Introduction to the Soft - Collinear Effective Theory

An effective field theory for energetic hadrons & jets

$E \gg \Lambda_{\rm QCD}$

Lecture 3

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So far

Lecture I

- Introduction to SCETI , SCETII
- Collinear & Soft degrees of freedom
- Construction of HQET
- SCETI propagators, field power counting
- Leading Lagrangian

Lecture II

- Heavy-light current and Wilson lines
- Gauge symmetry and reparameterizations in SCET
- Wilson coefficients & hard-collinear factorization
- Field redefinition & ultrasoft-collinear factorization
- One-Loop ultrasoft and collinear graphs, IR divergences

Lecture 3 Outline

- Renormalization group evolution & Sudakov logs
- $B \to X_s \gamma$ Factorization Theorem
- More on large logs, Evolution with Convolutions

- SCETII, building blocks, exploiting SCETI
- Factorization for $B \to D\pi$, $B \to \pi \ell \bar{\nu}$
- eg. of power corrections in SCETI

- Jet Production $e^+ e^- \to J_n J_{\bar{n}} X$

Renormalization in SCET
&
Summing Sudakov Logs

Renormalize Heavy to Light Current in SCET

$C(\omega,\mu)\big[(\bar{\xi}_n W)_\omega\,\Gamma h_v\big]$ $\qquad C^{\rm bare} = C + (Z_c - 1)C \qquad \omega = m_b$

graph sum = $-\frac{\alpha_s}{3\pi}\Big[\ln^2\Big(\frac{-p^2}{m_b^2}\Big) + \frac{3}{2}\ln\Big(\frac{-p^2}{m_b^2}\Big) + \frac{1}{\epsilon_{\rm IR}}$

$$-\frac{1}{\epsilon_{\rm UV}^2} - \frac{5}{2\epsilon_{\rm UV}} - \frac{2}{\epsilon_{\rm UV}}\ln\Big(\frac{\mu}{m_b}\Big) - 2\ln^2\Big(\frac{\mu}{m_b}\Big) - \frac{3}{2}\ln\Big(\frac{\mu^2}{m_b^2}\Big) + \text{constants}\Big]$$

need $\quad Z_c = 1 - \frac{\alpha_s(\mu)C_F}{4\pi}\Big(\frac{1}{\epsilon^2} + \frac{5}{2\epsilon} + \frac{2}{\epsilon}\ln\frac{\mu}{\omega}\Big)$ $\quad$ to remove UV divergences

Compute the Anomalous Dimension

$$\mu\frac{d}{d\mu}C^{\rm bare} = 0 \implies \underline{\mu\frac{d}{d\mu}C(\omega,\mu) = \gamma_c(\omega,\mu)C(\omega,\mu)}$$

$$\mu\frac{d}{d\mu}\alpha_s(\mu) = -2\epsilon\alpha_s(\mu) + \beta[\alpha_s]$$

$$\gamma_c = -Z_c^{-1}\mu\frac{d}{d\mu}Z_c \;= \mu\frac{d}{d\mu}\frac{\alpha_s(\mu)C_F}{4\pi}\Big(\frac{1}{\epsilon^2} + \frac{5}{2\epsilon} + \frac{2}{\epsilon}\ln\frac{\mu}{\omega}\Big)$$

$$= \frac{\alpha_s(\mu)C_F}{4\pi}\Big(\cancel{\frac{-2}{\epsilon}} - 5 - 4\ln\frac{\mu}{\omega} + \cancel{\frac{2}{\epsilon}}\Big) \;= -\frac{\alpha_s(\mu)C_F}{\pi}\Big(\ln\frac{\mu}{\omega} + \frac{5}{4}\Big)$$

LL $\qquad$ part of NLL

LL solution

cusp anomalous dimension

Solve $$\mu\frac{d}{d\mu}\ln C(\omega,\mu) = -\frac{\alpha_s(\mu)C_F}{\pi}\ln\frac{\mu}{\omega}\ , \qquad \mu\frac{d}{d\mu}\alpha_s(\mu) = -\frac{\beta_0}{2\pi}\alpha_s^2(\mu)$$

use $d\ln(\mu) = -\frac{2\pi}{\beta_0}\frac{d\alpha_s}{\alpha_s^2}$ and integrate to obtain the solution

$$C(\omega,\mu) = C(\omega,\mu_0)\exp\left[\frac{-4\pi C_F}{\beta_0^2\alpha_s(\mu_0)}\left(\frac{1}{z} - 1 + \ln z\right)\right]\left(\frac{\mu_0}{\omega}\right)^{2C_F\ln z/\beta_0}$$

boundary condition, no large logs for $\mu_0 \sim \omega$

$\sim \exp(\alpha_s \ln^2 + \alpha_s^2 \ln^3 + \ldots)$

$$z \equiv \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}$$

If $\beta_0 \to 0$ and $\alpha_s = \text{constant}$, then

$$C(\omega,\mu) = C(\omega,\mu_0)\exp\left[\frac{-\alpha_s C_F}{\pi}\left(\frac{1}{2}\ln^2\frac{\mu}{\mu_0} + \ln\frac{\mu}{\mu_0}\ln\frac{\mu_0}{\omega}\right)\right]$$

Sudakov double logs exponentiated

Exercise

SCET Loops for Two-Jet Production

Consider the two-jet production process through a virtual photon in SCET, namely $e^+e^- \to J_n J_{\bar{n}} X_{us}$ where J_n is a jet in the $n = (1, 0, 0, -1)$ direction, $J_{\bar{n}}$ is a jet in the $\bar{n} = (1, 0, 0, 1)$ direction, and any remaining particles in the final state are ultrasoft, contained in X_{us}.

a) Write down two collinear quark Lagrangians, one for ξ_n fields and one for $\xi_{\bar{n}}$ fields. Interactions between these two types of collinear fields are hard, and so do not effect your analysis. What are the Feynman rules for the ultrasoft gluon coupling to each of these collinear quarks?

b) Start with $J^{\rm QCD} = \bar{\psi}\gamma_\mu\psi$ and determine the appropriate LO SCET current $J^{\rm SCET} = \bar{\xi}_n \cdots \xi_{\bar{n}}$, ie. fill in the dots with appropriate collinear Wilson lines and Dirac structure.

c) Draw the five one-loop Feynman diagrams that are non-zero for $e^+e^- \to q_n \bar{q}_{\bar{n}}$ (use Feynman gauge for all gluons when determining which graphs are zero). Here q_n has n-collinear momentum p, and $\bar{q}_{\bar{n}}$ has $\bar{n}$-collinear momentum $\bar{p}$ and you should work in the CM frame. All graphs but one can be directly read off using the loop computations done in lecture (or given in the handout notes), as long as you use the same IR regulator. That is, you should keep both collinear quarks offshell, $p^2 \neq 0$ and $\bar{p}^2 \neq 0$. Compute the divergent terms in the one remaining ultrasoft graph using dimensional regularization in the UV.

d) Add up the $1/\epsilon$ terms from the graphs in c) and determine the lowest order anomalous dimension equation for C the Wilson coefficient of $J^{\rm SCET}$. Solve this equation keeping only the $\ln \mu/Q$ term and using a fixed coupling α_s, and then with a running coupling $\alpha_s(\mu)$. (Voilá, Sudakov double logs resummed.)

$\mathrm{SCET_I}$

Construction of operators (using power counting, ultrasoft & collinear gauge invariance, RPI)

We built gauge invariant operators with nice power counting:

eg. LO heavy-to-light current

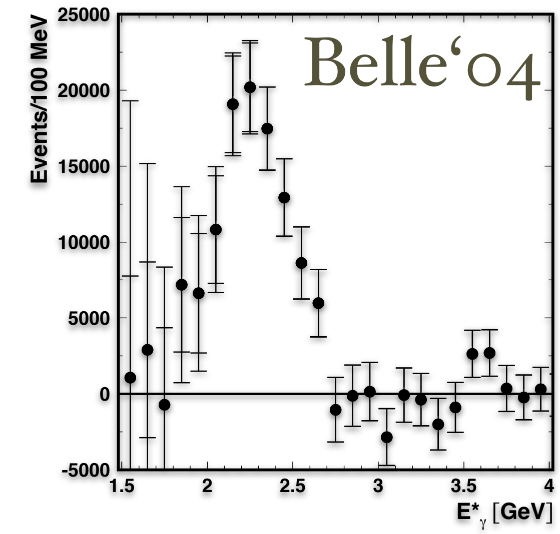
$$J^{(0)} = \int d\omega\, C(\omega,\mu) \Big[(\bar\xi_n W)\delta(\omega - \bar{\mathcal{P}}^\dagger)\Gamma(Y_n^\dagger h_v)\Big] = \int d\omega\, C(\omega,\mu)\ \bar\chi_{n,\omega}\,\Gamma\,\mathcal{H}_v^n$$

eg. a subleading current suppressed by λ

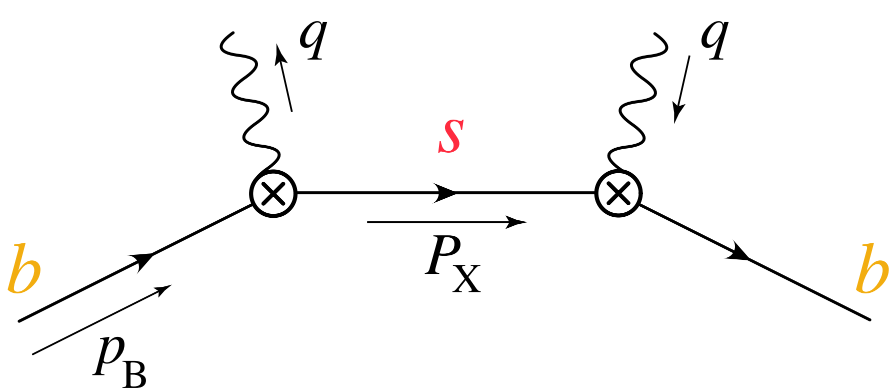
$$J^{(1)} = \int d\omega\, d\omega'\, C^{(1)}(\omega,\omega',\mu)\ \bar\chi_{n,\omega}\, ig\not{\mathcal{B}}^\perp_{\omega'}\,\Gamma\,\mathcal{H}_v^n$$

$$\begin{aligned} ig\mathcal{B}^{\perp\mu}_{\omega'} &= \frac{1}{\bar{\mathcal{P}}} W\big[i\bar n\!\cdot\! D_n, iD_n^{\perp\mu}\big]W^\dagger\ \delta(\omega' - \bar{\mathcal{P}}^\dagger) \\ &= gA^{\perp\mu}_{n,\omega'} + \dots \end{aligned}$$

Endpoint $B \to X_s\gamma$



Optical Thm: $\Gamma \sim \mathrm{Im} \int d^4x\, e^{-iq\cdot x} \langle B | T\{J_\mu^\dagger(x) J^\mu(0)\} | B \rangle$



	$P_X^2 = m_B(m_B - 2E_\gamma)$
standard OPE	$\sim m_B^2$
endpoint region	$\sim m_B \Lambda_{QCD}$
resonance region	$\sim \Lambda_{QCD}^2$

For EndPoint: $E_\gamma \gtrsim 2.2\,\mathrm{GeV}$, X_s collinear, B usoft, $\lambda = \sqrt{\dfrac{\Lambda_{QCD}}{m_B}}$

We want to prove that the
Decay rate is given by factorized form

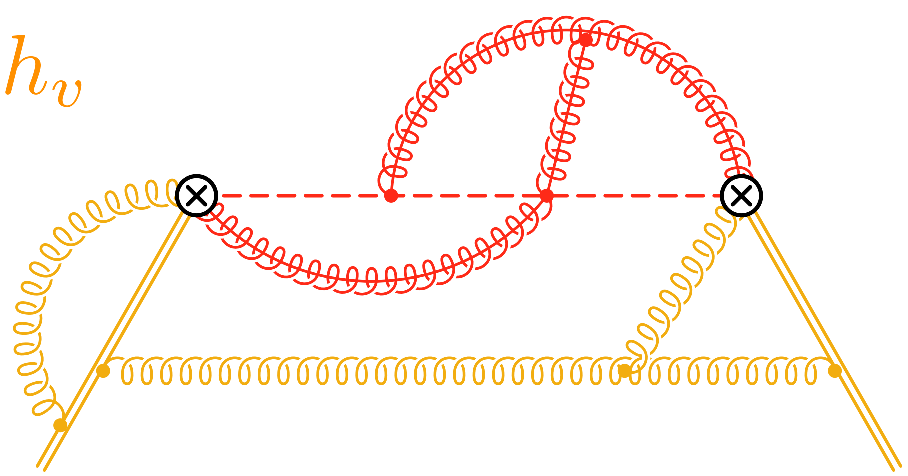
$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = H(m_b,\mu) \int_{2E_\gamma - m_b}^{\bar\Lambda} dk^+\, S(k^+,\mu)\, J(k^+ + m_b - 2E_\gamma, \mu)$$

Match: $\bar{s}\Gamma_\mu b \to e^{i(m_b v - \mathcal{P})\cdot x} C(\bar{\mathcal{P}}) \bar{\xi}_{n,p} W \gamma_\mu^\perp P_L h_v$

$$T_\mu^\mu = \int d^4x \, e^{i(m_b \frac{\bar{n}}{2} - q)\cdot x} \left\langle B \middle| T J^\dagger_{\text{eff}}(x) J_{\text{eff}}(0) \middle| B \right\rangle$$

label conservation
$\bar{\mathcal{P}} \to m_b$

Factor usoft: $\bar{\xi}_n W \Gamma_\mu h_v \to \bar{\xi}_n W \Gamma_\mu Y_n^\dagger h_v$



$$T_\mu^\mu = \left|C(m_b)\right|^2 \int d^4x e^{i(m_b \frac{\bar{n}}{2} - q)\cdot x} \left\langle B \middle| T[\bar{h}_v Y](x)[Y^\dagger h_v](0) \middle| B \right\rangle$$

$$\times \left\langle 0 \middle| T[W^\dagger \xi_n](x)[\bar{\xi}_n W](0) \middle| 0 \right\rangle \times [\Gamma_\mu \otimes \Gamma^\mu]$$

$$= \left|C(m_b)\right|^2 \int d^4x \int \frac{d^4k}{(2\pi)^4} e^{i(m_b \frac{\bar{n}}{2} - q - k)\cdot x} \left\langle B \middle| T[\bar{h}_v Y](x)[Y^\dagger h_v](0) \middle| B \right\rangle$$

$$\times J_P(k) \times [\Gamma_\mu \otimes \Gamma^\mu]$$

Convolution $J_P(k) = J_P(k^+)$

$$\mathrm{Im}\, T^\mu_\mu = \left|C(m_b)\right|^2 \int d^4x \int \frac{d^4k}{(2\pi)^4}\, e^{i(m_b \frac{\bar{n}}{2} - q - k)\cdot x} \left\langle B \middle| T[\bar{h}_v Y](x)[Y^\dagger h_v](0) \middle| B \right\rangle$$
$$\times \mathrm{Im} J_P(k^+)$$
$$= \left|C(m_b)\right|^2 \int dk^+ \left[\int \frac{dx^-}{4\pi}\, e^{i(m_b - 2E_\gamma - k^+)x^-/2} \left\langle B \middle| T[\bar{h}_v Y](x)[Y^\dagger h_v](0) \middle| B \right\rangle \right]$$
$$\times \mathrm{Im} J_P(k^+)$$
$$= \left|C(m_b)\right|^2 \int dk^+ S(2E_\gamma - m_b + k^+) \mathrm{Im} J_P(k^+)$$

as desired

calculable | calculable | nonpert. shape function

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = H(m_b, \mu) \int dk^+ J(k^+, \mu)\ S(2E_\gamma - m_b + k^+, \mu)$$

$p^2 \sim m_b^2$ $\quad$ $p^2 \sim m_b \Lambda_{\rm QCD}$ $\quad$ $p^2 \gtrsim \Lambda_{\rm QCD}^2$

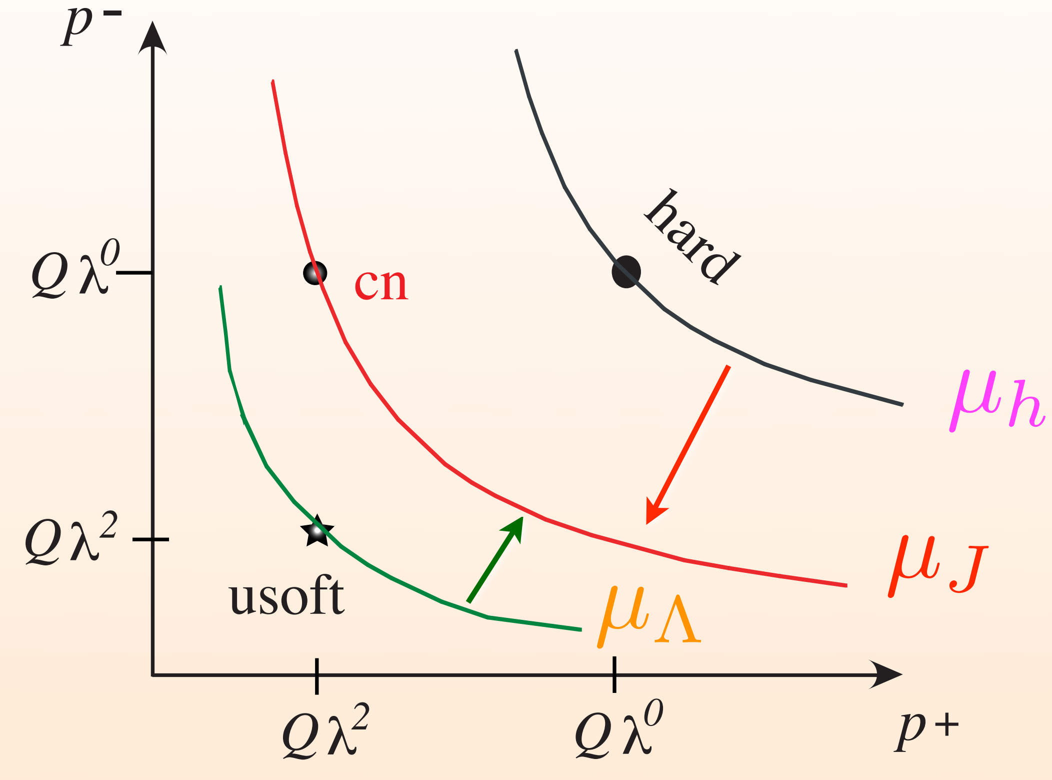
$\sim \mu_h^2$ $\quad$ $\sim \mu_J^2$ $\quad$ $\sim \mu_\Lambda^2$

To minimize large logs we want to evaluate these functions at different μ's

- our result for the RGE for C, allows us to write

$$H(m_b, \mu_J) = H(m_b, \mu_h)\, U_H(m_b, \mu_h, \mu_J)$$

- need to be able to run the shape function up to μ_J



or we could run the jet and hard functions down to μ_Λ

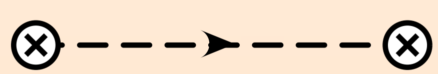
Lets consider the jet function & its RGE

The Jet Function

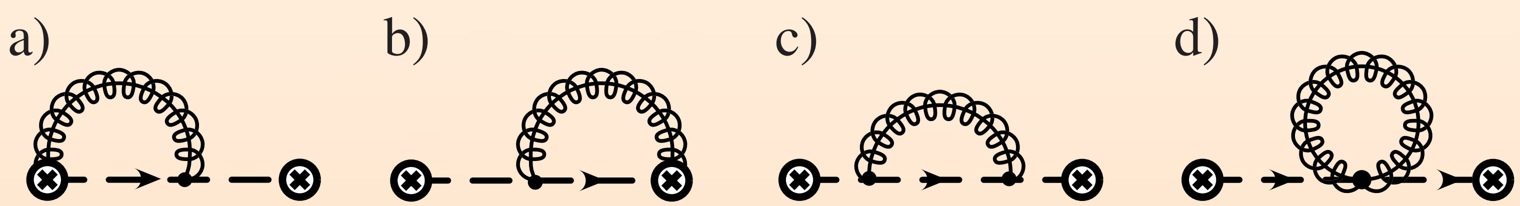
$$\sum_{X_n} \frac{1}{4N_c} \mathrm{tr}\, \langle 0 | \bar{n}\!\!\!/ \chi_n(x) | X_n \rangle \langle X_n | \chi_{n,Q}(0) | 0 \rangle = Q \int \frac{d^4 r_n}{(2\pi)^3} e^{-i r_n \cdot x} J_n(Q r_n^+, \mu)$$

$$J_n(Q r_n^+, \mu) = \frac{-1}{8\pi N_c Q} \, \mathrm{Disc} \int d^4x \; e^{i r_n \cdot x} \, \langle 0 | \mathrm{T} \; \bar{\chi}_{n,Q}(0) \bar{n}\!\!\!/ \chi_n(x) | 0 \rangle$$

tree level:



one loop:



RGE:

$$\mu \frac{d}{d\mu} J(s, \mu) = \int ds' \; \gamma_J(s - s', \mu) \; J(s', \mu)$$

solution

$$J(s, \mu) = \int ds' \; U_J(s - s', \mu, \mu_0) \; J(s', \mu_0)$$

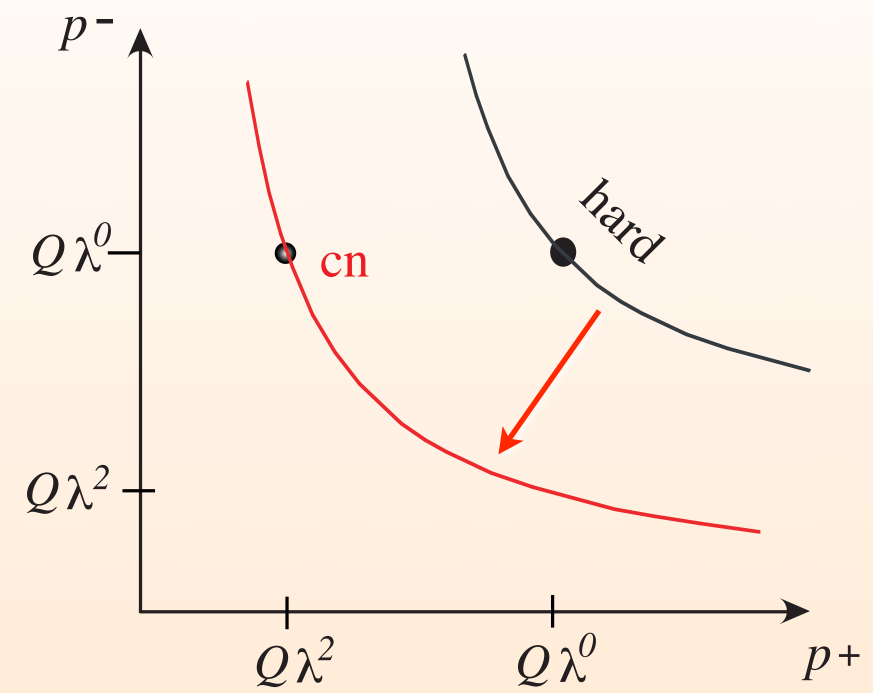
$$U_J(s - s', \mu, \mu_0) = \frac{e^K \left(e^{\gamma_E}\right)^{\omega}}{\mu_0^2 \, \Gamma(-\omega)} \left[\frac{(\mu_0^2)^{1+\omega} \theta(s - s')}{(s - s')^{1+\omega}} \right]_+$$

More examples which involve convolutions

twist 2 operators

label on 2nd block of fields is fixed by mom.cons. in m.elt.

$$J^{(0)} = \int d\omega\, C(\omega, \mu)\, \underbrace{\bar{\chi}_{n,\omega}\, \bar{n}\!\!\!/}_{\mathcal{O}(\omega)}\, \chi_n$$



Matrix Elements

- π light-cone distrib. $\langle \pi_n(p_\pi^-)|J^{(0)}|0\rangle = \int d\omega\, C(\omega,\mu)\, \phi_\pi(\omega/p_\pi^-, \mu) = p_\pi^- \int_0^1 dx\, C(xp_\pi^-, \mu)\, \phi_\pi(x,\mu)$

- DIS p.d.f

$$\langle p_n(p^-)|J^{(0)}|p_n(p^-)\rangle = \int d\omega\, C(\omega, Q, \mu)\, f_{i/p}(\omega/p^-, \mu) = \frac{Q}{x}\int_x^1 d\xi\, C\left(\frac{Q\xi}{x}, Q, \mu\right) f_{i/p}(\xi, \mu)$$

$$p^- = \frac{Q}{x}$$

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = H(m_b,\mu)\int dk^+ J(k^+,\mu)\ S(2E_\gamma - m_b + k^+,\mu)$$

$$+\ldots$$

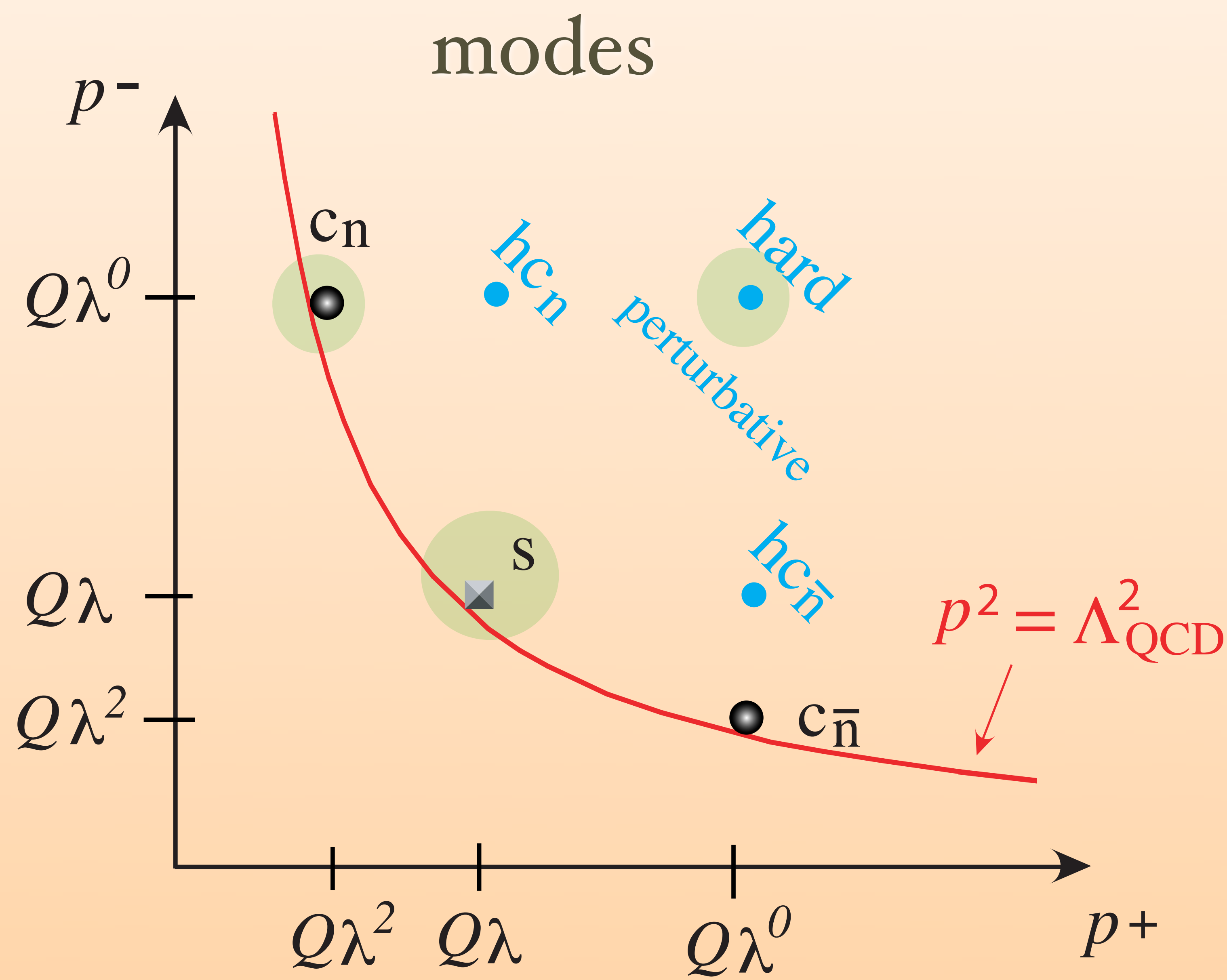
Factorization formulas of this type have also been derived for the power corrections using SCET

SCET$_{\text{II}}$

$$\lambda = \frac{\Lambda}{Q}$$

- So far we have considered inclusive processes with jets, or processes with only one identified hadron like DIS
- SCET$_{\text{II}}$ allows us to treat cases with two or more hadrons

  eg. $B \to D\pi$, $B \to \pi\ell\bar{\nu}$, $B \to \pi\pi$



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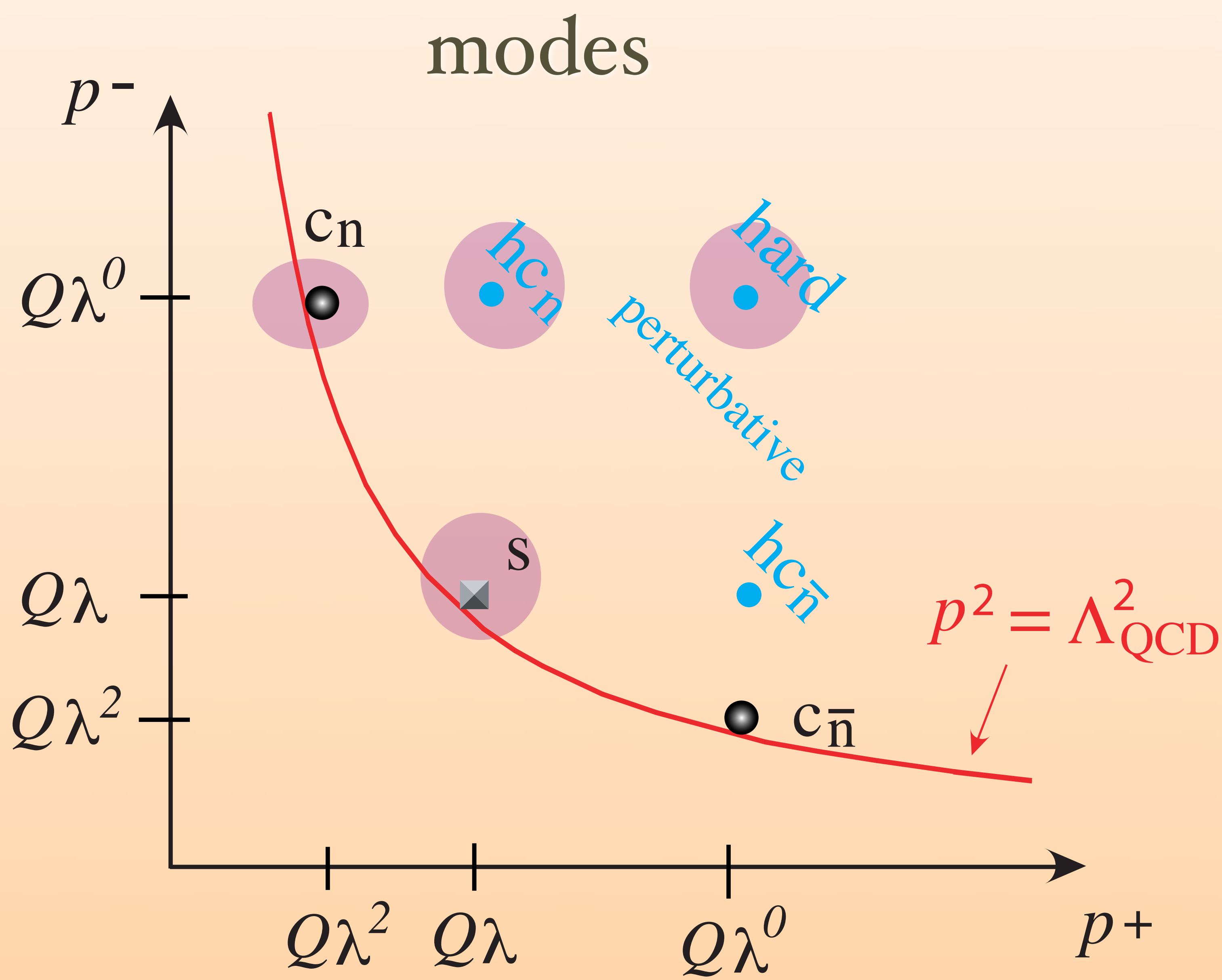

SCET_{II}

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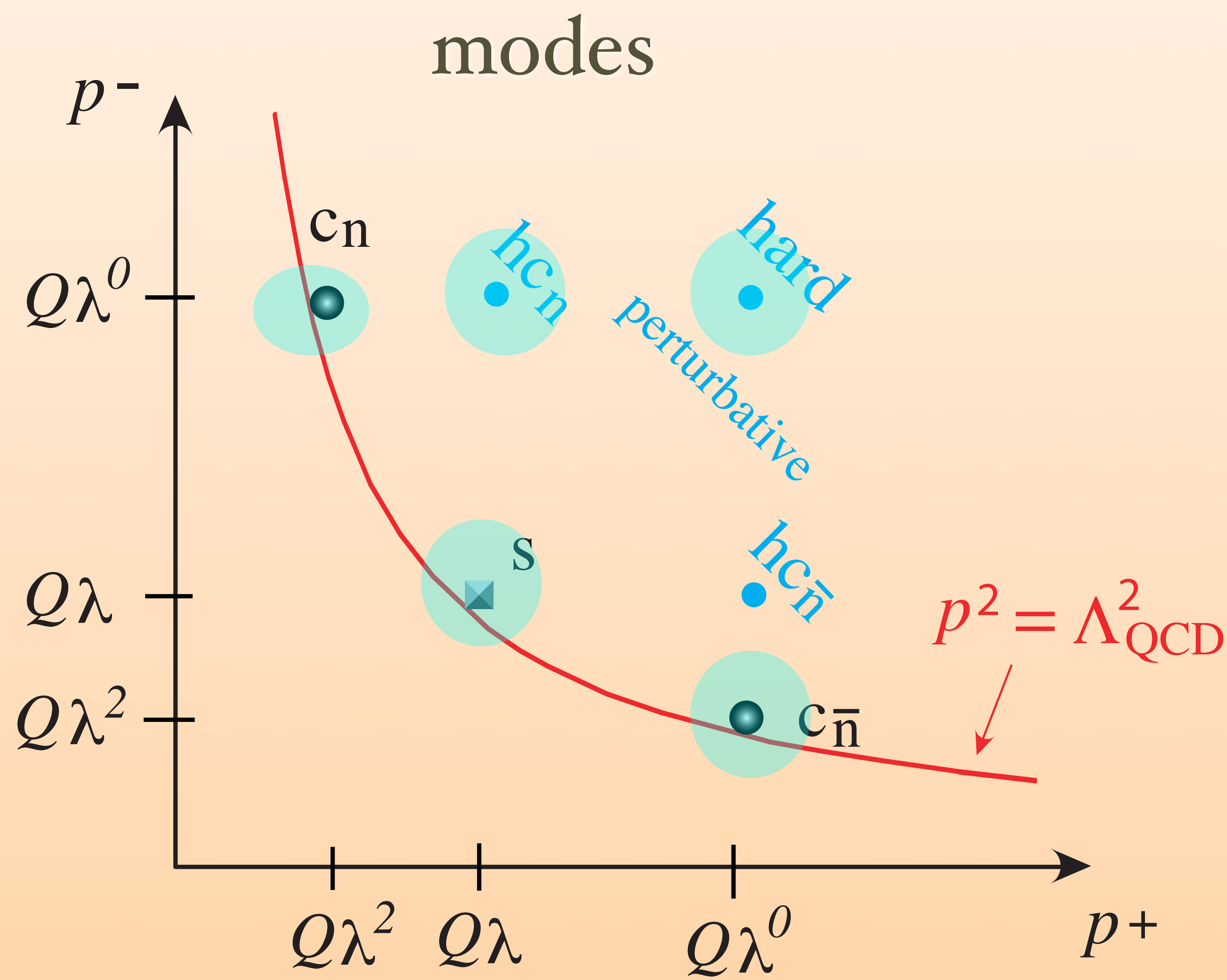
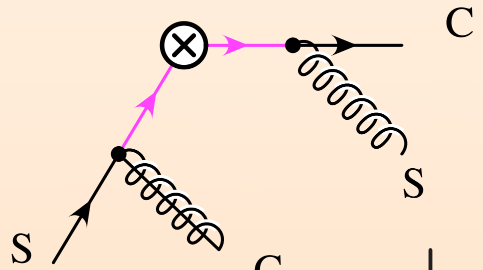


Constructing $\mathrm{SCET_{II}}$ Operators

- For simplicity consider a collinear (c_n) and a soft (s) mode

We can construct operators directly from QCD by integrating out the offshell modes

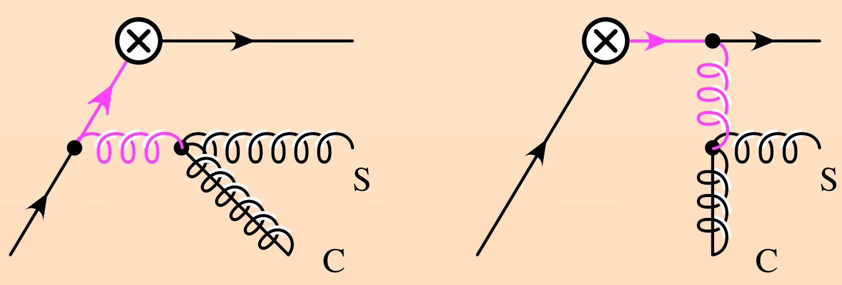
$$q = q_s + q_n \sim Q(\lambda, 1, \lambda) \quad \text{in h.c.} \qquad q^2 \sim Q\lambda \gg \Lambda^2$$



$+ \ldots$

builds up $\bar{\xi}_n \, S_n^\dagger \, \Gamma \, W \, q_s$

soft Wilson line



$+ \ldots$

switches order $\bar{\xi}_n \, W \, \Gamma \, S_n^\dagger \, q_s$

Soft & Collinear
Gauge Invariant

Soft-Collinear Factorization

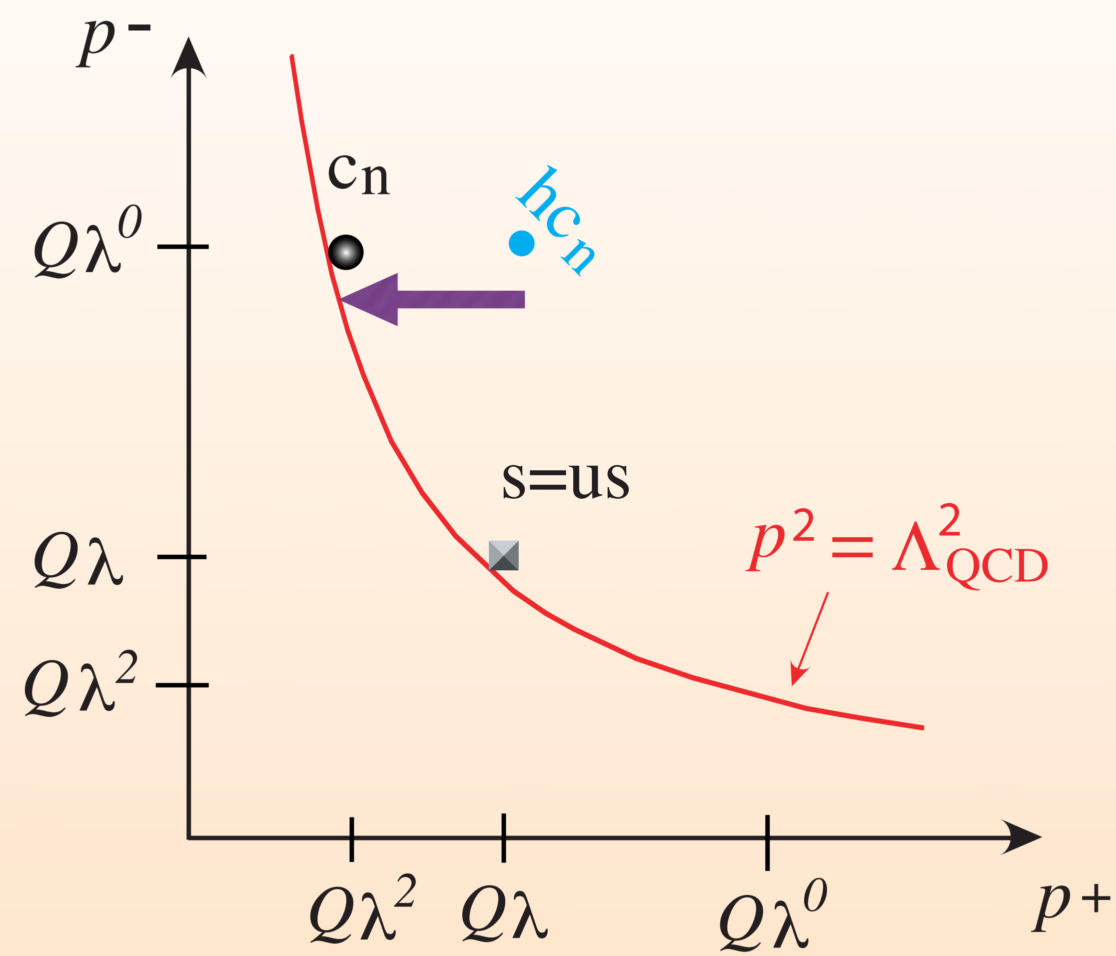
A Simpler Method: use factorization in SCETI

1) Match QCD onto $\mathrm{SCET_I}$

2) Factorize usoft with field redefinition

3) Match onto $\mathrm{SCET_{II}}$

$$\{\mathrm{hc}_n, \mathrm{us}\} \longrightarrow \{\mathrm{c}_n, \mathrm{s}\}$$

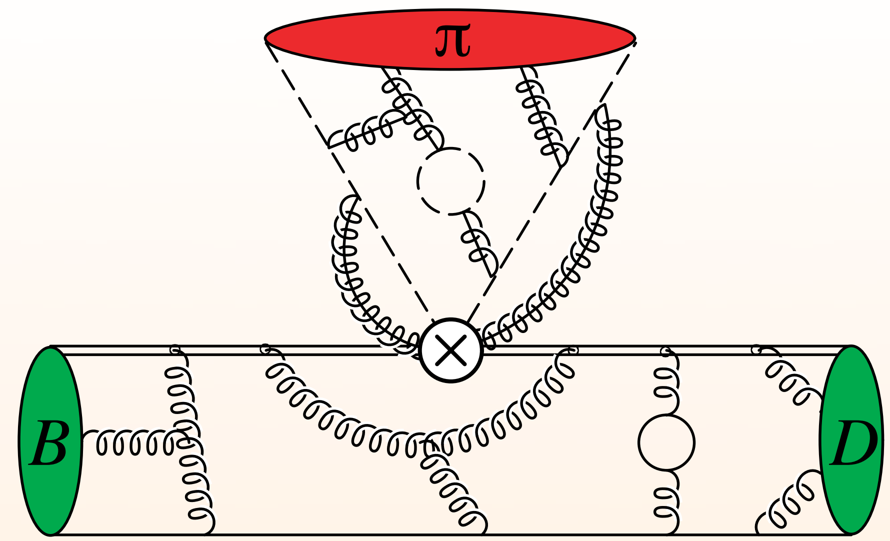


eg.

$$J = (\bar{\xi}_n W)\Gamma h_v$$
$$= (\bar{\xi}_n W)\Gamma(Y_n^\dagger h_v)$$
$$\longrightarrow J = (\bar{\xi}_n W)\Gamma(S_n^\dagger h_v)$$

In this matching, the power of λ can only increase and does so due to change in scaling to uncontracted fields

Exclusive Example $B \to D\pi^-$



Steps

- Match at $\mu^2 \sim Q^2$ onto SCET$_\mathrm{I}$ [Decouple $\xi \to Y\xi^{(0)}$]

$$\left.\begin{array}{c} [\bar{c}\, b]\, [\bar{d}\, u] \\ [\bar{c}\, T^A b]\, [\bar{d}\, T^A u] \end{array}\right\} \Longrightarrow \left\{\begin{array}{c} [\bar{h}_{v'}^{(c)}\;\; h_v^{(b)}]\, [\bar{\xi}^{(0)}_{n,p'} W^{(0)} C_0(\bar{\mathcal{P}}_+) W^{(0)\dagger} \xi^{(0)}_{n,p}] \\ [\bar{h}_{v'}^{(c)}\; Y T^A Y^\dagger\, h_v^{(b)}]\, [\bar{\xi}^{(0)}_{n,p'} W^{(0)} C_8(\bar{\mathcal{P}}_+) T^A W^{(0)\dagger} \xi^{(0)}_{n,p}] \end{array}\right.$$

- Match at $\mu^2 \sim Q\Lambda$ onto SCET$_\mathrm{II}$

$$[\bar{h}_{v'}^{(c)}\;\; h_v^{(b)}]\, [\bar{\xi}_{n,p'} W C_0(\bar{\mathcal{P}}_+) W^\dagger \xi_{n,p}]$$

$$[\bar{h}_{v'}^{(c)}\; S T^A S^\dagger\, h_v^{(b)}]\, [\bar{\xi}_{n,p'} W C_8(\bar{\mathcal{P}}_+) T^A W^\dagger \xi_{n,p}] \longleftarrow$$

Factorized!

octet m.elt. will vanish

- Take matrix elements

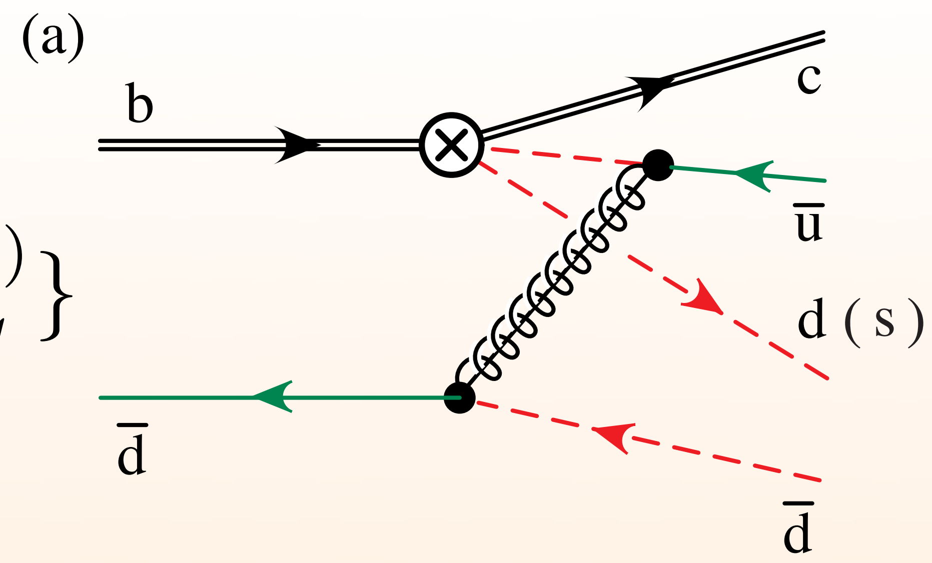
$$\langle \pi_n | \bar{\xi}^{(0)}_{n,p'} W^{(0)} C_0(\bar{\mathcal{P}}_+) W^{(0)\dagger} \xi^{(0)}_{n,p} | 0 \rangle = \frac{i}{2} f_\pi E_\pi \int dx\, C[2E_\pi(2x-1)] \phi_\pi(x)$$

$$\langle D_{v'} | \bar{h}_{v'} \Gamma_h h_v | B_v \rangle = F^{B\to D}(0)$$

$$\boxed{\langle D\pi | \bar{c} b \bar{u} d | B \rangle = N\, F^{B\to D} \int_0^1 dx\, T(x,\mu)\, \phi_\pi(x,\mu)}$$

+ power corrections

Power Corrections & Color Suppressed Decays



$$\bar{B}^0 \to D^0 \pi^0 \,,$$
$$\bar{B}^0 \to D^{*0} \pi^0$$

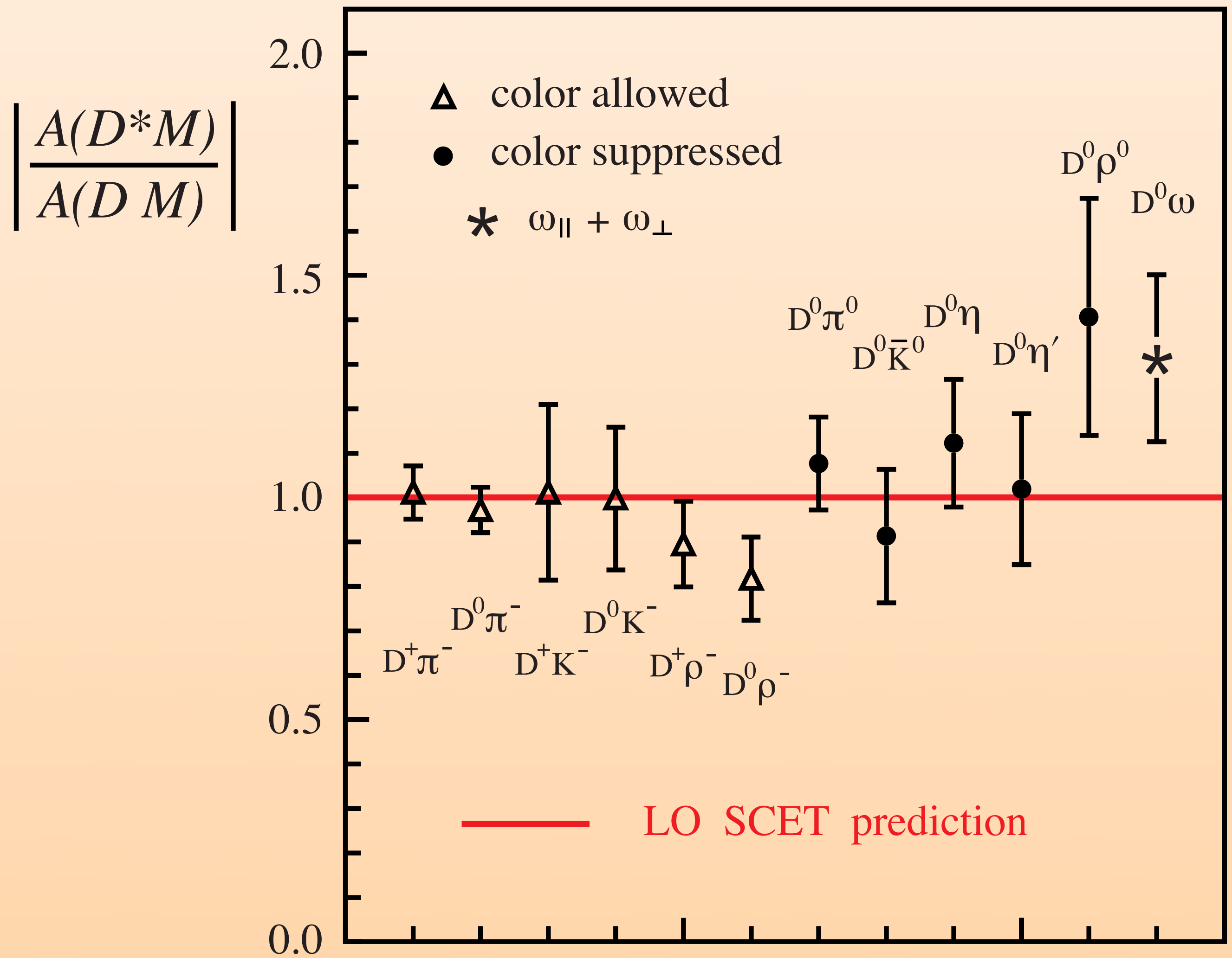
$$T\{O^{(0)} \mathcal{L}^{(1)}_{\xi q} \mathcal{L}^{(1)}_{\xi q}\}$$

$$\mathcal{L}^{(1)}_{\xi q} = (\bar{q} Y) ig \not{B}^{\perp}_{n,\omega'} \chi_n$$

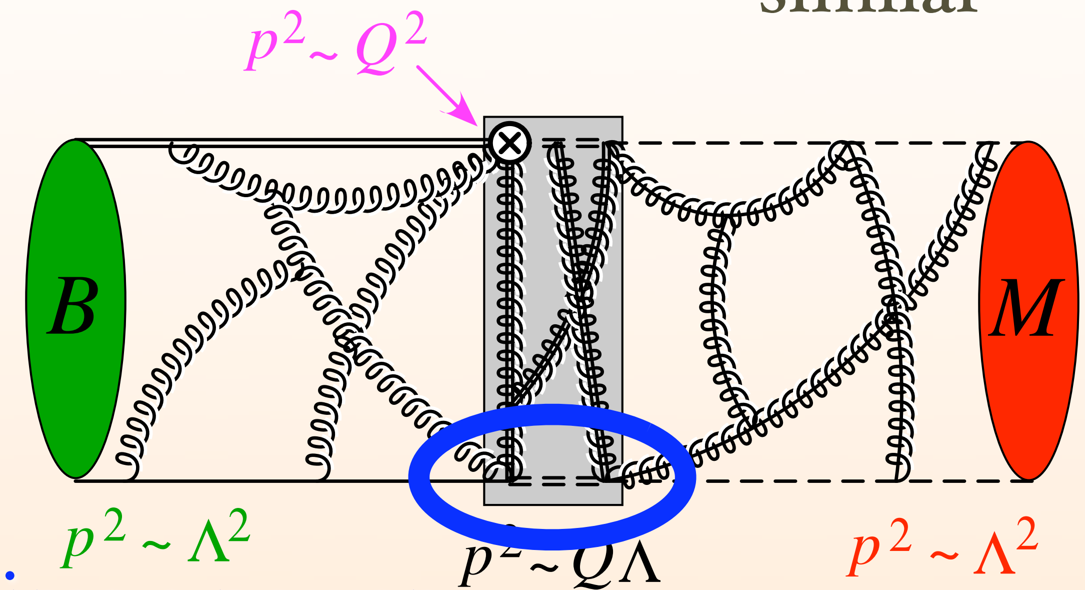
$$A^{D^{(*)}}_{00} = N^{(*)}_0 \int dx\, dz\, dk_1^+ dk_2^+ \, T^{(i)}(z)\, J^{(i)}(z, x, k_1^+, k_2^+)\, S^{(i)}(k_1^+, k_2^+)\, \phi_M(x)$$

Comparison to Data

$$\delta(D\pi) = 30.4 \pm 4.8^\circ$$
$$\delta(D^*\pi) = 31.0 \pm 5.0^\circ$$



Another Exclusive Example $B \to \pi \ell \bar{\nu}$ $(B \to \pi\pi)$ similar



Requires a power suppressed interaction

SCET$_{\rm I}$

needs time-ordered products of

$$Q^{(0)} = \bar{\chi}_{n,\omega} \Gamma \mathcal{H}_v^n$$

$$Q^{(1)} = \bar{\chi}_{n,\omega} ig \mathcal{B}^{\perp}_{n,\omega'} \Gamma \mathcal{H}_v^n$$

with

$$\mathcal{L}^{(1)}_{\xi q} = (\bar{q} Y) ig \mathcal{B}^{\perp}_{n,\omega'} \chi_n \quad ,\ldots$$

$$f(E) = \int dz\, T(z,E)\, \zeta_J^{BM}(z,E) + C(E)\, \zeta^{BM}(E)$$

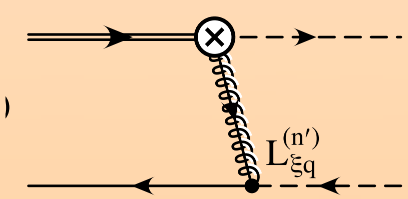
same functions in $B \to \pi\pi$

universality at $E\Lambda$

SCET$_{\rm II}$ (further factorization)

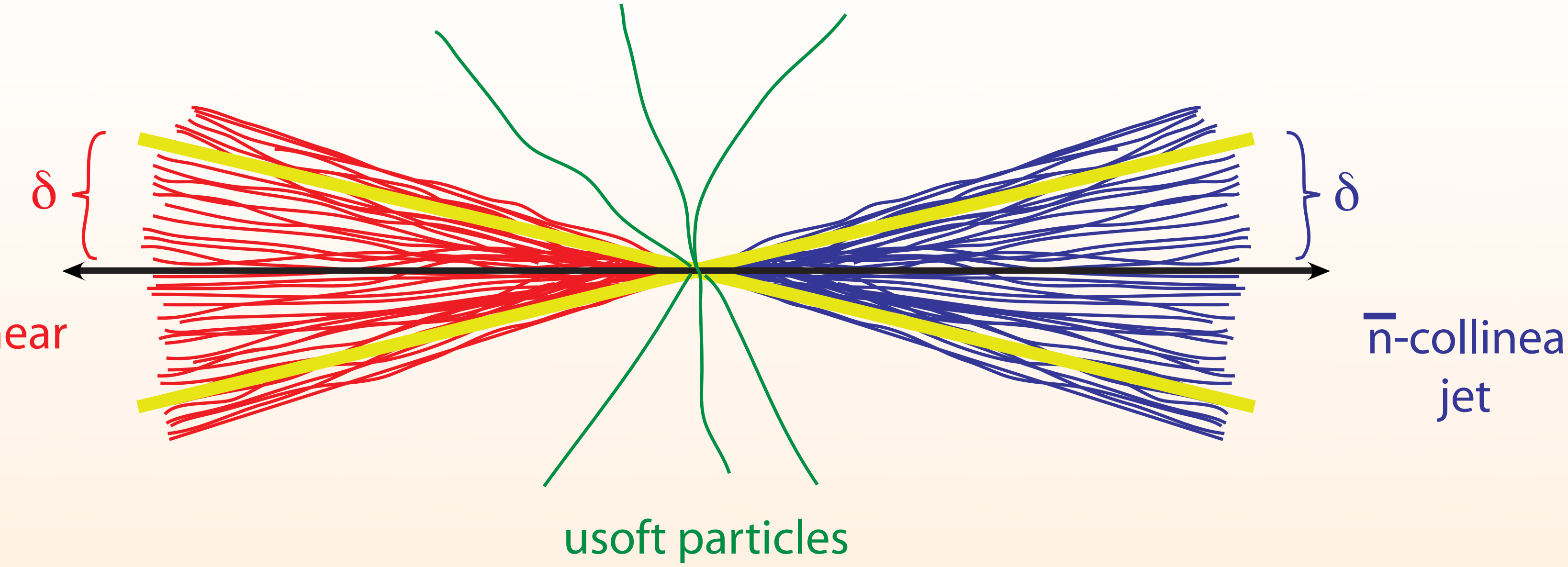
$$\zeta_J^{BM}(z) = f_M f_B \int_0^1 dx \int_0^\infty dk^+ J(z,x,k^+,E) \phi_M(x) \phi_B(k^+)$$

$\zeta^{BM} = \,?$ has endpoint singularities



$$\int_0^1 dx\, \frac{\phi_\pi(x)}{x^2}$$

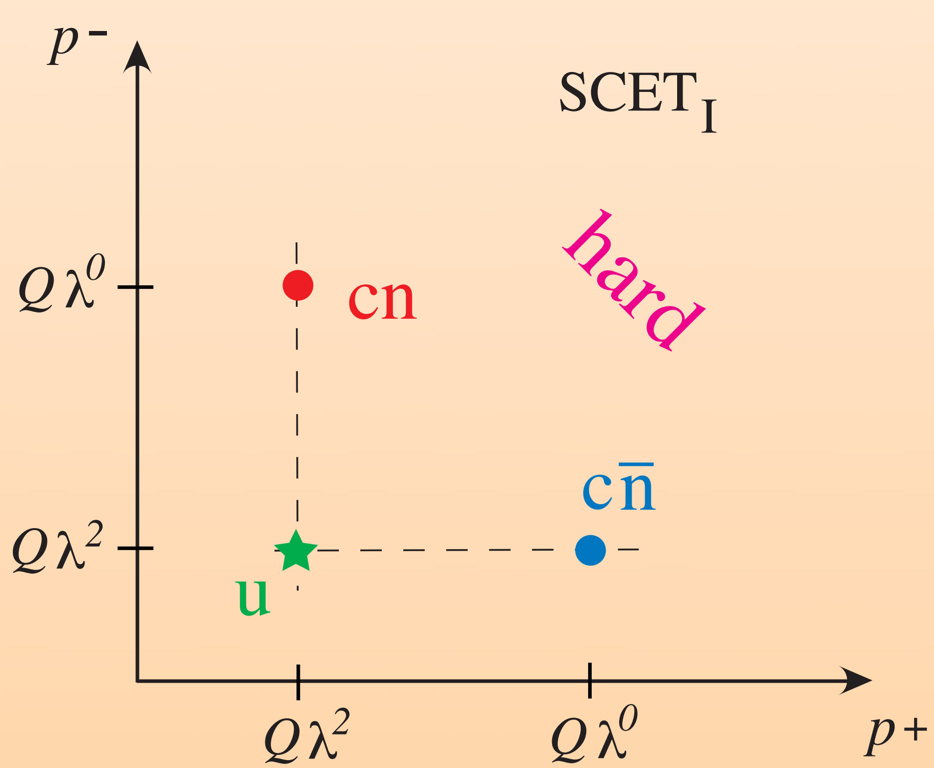
eg. $\quad e^+e^- \to 2$ jets



event shapes in two jet region

$$\frac{d\sigma}{de} = \frac{1}{Q^2}\sum_X \text{Ł}_{\mu\nu}\langle 0|J^{\dagger\nu}(0)|X\rangle\langle X|J^{\mu}(0)|0\rangle\delta(e-e(X))\delta^4(q-p_X)$$

$\mathrm{SCET_I}$

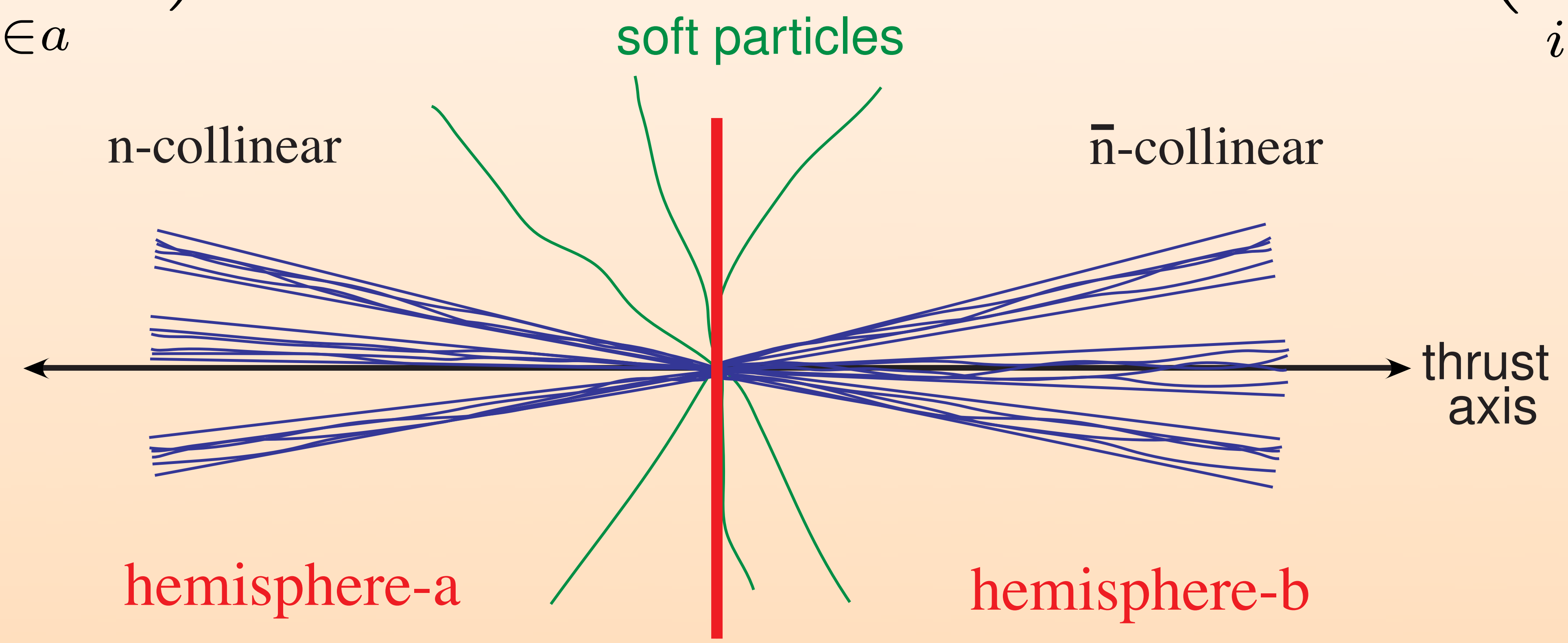


$$|X\rangle = |X_n X_{\bar n} X_{us}\rangle$$

What observable? $\frac{d^2\sigma}{dM^2\, d\overline{M}^2}$

Hemisphere Invariant Masses

$$M^2 = \Big(\sum_{i\in a} p_i^\mu\Big)^2 \qquad \overline{M}^2 = \Big(\sum_{i\in b} p_i^\mu\Big)^2$$



Dijet region: $M^2, \overline{M}^2 \ll Q^2$

Let: $s \equiv M^2$

$\bar{s} \equiv \overline{M}^2$

In QCD: The full cross-section is

a restricted set of states: $s \equiv M^2 \ll Q^2$

$$\sigma = \sum_X^{res.} (2\pi)^4\, \delta^4(q - p_X) \sum_{i=a,v} L^i_{\mu\nu}\, \langle 0|\mathcal{J}_i^{\nu\dagger}(0)|X\rangle\langle X|\mathcal{J}_i^{\mu}(0)|0\rangle$$

lepton tensor, γ & Z exchange

by using EFT's we will be able to move these restrictions into the operators

In SCET:

$$\mathcal{J}_i^{\mu}(0) = \int d\omega\, d\bar\omega\, C(\omega,\bar\omega,\mu) J_i^{(0)\mu}(\omega,\bar\omega,\mu)$$

Wilson coefficient

SCET current

$$(\bar\xi_n W_n)_\omega\, Y_n^\dagger \Gamma^\mu Y_{\bar n} (W_{\bar n}^\dagger \xi_{\bar n})_{\bar\omega}$$

$$\equiv \bar\chi_{n,\omega}\, Y_n^\dagger \Gamma^\mu Y_{\bar n} \chi_{\bar n,\bar\omega}$$

Momentum conservation:

$$\to C(Q, Q, \mu)$$

SCET cross-section:

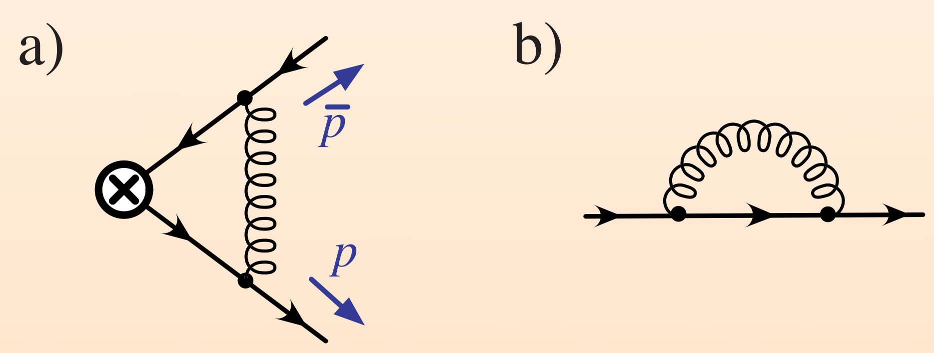
$$|X\rangle = |X_n X_{\bar n} X_s\rangle$$

$$\sigma = K_0 \sum_{\vec n} \sum_{X_n X_{\bar n} X_s}^{res.!} (2\pi)^4\, \delta^4(q - P_{X_n} - P_{X_{\bar n}} - P_{X_s}) \langle 0|\overline{Y}_{\bar n}\, Y_n|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger|0\rangle$$

$$\times\ |C(Q,\mu)|^2\, \langle 0|\bar{\not n}\chi_{n,\omega'}|X_n\rangle\langle X_n|\overline{\chi}_{n,\omega}|0\rangle\langle 0|\overline{\chi}_{\bar n,\bar\omega'}|X_{\bar n}\rangle\langle X_{\bar n}|\not n\chi_{\bar n,\bar\omega}|0\rangle$$

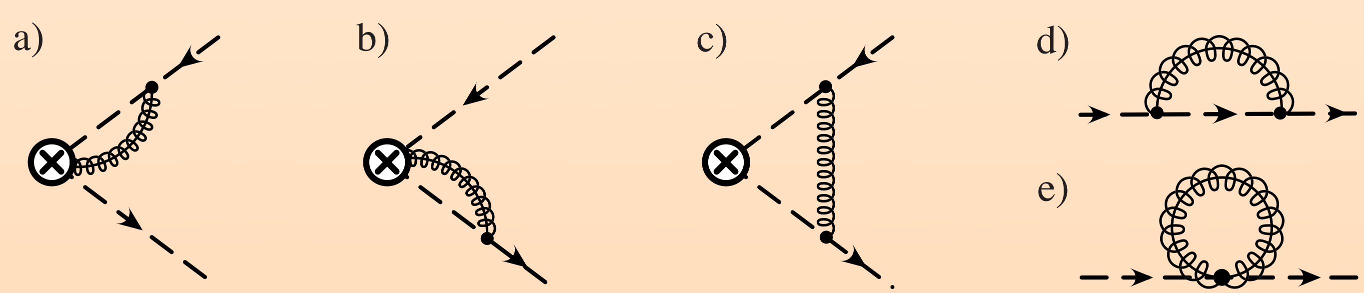
all-orders

QCD



one-loop

SCET



difference gives one-loop matching:

$$C(Q,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left[3\log\frac{-Q^2 - i0}{\mu^2} - \log^2\frac{-Q^2 - i0}{\mu^2} - 8 + \frac{\pi^2}{6}\right]$$

Specify hemisphere invariant masses for the jets:

total soft momentum is the sum of momentum in each hemisphere

$$K_{X_s} = k_s^a + k_s^b \qquad \hat{P}_a \, |X_s\rangle = k_s^a \, |X_s\rangle, \quad \hat{P}_b \, |X_s\rangle = k_s^b \, |X_s\rangle$$

hemisphere projection operators

Insert: $$1 = \int ds \, \delta\Big((p_n + k_s^a)^2 - s\Big) \int d\bar{s} \, \delta\Big((p_{\bar{n}} + k_s^b)^2 - \bar{s}\Big)$$

expand: $$\delta\Big((p_n + k_s^a)^2 - s\Big) = \frac{1}{Q} \, \delta\Big(k_n^+ + k_s^{+a} - \frac{s}{Q}\Big)$$

$$\delta\Big((p_{\bar{n}} + k_s^b)^2 - \bar{s}\Big) = \frac{1}{Q} \, \delta\Big(k_n^- + k_s^{-b} - \frac{\bar{s}}{Q}\Big)$$

... Some Algebra ...

$$\frac{d^2\sigma}{ds\,d\bar{s}} = \frac{\sigma_0}{Q^2}\left|C(Q,\mu)\right|^2 \int dk_n^+\, dk_{\bar{n}}^-\, d\ell^+\, d\ell^- \delta\Big(k_n^+ + \ell^+ - \frac{s}{Q}\Big)\delta\Big(k_{\bar{n}}^- + \ell^- - \frac{\bar{s}}{Q}\Big)$$

$$\times \sum_{X_n} \frac{1}{2\pi}\int d^4x\, e^{ik_n^+ x^-/2}\, \mathrm{tr}\langle 0|\bar{\not{n}}\chi_n(x)|X_n\rangle\langle X_n|\bar{\chi}_{n,Q}(0)|0\rangle$$

$$\times \sum_{X_{\bar{n}}} \frac{1}{2\pi}\int d^4y\, e^{ik_{\bar{n}}^- y^+/2}\, \mathrm{tr}\langle 0|\overline{\chi}_{\bar{n}}(y)|X_{\bar{n}}\rangle\langle X_{\bar{n}}|\not{n}\chi_{\bar{n},-Q}(0)|0\rangle$$

$$\times \sum_{X_s} \frac{1}{N_c}\delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\mathrm{tr}\langle 0|\overline{Y}_{\bar{n}}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\, \overline{Y}_{\bar{n}}^\dagger(0)|0\rangle$$

Factorization Theorem:

$$\boxed{\frac{d^2\sigma}{ds\,d\bar{s}} = \sigma_0\, H_Q(Q,\mu) \int_{-\infty}^{+\infty} d\ell^+ d\ell^-\, J_n(s - Q\ell^+,\mu)\, J_{\bar{n}}(\bar{s} - Q\ell^-,\mu)\, S_{\mathrm{hemi}}(\ell^+,\ell^-,\mu)}$$

Hard Function

$H_Q(Q,\mu) = |C(Q,\mu)|^2$

Quark Jet Function

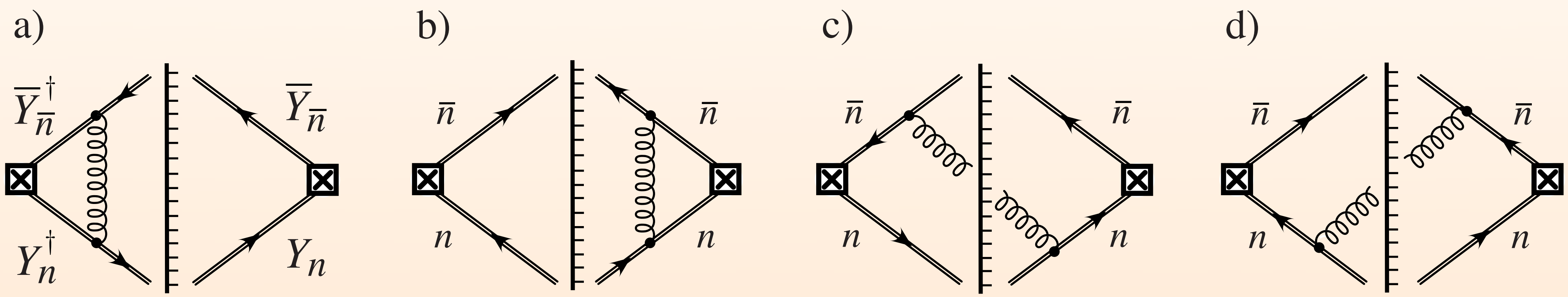
Anti-quark Jet Function

Soft radiation Function

universal

$S_{\rm hemi}(\ell^+, \ell^-, \mu)$

$$S_{\rm hemi}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b}) \langle 0 | \overline{Y}_{\bar n}\, Y_n(0) | X_s \rangle \langle X_s | Y_n^\dagger\, \overline{Y}_{\bar n}^\dagger(0) | 0 \rangle$$



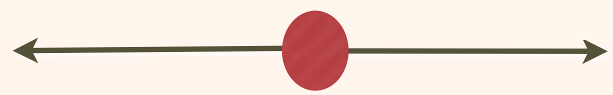
Soft function is perturbative if $\ell^+, \ell^- \gg \Lambda_{\rm QCD}$
and is nonperturbative if $\ell^+, \ell^- \sim \Lambda_{\rm QCD}$

It is also universal, it appears in many different event shapes (thrust, heavy-jet mass, ...) for both massless and massive jets

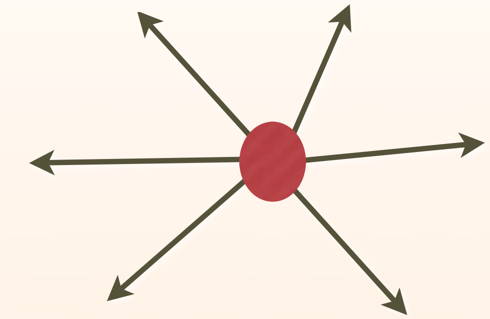
A very popular event shape is thrust $T = \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \mathbf{p}_i|}{Q}$

Thrust

$T = 1$ dijet



$T = \frac{1}{2}$



Insert: $$1 = \int dT \, \delta\Big(1 - T - \frac{s + \bar{s}}{Q^2}\Big)$$

Factorization theorem

$$\frac{d\sigma}{dT} = \sigma_0 \, H(Q,\mu) \int ds \; J_T(s,\mu) \; S_{\rm thrust}\Big(Q(1-T) - \frac{s}{Q}, \mu\Big)$$

with $$S_{\rm thrust}(\ell,\mu) = \int_0^\infty d\ell^+ d\ell^- \; \delta(\ell - \ell^+ - \ell^-) \; S_{\rm hemi}(\ell^+, \ell^-, \mu)$$

SCET is a field theory which:

- explains how soft & collinear degrees of freedom communicate with each other, and with hard interactions
- organizes the interactions in a series expansion in λ

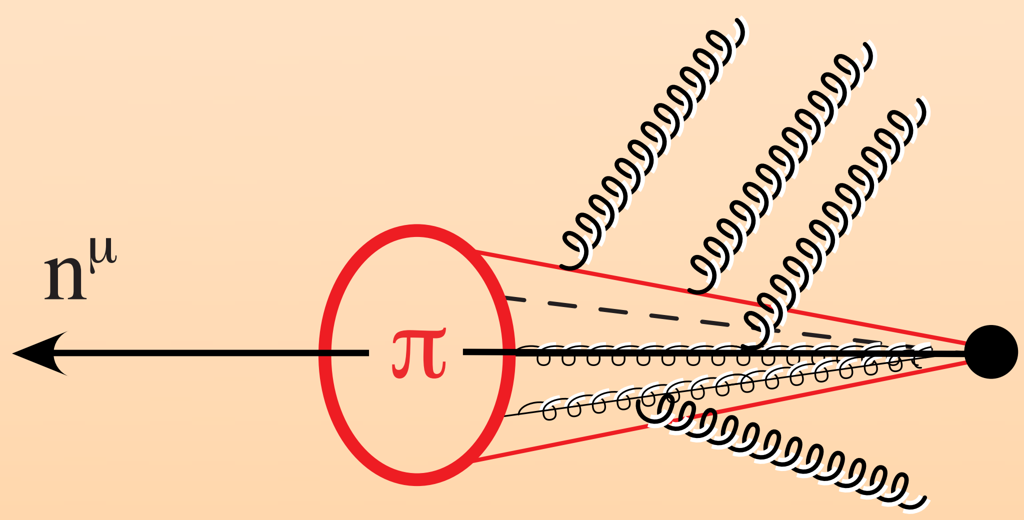
  which measures how collinear/soft the particles are

$$\lambda = \sqrt{\frac{\Lambda_{\rm QCD}}{m_b}} \qquad \lambda = \frac{\Lambda_{\rm QCD}}{m_b} \qquad \lambda^2 = \frac{m_X^2}{Q^2}$$

- provides a simple operator language to derive factorization theorems in fairly general circumstances

  eg. unifies the treatment of factorization for exclusive and inclusive QCD processes

- results are constrained by symmetries
- scale separation & decoupling



How is SCET used?

- cleanly separate short and long distance effects in QCD
  - ➡ derive new factorization theorems
  - ➡ find universal hadronic functions, exploit symmetries
  - ➡ predict decay rates and cross sections
- model independent, systematic expansion
  - ➡ study power corrections
- keep track of μ dependence
  - ➡ sum large logarithms

The End