Introduction to the Soft - Collinear Effective Theory

An effective field theory for energetic hadrons & jets

$E \gg \Lambda_{\rm QCD}$

Lecture 1

Heavy Quark Physics
Dubna International Summer School
August, 2008

Disclaimer:

There are no references in this talk, and it is not a guide to the literature. My goal is to make you familiar with the formalism and techniques that are commonly used, and to go through a few choice examples in detail.

As a reference for this material see my lecture notes (http://web.mit.edu/8.851/www/scet_2006.pdf).

Outline

- Review of Effective Field Theory Concepts
- Introduction to SCETI, SCETII
  - momentum scales and momentum regions
  - collinear & soft degrees of freedom
- Quick review of HQET
- Construction of SCET
  - propagators, field power counting
  - separating momenta & gauge fields
  - leading order collinear Lagrangian
- Wilson lines and the heavy-light current

EFT Concepts

1) The ingredients in any EFT: Relevant degrees of freedom, symmetries, scales & power counting

2) Renormalization: Meaning of parameters

3) Decoupling: Effects from heavy particles and offshell particles are suppressed

4) Matching: How we can encode dynamics of one theory into another

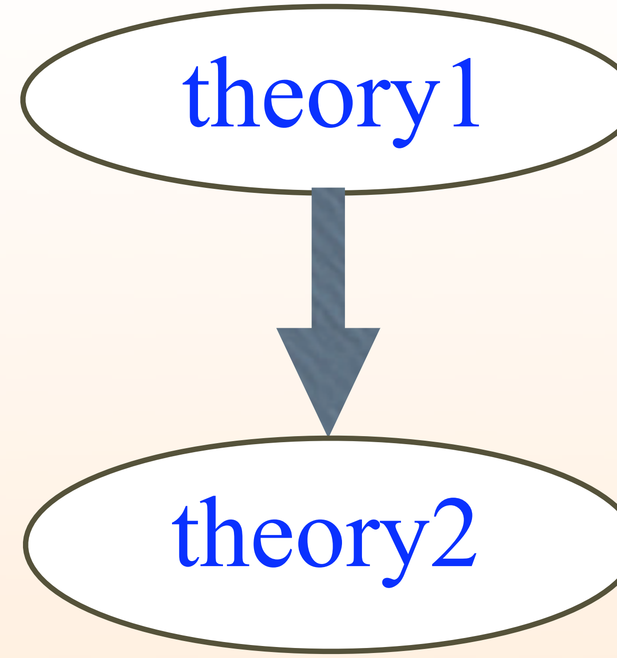
5) Running: Connecting physics at different momentum scales using renormalization group techniques

Top

Theory 1 is understood, but it is useful to have the simpler theory 2 at low energies.

Integrate out heavier particles in 1 and **match** onto 2

$$\mathcal{L}_{\mathrm{theory1}} \to \sum_n \mathcal{L}^{(n)}_{\mathrm{theory2}}$$

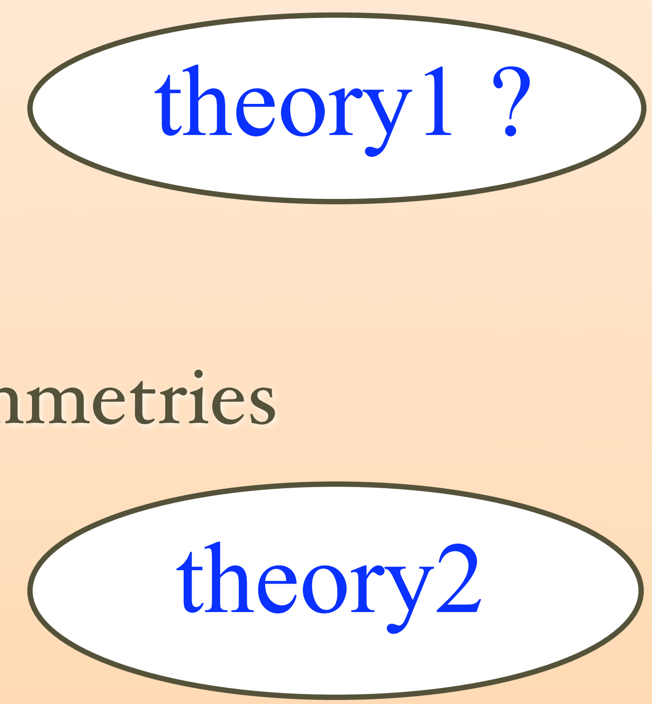


Theory 1 & theory 2 agree in the IR, differ in UV

eg. NRQED; Heavy Quark Effective Theory;
Remove t,W,Z: H_{weak} ; Soft Collinear Effective Theory

Bottom

Theory 1 is unknown or matching is too difficult to carry out analytically

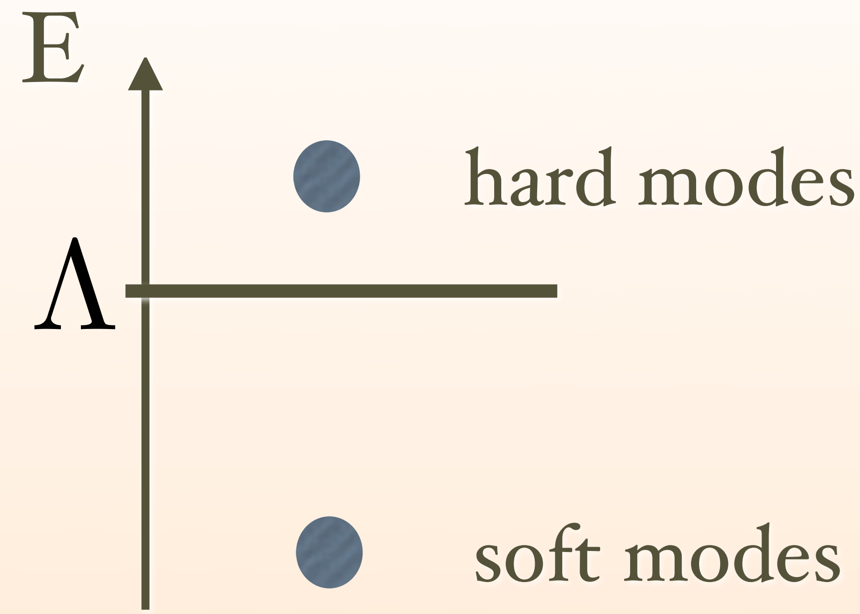


Construct $\sum_n \mathcal{L}^{(n)}_{\mathrm{theory2}}$ by writing down most general set of interactions consistent with symmetries

eg. Standard Model; Chiral Perturbation theory for low energy pion and Kaon interactions

Wilsonian vs. Continuum EFT

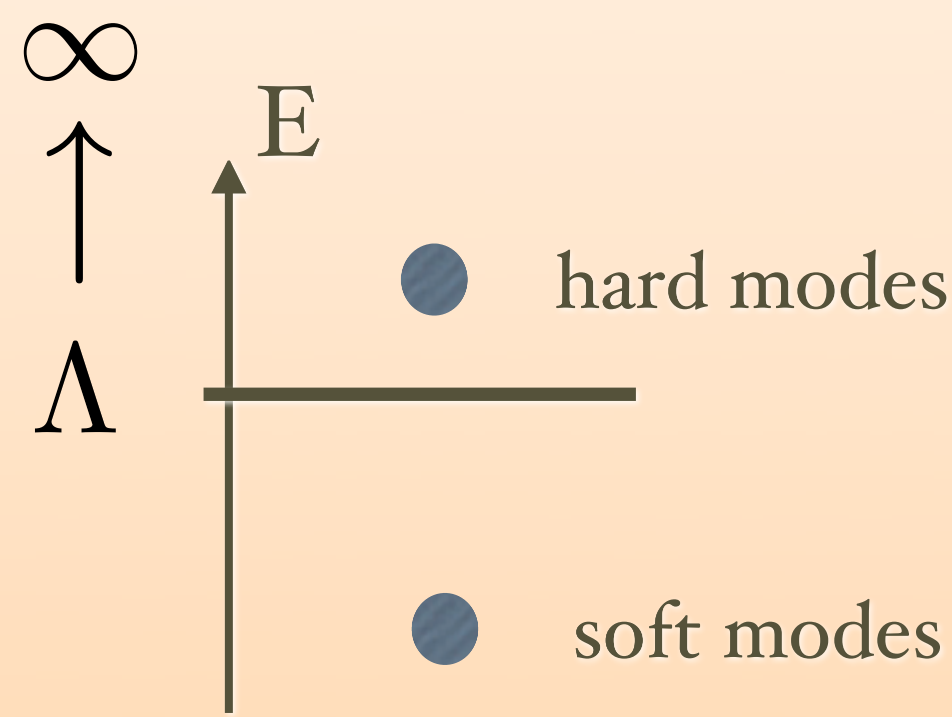
Wilson effective action for soft modes e^{-S_Λ}



removing modes with $\Lambda - \delta\Lambda < E < \Lambda$

$$e^{-S_{\Lambda-\delta\Lambda}} = \int_{\delta\Lambda} d\phi \; e^{-S_\Lambda}$$

Continuum $\mathcal{L}^{\rm EFT} = C(\mu)\mathcal{O}(\mu)$



operators for soft modes $\mathcal{O}(\mu)$

Wilson coefficients for hard modes $\mathrm{C}(\mu)$

Sending Λ to ∞ double counts the hard region in matrix elements of our operators, but we fix $\mathrm{C}(\mu)$ to correct for this. μ is the scale where this matching is done.

EFT Principles

1) Dynamics at low E does not depend on details of dynamics at high E

2) Build an EFT using the relevant d.o.f. and known symmetries.

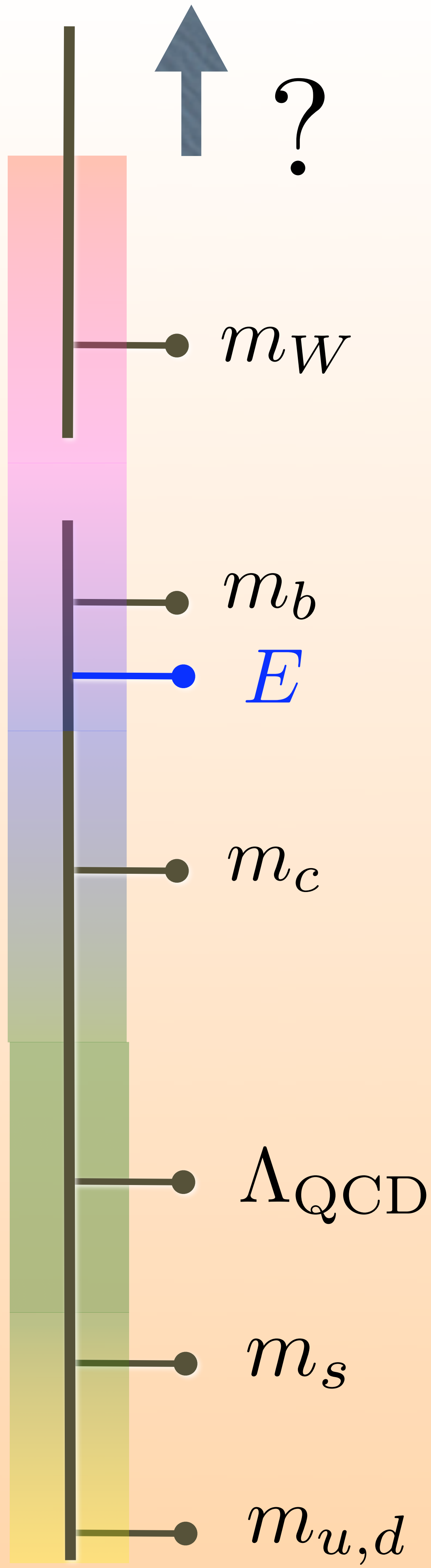
3) EFT has an infinite number of operators, but only a finite number are needed for a given precision as determined by the power counting. With this precision this set closes under renormalization.

$$\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots$$

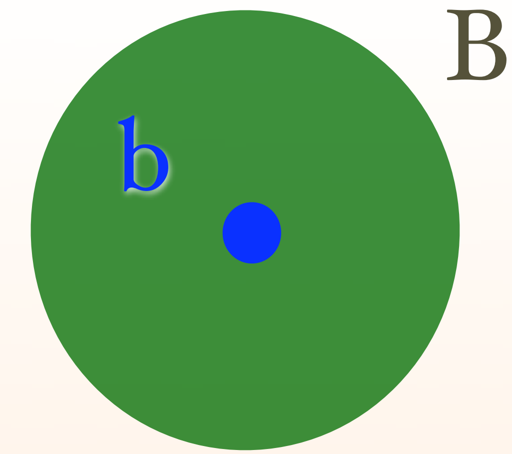
4) EFT has same infrared but different ultraviolet than the more fundamental theory.

5) Nature of high energy theory shows up as couplings and symmetries in the low energy EFT.

Our goal here is to apply these principles to encode hard perturbative QCD dynamics in Wilson coefficients & describe the collinear and soft physics associated with hadronization and jet production in SCET operators



SCET can be used for:



B - decays by weak interactions:

$B \to X_u \ell \bar{\nu}$ $\quad B \to D\pi$ $\quad B \to K^*\gamma$

$B \to \pi \ell \bar{\nu}$ $\quad B \to X_s \gamma$ $\quad B \to \rho\gamma$

$B \to D^*\eta'$ $\quad B \to \rho\rho$ $\quad B \to \pi\pi$ $\quad B \to K\pi$ $\quad B \to \gamma \ell \bar{\nu}$

The B is heavy, so many of its decay products are energetic, E

Any other QCD process with large energy transfer:

$e^- p \to e^- X$ $\quad p\bar{p} \to X \ell^+ \ell^-$

$e^-\gamma \to e^-\pi^0$ $\quad \gamma^* M \to M'$ $\quad \Upsilon \to X\gamma$

$e^+e^- \to \text{jets}$ $\quad e^+e^- \to J/\Psi X$

Degrees of Freedom, Power Counting and Scales, and Lagrangian

What makes SCET different from other simpler EFT's ?

- We will have multiple fields for the same particle

  ξ_n collinear quark field, q_s soft quark field

- We will integrate out offshell modes, but not entire d.o.f.

  eg. compare the electroweak Hamiltonian where the top,W,Z are integrated out completely, to HQET where high energy fluctuations of the bottom quark are integrated out, but low energy ones are kept

- SCET has convolutions $\sum_i C_i O_i \longrightarrow \int d\omega\, C(\omega)\, O(\omega)$

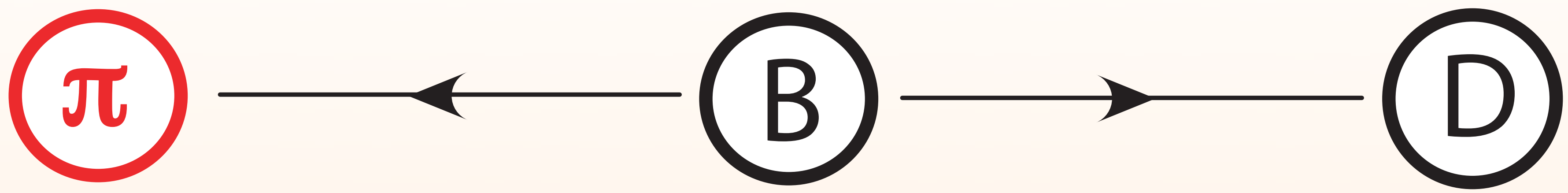
  eg. DIS $F_1(x, Q^2) = \int \frac{d\xi}{x}\, H\Big(\frac{\xi}{x}, \mu, Q\Big)\, f_{q/p}(x, \mu)$

- The power counting parameter λ is not the mass dimension of fields
- Wilson Lines $W = \mathrm{P}\exp\left(ig\int ds\ \bar{n}\cdot A(\bar{n}s)\right)$
- $\frac{1}{\epsilon^2}$ divergences appear at one loop that require UV counterterms

SCET degrees of freedom

Degrees of Freedom

eg.



Pion has: $p_\pi^\mu = (2.310\,\mathrm{GeV}, 0, 0, -2.306\,\mathrm{GeV}) = Q n^\mu$

$Q \gg \Lambda_{\mathrm{QCD}} \qquad n^\mu = (1, 0, 0, -1)$

Light - Cone coordinates: Basis vectors n^μ, $\bar{n}^\mu$ with $n^2 = 0$, $\bar{n}^2 = 0$, $n \cdot \bar{n} = 2$

$$p^\mu = \frac{n^\mu}{2} \bar{n} \cdot p + \frac{\bar{n}^\mu}{2} n \cdot p + p_\perp^\mu \qquad p^+ \equiv n \cdot p, \qquad p^- \equiv \bar{n} \cdot p$$

$$g^{\mu\nu} = \frac{n^\mu \bar{n}^\nu}{2} + \frac{\bar{n}^\mu n^\nu}{2} + g_\perp^{\mu\nu}$$

eg. $\bar{n}^\mu = (1, 0, 0, 1)$

Soft Constituents

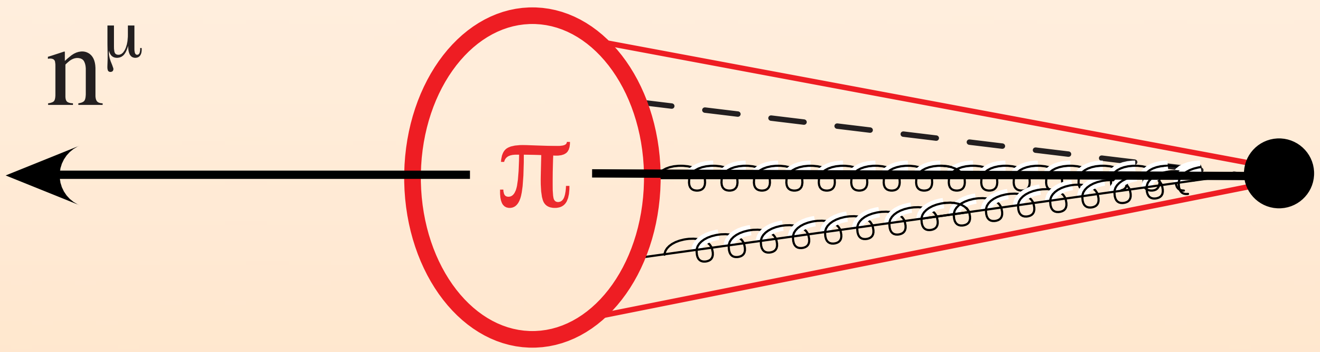
$$p_s^\mu = (p^+, p^-, p^\perp) \sim (\Lambda, \Lambda, \Lambda) \sim Q(\lambda, \lambda, \lambda)$$

D and 

Collinear constituents:

$$p_c^\mu = (p^+, p^-, p^\perp) \sim \left(\frac{\Lambda^2}{Q}, Q, \Lambda\right) \sim Q(\lambda^2, 1, \lambda)$$

Just a boost of the soft constituents, but necessary to describe π and B in the same frame.



SCET$_{\rm II}$ Energetic hadrons

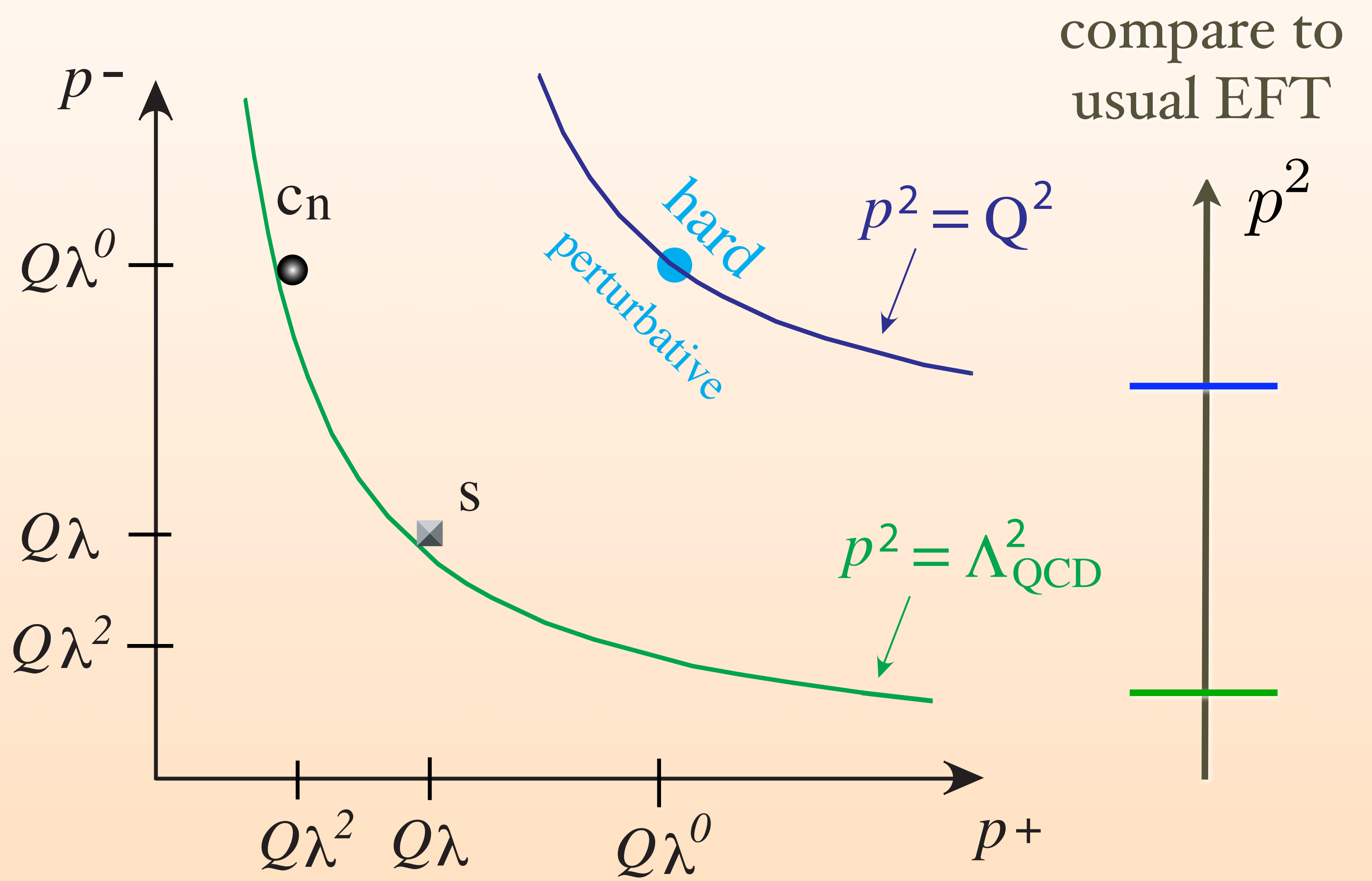
$$\lambda = \frac{\Lambda}{Q}$$

modes	$p^\mu = (+, -, \perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$

It is useful to have a picture to help remember what momentum regions these degrees of freedom occupy

$$p^2 = p^+ p^- - \vec{p}_\perp^{\,2}$$

Here $p^2 = p^+ p^-$
so the hyperbola's are lines of constant invariant mass



SCET_{II} Energetic hadrons

modes	$p^\mu = (+, -, \perp)$	p^2	fields
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soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$

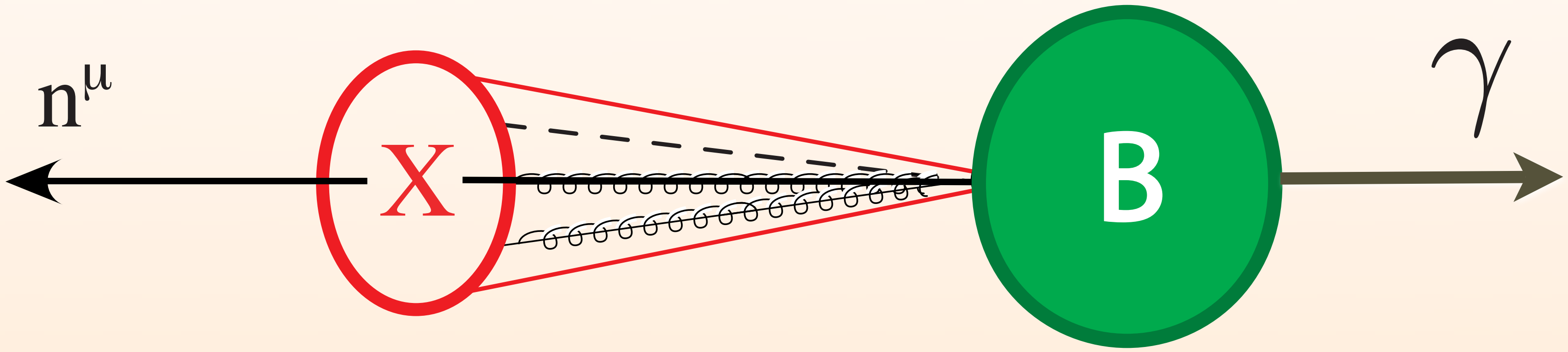
eg. $B \to X_s \gamma$

$m_X^2 \sim \Lambda^2$ not inclusive

$m_X^2 \sim m_b^2$ not a jet

$m_X^2 \sim \Lambda Q$

$\Lambda^2 \ll Q\Lambda \ll Q^2$



Jet constituents: $p^\mu \sim (\Lambda, Q, \sqrt{Q\Lambda}) \sim Q(\lambda^2, 1, \lambda)$

SCET$_\mathrm{I}$ Energetic jets

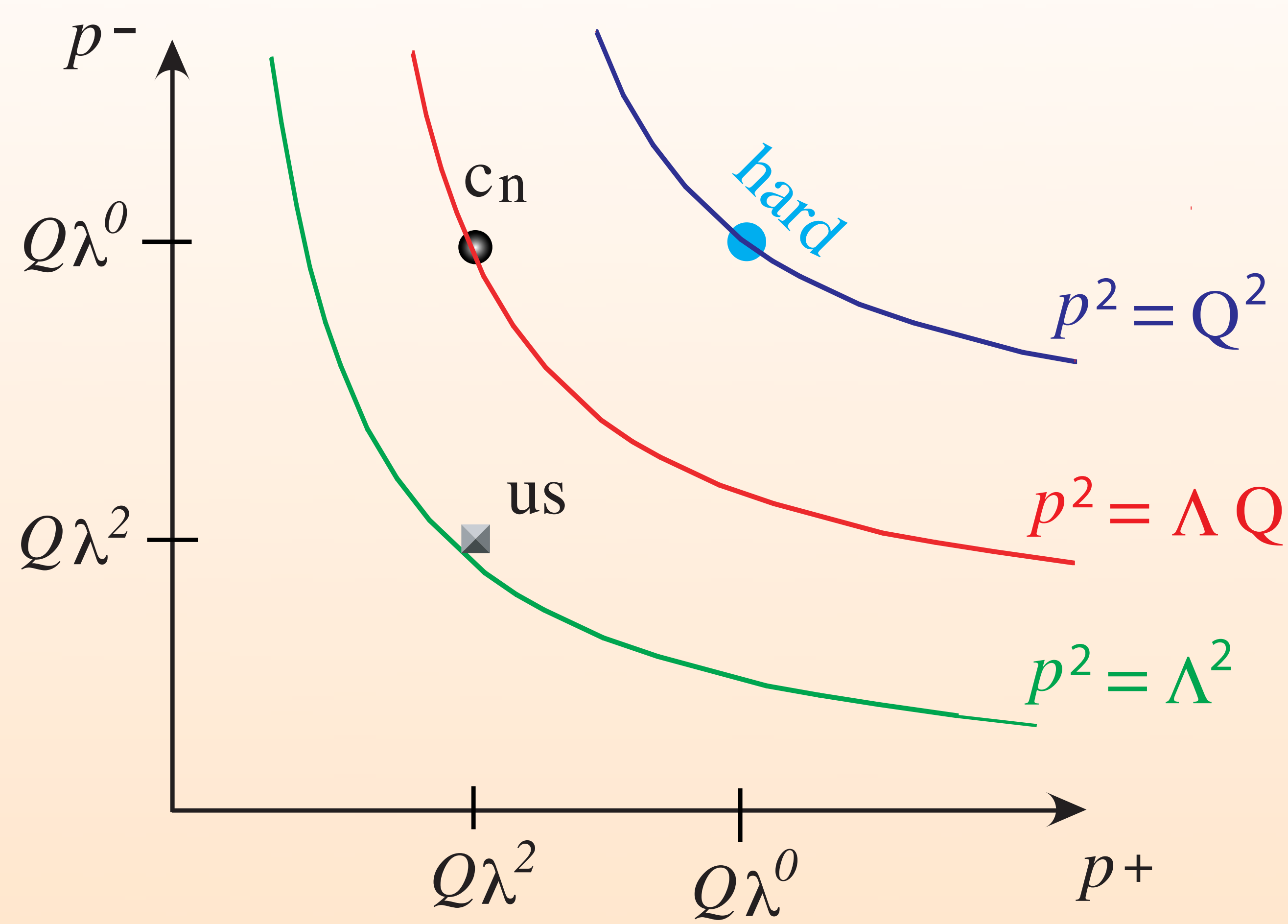
usoft $p^\mu \sim \Lambda$

collinear $p_c^2 \sim Q\Lambda$, $\lambda = \sqrt{\Lambda/Q}$

modes	$p^\mu = (+, -, \perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	ξ_n, A_n^μ
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	q_{us}, A_{us}^μ

The mode picture now becomes

The collinear modes in the jet have larger offshellness than those in an energetic hadron



SCET$_\text{I}$ Energetic jets

modes	$p^\mu = (+, -, \perp)$	p^2	fields
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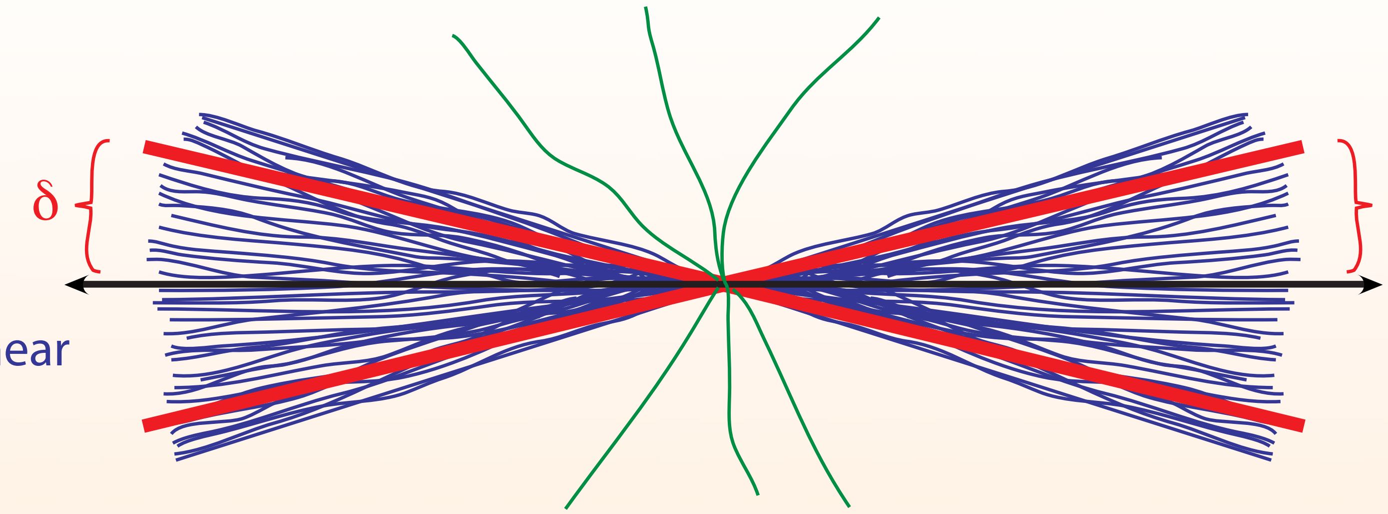
eg. $e^+e^- \to 2$ jets



$m_X^2 \sim \Delta^2$

$\Lambda^2 \ll \Delta^2 \ll Q^2$

$\lambda \sim \delta$

Jet constituents : $p^\mu \sim \left(\frac{\Delta^2}{Q}, Q, \Delta\right) \sim Q(\lambda^2, 1, \lambda)$

$\mathrm{SCET_I}$ Energetic jets

$$\lambda \sim \frac{\Delta}{Q}$$

modes	$p^\mu = (+, -, \perp)$	p^2	fields
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	ξ_n, A_n^μ
$\bar{n}$-collinear	$Q(1, \lambda^2, \lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}}, A_{\bar{n}}^\mu$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	q_{us}, A_{us}^μ

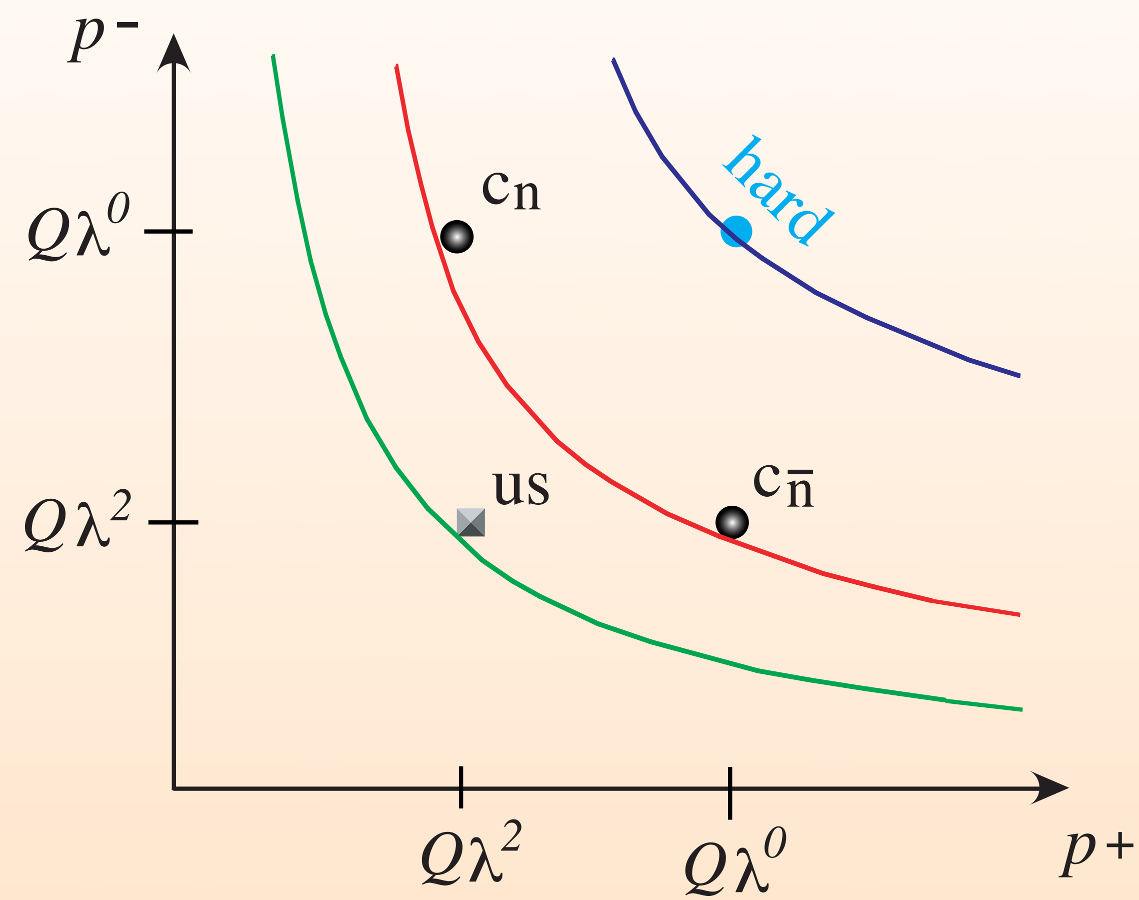
Two jets and usoft radiation

Comments:

1) multiple modes for IR

2) integrate out modes above a hyperbola

3) frame dependence



SCET_I Energetic jets

modes	$p^\mu = (+, -, \perp)$	p^2	fields
n-collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
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usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us},\ A_{us}^\mu$

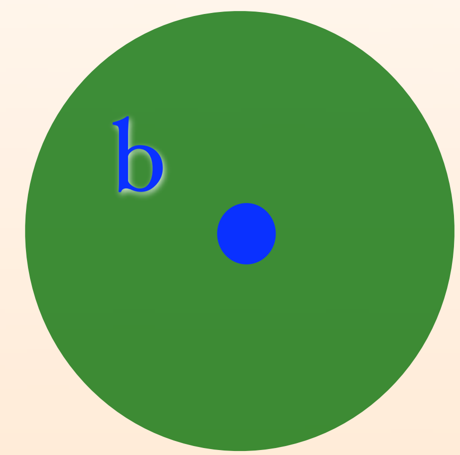
Review Construction of HQET

A low energy EFT for heavy particles that are **not** removed from the theory (static sources that perturbations can cause to wiggle)

$$v^\mu = (1,0,0,0)$$

Want to describe fluctuations of heavy quark Q, due to lighter degrees of freedom.

B-meson



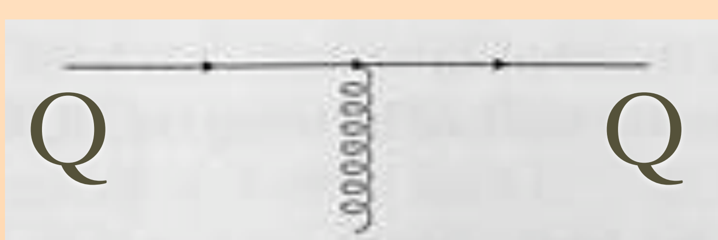
$m_b \gg \Lambda_{\rm QCD}$

- At LO, light d.o.f. have QCD Lagrangian
- $\displaystyle \lim_{m_Q\to\infty} \mathcal{L}_{\rm QCD} = \lim_{m_Q\to\infty} \bar{Q}(i\not{D} - m_Q)Q$?

Propagator: $p^\mu = m_Q v^\mu + k^\mu \qquad k^\mu \sim \Lambda_{\rm QCD}$

$$\frac{\not{p} + m_Q}{p^2 - m_Q^2 + i\epsilon} = \frac{m_Q(\not{v}+1) + \not{k}}{2m_Q\, v\cdot k + k^2 + i\epsilon} = \frac{1+\not{v}}{2}\,\frac{1}{v\cdot k + i\epsilon} + \mathcal{O}(1/m_Q)$$

Vertex:



$$\frac{(1+\not{v})}{2}\gamma^\mu\frac{(1+\not{v})}{2} = \underbrace{\frac{(1+\not{v})}{2}\,\frac{(1-\not{v})}{2}}_{0}\gamma^\mu + \frac{(1+\not{v})}{2}\,v^\mu \to v^\mu$$

$$-ig\gamma^\mu T^A = -igv^\mu T^A$$

$$\mathcal{L}_{\rm HQET} = \bar{Q}_v iv\cdot D Q_v\,, \qquad \frac{(1+\not{v})}{2} Q_v = Q_v$$

Direct Derivation

$$\frac{(1+\slashed{v})}{2}Q_v = Q_v \qquad \slashed{v}Q_v = Q_v$$

$$\frac{(1-\slashed{v})}{2}B_v = B_v \qquad \slashed{v}B_v = -B_v$$

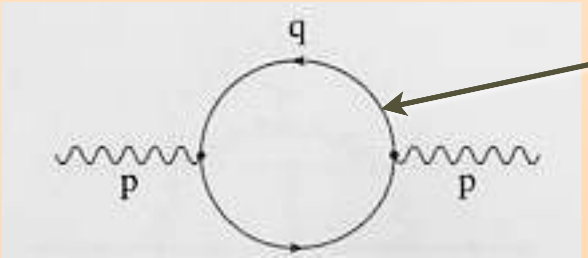
change variables $Q(x) = e^{-im_Q v\cdot x}\left[Q_v(x) + B_v(x)\right]$

$$\begin{aligned}
\mathcal{L}_{\rm QCD} &= [\bar{Q}_v + \bar{B}_v]e^{im_Q v\cdot x}\{\slashed{v} iv\cdot D + i\slashed{D}_T - m_Q\}e^{-im_Q v\cdot x}[Q_v + B_v] \\
&= [\bar{Q}_v + \bar{B}_v]\Big\{(\slashed{v}-1)m_Q + \slashed{v} iv\cdot D + i\slashed{D}_T\Big\}[Q_v + B_v] \\
&= \bar{Q}_v(iv\cdot D)Q_v - \bar{B}_v(iv\cdot D + 2m_Q)B_v + \bar{Q}_v(i\slashed{D}_T)B_v + \bar{B}_v(i\slashed{D}_T)Q_v
\end{aligned}$$

So far we've done nothing to QCD

- Take Q_v external particles, then as $m_Q \to \infty$ the B_v particles (ie. anti-particles) decouple

$$\frac{1+\gamma_0}{2}U_{\rm Dirac} = \begin{pmatrix} \psi_v \\ 0 \end{pmatrix}$$



offshell by $2m_Q$

Surviving term is HQET Lagrangian at LO

Comments

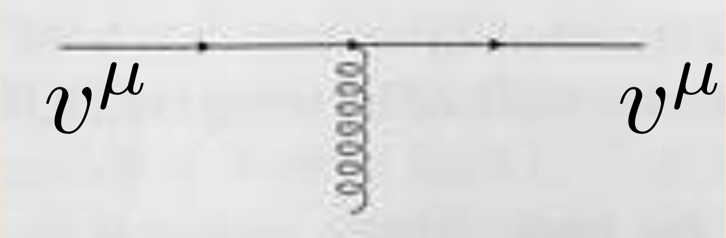
$$\mathcal{L}_{\rm HQET} = \bar{Q}_v iv\cdot DQ_v\,, \qquad \frac{(1+\not{v})}{2}Q_v = Q_v$$

1) Antiparticles are integrated out, number of heavy quarks is preserved, a U(1) symmetry

2) Heavy Quark Spin-Flavor Symmetry U(2 N_Q)

- no flavor (m_Q) dependence
- no dependence on remaining two spin components

3) Velocity v^μ is preserved by low energy QCD interactions "velocity superselection rule"



4) Power Counting in $1/m_Q$ is now simple!

$$Q_v(x) \sim e^{-ik\cdot x} \qquad i\partial^\mu Q_v(x) \sim \Lambda_{\rm QCD} Q_v(x)$$

all powers of $1/m_Q$ appear in prefactors

$$\mathcal{L}_{\rm HQET} = \mathcal{L}^{(0)}_{\rm HQET} + \sum_{n=1}^{\infty}\frac{1}{m_Q^n}\mathcal{L}^{(n)}_{\rm HQET} \qquad J_{\rm HQET} = J^{(0)}_{\rm HQET} + \sum_{n=1}^{\infty}\frac{1}{m_Q^n}J^{(n)}_{\rm HQET}$$

Lets now construct SCET_{I}

n-Collinear Propagators

$$p^2 + i\epsilon = \bar{n}\cdot p \;\; n\cdot p - \vec{p}_\perp^{\,2} + i\epsilon$$
$$\sim \;\; \lambda^0 * \lambda^2 \;\; - (\lambda)^2$$

same size

Collinear Fermions

$$\frac{i\,\not{p}}{p^2 + i\epsilon} = \frac{i\,\not{n}}{2}\,\frac{\bar{n}\cdot p}{p^2 + i\epsilon} + \ldots$$
$$= \frac{i\,\not{n}}{2}\,\frac{1}{n\cdot p - \frac{\vec{p}_\perp^{\,2}}{\bar{n}\cdot p} + i\epsilon\,\mathrm{sign}(\bar{n}\cdot p)} + \ldots$$

thus we expect

$$\underbrace{\int d^4x}_{\lambda^{-4}}\, e^{ip\cdot x}\,\langle 0|T\xi_n(x)\bar{\xi}_n(0)|0\rangle = \frac{i\,\not{n}}{2}\,\underbrace{\frac{\bar{n}\cdot p}{p^2 + i\epsilon}}_{\lambda^{-2}}$$

so $\boxed{\xi_n \sim \lambda}$

power counting for the field

Spin Projection

$$(1)\psi = \left(\frac{\not{n}\bar{\not{n}}}{4} + \frac{\bar{\not{n}}\not{n}}{4}\right)\psi$$

$$\psi = \xi_n + \chi_{\bar{n}}$$

big small

$$\boxed{\frac{\not{n}\bar{\not{n}}}{4}\xi_n = \xi_n \quad , \qquad \not{n}\xi_n = 0}$$

$$\frac{\bar{\not{n}}\not{n}}{4}\chi_{\bar{n}} = \chi_{\bar{n}} \quad , \qquad \bar{\not{n}}\chi_{\bar{n}} = 0$$

To check this look at spinors $\quad u_n = \frac{\not{n}\bar{\not{n}}}{4}\, u^{\mathrm{QCD}}$

$$\begin{aligned}
\sum_s u_n^s \bar{u}_n^s &= \frac{\not{n}\bar{\not{n}}}{4} \sum_s u^s \bar{u}^s \,\frac{\bar{\not{n}}\not{n}}{4} \\
&= \frac{\not{n}\bar{\not{n}}}{4}\, \not{p}\, \frac{\bar{\not{n}}\not{n}}{4} \\
&= \frac{\not{n}}{2}\, \bar{n}\cdot p
\end{aligned}$$

agrees with numerator of propagator

$$i\frac{\not{n}}{2}\,\frac{\bar{n}\cdot p}{p^2 + i\epsilon}$$

SCET Lagrangian

$$\mathcal{L}_{\rm QCD} = \bar\psi\, i\not{D}\psi$$

Write $\psi = \xi_n + \chi_{\bar n}$

$$\xi_n = \frac{\not{n}\not{\bar n}}{4}\,\psi$$

$$\chi_{\bar n} = \frac{\not{\bar n}\not{n}}{4}\,\psi$$

$$\mathcal{L} = (\bar\chi_{\bar n} + \bar\xi_n)\Big[i\frac{\not{\bar n}}{2}\, n\cdot D + i\frac{\not{n}}{2}\,\bar n\cdot D + i\not{D}_\perp\Big](\xi_n + \chi_{\bar n})$$

$$= \Big(\bar\xi_n\, \frac{\not{\bar n}}{2}\, in\cdot D\,\xi_n\Big) + \Big(\bar\chi_{\bar n}\,\frac{\not{n}}{2} i\bar n\cdot D\,\chi_{\bar n}\Big) + \Big(\bar\xi_n\, i\not{D}_\perp\,\chi_{\bar n}\Big) + \Big(\bar\chi_{\bar n}\, i\not{D}_\perp\,\xi_n\Big)$$

e.o.m: $\dfrac{\delta}{\delta\bar\chi_{\bar n}}$: $\quad i\bar n\cdot D\chi_{\bar n} + \dfrac{\not{n}}{2}\, i\not{D}_\perp\xi_n = 0$

$$\chi_{\bar n} = \frac{1}{i\bar n\cdot D} i\not{D}_\perp \frac{\not{n}}{2}\,\xi_n$$

$$\mathcal{L} = \bar\xi_n\Big(in\cdot D + i\not{D}_\perp \frac{1}{i\bar n\cdot D} i\not{D}_\perp\Big)\frac{\not{\bar n}}{2}\xi_n$$

- we will not consider a source for $\chi_{\bar n}$

Still need to:

- separate collinear and usoft gauge fields
- separate collinear and usoft momenta (derivatives)
- expand

Gauge Fields for $\mathrm{SCET_I}$

Collinear Gluons - same propagator as QCD

covariant gauges

$$\int d^4x\, e^{ip\cdot x}\, \langle 0|T A_n^\mu(x) A_n^\nu(0)|0\rangle = \frac{-i}{p^2}\Big(g^{\mu\nu} - \alpha \frac{p^\mu p^\nu}{p^2}\Big)$$

components scale differently

solution

$$(A_n^+, A_n^-, A_n^\perp) \sim (\lambda^2, 1, \lambda) \sim p^\mu$$

Usoft Gluon

$$A_{us}^\mu \sim (\lambda^2, \lambda^2, \lambda^2) \sim p_{us}^\mu$$

write

$$A^\mu = A_n^\mu + A_{us}^\mu + \ldots$$

like a classical background field to ξ_n, A_n^μ

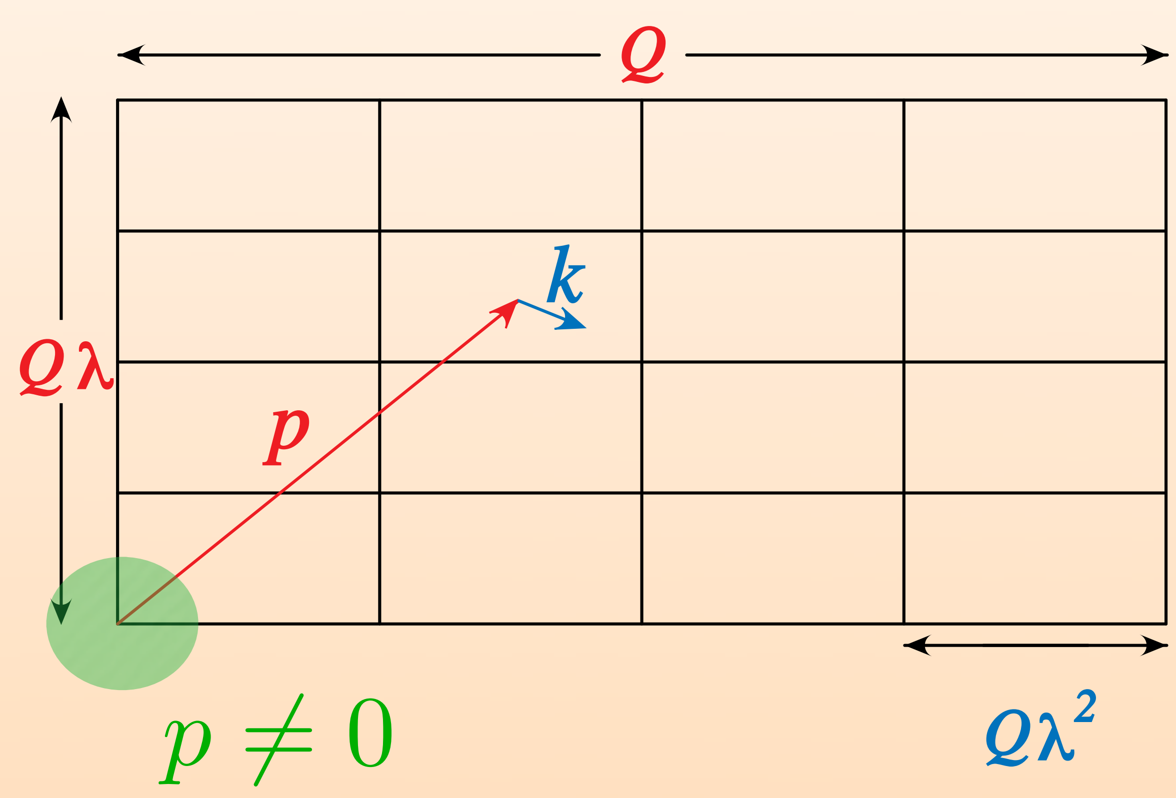
$p_{us}^2 \sim \lambda^4 \ll p_c^2 \sim \lambda^2$

dots are terms that matter for power corrections that we can ignore (fixed by gauge inv.)

Separate Momenta (multipole expansion)

		label	residual	
HQET	$P^\mu =$	$m_b v^\mu$	$+\ k^\mu$	$h_v(x)$
SCET	$P^\mu =$	p^μ	$+\ k^\mu$	$\xi_{n,p}(x)$

p^μ: $(1, \lambda)$ $\qquad$ k^μ: λ^2



Collinear Quarks

- ▷ $\psi(x) \to \sum_p e^{-ip\cdot x} \xi_{n,p}(x)$
- ▷ $\not{n}\, \xi_{n,p} = 0$
- ▷ $\partial^\mu\, \xi_{n,p} \sim (Q\lambda^2)\, \xi_{n,p}$

usual derivative

Introduce Label Operator

$$\mathcal{P}^\mu \left(\phi^\dagger_{q_1} \cdots \phi_{p_1} \cdots\right) = \left(p_1^\mu + \ldots - q_1^\mu - \ldots\right)\left(\phi^\dagger_{q_1} \cdots \phi_{p_1} \cdots\right)$$

derivative for labels

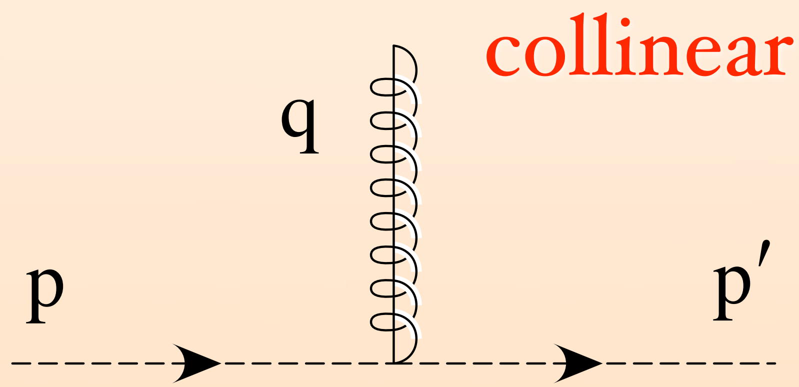
$$i\partial^\mu e^{-ip\cdot x}\, \phi_p(x) = e^{-ip\cdot x}(\mathcal{P}^\mu + i\partial^\mu)\phi_p(x)$$

$$\sum_{p'} e^{ip'\cdot x}\bar{\xi}_{n,p'} \sum_q e^{-iq\cdot x} A_{n,q} \sum_p e^{-ip\cdot x}\xi_{n,p}$$

$$= e^{-ix\cdot\mathcal{P}} \sum_{p,p',q} \bar{\xi}_{n,p'} A_{n,q}\, \xi_{n,p}$$

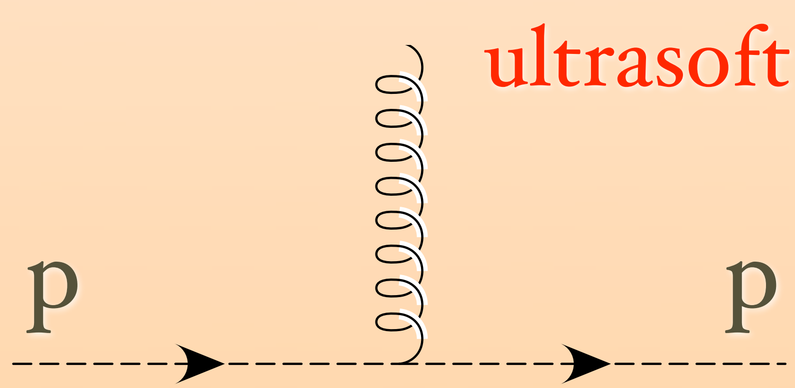
this phase and sum are often suppressed for simplicity

- Labels are changed by collinear interactions



$$p' = q + p$$

- Labels are preserved by ultrasoft interactions



Power Counting Summary

Type	$(p^+, p^-, p^\perp)$	Fields	Field Scaling
collinear	$(\lambda^2, 1, \lambda)$	$\xi_{n,p}$	λ
		$(A^+_{n,p}, A^-_{n,p}, A^\perp_{n,p})$	$(\lambda^2, 1, \lambda)$
soft	$(\lambda, \lambda, \lambda)$	$q_{s,p}$	$\lambda^{3/2}$
		$A^\mu_{s,p}$	λ
usoft	$(\lambda^2, \lambda^2, \lambda^2)$	q_{us}	λ^3
		A^μ_{us}	λ^2

Power counting of fields and derivatives gives a power counting for operators

Power counting of operators yields a power counting for any Feynman graph

The power counting can be associated entirely to vertices and is then gauge invariant

LO SCET Lagrangian

$$\mathcal{L} = \bar{\xi}_n \Big(in\cdot D + i\not{D}_\perp \frac{1}{i\bar{n}\cdot D} i\not{D}_\perp \Big) \frac{\not{\bar{n}}}{2} \xi_n$$

- expand

$$iD^c_\perp = \mathcal{P}_\perp + gA^\perp_n \sim \lambda \gg i\partial^\perp, gA^\perp_{us} \sim \lambda^2$$

$$\mathcal{L}^{(0)}_c = \bar{\xi}_n \Big\{ n\cdot iD_{us} + gn\cdot A_n + i\not{D}^c_\perp \frac{1}{i\bar{n}\cdot D_c} i\not{D}^c_\perp \Big\} \frac{\not{\bar{n}}}{2} \xi_n$$

both λ^2

$\lambda \times \lambda^0 \times \lambda = \lambda^2$

$\mathcal{L}^{(0)}_c \sim \lambda^4$, the leading order quark Lagrangian

$$\int d^4x \, \mathcal{L}^{(0)}_c \sim \lambda^0$$

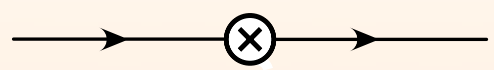
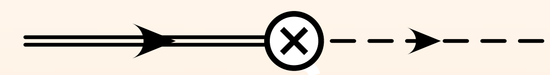
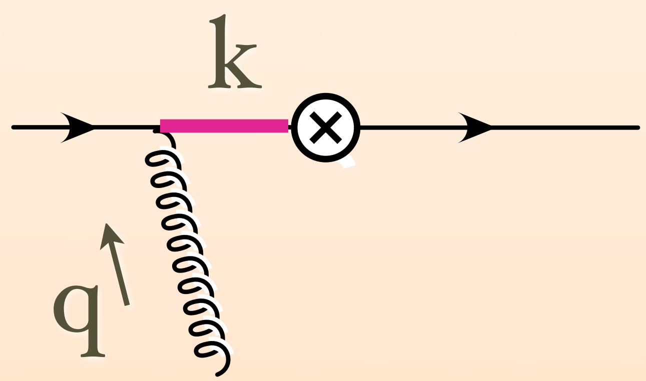
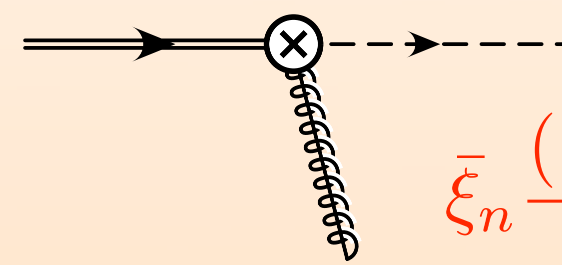
For gluons repeating this type of analysis gives:

$$\mathcal{L}^{(0)}_{cg} = \mathcal{L}^{(0)}_{cg}(A^\mu_n, n\cdot A_{us})$$

$$(A_n^+, A_n^-, A_n^\perp) \sim (\lambda^2, 1, \lambda) \sim p^\mu$$

Currents

eg. $\bar{u}\,\Gamma\,b$ involves both collinear and usoft objects

	QCD		SCET
no gluons	 $\bar{u}\,\Gamma\,b$	→	 $\bar{\xi}_n\;\Gamma\;h_v$
one gluon		→	 $\bar{\xi}_n \dfrac{(-g\bar{n}\cdot A_n)}{\bar{n}\cdot q}\Gamma h_v$

offshell
$$k^\mu = m_b v^\mu + \frac{n^\mu}{2}\bar{n}\cdot q + \ldots$$
$$k^2 - m_b^2 = n\cdot v\, m_b \bar{n}\cdot q + \ldots$$

graph = $$\bar{u}_n \Gamma \frac{i(\not{k} + m_b)}{k^2 - m_b^2} ig T^A \gamma^\mu u_v = \frac{-g}{n\cdot v\, m_b\, \bar{n}\cdot q}\bar{u}_n\Gamma\Big[m_b(1+\not{v}) + \frac{\not{n}}{2}\bar{n}\cdot q\Big]\Big(\frac{\not{n}}{2}\bar{n}^\mu\Big)T^A u_v$$

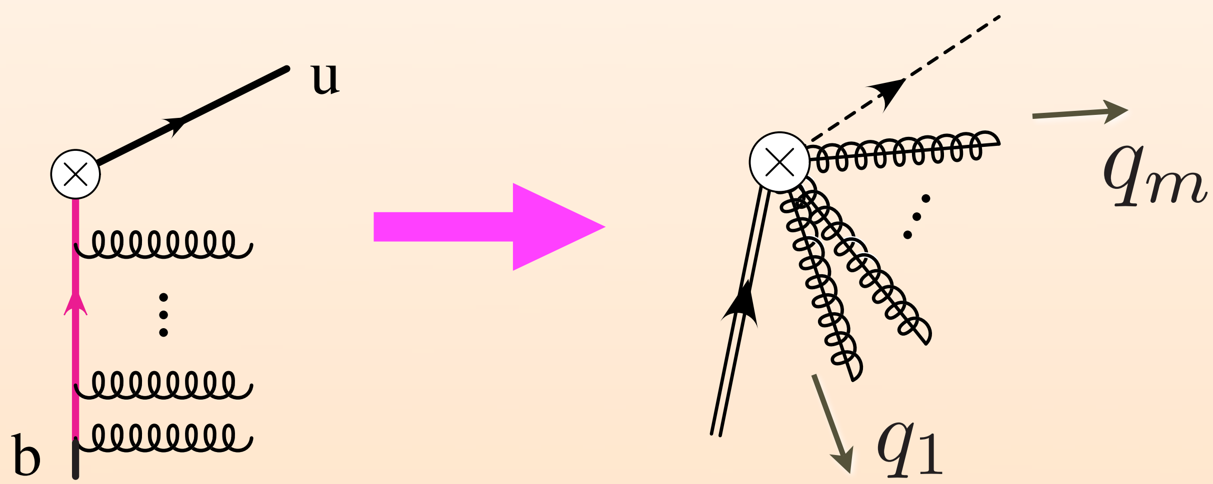
$$= \frac{-g\bar{n}^\mu}{\bar{n}\cdot q}\bar{u}_n \Gamma T^A \left\{\frac{\frac{\not{n}}{2}(1-\not{v}) + n\cdot v + 0}{n\cdot v}\right\} u_v \quad = \frac{-g\bar{n}^\mu}{\bar{n}\cdot q}\bar{u}_n\Gamma T^A u_v$$

Currents

eg. $\bar{u}\,\Gamma\,b$ involves both collinear and usoft objects

add any number of gluons

$\bar{u}\,\Gamma\,b \;\Rightarrow\; \bar{\xi}_n W\ \Gamma\ h_v$ get a Wilson line



$$= g^m \sum_{\text{perms}} \frac{(\bar{n}^{\mu_m} T^{A_m})\cdots(\bar{n}^{\mu_1} T^{A_1})}{[\bar{n}\cdot q_1][\bar{n}\cdot(q_1+q_2)]\cdots[\bar{n}\cdot\sum_{i=1}^{m} q_i]}$$

$\sim \lambda^0$ no cost to add these gluons

momentum space Wilson line
$$W = \sum_k \sum_{\text{perms}} \frac{(-g)^k}{k!}\left(\frac{\bar{n}\cdot A_{\bar{n},q_1}\cdots\bar{n}\cdot A_{\bar{n},q_k}}{[\bar{n}\cdot q_1][\bar{n}\cdot(q_1+q_2)]\cdots[\bar{n}\cdot\sum_{i=1}^{k} q_i]}\right)$$

position space Wilson line
$$W(y,-\infty) = P\exp\Big(ig\int_{-\infty}^{y} ds\,\bar{n}\cdot A_n(s\bar{n}^\mu)\Big)$$

Exercise

SCET Operators with Collinear Quarks and Wilson Lines

a) Start with the QCD Lagrangian for a massive quark and decompose $\not{D}$ in terms of n, $\bar{n}$, and $\perp$ components. As in lecture, write $\psi = \xi_n + \zeta_{\bar{n}}$ where $\not{n}\xi_n = 0$ and $\bar{\not{n}}\zeta_{\bar{n}} = 0$ and determine which products of fields are non-zero. Keeping all the non-zero terms, integrate out the field $\zeta_{\bar{n}}$ to generate an effective action for the massive collinear quark ξ_n.

[With power counting $m \sim p_\perp \sim Q\lambda \ll Q$ this is the starting point to derive the action for a massive collinear quark, ie. prior to decomposing the gluon field into collinear and ultrasoft pieces and prior to distinguishing between large and small momenta. The remaining steps are the same as those discussed in lecture except that you keep the mass. The mass terms that you have derived are important for considering how a collinear Lagrangian of light quarks u, d, s explicitly breaks chiral symmetry. They are also relevant for discussing an energetic jet initiated by a massive quark, when the jet energy $Q \gg m$.]

b) To get more familiar with Wilson lines lets consider the current for a $b \to u$ transition. In QCD $J = \bar{u}\Gamma b$. For SCET we did a matching calculation to find the leading order current

$$J^{(0)} = \bar{\xi}_n W \Gamma h_v \,, \tag{2}$$

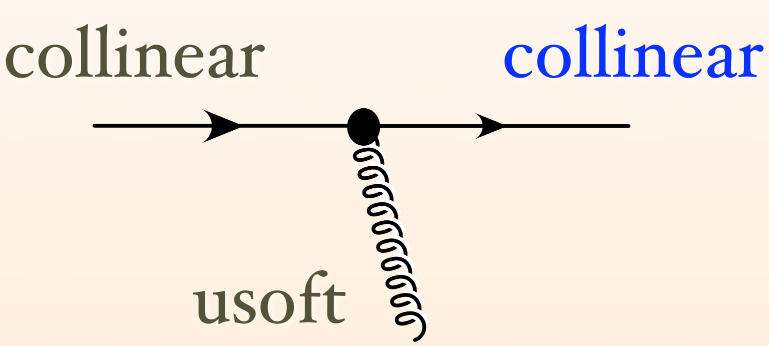
where W included terms involving the order λ^0 collinear gluon field $\bar{n}\cdot A_n$. In lecture we explicitly computed the term in W with one $\bar{n}\cdot A_n$ field and wrote down the result for any number of $\bar{n}\cdot A_n$ fields. Do the matching computation for two $\bar{n}\cdot A_n$ fields (by expanding QCD diagrams with offshell propagators). Verify that the result for one and two $\bar{n}\cdot A_n$ fields agree with the momentum space Feynman rules derived from the position space Wilson line

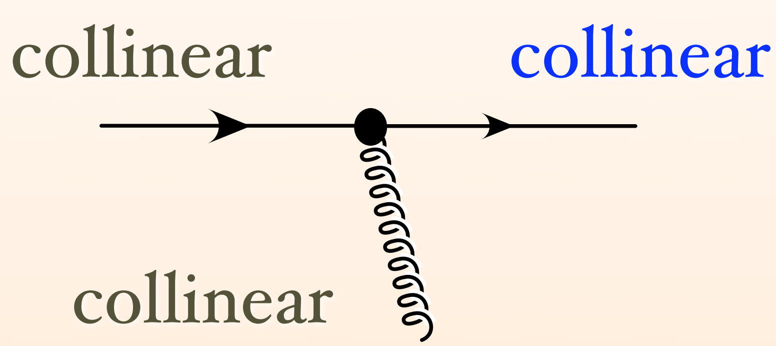
$$W(y^+) = P \exp\Big(ig \int_{-\infty}^{0} ds \, \bar{n}\cdot A_n(s\bar{n} + y^+) \Big) \,,$$

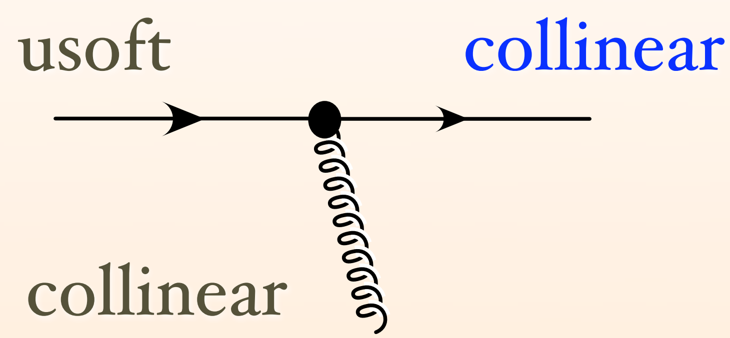
where P is path-ordering.

Interaction of modes: Offshell versus Onshell

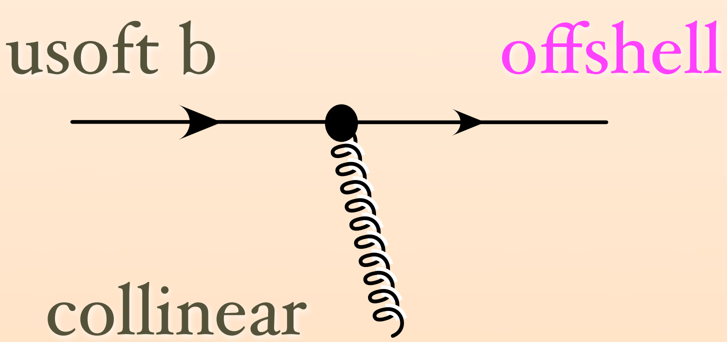
Which fields can interact in a local way?





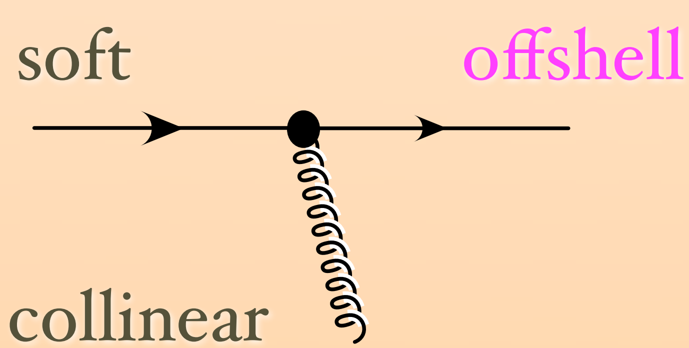


these three are all in $\mathrm{SCET}_{\mathrm{I}}$



this generated a Wilson line

$\mathrm{SCET}_{\mathrm{II}}$



This makes interactions in $\mathrm{SCET}_{\mathrm{II}}$ more complicated to construct, so we postponed further discussion to after fully developing $\mathrm{SCET}_{\mathrm{I}}$

That was tree level.

Rather than extending the matching to loops it is simpler to take a bottom up approach and USE SYMMETRIES of SCET

Next:

Gauge symmetry, Lorentz invariance (?)

Gauge symmetry

$$U(x) = \exp\left[i\alpha^A(x)T^A\right]$$

need to consider U's which leave us in the EFT

collinear $\quad i\partial^\mu \mathcal{U}_c(x) \sim p_c^\mu \mathcal{U}_c(x) \leftrightarrow A^\mu_{n,q}$

usoft $\quad i\partial^\mu U_{us}(x) \sim p_{us}^\mu U_{us}(x) \leftrightarrow A^\mu_{us}$

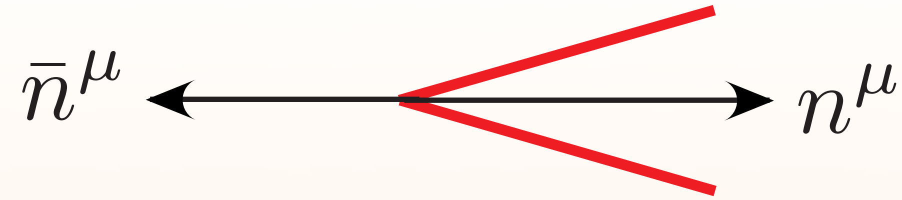
Object	Collinear $\mathcal{U}_c$	Usoft U_{us}
ξ_n	$\mathcal{U}_c\, \xi_n$	$U_{us}\, \xi_n$
gA_n^μ	$\mathcal{U}_c\, gA_n^\mu\, \mathcal{U}_c^\dagger + \mathcal{U}_c\left[i\mathcal{D}^\mu, \mathcal{U}_c^\dagger\right]$	$U_{us}\, gA_n^\mu\, U_{us}^\dagger$
W	$\mathcal{U}_c\, W$	$U_{us}\, W\, U_{us}^\dagger$
q_{us}	q_{us}	$U_{us}\, q_{us}$
gA_{us}^μ	gA_{us}^μ	$U_{us} gA_{us}^\mu U_{us}^\dagger + U_{us}[i\partial^\mu, U_{us}^\dagger]$
Y	Y	$U_{us}\, Y$

our current is invariant:

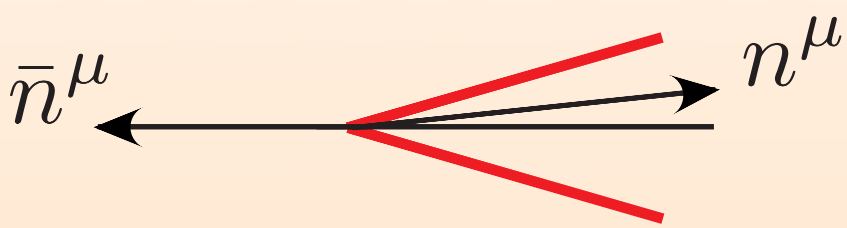
$$(\bar\xi_n W)\Gamma h_v \longrightarrow (\bar\xi_n \mathcal{U}_c^\dagger \mathcal{U}_c W)\Gamma h_v = (\bar\xi_n W)\Gamma h_v$$

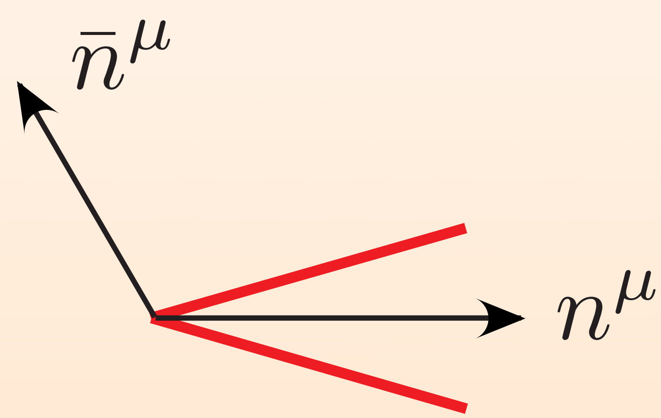
$$\longrightarrow (\bar\xi_n U_{us}^\dagger U_{us} W) U_{us}^\dagger \Gamma U_{us} h_v = (\bar\xi_n W)\Gamma h_v$$

Reparameterization Invariance (RPI)



$n\ ,\ \ \bar{n}$ break Lorentz invariance, restored within collinear cone by RPI, three types





longitudinal boost

$$(\mathrm{I})\ \begin{cases} n_\mu \to n_\mu + \Delta_\mu^\perp \\ \bar{n}_\mu \to \bar{n}_\mu \end{cases} \qquad (\mathrm{II})\ \begin{cases} n_\mu \to n_\mu \\ \bar{n}_\mu \to \bar{n}_\mu + \varepsilon_\mu^\perp \end{cases} \qquad (\mathrm{III})\ \begin{cases} n_\mu \to (1+\alpha)\, n_\mu \\ \bar{n}_\mu \to (1-\alpha)\, \bar{n}_\mu \end{cases}$$

$\Delta_\mu^\perp \sim \lambda$ $\qquad$ $\varepsilon_\mu^\perp \sim \lambda^0$ $\qquad$ $\alpha \sim \lambda^0$

unique

$$\mathcal{L}_c^{(0)} = \bar{\xi}_n \left\{ n \cdot iD_{us} + g n \cdot A_n + i \not{D}_\perp^c \frac{1}{i\bar{n} \cdot D_c} i \not{D}_\perp^c \right\} \frac{\not{\bar{n}}}{2} \xi_n$$