarrow \text{be careful !} \end{cases}$$

Rescale couplings to keep them dimensionless

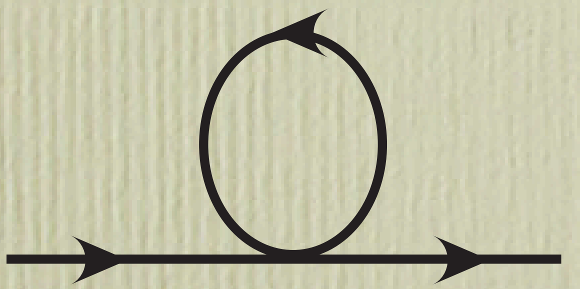
eg. $$g^{(0)} = Z_g\, \mu^\epsilon\, g(\mu)$$

μ is dim.reg. parameter and acts like a "resolution" in $\overline{\text{MS}}$ scheme



Mass independent regulator

eg. $$\mathcal{L} = \bar\psi(i\partial\!\!\!/ - m)\psi - \frac{a}{M^2}(\bar\psi\psi)^2 + \dots \qquad m \ll M,\, a \sim 1$$



gives $$\delta m \sim \frac{i\,a}{M^2}\int \frac{d^4k}{(2\pi)^4}\frac{k\!\!\!/ + m}{k^2 - m^2} = \frac{i\,a}{M^2}\int \frac{d^4k}{(2\pi)^4}\frac{m}{k^2-m^2}$$

$$\delta m^{\text{cutoff}} \sim \frac{a}{M^2}\,\Lambda^2 + \dots$$ the cutoff indep. terms are "hidden"

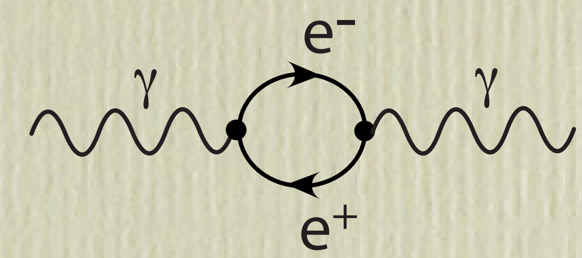
$$\delta m^{\text{dim.reg.}} \sim \frac{a}{M^2}\,m^2$$ small as expected, power counting manifest

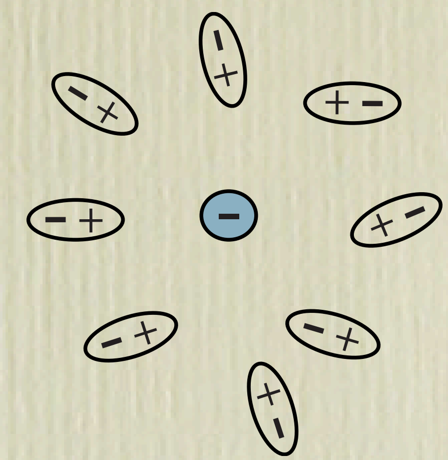
Decoupling Theorem

If remaining low energy theory is renormalizable then all effects due to heavy particles appear as changes in the coupling constants or are suppressed by 1/M

requires a “physical” renormalization scheme,

not true in $\overline{\mathrm{MS}}$ (board)

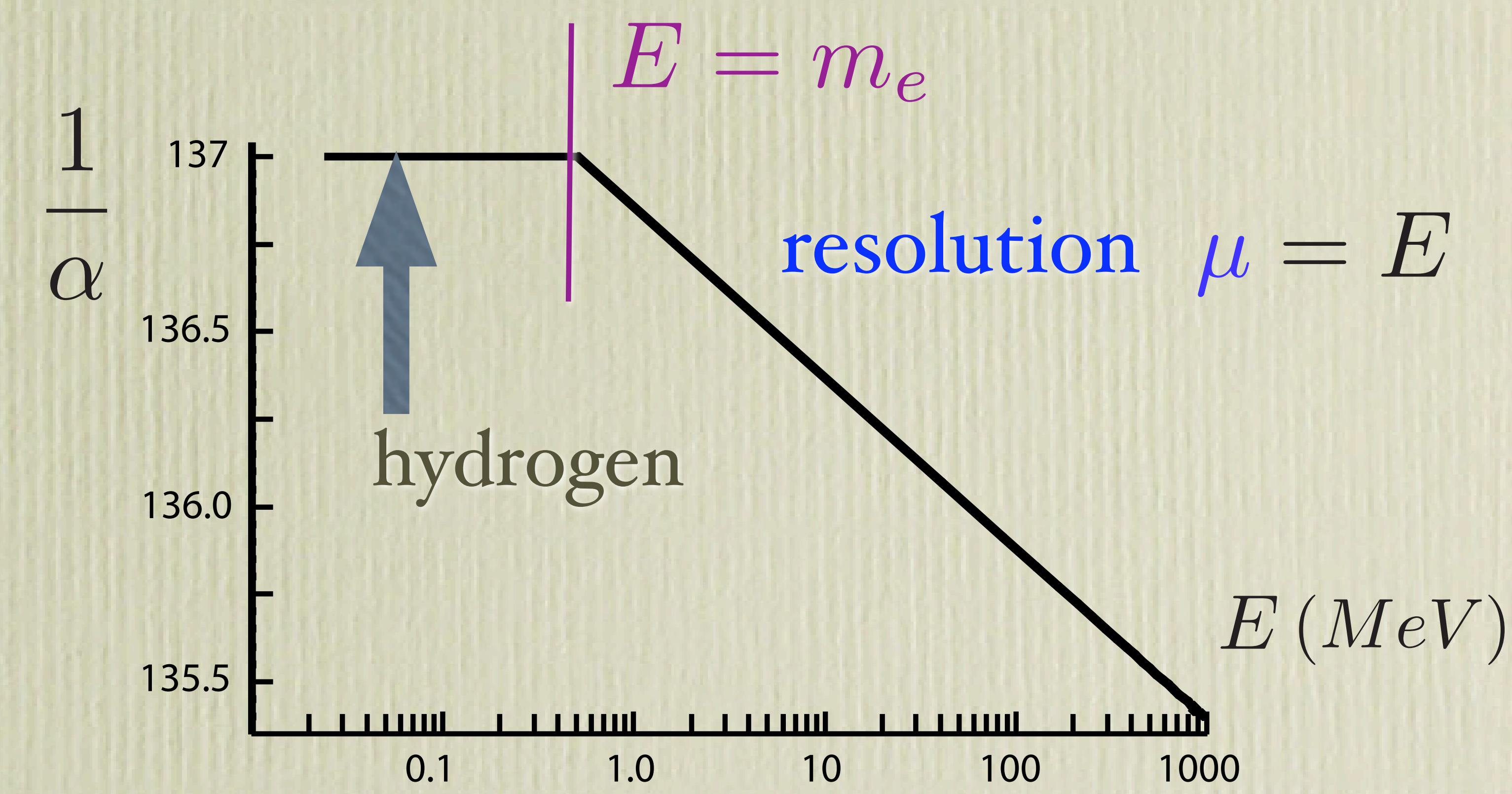




Solution: We must implement decoupling by hand at $\mu \approx m$

$$\beta_{\mathrm{QED}} = \frac{e^3}{12\pi^2}$$

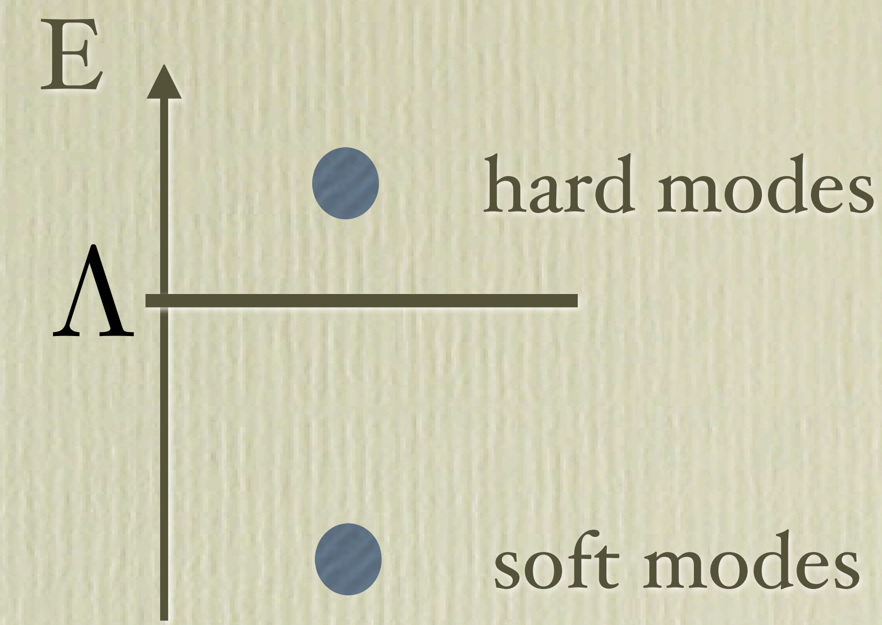
$$\beta_{\mathrm{QED}} = 0$$



Wilsonian vs. Continuum EFT

Wilson

effective action for soft modes e^{-S_Λ}

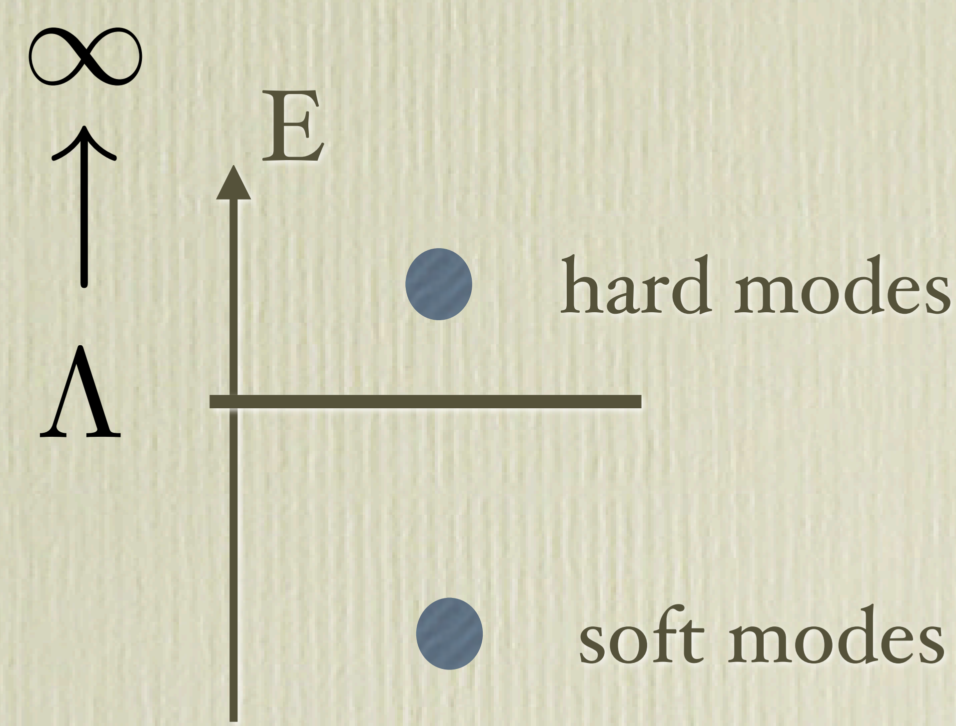


removing modes with $\Lambda - \delta\Lambda < E < \Lambda$

$$e^{-S_{\Lambda-\delta\Lambda}} = \int_{\delta\Lambda} d\phi \; e^{-S_\Lambda}$$

Continuum

$$\mathcal{L}^{\mathrm{EFT}} = C(\mu)\mathcal{O}(\mu)$$



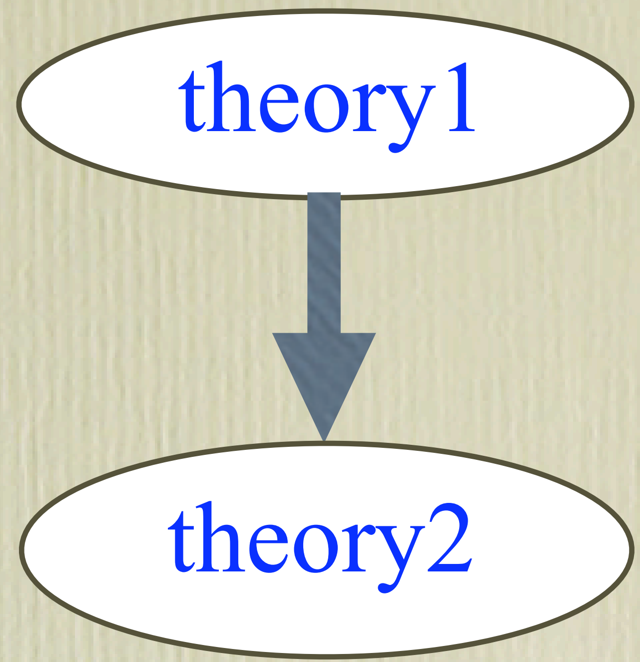
operators for soft modes $\mathcal{O}\,(\mu)$

Wilson coefficients for hard modes $\mathrm{C}\,(\mu)$

Sending Λ to ∞ double counts the hard region in matrix elments of our operators, but we fix $\mathrm{C}\,(\mu)$ to correct for this. μ is the scale where this matching is done.

Top

Theory 1 is understood, but it is useful to have the simpler theory 2 at low energies.



Integrate out heavier particles in 1 and match onto 2

$$\mathcal{L}_{\text{theory1}} \rightarrow \sum_n \mathcal{L}^{(n)}_{\text{theory2}}$$

Theory 1 & theory 2 agree in the IR, differ in UV

eg. NRQED; Heavy Quark Effective Theory;
Remove t,W,Z: H_{weak} ; Soft Collinear Effective Theory

Bottom

Theory 1 is unknown or matching is too difficult to carry out analytically

Construct $\sum_n \mathcal{L}^{(n)}_{\text{theory2}}$ by writing down most general set of interactions consistent with symmetries

eg. Standard Model; Chiral Perturbation theory for low energy pion and Kaon interactions

EFT Principles

1) Dynamics at low E does not depend on details of dynamics at high E

2) Build an EFT using the relevant d.o.f. and known symmetries.

3) EFT has an infinite number of operators, but only a finite number are needed for a given precision as determined by the power counting. With this precision this set closes under renormalization.

4) EFT has same infrared but different ultraviolet than the more fundamental theory.

5) Nature of high energy theory shows up as couplings and symmetries in the low energy EFT.

Effective Field Theories of QCD